Exercise 3.1

a)

$$\boxed{\begin{array}{c} K(p \lor q) \land K\neg p,\ K(p \lor q),\ K\neg p, \\ p \lor q, \neg p, p \end{array}} \qquad \boxed{\begin{array}{c} K(p \lor q) \land K\neg p,\ K(p \lor q),\ K\neg p, \\ p \lor q, \neg p, q \end{array}}$$

$$\boxed{p \lor q} \qquad \boxed{\neg p}$$

This yields the model $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) = \{(w_1, w_1)\}$, $V(p) =$ and $V(q) = \{w_1\}$.

Exercise 3.1 Variation

a)

$$\boxed{K\,(p \lor q) \land K\neg p}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c} K\,(p \lor q) \land K\neg p, \\ K\,(p \lor q),\ K\neg p \end{array}}$$

Apply Axiom T:

$$\boxed{\begin{array}{c} K\,(p \lor q) \land K\neg p, \\ K\,(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p \end{array}}$$

Apply OR-Rule:

$$\boxed{\begin{array}{c} K\,(p \lor q) \land K\neg p, \\ K\,(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ p \end{array}} \qquad \boxed{\begin{array}{c} K\,(p \lor q) \land K\neg p, \\ K\,(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ q \end{array}}$$

$\perp$-Rule:

$$\boxed{\begin{array}{c} K\,(p \lor q) \land K\neg p, \\ K\,(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ p, \\ \perp \end{array}} \qquad \boxed{\begin{array}{c} K\,(p \lor q) \land K\neg p, \\ K\,(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ q \end{array}}$$

No more rules can be applied and there is an open premodel left, thus the formula is satisfiable. Kripke Model: $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) =$

$\{(w_1, w_1)\}$, $V(p) = \{\}$ and $V(q) = \{w_1\}$. Pointed S5 model (M, w) with M from above and $w = \{\neg p, q\}$.

b)

$\boxed{\neg (K (p \land q) \to Kp)}$

Apply NotImpl-Rule:

$$\boxed{\begin{array}{c} \neg (K (p \land q) \to Kp), \\ (K (p \land q) \land \neg Kp) \end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c} \neg (K (p \land q) \to Kp), \\ (K (p \land q) \land \neg Kp), \\ K (p \land q), \neg Kp \end{array}}$$

Exercise 3.2

ϕ	$M, w_2 \models K_1\phi$	$M, w_2 \models K_2\phi$	$M, w_2 \models C\phi$	$M, w_2 \models D\phi$	$M, w_1 \models C\phi$	$M, w_1 \models D\phi$
p	☐	☒	☐	☒	☐	☐
q	☒	☐	☐	☒	☐	☒
$p \land q$	☐	☐	☐	☒	☐	☐
$p \lor q$	☒	☒	☒	☒	☒	☒