Macroeconomics II, Problem Set 1 (Individual)

Jingwan (Amadea) Luo [1]

January 27, 2023

[1] IDEA Graduate Programme in Economics, first year student

This document answers all questions in the problem set 1 of *Macroeconomics II* taught by *Prof. Alexander Ludwig*. Content in **blue** is for grading. Sidenote is for self-memo. For computational and empirical analysis, the software in use is `Matlab`. Codes are saved for Section Annex, the `.m` files can be found both accompanied with my submission to *TA. Mridula Duggal* before *27th Jan., 9 a.m.* and also my Github Repository [2].

The Solow Growth Model in Discrete Time

Consider a closed economy where in each period t output Y_t is produced using a neoclassical production function $F(\cdot)$ with inputs capital K_t and labour L_t. Labour is augmented by a technology level A_t so that $A_t L_t$ are the efficiency units of labour. Thus, output in the economy is given by

$$Y_t = F(K_t, A_t L_t) \tag{1}$$

In a closed economy we have that from national income accounting all goods produced must be consumed or invested. Aggregate consumption in the economy is C_t and aggregate investment is I_t and thus

$$Y_t = C_t + I_t \tag{2}$$

Installed capital depreciates in each period at constant rate δ. Over time, the capital stock therefore evolves as

$$K_{t+1} = K_t(1 - \delta) + I_t \tag{3}$$

We assume a constant saving rate s, so that aggregate savings in the economy are $S_t = sY_t$ and aggregate consumption is $C_t = (1 - s)Y_t$.

Throughout, we assume a Cobb-Douglas production function

$$Y_t = F(K_t, A_t L_t) = K_t^\alpha (A_t L_t)^{1-\alpha} \tag{4}$$

and we define by $k_t = \frac{K_t}{A_t L_t}$ the capital stock per efficiency unit.

1.1 Analysis of the model

QUESTION 1.1.1 Show that the aggregate income resource constraint can be rewritten as

$$K_{t+1} = F(K_t, A_t L_t) + (1 - \delta)K_t - C_t \tag{5}$$

Solution. Recall total amount of goods is either consumed or invested, by Equation (2) we rewrite investment I_t as

$$I_t = Y_t - C_t,$$

Further, total amount of goods in the economy, or output, is produced under Equation (1), thus investment I_t is also

$$I_t = F(K_t, A_t L_t) - C_t,$$

combine the re-written investment with law of motion of capital presented in Equation (3) gives the desired result which is

$$K_{t+1} = (1 - \delta)K_t + F(K_t, A_t L_t) - C_t.$$

∎

QUESTION 1.1.2 Use the Cobb-Douglas production function and show that you can rewrite the production function in intensive form as

$$y_t = f(k_t) = k_t^\alpha$$

where $y_t = \frac{Y_t}{A_t L_t}$ and $k_t = \frac{K_t}{A_t L_t}$.

Solution. Taking the functional form of production function given in Equation (4) and divided both sides with $A_t L_t$,

$$y_t \equiv \frac{Y_t}{A_t L_t} = K_t^\alpha \cdot \frac{(A_t L_t)^{1-\alpha}}{A_t L_t} = \left(\frac{K_t}{A_t L_t}\right)^\alpha \equiv k_t^\alpha, \quad \alpha \in (0,1),$$

Notice that with the assumption of the functional form in Equation (4), production function exhibits constant returns to scale, to see this, take $\lambda \in \mathbb{R}_+$

$$F(\lambda K_t, \lambda A_t L_t) = \lambda^\alpha \lambda^{1-\alpha} K_t^\alpha L_t^{1-\alpha} = \lambda K_t^\alpha (A_t L_t)^{1-\alpha} = \lambda F(K_t, A_t L_t),$$

Recall production function presented in Equation (1), divided both sides with $A_t L_t$ and write it in output per capita term gives [3]

$$y_t \equiv \frac{Y_t}{A_t L_t} = F(\frac{K_t}{A_t L_t}, 1) = f(k_t),$$

[3] Here the validity of this step is relying on the assumption of production function being constant returns to scale, which is proved earlier.

So now that with constant returns to scale of the production function, output per capita is simply a function of the capital-effective labour ratio.

In conclusion,

$$y_t = f(k_t) = k_t^\alpha.$$

as requested to prove. ∎

QUESTION 1.1.3 Show that $f(k_t)$ satisfies *Inada Condition* and is globally concave.

Solution. For $k_t \in \mathbb{R}^+$ and $f(k_t) = k_t^\alpha$, it's evidently *twice differentiable* on $(0, \infty)$, since $k_t \in (0, \infty)$,

$$\lim_{k_t \to 0} f(k_t) = 1 \quad \lim_{k_t \to \infty} f(k_t) = 0,$$

for all $\alpha \in (0, 1)$, therefore it satisfies Inada condition;

Let's look at the Hessian of $f(k_t)$

$$f'(k_t) = \alpha k_t^{\alpha-1} > 0, f''(k_t) = \alpha(\alpha-1)k_t^{\alpha-2} < 0, \quad \alpha \in (0, 1)$$

So the Hessian is negative-definite for any k_t, meaning it's globally concave. ■

QUESTION 1.1.4 Let profits of the representative firm be

$$\Pi_t = f(k_t)A_tL_t - w_tL_t - (r_t + \delta)K_t.$$

Show that profit maximisation implies that [4]

$$r_t = f'(k_t) - \delta \tag{5a}$$
$$w_t = A_t f(k_t) - f'(k_t)k_t \tag{5b}$$
$$\Pi_t = 0 \tag{5c}$$

Solution. Notice that firstly

$$f(k_t)A_tL_t = F(k_t \cdot A_tL_t, A_tL_t) = F(K_t, A_tL_t) = Y_t,$$

We first set up the firms' profit maximisation problem and check its first order conditions

$$\max_{K,L} \Pi_t = F(K_t, A_tL_t) - w_tL_t - (r_t + \delta)K_t$$

$$[L_t] : \frac{\partial F_L(K_t, A_tL_t)}{\partial L_t} - w_t =_{\text{set}} 0$$

$$(1 - \alpha)K_t^\alpha A_t^{1-\alpha}L_t^{-\alpha} = w_t$$

$$(1 - \alpha)\frac{Y_t}{L_t} = w_t;$$

$$[K_t] : \frac{\partial F_K(K_t, A_tL_t)}{\partial K_t} - (r_t + \delta) =_{\text{set}} 0$$

$$\alpha K_t^{\alpha-1}(A_tL_t)^{1-\alpha} = r_t + \delta$$

$$\alpha \frac{Y_t}{K_t} = r_t + \delta;$$

Substitute the first order conditions back into the objective function, then by Euler's Theorem [5]

$$\Pi_t = F(K_t, \ A_tL_t) - w_tL_t - (r_t + \delta)K_t$$

$$= F(K_t, A_tL_t) - \underbrace{(1 - \alpha)\frac{Y_t}{L_t}}_{=w_t = \frac{\partial F_L(K_t, \ L_t)}{\partial L_t}} L_t - \underbrace{\alpha \frac{Y_t}{K_t}}_{=r_t + \delta = \frac{\partial F_K(K_t, \ L_t)}{\partial K_t}} K_t$$

$$= 0.$$

Therefore Equation (5c) is proved. In turn, profit is zero also in effective labour terms, $\pi_t = 0$. Transform the profit function into per effective labour units,

$$\frac{\Pi_t}{A_t L_t} = f(k_t) - w_t \cdot \frac{1}{A_t} - (r_t + \delta)\frac{K_t}{A_t L_t},$$

$$\pi_t = f(k_t) - \underbrace{w_t \cdot \frac{1}{A_t}}_{\text{detrended wage rate}} - (r_t + \delta)k_t.$$

Firms' profit maximisation problem is then

$$\max_{k_t} \pi_t,$$

Compute the first order condition with respect to k_t and set it to zero

$$f'(k_t) = (r_t + \delta),$$

So we reached the condition required in Equation (5a). Together with *zero profit condition* and substitute the above $\mathcal{F.O.C.}$ into the objective function in effective labour terms

$$f(k_t) - \frac{w_t}{A_t} - (r_t + \delta)k_t = 0,$$

$$f(k_t) - f'(k_t)k_t = \frac{w_t}{A_t}. \tag{1.1.4.1}$$

Therefore, condition presented in Equation (5b) is also reached.   ■

QUESTION 1.1.5 Assume that $A_t = A_0(1+g)^t$, $L_t = L_0(1+n)^t$ for some initial levels A_0, L_0. Show that you can rewrite Equation (5) as

$$k_{t+1} = \frac{1}{(1+g)(1+n)}(sf(k_t) + (1-\delta)k_t) \tag{6}$$

and argue why this implies a unique long-run equilibrium capital stock k^*.

Solution. Notice that we can re-write

$$K_{t+1} = (1-\delta)K_t + F(K_t, A_t L_t) - C_t$$
$$= (1-\delta)K_t + Y_t - (1-s)Y_t$$
$$= (1-\delta)K_t + sY_t,$$

Or in per effective labour

$$\frac{K_{t+1}}{A_{t+1}L_{t+1}}\frac{A_{t+1}L_{t+1}}{A_t L_t} = (1-\delta)k_t + sy_t,$$

$$k_{t+1}(1+g)(1+n) = (1-\delta)k_t + sy_t. \tag{1.1.5.1}$$

Recall that $y_t = f(k_t)$ substitute it in, then

$$k_{t+1} = \frac{1}{(1+g)(1+n)}\left(sf(k_t) + (1-\delta)k_t\right)$$

is reached.

Reason: Firstly, the RHS of Equation 6) is continuous, strictly concave and satisfies Inada condition, it's evidently that we will have a fixed point for which $RHS(k^*) = k^*$ can be proved by Banach fixed point theorem; secondly, functions with Inada condition satisfied can guarantee the existence of interior equilibria. ∎

QUESTION 1.1.6 Show that you can rewrite Equation (6) further as

$$\Delta k_t(n + g + ng) = sf(k_t) - (\delta + n + g + ng)k_t \qquad (7)$$

where $\Delta k_t = k_{t+1} - k_t$, and plot $f(k_t)$, $sf(k_t) - (\delta + n + g + ng)k_t$ as well as the equilibrium capital stock k^*. Assume that $n \cdot g = 0$ and provide an interpretation of k^*.

Solution. By Equation (1.1.5.1),

$$k_{t+1}(1+g)(1+n) - (1+g)(1+n)k_t = k_t(1 - \delta - (1+g)(1+n)) + sy_t,$$
$$\Delta k_t(1+g)(1+n) = -k_t(n + g + ng + \delta) + sf(k_t),$$

as requested. At steady state, $\Delta k_t = 0$,

$$k^*(n + g + ng + \delta) = sk^*,$$

$$k^* = \left(\frac{s}{n + g + ng + \delta}\right)^{\frac{1}{1-\alpha}}.$$

Figure (1) depicts the Solow dynamics as requested. The calibration parameters in use are as followed, values in use are from (Gomme and Lkhagvasuren, 2013).

Figure 1: PRODUCTION FUNCTION, BREAKEVEN INVESTMENT AND EQUILIBRIUM CAPITAL

Meaning	Notation	Value
Saving Rate	s	0.2
Capital Share	α	0.3
Population Growth Rate	n	0.01
Depreciation Rate	δ	0.05
Technology Growth Rate	g	0.05

The codes to generate the plot can be found at Annex. ∎
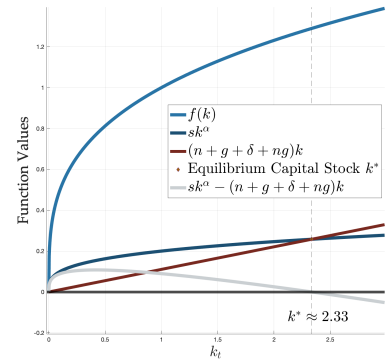
1.2 Computational Analysis of the Model

Assume that the initial equilibrium, i.e., for all $t \leq 0$, saving rate is $s^* = 0.2$:

QUESTION 1.2.1 Assume that a social planner implements the saving rate s^{GR} in period $t = 1$. Argue and verbally characterise the equilibrium dynamics of the capital intensity k_t over time.

Solution. At golden rule saving rate, capital's dynamics is on balanced growth path, growth rate of capital is consistent with the growth rate of consumption and output. ∎

QUESTION 1.2.2 Analytically compute the labour share $LS_t = \frac{w_t L_t}{Y_t}$ for the Cobb-Douglas production function.

Solution. Recall in Equation (4) the functional form of Cobb-Douglas function,

$$
\begin{aligned}
LS_t &= \frac{w_t L_t}{Y_t} \\
&= \frac{w_t L_t}{K_t^\alpha (A_t L_t)^{1-\alpha}} \\
&= w_t \cdot \left(\frac{K_t}{A_t L_t} \right)^{-\alpha} \\
&= w_t k_t^{-\alpha},
\end{aligned}
$$

Also recall that in Equation (1.1.4.1),

$$
\begin{aligned}
\frac{w_t}{A_t} &= f(k_t) - f'(k_t) k_t \\
&= k_t^\alpha - \alpha k_t^{\alpha-1} k_t \\
&= (1 - \alpha) k_t^\alpha \\
w_t &= A_t[(1 - \alpha) k_t^\alpha];
\end{aligned}
$$

Substitute w_t in, [6]

$$
LS_t = A_t[(1 - \alpha) k_t^\alpha] \cdot k_t^{-\alpha} = A_t(1 - \alpha).
$$

∎

[6] Since the labour cost $w_t L_t$ is not in effective labour units, in the final analytical expression, there's an extra A_t comparing to the standard result, which is $1 - \alpha$.

QUESTION 1.2.3 Go to the NIPA (National Income and Product Accounts) tables and obtain the following annual time series starting in 1960 [7]:

+ "Gross domestic product" from Table 1.1.5 (BEA, 2023d);

+ "Compensation of employees" from Table 1.12 (BEA, 2023a) ;

+ "Proprietors' income with IVA and CCAdj", "Taxes on production and imports", "Business current transfer payments (net)", "Current Surplus of government enterprises" from Table 1.12. (BEA, 2023e) Sum these up and call them "Adjustment to GDP".

[7] Here, duration of study is 1960-2022, frequency in use is *annual*, unit in use is *billions*.

Solution. Please find the code in annex as well as descriptive statistics of the extracted variables in Table (1). ■

	Mean	Median	S.d.	Min	Max
GDP	7935.35	6060.60	6739.06	542.40	23315.10
Labour Cost	4321.26	3395.45	3601.39	301.30	12538.50
Adjustment to GDP	1201.91	839.75	1049.32	97.30	3590.10

Table 1: DESCRIPTIVE STATISTICS FOR LABOUR COST AND GDP

QUESTION 1.2.4 Compute "Net GDP" as "Gross Domestic Product" minus "Adjustment to GDP" and the ratio of "Compensation to employees" to "Net GDP". Plot the ratio you obtained over time. Interpret your findings with the background of the Cobb-Douglas production function. Report the average of the ratio.

Solution. INTERPRETATION: Here the average of the ratio is

$$\mu_{\text{Labour Share}} = 0.6523.$$

Figure (2) along with the above average depicts that labour expenditure constitutes larger proportion relative to other factors of inputs (over 50 %). Under the assumption of the Cobb-Douglas production function [8], by QUESTION 1.2.2, we know:

+ (If we define labour cost in effective labour terms, namely, $\frac{w_t A_t L_t}{Y_t}$), we could say that the average of labour share is $1 - \alpha = 0.6523$,

+ the **output elasticity with respect to labour** should be about 0.65, namely,

$$\epsilon_L = \frac{\partial Y_t}{\partial L_t} \frac{L_t}{Y_t} = (1 - \alpha) \left(\frac{K_t}{L_t} \right)^\alpha \frac{L_t}{Y_t} = 1 - \alpha.$$

Moreover, by recent empirical findings, the calibration parameter for capital share is about 0.2852 (Gomme and Lkhagvasuren, 2013), so capital share together with $\mu_{\text{Labour Share}}$ adds up to one, implies that our findings about the labour share approximately coincides with the literature.

Further, in Figure (3) presents the ratio plot requested. From this graph, we could see the ratio is in decreasing trend, meaning labour share in production has been declining. One possible reason for this is that the growth rate of compensation of labour is smaller than the growth of productivity, to see this:

$$g_{LS} \equiv \frac{d \ln w_t L_t}{d w_t L_t} \frac{d w_t L_t}{dt} - \frac{d \ln Y_t}{d Y_t} \frac{d Y_t}{dt} < 0$$
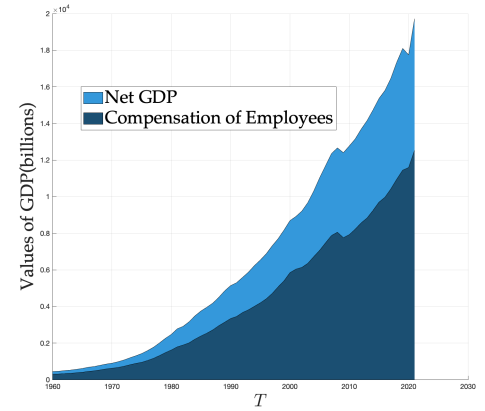
$$\text{iff} \quad g_{coe} < g_Y.$$

Figure 2: LABOUR SHARE IN AREAS

[8] Here, not necessarily $K_t^\alpha (A_t L_t)^{1-\alpha}$, but also more generic functional form, $K_t^\alpha L_t^{1-\alpha}$, with constant returns to scale property holds.
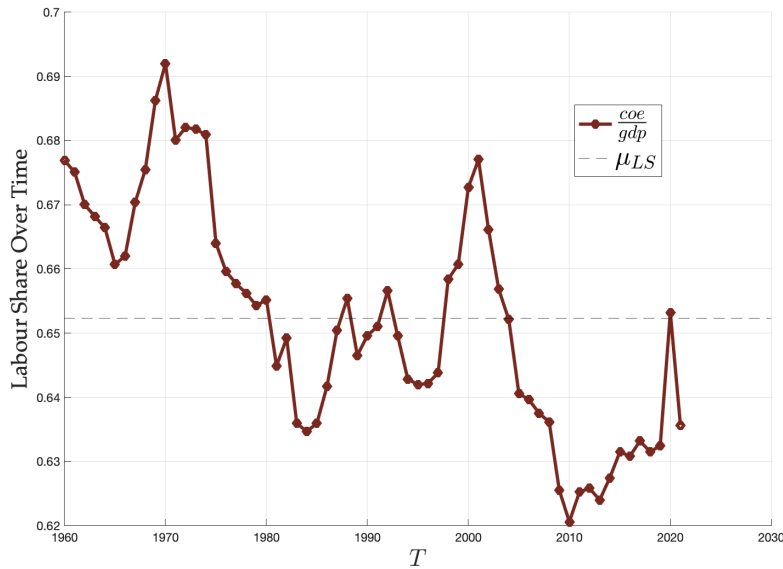
QUESTION 1.2.5 Obtain "Fixed Assets" from Table 1.1 (BEA, 2023*b*) [9] as "Capital Stock", K_t. Compute and plot the "Capital-Output-Ratio" by relating this to "Gross Domestic Product" and obtain the average.

[9] For this, go to the National Data on Fixed Assets, duration of study is 1960-2021, frequence in use is *annual*, unit in use is *billions*.

Solution. Here the average of the ratio is

$$\mu_{k/y(\text{unadj.}) \text{ ratio}} = 2.9299;$$

$$\mu_{k/y(\text{net}) \text{ ratio}} = 3.4564.$$

The plot requested can be found at Figure (5). [10]

[10] in (Gomme and Lkhagvasuren, 2013), the calibrated parameter for capital-output ratio is 1.6590.

QUESTION 1.2.6 Go to NIPA Table 1.3 (BEA, 2023*f*) [11] and download the series of depreciation of fixed assets and use the capital stock data to compute the depreciation rate δ_t. Plot it and compute its mean.

[11] Also in the National Data on Fixed Assets table.

Solution. Here the depreciation rate is computed based on

$$\delta = \frac{\text{Fixed Asset Depreciation}}{\text{Fixed Asset (Capital Stock)}},$$

Depreciation rate's mean value is

$$\mu_\delta = 0.0495.$$

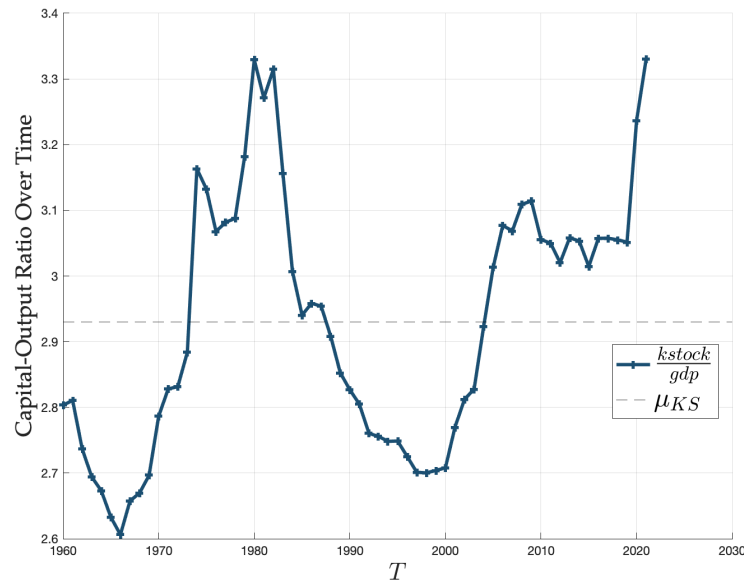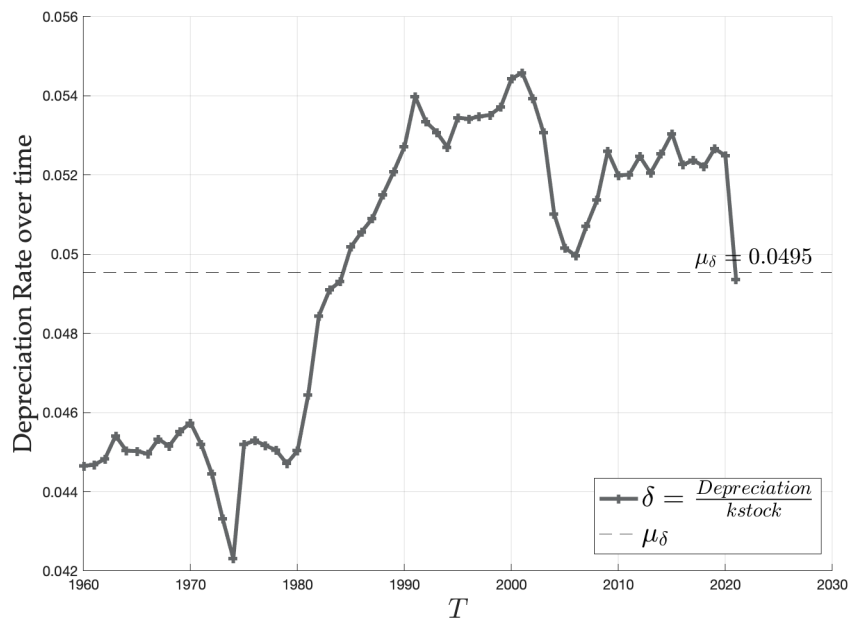Figure 4: CAPITAL-OUTPUT RATIO OVER TIME

Figure 5: DEPRECIATION RATE OVER TIME

QUESTION 1.2.7 Deflate "GDP" using the "GDP deflator" and "Capital Stock" by the price index for "Gross private domestic investment" (BEA, 2023c) to obtain real variables [12]. Download also "Total hours worked in domestic industries" (BEA, 2023g,h,i) [13] and label it as L_t. From

$$Y_t^r = (K_t^r)^{\alpha_t} (A_t L_t)^{1-\alpha_t}$$

where the superscript r denotes real variables and α_t is obtained from the time series of the labour share above. Compute A_t. Compute the plot its growth rate g_t [14] as well as its mean g and interpret it.

[14] Throughout you may compute growth rates as $x_t = \frac{X_t - X_{t-1}}{X_{t-1}}$. Then $x = \bar{x}_t$ is the average growth rate.

Solution. Computed $g_{A_t} = 0.1961$, namely the growth rate of technology. Analytically, $A_t^{1-\alpha_t}$ is the Solow residual, and also called total factor productivity. The growth rate of TFP is then $(1 - \alpha) \cdot g_{A_t}$.

Figure 6: GROWTH RATE OF TECHNOLOGY

Figure (6) demonstrates the evolution of g_{A_t}. From this graph, we can observe that the growth rate of technology approximately fluctuates around its mean value.   ■

QUESTION 1.2.8 From your sequence of L_t compute the growth rate of aggregate hours worked n_t and plot it. Also compute its mean n.

Solution. Similarly to the previous analysis and method, Figure (7) demonstrates the growth rate of hours worked. Computed mean is $n = 0.0133$.   ■

QUESTION 1.2.9 From now on, base the mean computations of n, g, δ on the data for the past 20 years. Using this data, compute s^* [15]. How does it mean compare to s^{GR}?

[15] $s^* = 0.2$ is given.

Figure 7: GROWTH RATE OF L_t

Solution. So far, $n = 0.0133$, $\delta = 0.0495$, $g = 0.1961$. Plug in to the previously written model, $s = 0.2453$. In Comparison, very close to steady state value of saving rate.

QUESTION 1.2.10 Simulate k_t, y_t, c_t for some periods using a programming language of your choice. Interpret it.

Solution. | INTERPRETATION: | With the set of parameters,

+ it converges;

+ the periods needed for k_t to converge is approximately $T = 10$.

+ as T grows larger, the degree of concavity for the capital's evolution is larger.

Figure (8) is for demonstrating k_t, since both y_t and c_t are monotonic transformation of k_t, it is sufficient. ■
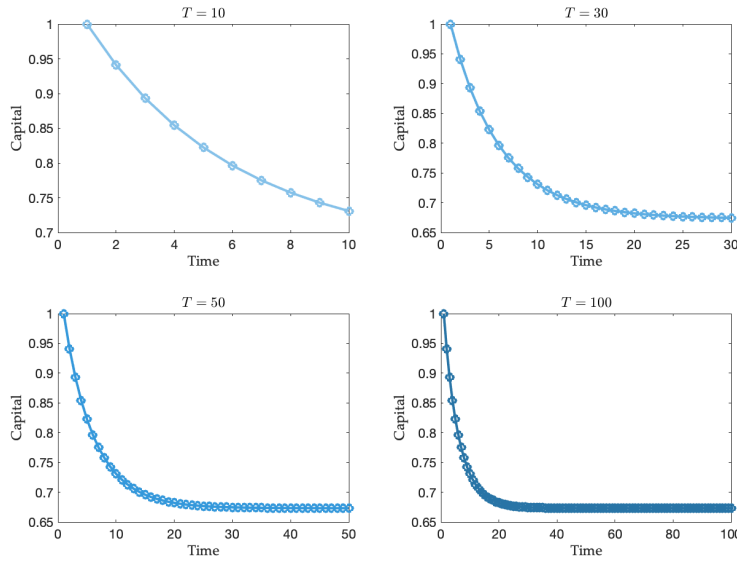
1.3 Empirical Analysis

Go to the AMECO database and download, for all available countries, *i)* Civilian employment, *ii)* GDP, *iii)* the Solow residual, *iv)* gross domestic savings, *v)* gross domestic consumption.

QUESTION 1.3.1 Compute the gross domestic saving rate $s_t = \frac{S_t}{Y_t}$ for all countries and detrended domestic consumption using the sequences for $A_t L_t$.

Solution. The study scope after excluding the missing is 1994-2021, while 22 countries included in the sample. In Table (2) [16], mean values of the requested computed values for each countries are presented. ∎

[16] See Annex.

QUESTION 1.3.2 Construct a scatter plot of $\ln c_t$ on s_t.

QUESTION 1.3.3 Run a pooled regression

$$\ln c_t = \beta_0 + \beta_1 s_t + \beta_2 s_t^2$$

and interpret your findings.

Solution. `p = polyfit(x,y,n)` returns the coefficients for a polynomial $p(x)$ of degree n that is a best fit (in a least-squares sense) for the data in y.

$$p = [\beta_2, \beta_1, \beta_0] = [0.000064, -0.0083, -7.833]$$
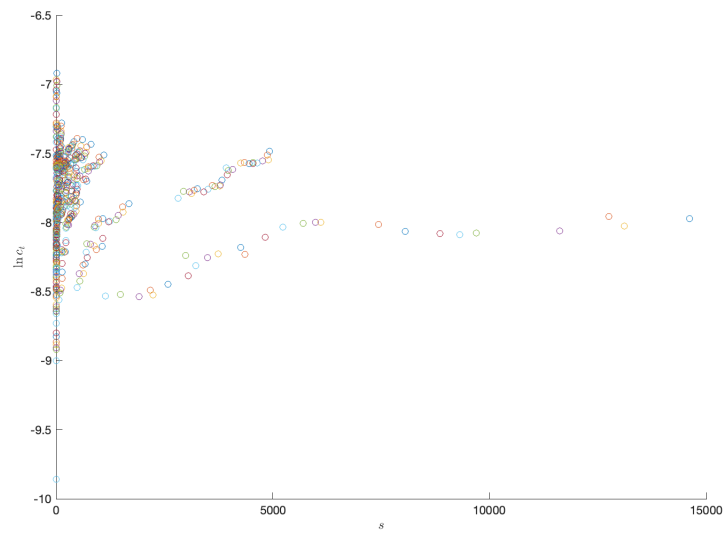
Solution.

INTERPRETATION : hump shape [17] relationship between log of
detrended consumption and saving rate, while taking out of the effect
$A_t L_t$ makes the level of the parameter small.   ∎

[17] upside-down U shape

Literature

Read and briefly [18] summarise the following papers:

+ (Solow, 1956), "A Contribution to the Theory of Economic Growth";

+ (Mankiw et al., 1992), "A Contribution to the Empirics of Economic Growth".

LITERATURE REVIEW ON SOLOW (1956) In the paper which is based on Harrod-Domar model, contribution manifests itself in the following ways, firstly, this version of growth model forsaken fixed proportion assumptions; secondly, it's the very foundation of the concept, "the state of the balanced growth ", which characterised a sense of convergence to a natural rate.

Regarding assumptions, Solow-Swan growth model posits that as more capital is added to a closed economy, the additional output from each unit of capital will decrease. This is known as diminishing returns. In the absence of technological advancements or an increasing workforce, at some point the amount of capital being produced will only be enough to replace capital that has been lost through depreciation. At this point, the economy will stop growing. If there are non-zero rates of labor growth, the situation becomes more complex but the basic concept still applies. The short-term rate of growth will slow as diminishing returns take effect, eventually reaching a constant, steady-state rate of growth. If there is also technological progress, output per worker-hour will remain constant but per-capita output will grow at the rate of technological progress in the steady-state.

The paper also mentions about its extensions such as technology changes, supply of labor, changeable saving ratio, taxation and variable population growth. It expands upon the Harrod-Domar model by including labor as a factor of production and allowing for capital-output ratios that are not fixed. This allows for the distinction between increasing capital intensity and technological progress. Solow considers the fixed proportions production function to be a key assumption in the instability results of the Harrod-Domar model and examines the implications of alternative specifications, such as the Cobb-Douglas and constant elasticity of substitution. While this model has become a central and celebrated concept in economics, recent reappraisals of Harrod's work have questioned its origins and focus on economic growth and fixed proportions production function. ∎

LITERATURE REVIEW ON MANKIW ET AL. (1992) This paper bases itself again on "A Contribution to the Theory of Economic Growth"(Solow, 1956), a contribution on top of the contribution, in the sense that the authors includes the concept of human capital in addition to physical capital in order to better understand the reasons for the lack of investment in poorer countries. According to this model, low levels of human capital in such countries leads to lower levels of output and a decrease in the marginal productivity of capital, which in turn makes these countries less attractive for investment.

Two fundamental dynamic equations in this version is brought up:

$$\dot{k} = s_K k^\alpha h^\beta - (n + g + \delta)k$$
$$\dot{h} = s_H k^\alpha h^\beta - (n + g + \delta)h$$

By parallel comparing, previously, in the Solow-Swan model, the rate of growth in productivity is determined by factors outside the model, represented by the residual after accounting for capital accumulation. However, this paper proposed an augmented version of the model that takes into account human capital as a factor of production (Mankiw et al., 1992). This modification allows for a more accurate explanation of the differences in income across countries, as the effect of human capital is now accounted for in the model. As a result, the residual TFP growth rate is lower in this model than in the basic Solow-Swan model.

■

Annex

The following summarises files, "`Q116_SolowDynamics.m`", "", and "", can be found at the `.zip` folder in the submission. [19]

To accompany QUESTION 1.1.6

```matlab
1   %% Question 1.1.6
2   clc;clear;
3   %%
4   %Before anything, set the graph aesthetics
5   PS = PLOT_STANDARDS();
6
7   %Define parameter values
8   s = 0.2;
9   alpha = 0.3;
10  n = 0.01;
11  ∆ = 0.05;
12  g = 0.05;
13
14  %Create a number of 1000 k values from 1 to 3
15  k = linspace(0, 3, 1000);
16
17  %Compute sk^alpha and (n+g+∆)k
18  k_alpha = k .^ alpha;
19  sk_alpha = s .* k .^ alpha;
20  ngd_k = (n + g + ∆ + n*g) .* k;
21  line = s .* k .^ alpha - (n + g + ∆ + n*g) .* k;
22
23  %Find the intersection point of the two functions except ...
        for k=0
24  intersection_point = fzero(@(k) s*k^alpha-(n+g+∆+n*g)*k, ...
        [1,3]);
25
26  %Create a figure
27  figure(1);
28  fig1_comps.fig = gcf;
29  grid on;
30  hold on;
31
32  %Plot the requested functions
33  fig1_comps.p0 = plot(k, k_alpha);
34  fig1_comps.p1 = plot(k, sk_alpha);
35  fig1_comps.p2 = plot(k, ngd_k);
36  fig1_comps.p3 = scatter(intersection_point,...
37      s*intersection_point^alpha);
38  fig1_comps.p5 = plot(k, line);
39
40  %Graph aesthetics
41  set(fig1_comps.p0, 'Color', PS.Blue4, 'LineWidth', 6);
42  set(fig1_comps.p1, 'Color', PS.Blue5, 'LineWidth', 6);
43  set(fig1_comps.p2, 'Color', PS.Red5, 'LineWidth', 5.5);
44  set(fig1_comps.p3, 'Marker','diamond', ...
45      'MarkerFaceColor', PS.Orange5)
46  set(fig1_comps.p5, 'Color', PS.Grey2, 'LineWidth', 6);
47
48  xline(intersection_point, 'Color','black','LineStyle','--');
```

```matlab
49  yline(0, 'Color', 'black', 'LineWidth',5);

50

51  %Label the axis and the title
52  xlabel('$k_t$','FontSize',28,'Interpreter','latex');
53  ylabel('Function Values', 'FontSize',28, ...
        'FontName','Palatino');
54  legend('$f(k)$',...
55      '$sk^\alpha$',...
56      '$(n+g+\a+ng)k$', ...
57      'Equilibrium Capital Stock $k^*$',...
58      '$sk^\alpha-(n+g+\a+ng)k$',...
59      'Location','best',...
60      'Interpreter','latex',...
61      'FontSize', 28)
62  text(2.3, -0.01, '$k^* \approx 2.33$', ...
        'Interpreter','latex', 'FontSize', 28);
63  hold off;
```

To accompany QUESTION 1.2.3

```matlab
1  %% Question 1.2.3
2  clc;clear;
3
4  % Set the working directory to the place where the ...
        current file is saved
5  tmp = matlab.desktop.editor.getActive;
6  cd(fileparts(tmp.Filename));
```

```matlab
1  % Read and extract Gross Domestic Product Data from Table ...
        1.1.5
2
3  T115 = readtable('Table 1.1.5.xlsx', ...
        'VariableNamingRule','preserve');
4  gdp = T115{1,3:end};
5
6  % Read and extract Compensation of Employees from Table 1.12
7
8  T112 = readtable('Table 1.12.xlsx', ...
        'VariableNamingRule','preserve');
9  coe = T112{2,3:end};
10
11 % Extract and munipulate for ... from Table 1.12
12
13 % Proprietors' income with IVA and CCAdj
14 % Taxes on production and imports
15 % Business current transfer payments (net)
16 % Current Surplus of government enterprises
17
18 adj1 = T112{32, 3:end};
19 adj2 = T112{19, 3:end};
20 adj3 = T112{21, 3:end};
21 adj4 = T112{25, 3:end};
22
23 adjgdp = adj1+adj2+adj3+adj4;
```

```matlab
1   %% A quick descriptive stats for variables in 1.2.3
2
3   variables = {'gdp', 'coe', 'adjgdp'};
4   mean_values = zeros(1,3);
5   median_values = zeros(1,3);
6   std_values = zeros(1,3);
7   min_values = zeros(1,3);
8   max_values = zeros(1,3);
9
10  for i = 1:length(variables)
11      mean_values(i) = mean(variables{i});
12      median_values(i) = median(variables{i});
13      std_values(i) = std(variables{i});
14      min_values(i) = min(variables{i});
15      max_values(i) = max(variables{i});
16  end
17  stats = {'mean', 'median', 'std', 'min', 'max'};
18
19  summary_table = ...
        array2table(cell(length(variables),length(stats)),...
20      'VariableNames',stats);
21  for i = 1:length(variables)
22      for j = 1:length(stats)
23          summary_table{i,j} = arrayfun(@(x) ...
                sprintf('%.2f',x),...
24              eval([stats{j} '(' variables{i} ...
                    ')']),'UniformOutput',false);
25      end
26  end
27  summary_table.Properties.RowNames = variables;
28
29  %% (Please use the add-on 'MATLAB Table to LaTeX converter')
30
31  % Exporting Summary Stats as solution
32  table2latex(summary_table, 'Q123summary_stats.tex')
```

To Accompany Question 1.2.4

```matlab
1   %% Question 1.2.4 Compute Net GDP
2
3   %Before anything, set the graph aesthetics
4   PS = PLOT_STANDARDS();
5
6   netgdp = gdp - adjgdp;
7   labshare = coe ./ netgdp;
8   m_labshare = mean(labshare);
9
10  %% An Area Plot for Labshare
11
12  T = 1960:2021;
13  figure(2);
14  fig1_comps.fig = gca;
15  grid on;
16  hold on;
17
18  fig2_comps.p0 = area(T, netgdp);
19  fig2_comps.p1 = area(T, coe);
```

```matlab
20
21  set(fig2_comps.p0, 'FaceColor', PS.Blue3);
22  set(fig2_comps.p1, 'FaceColor', PS.Blue5);
23
24  xlabel('$T$','FontSize',18,'Interpreter','latex');
25  ylabel('Values of GDP(billions)', 'FontSize',22, ...
        'FontName','Palatino');
26  legend('Net GDP','Compensation of Employees',...
27      'FontSize',18, 'FontName','Palatino', 'Location', ...
            'Best');
28
29  hold off;
```

```matlab
1  %% A Line Plot for Labshare (requested by the Q)
2
3  T = 1960:2021;
4  figure(3);
5  fig3_comps.fig = gcf;
6  grid on;
7  hold on;
8
9  fig3_comps.p0 = plot(T, labshare);
10  fig3_comps.p1 = yline(m_labshare, 'Color','black',...
11      'LineStyle','--');
12
13  set(fig3_comps.p0, 'Color', PS.Red5, 'LineWidth', 3, ...
        'Marker','diamond');
14  text(2020, 0.6, '$\mu_{LS}=0.6523$', ...
15      'Interpreter','latex', 'FontSize', 18);
16
17  xlabel('$T$','FontSize',22,'Interpreter','latex');
18  ylabel('Labour Share Over Time', 'FontSize',20, ...
        'FontName','Palatino');
19  legend('$\frac{coe}{gdp}$', '$\mu_{LS}$', ...
20      'Interpreter', 'latex',...
21      'location', 'best','Fontsize', 24);
22
23  hold off;
```

To Accompany Question 1.2.5

```matlab
1  %% Question 1.2.5 Plot for kyratio
2
3  T1 = readtable('Table 1.1.xlsx', ...
        'VariableNamingRule','preserve');
4  kstock = T1{2, 3:end};
5
6  %capital-output ratio:
7  kyratio = kstock ./gdp;
8  m_kyratio = mean(kyratio);
9  kyratio2 = kstock ./netgdp;
10  m_kyratio2 = mean(kyratio2);
11  %figure
12  T = 1960:2021;
13
```

```matlab
14  figure(4);
15  fig4_comps.fig = gcf;
16  grid on;
17  hold on;
18
19  fig4_comps.p0 = plot(T, kyratio);
20  fig4_comps.p1 = yline(m_kyratio, 'Color','black',...
21      'LineStyle','--');
22
23  set(fig4_comps.p0, 'Color', PS.Blue5, 'LineWidth', 3, ...
        'Marker','+');
24  text(2020, 0.6, '$\mu_{LS}=0.6523$', ...
        'Interpreter','latex', ...
25      'FontSize', 18);
26
27  xlabel('$T$','FontSize',22,'Interpreter','latex');
28  ylabel('Capital-Output Ratio Over Time', ...
29      'FontSize',20, 'FontName','Palatino');
30  legend('$\frac{kstock}{gdp}$', '$\mu_{KS}$', ...
31      'Interpreter', 'latex',...
32      'location', 'best','Fontsize', 24);
33
34  hold off;
```

To Accompany Question 1.2.6

```matlab
1  %% Question 1.2.6 Compute depreciation rate
2
3  T13 = readtable('Table 1.3.xlsx', ...
       'VariableNamingRule','preserve');
4  depreciation = T13{2, 3:end};
5
6  Δ = depreciation ./ kstock;
7  m_Δ = mean(Δ);
8
9  T = 1960:2021;
10
11  figure(5);
12  fig5_comps.fig = gcf;
13  grid on;
14  hold on;
15
16  fig5_comps.p0 = plot(T, Δ);
17  fig5_comps.p1 = yline(m_Δ, 'Color','black',...
18      'LineStyle','--');
19
20  set(fig5_comps.p0, 'Color', PS.Grey5, 'LineWidth', 3, ...
        'Marker','+');
21  text(2000, 0.05, '$\mu_{\Delta}=0.0495$', ...
22      'Interpreter','latex', 'FontSize', 18);
23
24  xlabel('$T$','FontSize',22,'Interpreter','latex');
25  ylabel('Depreciation Rate over time', ...
26      'FontSize',20, 'FontName','Palatino');
27  legend('$\Delta = \frac{Depreciation}{kstock}$', ...
       '$\mu_{\Delta}$', ...
28      'Interpreter', 'latex',...
29      'location', 'best','Fontsize', 22);
```

```
30
31  hold off;
```

To Accompany Question 1.2.7

```matlab
1   %% Question 1.2.7
2
3   % Fetch price indexes for gdp and capital stock
4   T114 = readtable('Table 1.1.4.xlsx', ...
        'VariableNamingRule','preserve');
5   gdpd = T114{1, 3:end};
6   capp = T114{7, 3:end};
7
8   % Deflate gdp and capital stock
9   realgdp = gdp ./ gdpd;
10  realk = kstock ./capp;
11
12  % Fetch Total hours worked in domestic industries
13  % and glue them together from three Tables
14  T69B = readtable('Table 6.9B.xlsx', ...
        'VariableNamingRule','preserve');
15  h6087 = T69B{1, 3:end};
16  T69C = readtable('Table 6.9C.xlsx', ...
        'VariableNamingRule','preserve');
17  h8800 = T69C{1, 4:end};
18  T69D = readtable('Table 6.9D.xlsx', ...
        'VariableNamingRule','preserve');
19  h0121 = T69D{1, 4:end};
20  hw = horzcat(h6087, h8800, h0121);
21  %
22  % Compute A_t
23  capcomponent = realk.^(1-labshare);
24  hwcomponent = hw.^labshare;
25  power = 1./(1-labshare);
26  A = (realgdp ./ (capcomponent .* hwcomponent)).^power;
27
28  % Compute A_t growth rate
29  gA = diff(A) ./ A(1:end-1);
30  m_gA = mean(gA);
31
32  % Graph¬
33  figure(6);
34  fig5_comps.fig = gcf;
35  grid on;
36  hold on;
37
38  T = 1960:2020;
39  fig6_comps.p0 = plot(T, gA);
40  fig6_comps.p1 = yline(m_gA, 'Color','black',...
41      'LineStyle','--');
42  set(fig6_comps.p0, 'Color', PS.Red3, 'LineWidth', 3, ...
        'Marker','+');
43
44  xlabel('$T$','FontSize',22,'Interpreter','latex');
45  ylabel('Growth Rate of $A_t$', ...
46      'FontSize',20, ...
            'FontName','Palatino','Interpreter','latex');
47  legend('$g_{A_t}$', '$\mu_{g_{A_t}}$', ...
```

```
48      'Interpreter', 'latex',...
49      'location', 'best','Fontsize', 22);
50
51  hold off;
```

To Accompany Question 1.2.8

```
1   %% Question 1.2.8 Growth rate of L_t
2
3   n = diff(hw) ./ hw(1:end-1);
4   m_n = mean(n);
5
6   % Graph¬
7   figure(7);
8   fig7_comps.fig = gcf;
9   grid on;
10  hold on;
11
12  T = 1960:2020;
13  fig7_comps.p0 = plot(T, n);
14  fig7_comps.p1 = yline(m_n, 'Color','black',...
15      'LineStyle','--');
16  set(fig7_comps.p0, 'Color', PS.Green5, 'LineWidth', 3, ...
        'Marker','o');
17
18  xlabel('$T$','FontSize',22,'Interpreter','latex');
19  ylabel('Growth Rate of $L_t$', ...
20      'FontSize',20, ...
            'FontName','Palatino','Interpreter','latex');
21  legend('$g_{L_t}$', '$\mu_{g_{L_t}}$', ...
22      'Interpreter', 'latex',...
23      'location', 'best','Fontsize', 22);
24
25  hold off;
```

To Accompany Question 1.2.10

```
1   %% Question 1.2.10
2   clc;clear;
3
4   % Set the working directory to the place where the ...
        current file is saved
5   tmp = matlab.desktop.editor.getActive;
6   cd(fileparts(tmp.Filename));
7
8   % Before anything, set the graph aesthetics
9   PS = PLOT_STANDARDS();
10  %% Simulation for T = 10, 30, 50 and 100
11
12  % Parameters
13  alpha = 1-0.6523;
14  s = 0.2;
15  n = 0.0133;
16  Δ = 0.0495;
```

```matlab
17  g = 0.1961;
18
19  % Define time periods
20  T = [10 30 50 100];
21
22  % Initialize variables
23  k = zeros(length(T), max(T));
24  y = zeros(length(T), max(T));
25  c = zeros(length(T), max(T));
26
27  % Loop through time periods
28  for i = 1:length(T)
29      % Initialize starting point
30      k(i,1) = 1;
31      for t = 2:T(i)
32          k(i,t) = s*k(i,t-1)^alpha + (1-Δ)*k(i,t-1) - ...
                  (n+g)*k(i,t-1);
33          y(i,t) = k(i,t)^alpha;
34          c(i,t) = (1-s)*y(i,t);
35      end
36  end
37
38  % Plot results in four subplots
39  figure(8);
40  hold on;
41  subplot(2,2,1);
42  k1 = plot(1:T(1), k(1,1:T(1)));
43  set(k1, 'Color', PS.Blue1, 'LineWidth', 2, 'Marker','o')
44  title('$T=10$', 'Interpreter','latex');
45  xlabel('Time', 'FontName','Palatino');
46  ylabel('Capital','FontName','Palatino');
47
48  subplot(2,2,2);
49  k2 = plot(1:T(2), k(2,1:T(2)));
50  set(k2, 'Color', PS.Blue2, 'LineWidth', 2, 'Marker','o')
51  title('$T=30$', 'Interpreter','latex');
52  xlabel('Time', 'FontName','Palatino');
53  ylabel('Capital','FontName','Palatino');
54
55  subplot(2,2,3);
56  k3 = plot(1:T(3), k(3,1:T(3)));
57  set(k3, 'Color', PS.Blue3, 'LineWidth', 2, 'Marker','o')
58  title('$T=50$', 'Interpreter','latex');
59  xlabel('Time', 'FontName','Palatino');
60  ylabel('Capital','FontName','Palatino');
61
62  subplot(2,2,4);
63  k4 = plot(1:T(4), k(4,1:T(4)));
64  set(k4, 'Color', PS.Blue4, 'LineWidth', 2, 'Marker','o')
65  title('$T=100$', 'Interpreter','latex');
66  xlabel('Time', 'FontName','Palatino');
67  ylabel('Capital','FontName','Palatino');
68
69  hold off;
```

To Accompany Question 1.3

```matlab
1   %% Question 1.3.1
2   clc;clear;
3
4   % Set the working directory to the place where the ...
         current file is saved
5   tmp = matlab.desktop.editor.getActive;
6   cd(fileparts(tmp.Filename));
7
8   %% 1.3.1 Data Import, Saving rate and
9   ameco = readtable('AMECOclean.xlsx', ...
        'ReadVariableNames',true,...
10       'VariableNamingRule','preserve');
11
12  saving = ameco{1:22, 3:end};
13  gdp = ameco{23:44, 3:end};
14  consumption = ameco{45:66, 3:end};
15  TFP = ameco{67:88, 3:end};
16  employment = ameco{89:110, 3:end};
17
18  % saving rate
19  s = saving./gdp;
20  m_s = mean(s, 2);
21  countries = ameco{1:22, 2};
22  saving_T = table(countries, m_s);
23  %
24
25  %
26  AL = TFP .* employment;
27  dconsumption = consumption./AL;
28  m_dc = mean(dconsumption, 2);
29  final_T = table(countries, m_s, m_dc);
30
31  table2latex(final_T, 'Q131.tex');
32
33
34  %% 1.3.2 Scatter Plot
35  lconsumption = log(dconsumption);
36  sca = scatter(saving, lconsumption);
37  xlabel('$s$', 'Interpreter','latex');
38  ylabel('$\ln c_t$', 'Interpreter','latex')
39
40
41  %% 1.3.3 Pooled Regression
42
43  newdconsumption = reshape(lconsumption, [594,1]);
44  s2 = s.^2;
45  news = reshape(s, [594,1]);
46  news2 = reshape(s2, [594,1]);
47  p = polyfit(news, newdconsumption, 2);
```

countries	μ_s	$\mu_{\frac{C}{AL}}$
Belgium	0.29037	0.00054546
Czechia	4.6013	0.00030112
Denmark	2.6815	0.00045623
Germany	0.28424	0.00046762
Estonia	0.16856	0.00023488
Ireland	0.29032	0.00052855
Greece	0.10254	0.00041756
Spain	0.19484	0.00044101
France	0.254	0.00049088
Italy	0.20243	0.0005158
Latvia	0.12673	0.00026489
Lithuania	0.10013	0.00028947
Luxembourg	0.29588	0.00072485
Hungary	35.2106	0.00028018
Netherlands	0.29502	0.00046761
Austria	0.28816	0.00046788
Poland	0.42925	0.00029502
Portugal	0.13332	0.00034247
Slovenia	0.18558	0.00032245
Slovakia	0.15113	0.00030712
Finland	0.30962	0.00044559
Sweden	14.173	0.00047297

Table 2: Mean Value of European Countries' saving rate

References

Aalok, A. (2023), 'Professional plots', https://www.
  mathworks.com/matlabcentral/fileexchange/
  100766-professional_plots. [Online; accessed January 22,
  2023].

BEA (2023*a*), 'Compensation of employees'. Data retrieved from U.S.
  Bureau of Economic Analysis, National Data Fixed Assets Accounts
  Tables, Table 1.12 National Income by Type of Income.

BEA (2023*b*), 'Fixed assets'. Data retrieved from U.S. Bureau of Eco-
  nomic Analysis, National Data Fixed Assets Accounts Tables, Table
  1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable
  Goods.

BEA (2023*c*), 'Gdp deflator, gdp private domestic investment'. Data
  retrieved from U.S. Bureau of Economic Analysis, National Data
  Fixed Assets Accounts Tables, Table 1.1.4. Price Indexes for Gross
  Domestic Product.

BEA (2023*d*), 'Gross domestic product'. Data retrieved from U.S.
  Bureau of Economic Analysis, National Data Fixed Assets Accounts
  Tables, Table 1.1.5 Gross Domestic Product.

BEA (2023*e*), 'Proprietors' income with iva and ccadj, taxes on produc-
  tion and imports, business current transfer payments (net), current
  surplus of government enterprises'. Data retrieved from U.S. Bu-
  reau of Economic Analysis, National Data Fixed Assets Accounts
  Tables, Table 1.12 National Income by Type of Income.

BEA (2023*f*), 'Series of depreciation of fixed assets'. Data retrieved
  from U.S. Bureau of Economic Analysis, National Data Fixed As-
  sets Accounts Tables, Table 1.3 Current-Cost Depreciation of Fixed
  Assets and Consumer Durable Goods.

BEA (2023*g*), 'Total hours worked in domestic industries'. Data re-
  trieved from U.S. Bureau of Economic Analysis, National Data
  Fixed Assets Accounts Tables, Table 6.9B. Hours Worked by Full-
  Time and Part-Time Employees by Industry.

BEA (2023*h*), 'Total hours worked in domestic industries'. Data re-
  trieved from U.S. Bureau of Economic Analysis, National Data
  Fixed Assets Accounts Tables, Table 6.9C. Hours Worked by Full-
  Time and Part-Time Employees by Industry.

BEA (2023*i*), 'Total hours worked in domestic industries'. Data re-
  trieved from U.S. Bureau of Economic Analysis, National Data

Fixed Assets Accounts Tables, Table 6.9D. Hours Worked by Full-Time and Part-Time Employees by Industry.

Cagigal, V. M. (2023), 'Matlab table to latex converter', https://www.mathworks.com/matlabcentral/fileexchange/69063-matlab-table-to-latex-convertor. [Online; accessed January 22, 2023].

Gomme, P. and Lkhagvasuren, D. (2013), 'Calibration and simulation of dsge models', pp. 575–592.

Mankiw, N. G., Romer, D. and Weil, D. N. (1992), 'A contribution to the empirics of economic growth', *The quarterly journal of economics* **107**(2), 407–437.

Solow, R. M. (1956), 'A contribution to the theory of economic growth', *The quarterly journal of economics* **70**(1), 65–94.