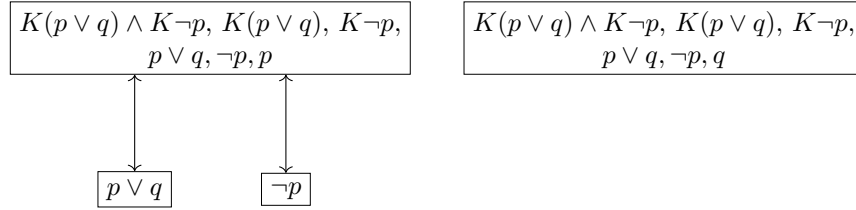


Exercise 3.1

a)



This yields the model $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) = \{(w_1, w_1)\}$, $V(p) = \{w_1\}$ and $V(q) = \{w_1\}$.

Exercise 3.1 Variation

a)

$K(p \vee q) \wedge K\neg p$

Apply AND-Rule:

$K(p \vee q) \wedge K\neg p,$
 $K(p \vee q), K\neg p$

Apply Axiom T:

$K(p \vee q) \wedge K\neg p,$
 $K(p \vee q), K\neg p,$
 $p \vee q, \neg p$

Apply OR-Rule:

$K(p \vee q) \wedge K\neg p,$
 $K(p \vee q), K\neg p,$
 $p \vee q, \neg p,$
 p

$K(p \vee q) \wedge K\neg p,$
 $K(p \vee q), K\neg p,$
 $p \vee q, \neg p,$
 q

\perp -Rule:

$K(p \vee q) \wedge K\neg p,$
 $K(p \vee q), K\neg p,$
 $p \vee q, \neg p,$
 $p,$
 \perp

$K(p \vee q) \wedge K\neg p,$
 $K(p \vee q), K\neg p,$
 $p \vee q, \neg p,$
 q

No more rules can be applied and there is an open premodel left, thus the formula is satisfiable. Kripke Model: $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) = \{(w_1, w_1)\}$, $V(p) = \{w_1\}$ and $V(q) = \{w_1\}$.

$\{(w_1, w_1)\}$, $V(p) = \{\}$ and $V(q) = \{w_1\}$. Pointed S5 model (M, w) with M from above and $w = \{\neg p, q\}$.

b)

$$\neg (K (p \wedge q) \rightarrow Kp)$$

Apply NotImpl-Rule:

$$\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp) \end{array}$$

Apply AND-Rule:

$$\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp \end{array}$$

Apply Axiom T:

$$\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp, \\ p \wedge q \end{array}$$

Apply AND-Rule:

$$\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp, \\ p \wedge q, \\ p, q \end{array}$$

Duality:

$$\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp, \\ p \wedge q, \\ p, q, \\ \neg(\neg \hat{K} \neg p) \end{array}$$

$$\begin{array}{c}
 \neg (K(p \wedge q) \rightarrow Kp), \\
 (K(p \wedge q) \wedge \neg Kp), \\
 K(p \wedge q), \neg Kp, \\
 p \wedge q, \\
 p, q, \\
 K\neg p
 \end{array}$$

c)

$$(K_ap \vee K_a\neg p) \wedge K_b(K_ap \vee K_a\neg p)$$

Apply AND-Rule:

$$\begin{array}{c}
 (K_ap \vee K_a\neg p) \wedge K_b(K_ap \vee K_a\neg p), \\
 (K_ap \vee K_a\neg p), K_b(K_ap \vee K_a\neg p)
 \end{array}$$

d)

$$\neg (O(Op \rightarrow p) \rightarrow (OOp \rightarrow Op))$$

e)

$$\neg (OKp \rightarrow Op)$$

Exercise 3.2

ϕ	$M, w_2 \models K_1\phi$	$M, w_2 \models K_2\phi$	$M, w_2 \models C\phi$	$M, w_2 \models D\phi$	$M, w_1 \models C\phi$	$M, w_1 \models D\phi$
p	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
q	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
$p \wedge q$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$p \vee q$	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>