

Welcome Tutorial :-)

Tutorial 4

GAO Ming

SE & DaSE @ ECNU

Foundations of Data Science, 2016

Tutorial 4

- ① Let X be a r.v., $\mu = E(X)$ and $\sigma^2 = E[(X - \mu)^2]$. If $X^* = \frac{X - \mu}{\sigma}$, please prove $P[|X^*| \geq c] \leq \frac{1}{c^2}$.
- ② Let X_i ($i = 1, 2, \dots, n$) be i.i.d., $\mu = E(X_i)$ and $\sigma^2 = E[(X_i - \mu)^2]$. If $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$, please prove $P[|\bar{X} - \mu| \geq \varepsilon] \leq \frac{\sigma^2}{n\varepsilon^2}$.
- ③ In n tosses of a fair coin, let X be the number of heads, what's the probability of $X < \frac{n}{2}$ heads?
- ④ Let X_i be a sequence of independent r.v.s with $P(X_i = 1) = p_i$ and $P(X_i = 0) = 1 - p_i$. r.v. $X = \sum_{i=1}^n X_i$ and $\mu = \sum_{i=1}^n p_i$. Please prove the following conclusions.
 - $P(X > (1 + \delta)\mu) < \left(\frac{e^\delta}{(1 + \delta)^{(1 + \delta)}} \right)^\mu$
 - $P(X > (1 + \delta)\mu) < \exp(-\mu\delta^2/4)$