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Exercise 4.1

a)

$$
\begin{aligned}
Q &= \{0, 1, ..., 17\} \\
Q_1 &= \{0, 2, ..., 16\} \\
Q_2 &= \{1, 3, ..., 17\} \\
S &= \{0, 1, ..., 9\} \\
M &= (W, R, V) \\
W &= \{w_i \quad \forall i \in Q\} \\
R(1) &= \{(w_i, w_i) \quad \forall i \in Q\} \cup \{(w_i, w_{i+1}) \quad \forall i \in Q_1 \setminus \{8\}\} \cup \{(w_i, w_{i-1}) \quad \forall i \in Q_2 \setminus \{9\}\} \\
R(2) &= \{(w_i, w_i) \quad \forall i \in Q\} \cup \{(w_i, w_{i-1}) \quad \forall i \in Q_1 \setminus \{0\}\} \cup \{(w_i, w_{i+1}) \quad \forall i \in Q_2 \setminus \{17\}\} \\
P &= \{a_i \forall i, j \in S\} \cup \{b_i \forall i, j \in S\} \\
V(a_0) &= \{w_0\} \\
V(b_1) &= \{w_0, w_1\} \\
V(a_2) &= \{w_1, w_2\} \\
V(b_3) &= \{w_2, w_3\} \\
&... \\
V(b_9) &= \{w_8\} \\
V(b_0) &= \{w_9\} \\
V(a_1) &= \{w_9, w_{10}\} \\
V(b_2) &= \{w_{10}, w_{11}\} \\
V(a_3) &= \{w_{11}, w_{12}\} \\
&... \\
V(a_9) &= \{w_{17}\}
\end{aligned}
$$

b)

$$
\begin{aligned}
\alpha_1 &= \neg \bigvee_{i \in S} K_1 a_i \\
\alpha_2 &= [!\alpha_1] \neg \bigvee_{i \in S} K_2 b_i \\
\alpha_3 &= [!\alpha_2] \neg \bigvee_{i \in S} K_1 a_i \\
\alpha_4 &= [!\alpha_3] \neg \bigvee_{i \in S} K_2 b_i \\
\alpha_5 &= [!\alpha_4] \bigvee_{i \in S} K_1 a_i
\end{aligned}
$$

c)

In the beginning both agents know whether their number is even or odd, since the numbers are consecutive. Both agents know they can see each others numbers, so this is common knowledge. If agent 1 sees the number 0 (or 9) he can conclude his number is 1 (or 8 respectively), since these numbers are at the edge of the spectrum. Announcing he does not know his number narrows the spectrum of possible numbers for agent b. This continues until one agent knows his number.

$$C_{\{1,2\}}\Big(\bigvee_{i\in\{1,3,\ldots,9\}} a_i \wedge \bigvee_{i\in\{0,2,\ldots,8\}} b_i\Big)$$

$$[!\alpha_1]\neg b_0$$

$$[!\alpha_2]\neg a_1 \wedge \neg a_9$$

$$[!\alpha_3]\neg b_2 \wedge \neg b_8$$

$$[!\alpha_4]\neg a_3 \wedge \neg a_7$$

$$[!\alpha_5] b_4 \vee b_6$$

d)

The number of Agent 1 is 5. Agent 2 does not know his number, since Agent 1 would have acted the same way if it was 6.