1

Hello. Today we are gonna be talking about the Taylor's theorem, and we will get to learn how to derive Taylor polynomials and see how to derive some numerical formulas by using Taylor's theorem.

2

Upon end of this class, you should be able to

1. Describe the definition of Taylor's theorem
2. Apply Taylor's theorem to derive Taylor polynomial
3. Utilize Taylor's theorem to derive finite difference formulas

Be careful that since you are year 1 students, our course just focus on the application of Taylor's theorem, the proof of theorem is optional, if you have any interests, you can read it after class.

3

Taylor's theorem is named after the mathematician Brook Taylor who stated a version of it in 1715. It gives simple arithmetic formulas to accurately compute values of many transcendental functions. Could you give me some examples about transcendental functions? Be free to share your ideas. Yes, as you mentioned, transcendental functions are functions not expressible as a finite combination of the algebraic operations (such as the exponential function)

You may wonder how can we approximate those functions. That's why we study Taylor's theorem.

Taylor's theorem gives an approximation of a k times differentiable function
around a given point by a k-th order Taylor polynomial.(Don't be worry about these definitions, i will show you later.) It is the starting point of
the study of analytic functions, and is fundamental in various areas of
mathematics, as well as in numerical analysis.

4

So, let's look at the theorem' definition. Taylor theorem states that

Let k larger than 1 be an integer and let the function f mapping R space to R space be k times differentiable. Be careful, the differentiable is the key point of theorem.

Then there exists a function R_k mapping R space to R space such that

The function f(X) is equal to f(A) plus f prime of A, X minus A, plus f prime prime of A over 2 factorial, (X minus A squared) and plus such items until k plus R_k.

The R_k is equal to the K plus one th derivative of f c ,over, K plus one factorial, (X minus A) to, K plus One, for some constant c between A and X with the limit of R_k(x) is equal to zero as X tends to A.

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Okay, as we saw. The polynomial appearing in the theorem which has the form of

f(A) plus f prime of A, X minus A, plus f prime prime of A over 2 factorial, (X minus A squared) and plus such items until k plus R_k

We denote it by P_k, and we call it Taylor's polynomial. Do you remember what i said at the beginning of the course? Yes, we are going to use the Taylor' theorem to approximate functions like transcendental functions.

And Taylor's polynomial plays an important role in approximation, so you should memorize this polynomial carefully.

The proof of theorem is optional for this course, you can find the proof on text book.

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Now lets have a practice. This practive will help you to understand how to apply Taylor theorem to approximate functions.

Find the Taylor polynomial of order 4 for the function at x equals to zero.

Use the polynomial to approximate the value of the function at x equals to zero point 2.

The function is given below, I will give you 5 minutes for this easy questions.

7

Okay, anyone wants to show your solution? raise up your hands please. Good job, the answer is correct. If someone still don't know how to get it, dont be worried. I will explain the solution.

Now, Look at the theorem, First we need to compute different orders of derivatives to get Taylor's polynomial. Its easy to get it, or if you have problem with it, you can come to me after class.

So we get f zero equals to 1, f prime zero equals to 0, f prime prime zero equals to minus pi squared over 4, third derivative of f zero equals to 0, fourth derivative of f zero equals to pi to 4 over 16.

Then we put those values of derivatives into the formula, we get the polynomial is that P X equals to one plus minus pi squared over 4 X squared plus pi to 4 over 16 X to 4.

After getting the fourth order Taylor polynomial, you can immediately approximate the f(zero point two), which is equal to point nine five one.

8.

Okay, lets move on to the next part: How to use Taylor's theorem in numerical analysis. The finite difference formulas in numerical analysis will be a good example of this.

Sice Taylor's theorem says that if function f is twice continuously differentiable, then we have the formula below.

f X plus H is equal to f X plus H f prime X plus H squared over two f prime prime C where C is between X and X plus H.

Can you get this formula immediately, if not please review the Taylor's theorem.

The formula above implies the folloing formula:

f prime X euqals to f X plus H minus f X over H minus half H f prime prime C

which is called Two-point forward-difference formula in numerical analysis.

Then we get a formula in numerical analysis by Taylor's theorem.

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As we saw, Taylor theorem works as a tool in deriving some difference formulas in numerical analysis. Can anyone give me other formulas?

Okay here is an exercise, the Three-point centered-difference formula for second derivative. Plesase use Taylor's theorem to derive this formula.

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Thats all about Taylor's theorem, lets quickly reviewed what we have learned today.

First the definition of the theorem, Second the process to derive Taylor polynomials and application in numerical analysis.

There's a quiz for you in the end of our class.Sum up what we ve learnt today first and finish the quiz quickly. Write down your answer and hand it to me when u leave the classroom. If you found this class interesting and want to discuss more about the material, make sure to come up to me at the office hour.