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Exercise 3.1

a)

$\boxed{\begin{array}{c} K(p \vee q) \wedge K\neg p,\ K(p \vee q),\ K\neg p, \\ p \vee q, \neg p, p \end{array}}$ $\boxed{\begin{array}{c} K(p \vee q) \wedge K\neg p,\ K(p \vee q),\ K\neg p, \\ p \vee q, \neg p, q \end{array}}$

$\boxed{p \vee q}$ $\boxed{\neg p}$

This yields the model $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) = \{(w_1, w_1)\}$, $V(p) =$ and $V(q) = \{w_1\}$.

Exercise 3.1 Variation

a)

$\boxed{K\ (p \vee q) \wedge K\neg p}$

Apply AND-Rule:

$\boxed{\begin{array}{c} K\ (p \vee q) \wedge K\neg p, \\ K\ (p \vee q),\ K\neg p \end{array}}$

Apply Axiom T:

$\boxed{\begin{array}{c} K\ (p \vee q) \wedge K\neg p, \\ K\ (p \vee q),\ K\neg p, \\ p \vee q,\ \neg p \end{array}}$

Apply OR-Rule:

$\boxed{\begin{array}{c} K\ (p \vee q) \wedge K\neg p, \\ K\ (p \vee q),\ K\neg p, \\ p \vee q,\ \neg p, \\ p \end{array}}$ $\boxed{\begin{array}{c} K\ (p \vee q) \wedge K\neg p, \\ K\ (p \vee q),\ K\neg p, \\ p \vee q,\ \neg p, \\ q \end{array}}$

$\perp$-Rule:

$\boxed{\begin{array}{c} K\ (p \vee q) \wedge K\neg p, \\ K\ (p \vee q),\ K\neg p, \\ p \vee q,\ \neg p, \\ p, \\ \perp \end{array}}$ $\boxed{\begin{array}{c} K\ (p \vee q) \wedge K\neg p, \\ K\ (p \vee q),\ K\neg p, \\ p \vee q,\ \neg p, \\ q \end{array}}$

No more rules can be applied and there is an open premodel left, thus the formula is satisfiable. Kripke Model: $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) =$

$\{(w_1, w_1)\}$, $V(p) = \{\}$ and $V(q) = \{w_1\}$. Pointed S5 model (M, w) with M from above and $w = \{\neg p, q\}$.

b)

$\boxed{\neg (K (p \wedge q) \rightarrow Kp)}$

Apply NotImpl-Rule:

$$\boxed{\begin{array}{c}\neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp)\end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c}\neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp\end{array}}$$

Apply Axiom T:

$$\boxed{\begin{array}{c}\neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp, \\ p \wedge q\end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c}\neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp, \\ p \wedge q, \\ p, q\end{array}}$$

Duality:

$$\boxed{\begin{array}{c}\neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp, \\ p \wedge q, \\ p, q, \\ \neg(\neg \hat{K} \neg p)\end{array}}$$

$$\boxed{\begin{array}{c} \neg (K(p \wedge q) \rightarrow Kp), \\ (K(p \wedge q) \wedge \neg Kp), \\ K(p \wedge q),\ \neg Kp, \\ p \wedge q, \\ p, q, \\ \hat{K}\neg p \end{array}}$$

c)

$$\boxed{(K_a p \vee K_a \neg p) \wedge K_b (K_a p \vee K_a \neg p)}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c} (K_a p \vee K_a \neg p) \wedge K_b (K_a p \vee K_a \neg p), \\ (K_a p \vee K_a \neg p),\ K_b (K_a p \vee K_a \neg p) \end{array}}$$

d)

$$\boxed{\neg (O(Op \rightarrow p) \rightarrow (OOp \rightarrow Op))}$$

e)

$$\boxed{\neg (OKp \rightarrow Op)}$$

Exercise 3.2

ϕ	$M, w_2 \models K_1\phi$	$M, w_2 \models K_2\phi$	$M, w_2 \models C\phi$	$M, w_2 \models D\phi$	$M, w_1 \models C\phi$	$M, w_1 \models D\phi$
p	☐	☒	☐	☒	☐	☐
q	☒	☐	☐	☒	☐	☒
$p \wedge q$	☐	☐	☐	☒	☐	☐
$p \vee q$	☒	☒	☒	☒	☒	☒