CHAPTER 6 Significance Tests for Means and Proportions

The average Chemistry Final exam score at Apex University is believed to be 76. Is their statistical evidence to suggest that the average Chemistry Final exam score is greater than 76? less than 76? or more generally not equal to 76?

Assumptions Hypothesis Test Statistic P value Conclusion

Assumptions:

- Quantitative Data or Categorical Data
- Randomization (elements of a Sample are randomly selected)
- Distribution of Population (Preferably Normal)
- Sample Size (Generally the larger the better; especially if the Population is not normal)

Hypothesis:

- Null hypothesis H_o (Established, Conventional, Status Quo belief about a population parameter)
- Alternative hypothesis H_a (Competing belief against the null hypothesis; H_o: $\mu = k$) can take one of three forms ;
    - H_a: $\mu > k$ (right tail test)
    - H_a: $\mu < k$ (left tail test)
    - H_a: $\mu \neq k$ (two tail test)

Test Statistic: The test statistic summarizes how far the estimate falls from the parameter value in H_o

P value: The P value is the probability that the test statistic equals the observed value or a value as extreme in the direction of H_a under the assumption that H_o is true

Conclusion: We decide if H_o should be rejected or not. The P value summarizes the evidence against H_o. Generally, if P is extremely small we will reject H_o.

The significance Test for a Mean will be executed by conducting a one sample t test. That is based on a t distribution. (google and/or read your textbook regarding the difference between and relationship of a t distribution and a normal distribution)

Example 1

Twenty chemistry final exam scores are randomly selected. The scores are provided below,

72, 77, 71, 68, 56, 70, 80, 60, 92, 66, 93, 84, 55, 68, 70, 84, 88, 95, 78, 64

Remember: The established belief is that the mean test score is 76.

Therefore H_o : $\mu = 76$ (null hypothesis)

We now consider a competing belief, specifically that the mean is greater than 76

Therefore H_a : $\mu > 76$ (alternative hypothesis)

Lets create a vector for the data

```
CTS <- c(72, 77, 71, 68, 74, 70, 80, 60, 73, 66, 77, 84, 73, 73, 70, 84, 88, 74, 78, 64)
CTS
```

Check for normality

```
hist(CTS)
boxplot(CTS)
summary(CTS)
```

`qqnorm(CTS)` (a quartile plot; if the points line up significantly the data set is normal or close to being normal)

`qqline(CTS)` (delivers a line through the quartile plot)

We now start the hypothesis testing process

H_o : $\mu = 76$

H_a : $\mu > 76$

Set the threshold for the p value (we are setting the significance level for the test) $\alpha = .05$ (if p is less than .05 we will reject H_o . if p is greater than .05, we fail to reject H_o)

Run the test using R code: `t.test(CTS, mu = 76, alternative = "greater", conf.level = .95)`

R output :

```
One Sample t-test    data:  CTS
t = -1.4223, df = 19, p-value = 0.9144
alternative hypothesis: true mean is greater than 76
95 percent confidence interval:      71.12535      Inf
sample estimates:     mean of x   73.8
```

Conclusion: Since the p value is greater than .05, we fail to reject H_o

Example 2

Twenty chemistry final exam scores are randomly selected. The scores are provided below,

72, 77, 71, 68, 56, 70, 80, 60, 92, 66, 93, 84, 55, 68, 70, 84, 88, 95, 78, 64

Remember: The established belief is that the mean test score is 76.

Therefore H_o : $\mu = 76$ (null hypothesis)

We now consider a competing belief, specifically that the mean is less than 76

Therefore H_a : $\mu < 76$ (alternative hypothesis)

```
CTS <- c(72, 77, 71, 68, 74, 70, 80, 60, 73, 66, 77, 84, 73, 73, 70, 84, 88, 74, 78, 64)
CTS
```

Check for normality (already done)

We now start the hypothesis testing process

H_o : $\mu = 76$

H_a : $\mu < 76$

Set the threshold for the p value (we are setting the significance level for the test) $\alpha = .05$
(if p is less than .05 we will reject H_o . if p is greater than .05, we fail to reject H_o)

Run the test using R code : `t.test(CTS, mu = 76, alternative = "less", conf.level = .95)`

R output:

```
One Sample t-test    data:  CTS
t = -1.4223, df = 19, p-value = 0.08558
alternative hypothesis: true mean is less than 76
95 percent confidence interval:
    -Inf   76.47465
sample estimates:   mean of x   73.8
```

Conclusion: Since the p value is greater than .05, we fail to reject H_o

Example 3

Twenty chemistry final exam scores are randomly selected. The scores are provided below,

72, 77, 71, 68, 56, 70, 80, 60, 92, 66, 93, 84, 55, 68, 70, 84, 88, 95, 78, 64

Remember: The established belief is that the mean test score is 76.

Therefore H_o : $\mu = 76$ (null hypothesis)

We now consider a competing belief, specifically that the mean is not equal to 76

Therefore H_a : $\mu \neq 76$ (alternative hypothesis)

```
CTS <- c(72, 77, 71, 68, 74, 70, 80, 60, 73, 66, 77, 84, 73, 73, 70, 84, 88, 74, 78, 64)
CTS
```

Check for normality (already done)

We now start the hypothesis testing process

H_o : $\mu = 76$

H_a : $\mu \neq 76$

Set the threshold for the p value (we are setting the significance level for the test) $\alpha = .05$
(if p is less than .05 we will reject H_o . if p is greater than .05, we fail to reject H_o)

Run the test using R code : `t.test(CTS, mu = 76, alternative = "two.sided", conf.level = .95)`

R output:

```
One Sample t-test

data:  CTS

t = -1.4223, df = 19, p-value = 0.1712

alternative hypothesis: true mean is not equal to 76

95 percent confidence interval:

 70.56248   77.03752

sample estimates:

mean of x

    73.8
```

Conclusion: Conclusion: Since the p value is greater than .05, we fail to reject H_o

Example 4

Supposed that it is believed by most that the mean miles per gallon for vehicles in the **mtcars** data table is 23. That is the mean mpg is 23. Is there statistical evidence that the mean mpg is actually less than 23?

```
library(tidyverse)
```

```
mtcars
```

Check for normality

`qqnorm(mtcars$mpg)` (very good; strong suggestion of normality)

`hist(mtcars$mpg)` (ok, moderate suggestion of normality. Good enough to continue)

State the null and alternative hypotheses

H_o : $\mu = 23$

H_a : $\mu < 23$

Set α (alpha) = .05

Execute the one sample t test by Running the R code:

```
t.test(mtcars$mpg, mu=23, alternative = "less", conf.level = .95)
```

R output

```
One Sample t-test

data:  mtcars$mpg

t = -2.7307, df = 31, p-value = 0.005165

alternative hypothesis: true mean is less than 23

95 percent confidence interval:

    -Inf    21.89707

sample estimates:

mean of x      20.09062
```

Conclusion: Since the p value is less than .05, we reject the null hypothesis H_o.

Example 5

Suppose we are given summary data for our Example 1 instead of the raw data. That is we are told that the mean of the sample **y(bar)** = 73.8, sample size **n** = 20, and the standard deviation of the sample **s** = 6.9176.

Consider the following null vs alternative hypothesis statements:

$H_o : \mu = 76$

$H_a : \mu > 76$

We will execute the one sample t test manually.

Step 1

Find the **t statistic**

t = (y(bar) - μ)/ se, se = s/sqrt(n)

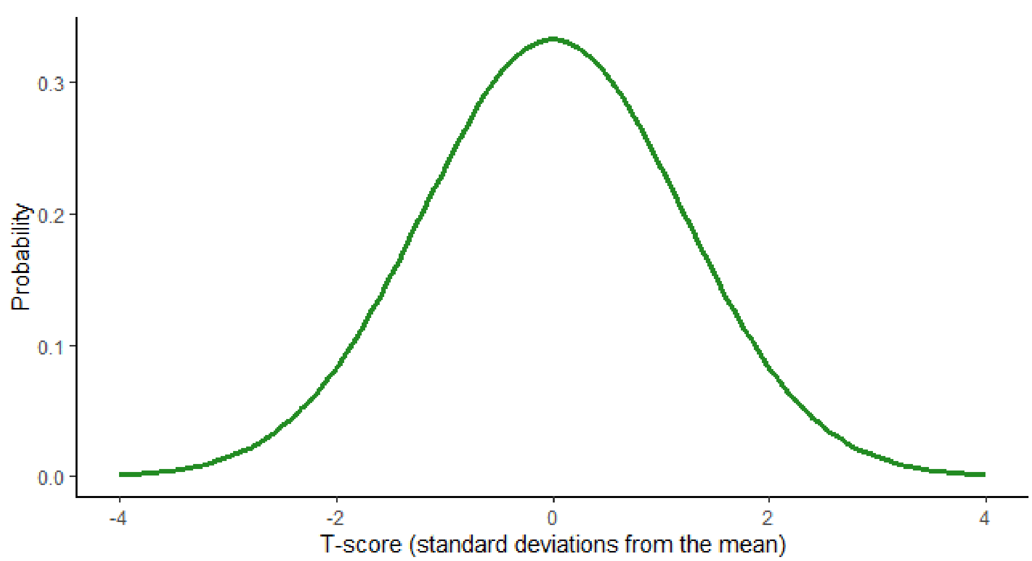
y(bar) = 73.8 , μ = 76 , s = 6.9176 , n = 20

se = 6.9176/ sqrt(20) = 1.5468

t = (73.8 – 76)/ 1.5468 = **-1.4223**

Step 2

Provide a sketch and locate t and the area for p



Go to this link to get you p value (Using a table to get a p value will take too long and you only get an estimate anyway) The link is posted on blackboard

https://www.danielsoper.com/statcalc/calculator.aspx?id=8

Enter the t statistic and the degrees of freedom.

$p = .914425$

Conclusion:

Since $p > .05$, we fail to reject the null hypothesis H_o. The p value of .914425 is not significant at the .05 level of significance.

Note: For **summary data**, it may be better to use another software option such as SPSS, STATA or STATCRUNCH

CHAPTER 6 Significance Tests for Means and Proportions

Assumptions: samples are randomly generated; sample sizes are reasonably large; sampling distribution of π(hat) is approximately normal;

Hypothesis: H_o: $\pi = \pi_o$ vs H_a: $\pi > \pi_o$ or H_a: $\pi < \pi_o$ or H_a: $\pi \neq \pi_o$

Test Statistic: **z = (**π(hat) - π_o)/se , se = sqrt (π_o (1-π_o)/n)

Example 6

H_o: $\pi = .3$ vs H_a: $\pi > .3$

$n = 200$, $x = 75$, $\alpha = .05$

Find the z statistic, p, and make a decision to reject or not to reject the null hypothesis

Solution:

R code:

```
prop.test (x = 75, n= 200, p = .3,  alternative ="greater", conf.level = .95, correct = FALSE)
```

```
prop.test (x = 75, n= 200, p = .3,  alternative ="greater", conf.level = .95, correct = FALSE)
```

R output:

```
1-sample proportions test without continuity correction

data:  75 out of 200, null probability 0.3

X-squared = 5.3571, df = 1, p-value = 0.01032
```

Note: z statistic is sqrt(5.3571)

```
alternative hypothesis: true p is greater than 0.3

95 percent confidence interval:

 0.3207128    1.0000000    sample estimates:       p  =  0.375
```

π(hat)

Conclusion: Since $p < .05$, we will reject the null hypothesis. Also note that you reject H_o since .3 is outside of you interval.

Example 7

Worried about Retirement? In April 2009, the Gallop organization surveyed 676 adults aged 18 and older and found that 352 believed they would not have enough money to live comfortably in retirement. Does the sample evidence suggest that a majority of adults in the United States believe they will not have enough money? Use the $\alpha = 0.05$ level of significance.

H_o: $\pi = .5$ vs H_a: $\pi > .5$

R code:

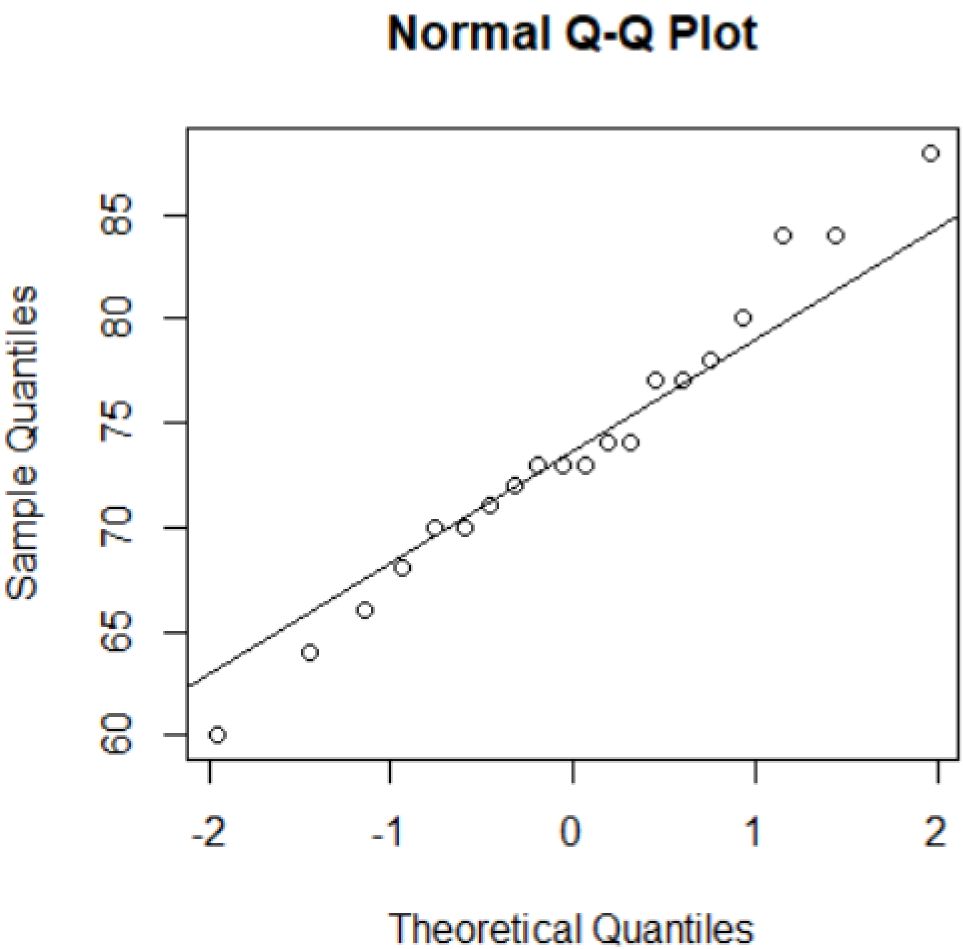
```
prop.test (x = 352, n= 676, p = .5,  alternative ="greater", conf.level = .95, correct = FALSE)
```

R output

```
1-sample proportions test without continuity correction

data:  352 out of 676, null probability 0.5
X-squared = 1.1598, df = 1, p-value = 0.1408
alternative hypothesis: true p is greater than 0.5
95 percent confidence interval:
 0.4890858 1.0000000
sample estimates     p = .5207101  (Sample Proportion)
```

Conclusion: Since $p = .1408 > .05$, we fail to reject H_0


Normal Q-Q Plot
Sample Quantiles
60
65
70
75
80
85
-2
-1
0
1
2
Theoretical Quantiles