Exercise 1.1

a)

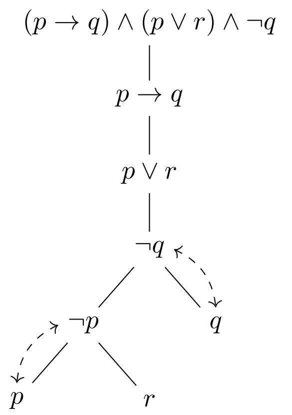
Each clause is a disjunction containing at least one negated atom. Therefore, the interpretation $p_i \mapsto \top \quad \forall i$ satisfies S.

b)

$$
\begin{aligned}
& (p \wedge (q \vee \neg p)) \vee (r \wedge \neg(s \vee r)) && \text{Commutativity} \\
\Longrightarrow\ & (p \wedge (\neg p \vee q)) \vee (r \wedge \neg(r \vee s)) && \text{De Morgan} \\
\Longrightarrow\ & (p \wedge (\neg p \vee q)) \vee (r \wedge \neg r \wedge \neg s)) && \text{Contradiction} \\
\Longrightarrow\ & (p \wedge (\neg p \vee q)) \vee (\bot \wedge \neg s)) && \text{Falsity} \\
\Longrightarrow\ & (p \wedge (\neg p \vee q)) \vee \bot && \text{Falsity} \\
\Longrightarrow\ & p \wedge (\neg p \vee q) && \text{Distributivity} \\
\Longrightarrow\ & (p \wedge \neg p) \vee (p \wedge q) && \text{Contradiction} \\
\Longrightarrow\ & \bot \vee (p \wedge q) && \text{Falsity} \\
\Longrightarrow\ & p \wedge q &&
\end{aligned}
$$

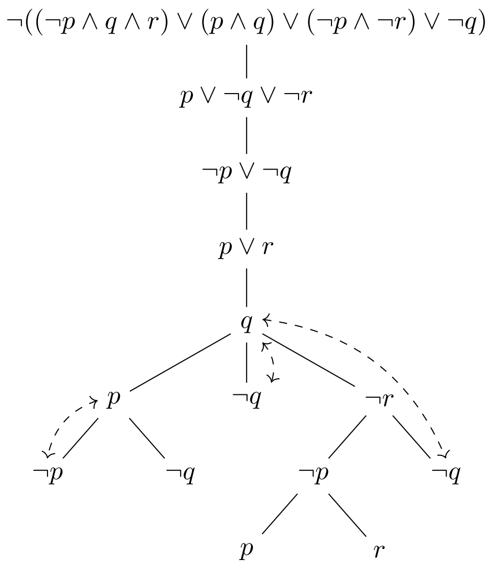
Exercise 1.2

a)



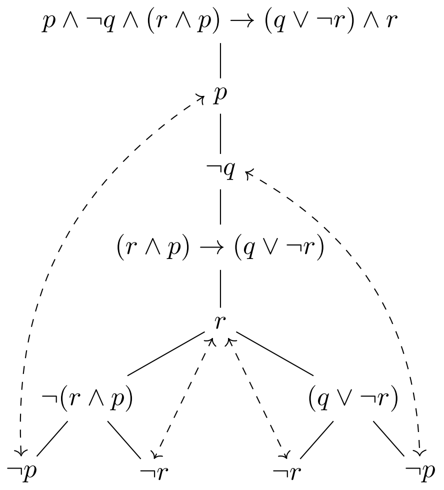
All branches are either saturated or closed. The only open branch provides the model $p \mapsto F, q \mapsto F, r \mapsto T$.

b)



Since its negation is unsatisfiable, $(\neg p \wedge q \wedge r) \vee (p \wedge q) \vee (\neg p \wedge \neg r) \vee \neg q$ is a tautology.

c)



$p \wedge \neg q \wedge (r \wedge p) \rightarrow (q \vee \neg r) \wedge r$ is unsatisfiable, therefore
$\{p \wedge \neg q, (r \wedge p) \rightarrow (q \vee \neg r)\} \models r$