

小写字母：向量；大写字母：矩阵

$$\begin{aligned}\frac{d\mathbf{x}^T}{d\mathbf{x}} &= \mathbf{I} & \frac{d\mathbf{x}}{d\mathbf{x}^T} &= \mathbf{I} \\ \frac{d\mathbf{x}^T \mathbf{A}}{d\mathbf{x}} &= \mathbf{A} & \frac{d\mathbf{A}\mathbf{x}}{d\mathbf{x}^T} &= \mathbf{A} \\ \frac{d\mathbf{A}\mathbf{x}}{d\mathbf{x}} &= \mathbf{A}^T & \frac{d\mathbf{x}\mathbf{A}}{d\mathbf{x}} &= \mathbf{A}^T & \frac{\partial \mathbf{u}}{\partial \mathbf{x}^T} &= \left(\frac{\partial \mathbf{u}^T}{\partial \mathbf{x}} \right)^T \\ \frac{\partial \mathbf{x}^T \mathbf{y}}{\partial \mathbf{x}} &= \mathbf{y} \text{ (定义推)}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{u}^T \mathbf{v}}{\partial \mathbf{x}} &= \frac{\partial \mathbf{u}^T}{\partial \mathbf{x}} \mathbf{v} + \frac{\partial \mathbf{v}^T}{\partial \mathbf{x}} \mathbf{u} \\ \frac{\partial \mathbf{u} \mathbf{v}^T}{\partial \mathbf{x}} &= \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \mathbf{v}^T + \mathbf{u} \frac{\partial \mathbf{v}^T}{\partial \mathbf{x}} \\ \frac{d\mathbf{x}^T \mathbf{x}}{d\mathbf{x}} &= 2\mathbf{x} \\ \frac{d\mathbf{x}^T \mathbf{A} \mathbf{x}}{d\mathbf{x}} &= (\mathbf{A} + \mathbf{A}^T) \mathbf{x} & \frac{\partial \mathbf{A} \mathbf{B}}{\partial \mathbf{x}} &= \frac{\partial \mathbf{A}}{\partial \mathbf{x}} \mathbf{B} + \mathbf{A} \frac{\partial \mathbf{B}}{\partial \mathbf{x}}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{u}^T \mathbf{X} \mathbf{v}}{\partial \mathbf{X}} &= \mathbf{u} \mathbf{v}^T \\ \frac{\partial \mathbf{u}^T \mathbf{X}^T \mathbf{X} \mathbf{u}}{\partial \mathbf{X}} &= 2\mathbf{X} \mathbf{u} \mathbf{u}^T \\ \frac{\partial \left[(\mathbf{X} \mathbf{u} - \mathbf{v})^T (\mathbf{X} \mathbf{u} - \mathbf{v}) \right]}{\partial \mathbf{X}} &= 2(\mathbf{X} \mathbf{u} - \mathbf{v}) \mathbf{u}^T\end{aligned}$$