STAT 614 **CHAPTERS 6 and 7** HYPOTHESIS TESTING ; CONFIDENCE INTERVALS ; TYPE I ERROR ;TYPE II ERROR POWER OF A TEST ; TWO-SAMPLE T TEST FOR MEANS

Twenty chemistry final exam scores are randomly selected. The scores are provided below,

72, 77, 71, 68, 56, 70, 80, 60, 92, 66, 93, 84, 55, 68, 70, 84, 88, 95, 78, 64

CTS <- c(72, 77, 71, 68, 74, 70, 80, 60, 73, 66, 77, 84, 73, 73, 70, 84, 88, 74, 78, 64)

CTS

H_o : $\mu = 76$

H_a : $\mu < 76$ $\alpha = .05$

Run the test using R code : t.test(CTS, mu = 76, alternative = "less", conf.level .95)

R output:

One Sample t-test data: CTS t = -1.4223, df = 19, p-value = 0.08558

alternative hypothesis: true mean is less than 76

95 percent confidence interval: -Inf 76.47465

sample estimates: mean of x 73.8

Observations:

The p-value is the probability of obtaining a sample statistic or a value more extreme in the direction of the alternative hypothesis, if the null hypothesis is true.

For our example;

If H_o : $\mu = 76$ is true, what is the probability of getting 73.8 or a value less than 73.8? That probability is the p value 0.08558

Your hypothesis test does not establish that H_o is true!! If we fail to reject that $\mu = 76$, we are saying that 76 is one of the possible values for μ, but so is 75, 74, 73.5, moreover any value that is in your interval and reasonably close 73.8. is a possible value for H_o. For these reasons, you should not say we “accept H_o”.

And in general,

If you reject H_o, you accept H_a

If you fail to reject H_o, you do not accept H_a

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```
CTS <- c(72, 77, 71, 68, 74, 70, 80, 60, 73, 66, 77, 84, 73, 73, 70, 84, 88, 74, 78, 64)
CTS
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H_o : $\mu = 76$

H_a : $\mu < 76$ $\alpha = .05$

Run the test using R code : `t.test(CTS, mu = 76, alternative = "less", conf.level .95)`

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One Sample t-test    data:  CTS         t = -1.4223, df = 19, p-value = 0.08558
alternative hypothesis: true mean is less than 76
95 percent confidence interval:       -Inf      76.47465
sample estimates:   mean of x   73.8
```

If H_o is rejected when it is in fact true, a Type 1 error has been committed. The α setting is the probability of committing a Type 1 error. For the test executed above the Type 1 error is .05

Again, a Type 1 error occurs when the null hypothesis $\mathbf{H_o}$ is rejected when it is true.

A Type 2 error, however, occurs when you fail to reject the null hypothesis $\mathbf{H_o}$ when it is not true.

Practical Example:

In the court of law, you are assumed innocent until you are proven guilty. Hence, we can consider null hypothesis to be that the defendant is innocent. And the alternative hypothesis is that the defendant is guilty.

H_o : innocence

H_a : guilt

If a defendant is sent to jail, even though he/she did not commit the crime a Type 1 error has occurred.

If a defendant is set free even though he/she committed the crime a Type 2 error has occurred.

Finding a TYPE 2 ERROR (failing to reject **Ho** when it is not true)

H_o : $\mu = 76$

H_a : $\mu < 76$ $\alpha = .05$

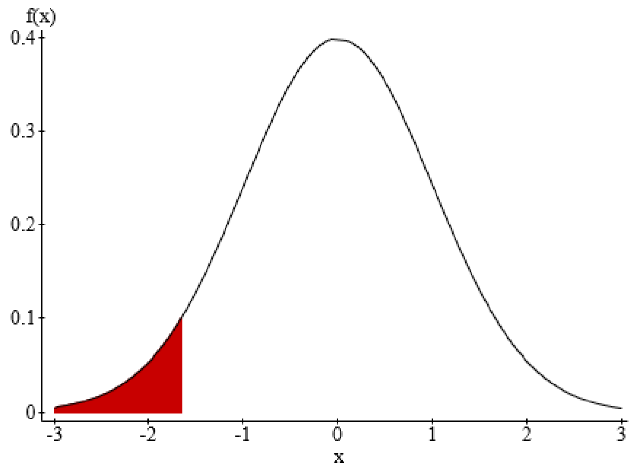
$n = 20$, $\sigma = 7.4$

$\mu_t =$ 72 (the true and exact mean; H_o should be rejected)

Step 1 Find the z score for the Type 1 error .05

qnorm(p = .05, mean = 0, sd = 1, lower.tail = TRUE) -1.645

We will reject H_0 if z is ≤ -1.645



Normal Distribution
Mean:0 Std. Dev.:1
P(X≤-1.6448536) = 0.05

Step 2 Find the values of y(bar) for which your reject H_o. (The y(bar)s are sample means)

Z = (y(bar) – μ_o) / (σ /sqrt(n))

= (y(bar) – 76) / (7.4/sqrt(20))

Solve for y(bar)

-1.645 = (y(bar) – 76) / (7.4/sqrt(20))

y(bar) = 73.28

Now, we also reject H_o for y(bar) ≤ 73.28

Step 3 Normalize 73.28 (73.28 – 72) /(7.4/sqrt(20)) = .774

If we reject for z ≤ .774, we then fail to reject for z > .774

Step 4 pnorm(..774, mean = 0, sd = 1, lower.tail = F) .219

The probability of a Type 2 error is .219.

Let's find the Power of our test.

The Power of a test is the probability that our test will reject a null hypothesis that is not true.

(Do you have a good test !! Generally if your power probability is 80 or above you have a very good test)

You find the power of a test by subtracting the Type 2 error from 1.

Power of a test = $1 - \beta$, where β is the probability of the Type 2 error.

Power = $1 - \beta = 1 - .219 =$ **.78**

Type 2 and Type 1 Error Summary Facts

- For a fixed α value, the Type 2 Error decreases as the sample size increases
- Generally large sample sizes will keep Type 1 and Type 2 errors low.
- The smaller the α value (Type 1 error) is in a test, the larger the Type 2 Error is
- The Type 2 error decreases as the Type 1 error increases
- The Type 2 error depends on the distance between the true parameter and the hypothesized parameter. The Type 2 error increases the closer they are.

Text book problem

A decision is planned to in a test $H_o : \mu = 0$ vs $H_a : \mu > 0$, using $\alpha = .05$. If $\mu = 5$, P(Type 2 error) = .17

a) Explain the meaning of the statement : **If $\mu = 5$, P(Type 2 error) = .17**

b) If the test used $\alpha = .01$, would the P(Type 2 error) be less than, equal to , or greater than .17?

c) If $\mu = 10$, would the P(Type 2 error) be less than, equal to , or greater than .17?

CHAPTER 7 COMPARING TWO MEANS THE TWO SAMPLE T TEST FOR MEANS

POPULATION 1

SAMPLE 1 8.59, 8.64, 7.43, 7.21, 6.87, 7.89, 9.79, 6.85, 7.00, 8.80, 9.30, 8.03, 6.39, 7.54

POPULATION 2

SAMPLE 2 8.65, 6.99, 8.40, 9.66, 7.62, 7.44, 8.55, 8.70, 7.33, 8.58, 9.88, 9.94, 7.14, 9.14

(Assume that the conditions for executing the test are met; Samples are independent and randomly generated; Populations are normal or the sample sizes are reasonably large and have no extreme ouliers)

Question/Investigation: Is there a significant difference between the means of Population 1 and Population 2?

H_o : $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$ **(There is no significant difference between the means)**

H_a : $\mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$ **(There is a significant difference between the means)**

$\alpha = .05$

```
SAMPLE1<- c(8.59, 8.64, 7.43, 7.21, 6.87, 7.89, 9.79, 6.85, 7.00, 8.80, 9.30, 8.03, 6.39, 7.54)
SAMPLE1
SAMPLE2<- c(8.65, 6.99, 8.40, 9.66, 7.62, 7.44, 8.55, 8.70, 7.33, 8.58, 9.88, 9.94, 7.14, 9.14)
SAMPLE2
t.test(SAMPLE1,SAMPLE2, mu = 0, alternative= "two.sided", var.equal = TRUE, conf.level=.95)
```

```
Two Sample t-test
data:  SAMPLE1 and SAMPLE2
t = -1.4368, df = 26, p-value = 0.1627
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.335120    0.236549
sample estimates:   mean of x mean of y      7.880714  8.430000
```

Conclusion: Since the p-value is greater than .05, we will reject the null hypothesis. Also note that the difference for H_o **0** is inside of your confidence interval. Additional justification to not reject the null hypothesis.

Elementary School children are divided into two groups. A special treatment is applied in order to increase reading abilities to one of the groups. The other group that did not receive the treatment is designated the control group. Test scores for each group are given below.

Treatment Group

24,43,58,71,43,49,61,44,67,49,53,56,59,52,62,54,57,33,46,43,57

Control Group

42,43,55,26,62,37,33,41,19,54,20,85,46,10,17,60,53,42,37,42,55,28,48

H_o : $\mu_T = \mu_C$

H_a : $\mu_T > \mu_C$

```
TG <- c(24,43,58,71,43,49,61,44,67,49,53,56,59,52,62,54,57,33,46,43,57)

TG

CG <- c(42,43,55,26,62,37,33,41,19,54,20,85,46,10,17,60,53,42,37,42,55,28,48)

CG

t.test(TG,CG, mu = 0, alternative= "greater", var.equal = FALSE, conf.level=.95)
```

```
Welch Two Sample t-test

data:  TG and CG

t = 2.3109, df = 37.855, p-value = 0.01319

alternative hypothesis: true difference in means is greater than 0

95 percent confidence interval:

 2.691293      Inf

sample estimates:

mean of x mean of y

 51.47619  41.52174
```

Conclusion: Since the p-value (.01319) < .05, we reject the H_o. Also note that 0 is not in your interval.