Exercise 5.1

a)

$K(\phi \wedge \neg K\phi) \models_{N,K} K\phi \wedge K\neg K\phi \models_T K\phi \wedge \neg K\phi \models \bot$
An agent cannot know know ϕ while not knowing ϕ.
Rather, $[!\phi \wedge \neg K\phi]K\phi \wedge K\phi$ holds.

b)

$K_j K_i\phi \models K_j K_i\phi \wedge (K_i\phi \rightarrow_T \phi) \models_N K_j K_i\phi \wedge K_j(K_i\phi \rightarrow_T \phi) \models_K K_j\phi$ An agent knows all logical implications of his knowledge, including the use of axioms.

c)

$O(\phi \wedge \psi) \models O(\phi \wedge \psi) \wedge (\phi \wedge \psi \rightarrow \psi) \models_N O(\phi \wedge \psi) \wedge O(\phi \wedge \psi \rightarrow \psi) \models_K O(\psi)$
An agent is obliged to all generalizations of his obligations.

d)

$P\phi \wedge \neg P\psi \models P\phi \models_{N,K} P(\phi \vee \psi)$ If only ϕ is permitted while ψ is not, $\phi \vee \psi$ is permitted (weakening rule). This is a counterexample to the given statement.

e)

$\models_T O(\phi) \rightarrow \phi \models_N O(O(\phi) \rightarrow \phi)$