Exercise 1.1

a)

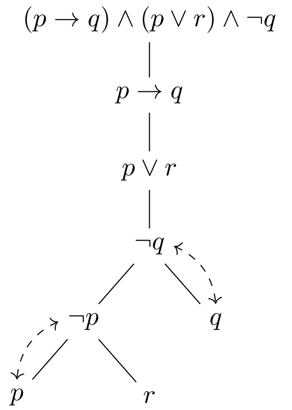
Each clause is a disjunction containing at least one negated atom. Therefore, the interpretation $p_i \mapsto \top \quad \forall i$ satisfies S.

b)

$$
\begin{array}{lr}
\quad\; (p \land (q \lor \neg p)) \lor (r \land \neg(s \lor r)) & \text{Commutativity} \\
\Longrightarrow (p \land (\neg p \lor q)) \lor (r \land \neg(r \lor s)) & \text{De Morgan} \\
\Longrightarrow (p \land (\neg p \lor q)) \lor (r \land \neg r \land \neg s)) & \text{Contradiction} \\
\Longrightarrow (p \land (\neg p \lor q)) \lor (\bot \land \neg s)) & \text{Falsity} \\
\Longrightarrow (p \land (\neg p \lor q)) \lor \bot & \text{Falsity} \\
\Longrightarrow p \land (\neg p \lor q) & \text{Distributivity} \\
\Longrightarrow (p \land \neg p) \lor (p \land q) & \text{Contradiction} \\
\Longrightarrow \bot \lor (p \land q) & \text{Falsity} \\
\Longrightarrow p \land q &
\end{array}
$$

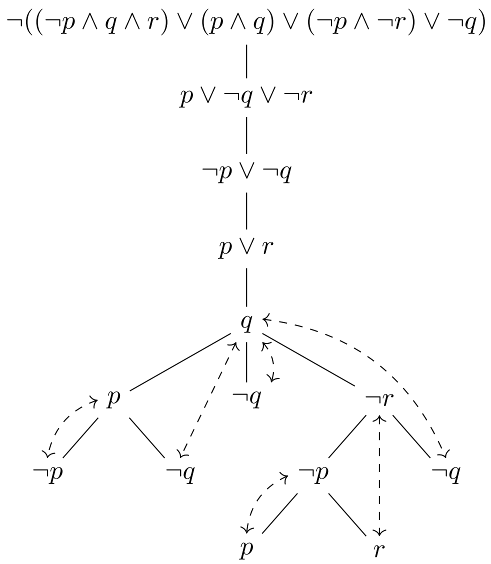
Exercise 1.2

a)



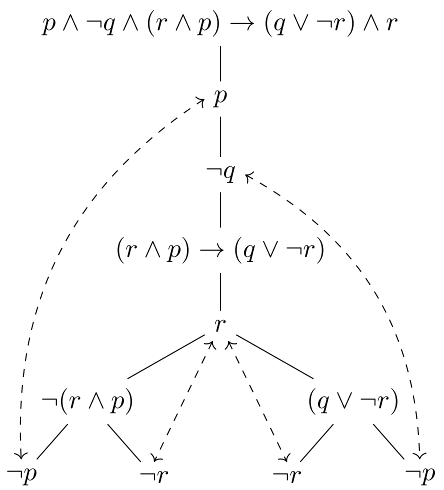
All branches are either saturated or closed. The only open branch provides the model $p \mapsto F, q \mapsto F, r \mapsto T$.

b)



Since its negation is unsatisfiable, $(\neg p \wedge q \wedge r) \vee (p \wedge q) \vee (\neg p \wedge \neg r) \vee \neg q$ is a tautology.

c)



$p \wedge \neg q \wedge (r \wedge p) \rightarrow (q \vee \neg r) \wedge r$ is unsatisfiable, therefore
$\{p \wedge \neg q, (r \wedge p) \rightarrow (q \vee \neg r)\} \models r$

Exercise 1.3

Detailed transformations for the first statement, with C meaning that Cyclope is guilty and $\neg C$ that Cyclope is not guilty. Other abbreviations analogous.

$$
\begin{aligned}
& (\neg C \wedge E \wedge \neg A) \vee (C \wedge \neg (E \wedge \neg A)) \\
\equiv\ & (\neg C \wedge E \wedge \neg A) \vee (C \wedge (\neg E \vee A)) \\
\equiv\ & (\neg C \vee (C \wedge (\neg E \vee A))) \wedge (E \vee (C \wedge (\neg E \vee A))) \wedge (\neg A \vee (C \wedge (\neg E \vee A))) \\
\equiv\ & (\neg C \vee C) \wedge (\neg C \vee \neg E \vee A) \wedge (E \vee C) \wedge (E \vee \neg E \vee A) \wedge (\neg A \vee C) \wedge (\neg A \vee \neg E \vee A) \\
\equiv\ & (\neg C \vee \neg E \vee A) \wedge (E \vee C) \wedge (\neg A \vee C)
\end{aligned}
$$

This leads to the following set of formulae:

$$\{\neg C, \neg E, A\} \tag{1.1}$$
$$\{C, E\} \tag{1.2}$$
$$\{C, \neg A\} \tag{1.3}$$

$$\{\neg E, \neg G, \neg L\} \tag{2.1}$$
$$\{E, G\} \tag{2.2}$$
$$\{E, L\} \tag{2.3}$$

$$\{\neg A, \neg Sh, L\} \tag{3.1}$$
$$\{A, Sh\} \tag{3.2}$$
$$\{A, \neg L\} \tag{3.3}$$

$$\{\neg G, \neg W, P\} \tag{4.1}$$
$$\{G, W\} \tag{4.2}$$
$$\{G, \neg P\} \tag{4.3}$$

$$\{\neg I, E, St\} \tag{5.1}$$
$$\{I, \neg E\} \tag{5.2}$$
$$\{I, \neg St\} \tag{5.3}$$

$$\{\neg L, \neg I, W\} \tag{6.1}$$
$$\{L, I\} \tag{6.2}$$
$$\{L, \neg W\} \tag{6.3}$$

$$\{\neg M, \neg P, C\} \tag{7.1}$$
$$\{M, P\} \tag{7.2}$$

$$\{M, \neg C\} \tag{7.3}$$

$$\{\neg P, \neg E, \neg G\} \tag{8.1}$$

$$\{M, P\} \tag{8.2}$$

$$\{M, \neg C\} \tag{8.3}$$

$$\{\neg Sh, \neg C, I\} \tag{9.1}$$

$$\{Sh, C\} \tag{9.2}$$

$$\{Sh, \neg I\} \tag{9.3}$$

$$\{\neg St, \neg G, M\} \tag{10.1}$$

$$\{St, G\} \tag{10.2}$$

$$\{St, \neg M\} \tag{10.3}$$

$$\{\neg V, \neg M, Sh\} \tag{11.1}$$

$$\{V, M\} \tag{11.2}$$

$$\{V, \neg Sh\} \tag{11.3}$$

$$\{\neg W, \neg St, \neg V\} \tag{12.1}$$

$$\{W, St\} \tag{12.2}$$

$$\{W, V\} \tag{12.3}$$

Now apply resolution:

$$(1.2) + (8.1): \quad \{C, \neg P, \neg G\} \tag{13}$$

$$(4.3) + (13): \quad \{C, \neg P\} \tag{14}$$

$$(7.1) + (14): \quad \{\neg M, \neg P\} \tag{15}$$

$$(7.3) + (15): \quad \{\neg P, \neg C\} \tag{16}$$

$$(16) + (14): \quad \{\neg P\} \tag{17}$$

$$(17) + (8.2): \quad \{E\} \tag{18}$$

$$(17) + (8.3): \quad \{G\} \tag{19}$$

$$(18) + (5.2): \quad \{I\} \tag{20}$$

$$(16) + (7.2): \quad \{M\} \tag{21}$$

$$(21) + (10.3): \quad \{St\} \tag{22}$$

$$(2.1) + (18): \quad \{\neg G, \neg L\} \tag{23}$$

$$(23) + (19): \quad \{\neg L\} \tag{24}$$

$$(24) + (6.3): \quad \{\neg W\} \tag{25}$$

$$(25) + (12.3): \quad \{V\} \tag{26}$$

$$(20) + (9.3): \quad \{Sh\} \tag{27}$$

$$(3.1) + (27): \quad \{\neg A, L\} \tag{28}$$

$$(28) + (24): \quad \{\neg A\} \tag{29}$$

$$(29) + (1.1): \quad \{\neg C, \neg E\} \tag{30}$$

$$(30) + (18): \quad \{\neg C\} \tag{31}$$

This leads to the following set of guilty people: Emma Frost, Gambit, Iceberg, Malicia, Shadowcat, Storm, and Vega