ANOVA approach to hypothesis testing

The F Test

- In this section, we will talk about a different strategy to testing $H_0 : \beta_1 = 0$

In simple linear regression, this results in the *exact same-value* as the test that uses the t-statistic.

However, this strategy is more applicable to general linear hypotheses that we'll discuss in multiple linear regression.

- Testing $H_0 : \beta_1 = 0$

is really a comparison between the two models:

$$H_0 : \; Y_i = \beta_0 + \epsilon_i$$
$$H_A : \; Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

- We will call the first model the **reduced** model and the second model the **full** model. This is because the reduced model is a subset of the full model (you get the reduced from the full by setting $\beta_1 = 0$).

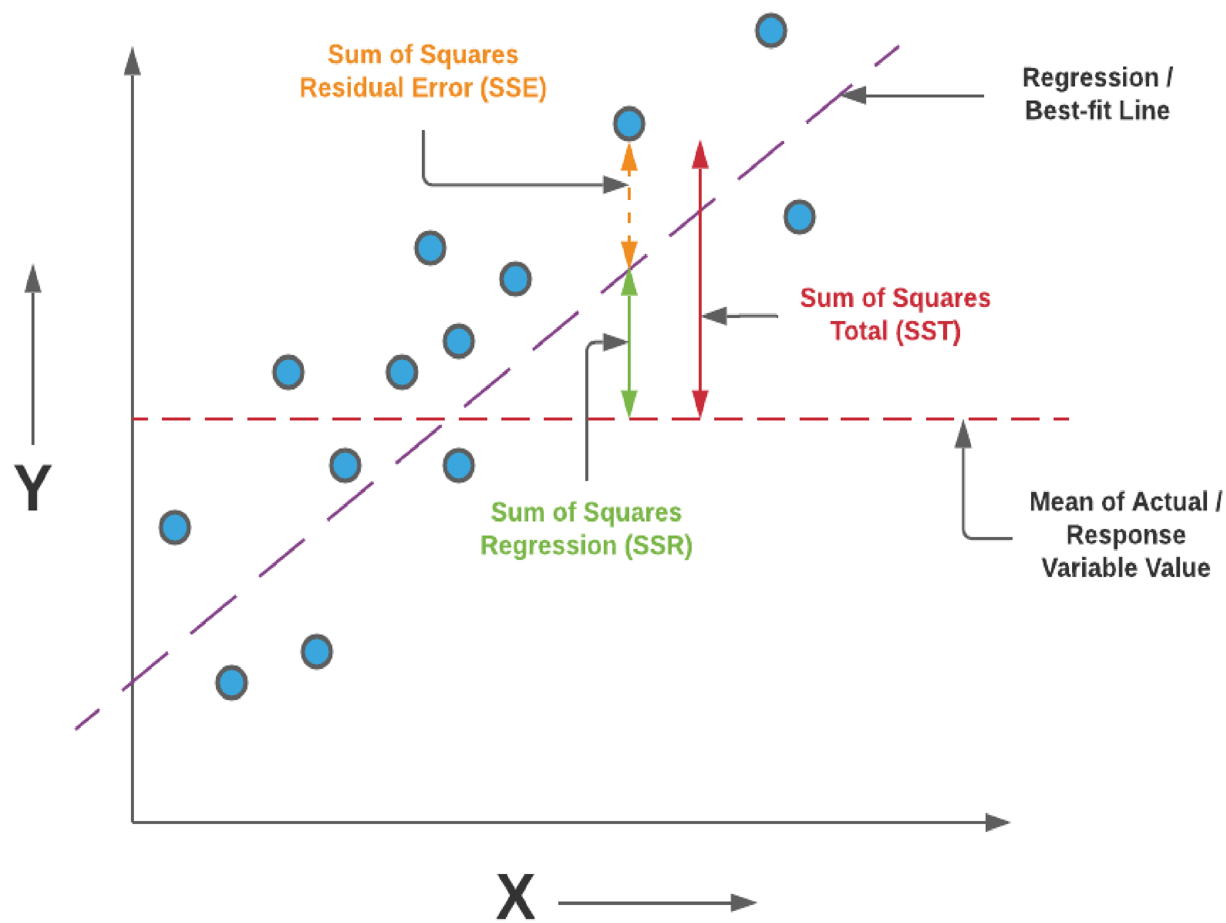
- Our strategy will be to compare the residuals under H_0 and H_A. If H_A were true, we would expect the those residuals to be much smaller than the residuals under H_0 (because the line fits a lot better).

If H_0 were true, then we would expect the residuals under H_A to only be a little bit smaller than those under H_0

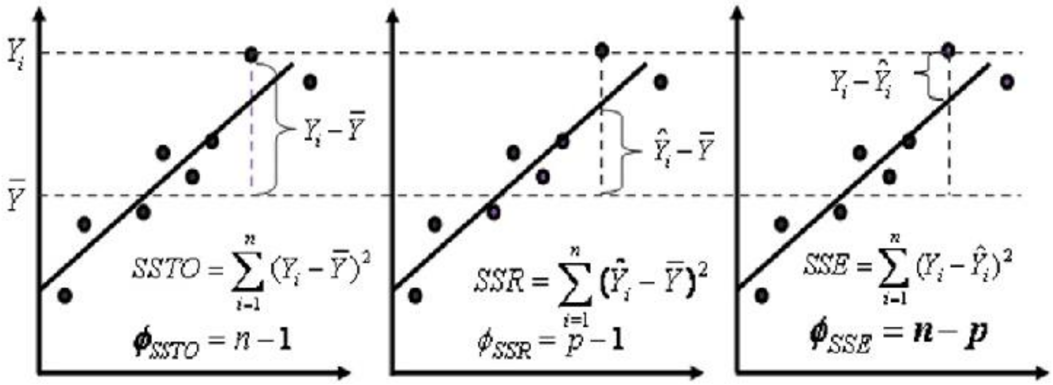
- We fit H_A by the method of least squares, obtaining the OLS estimates and the corresponding residuals.

- We fit H_0 also by least squares. It turns out that under H_0, the OLS estimate is just $\bar{Y}$ We measure how small the residuals are by the sum of squared residuals.

PICTORIAL and PLOT representation SSTO, SSE, and SSR



Example:



X	Y	Y_i(hat) Predicted Value	Y_I - Y_i(hat) Residual	(Y_I - Y(bar))^2	(Y_I - Y(hat))^2	(Y_i(hat) - Y(bar))^2
34	5	4.1505	.8496	25	.72165	34.21665
108	17	14.9693	2.0307	49	4.12374	24.69394
64	11	8.5365	2.4635	1	6.06883	2.14183
88	8	12.0453	-4.0453	4	16.3644	4.18325
99	14	13.6535	.3465	16	.12006	13.34906
51	5	6.6359	-1.6359	25	2.6762	11.31717
TOTALS				120	30.07488	89.9019

Y(bar) = 10

The following Regression Model for the data is found by using R coding: lm(y ~ x)

Y(hat) = -0.8203 + 0.1462X

The following ANOVA table is generated by using R coding: anova(lm(y ~ x)

```
Analysis of Variance Table

Response: y

          Df   Sum Sq   Mean Sq   F value  Pr(>F)

x          1   89.925   89.925    11.96    0.02586 *

Residuas   4   30.075   7.519

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Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Now let us conduct the F test to determine if H_o: *$Yi = \beta_o + \epsilon_i$ should be rejected.*

If the $F_{value,}$ is greater than the $F_{critical,}$ we reject H_o and conclude H_a
If the $F_{value,}$ is less than or equal to $F_{critical,}$ we fail to reject H_a
The $F_{value,}$ is found in the ANOVA table.
The $F_{critical}$ value if found in the F distribution table, using the construction F(1 – alpha; 1, n -2) or by using
For our case, $F_{value,}$ is 11.96 (found in the ANOVA output table).

The $F_{critical}$ value can be found as follows;

Let alpha = .05, and our designated degrees of freedom are 1 and 6.

We therefore require F(.95,1, 4), now going to the F table in the back of the we get 7.71

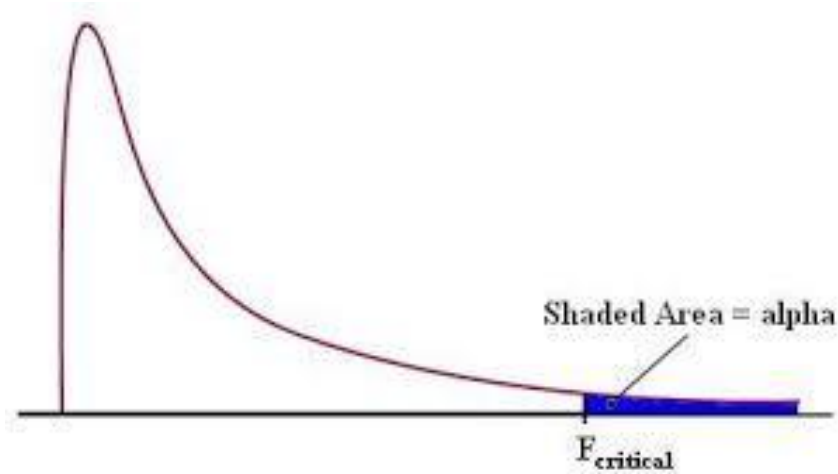
We can also use R code as follows to get the $F_{critical}$ value.

```
qf(p=.05, df1=1, df2=4, lower.tail=FALSE)
```

```
7.708647
```

Since $F_{value,}$(11.96) is greater than $F_{critical,\,,}$(7.71), we will reject the null Hypothesis that $B_1 = 0$ and conclude that B_1 does not = 0. Moreover, a linear relationship does exist between Y and X.

Graphical Interpretation: F Distribution Curve



If F_{value} is greater than (to the right of $F_{critical}$ you are to reject H_o

If F_{value} is less than (to the left)of $F_{critical}$ or equalt to $F_{critical}$, you fail to reject H_o

F Distribution Table (Percentiles)

		1	2	3	4	5	6	7	8	9	10
0.95	1	161.5	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
0.975		647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.7
0.99		4052.2	4999.5	5403.4	5624.6	5763.7	5859.0	5928.4	5981.07	6022.4	6055.9
0.95	2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
0.975		38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
0.99		98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
0.95	3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
0.975		17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
0.99		34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
0.95	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
0.975		12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
0.99		21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
0.95	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
0.975		10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
0.99		16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
0.95	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
0.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
0.99		13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
0.95	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
0.975		8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
0.99		12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
0.95	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
0.975		7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
0.99		11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
0.95	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
0.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
0.99		10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
0.95	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
0.975		6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
0.99		10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
0.95	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
0.975		6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53
0.99		9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
0.95	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
0.975		6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37
0.99		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
0.95	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
0.975		6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25
0.99		9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
0.95	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
0.975		6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15
0.99		8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
0.95	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
0.975		6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06
0.99		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80