Welcome Tutorial :-)

Tutorial 7

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1. Let following matrix A be adjacency matrix of graph G. Please write down the transition probability of the random walk corresponding to G, Laplacian matrix and normalized Laplacian matrix.

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

2. Given graph G with n vertices, let n eigenvalues of the normalized Laplacian matrix be $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Please prove the following properties.
   - The mean of $\lambda_2, \lambda_3, \cdots, \lambda_n$ is $\frac{n}{n-1}$.
   - $0 \leq \lambda_2 \leq \frac{n}{n-1} \leq \lambda_n$.
3. Let $\mathscr{L}$ be normalized Laplacian matrix of graph G, and P be the transition probability matrix of the random walk corresponding to graph G. If λ is an eigenvalue of P, please prove that $1-\lambda$ is an eigenvalue of $\mathscr{L}$.