

Welcome Tutorial :-)

Tutorial 5

GAO Ming

SE & DaSE @ ECNU

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- 1 Please prove $\frac{\partial A\mathbf{x}}{\partial \mathbf{x}} = A$ and $\frac{\partial \mathbf{x}^T A}{\partial \mathbf{x}} = A^T$.
- 2 Please prove that $\lim_{k \rightarrow \infty} (A^T A)^k = v_1 \sigma_1^2 v_1^T$ (v_1 is the eigenvector of the largest singular value σ_1) if $\sigma_1 \neq \sigma_2$.
- 3 Let UDV^T be the SVD of matrix A ($\sigma_1 \geq \sigma_2 \geq \dots$, $\text{rank}(A) = n$) and $B = USV^T$, where S is diagonal $n \times n$ matrix where $s_i = \sigma_i$ if $1 \leq i \leq k$, else $s_i = 0$. Please calculate reconstruction error $\|A - B\|_F$, where $\|A\|_F$ is Frobenius norm of matrix, i.e.,

$$\|A\|_F = \sqrt{\sum_{i=1}^m \sum_{j=1}^n a_{ij}^2} = \sqrt{\text{Trace}(A^T A)} = \sqrt{\sum_{i=1}^{\min\{m,n\}} \sigma_i^2}.$$