

## Exercise 1.1

a)

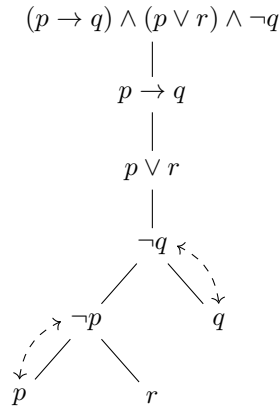
Each clause is a disjunction containing at least one negated atom. Therefore, the interpretation  $p_i \mapsto T \quad \forall i$  satisfies S.

b)

$(p \wedge (q \vee \neg p)) \vee (r \wedge \neg(s \vee r))$	Commutativity
$\implies (p \wedge (\neg p \vee q)) \vee (r \wedge \neg(r \vee s))$	De Morgan
$\implies (p \wedge (\neg p \vee q)) \vee (r \wedge \neg r \wedge \neg s)$	Contradiction
$\implies (p \wedge (\neg p \vee q)) \vee (\perp \wedge \neg s)$	Falsity
$\implies (p \wedge (\neg p \vee q)) \vee \perp$	Falsity
$\implies p \wedge (\neg p \vee q)$	Distributivity
$\implies (p \wedge \neg p) \vee (p \wedge q)$	Contradiction
$\implies \perp \vee (p \wedge q)$	Falsity
$\implies p \wedge q$	

## Exercise 1.2

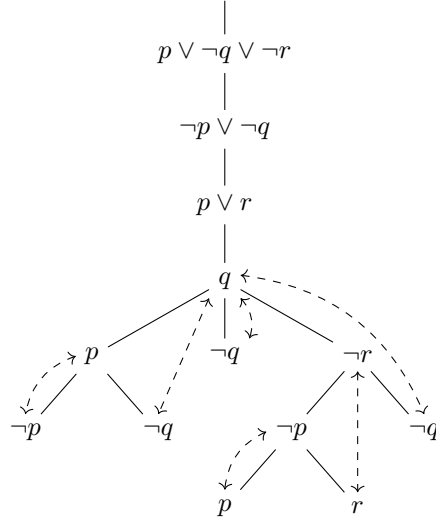
a)



All branches are either saturated or closed. The only open branch provides the model  $p \mapsto F, q \mapsto F, r \mapsto T$ .

**b)**

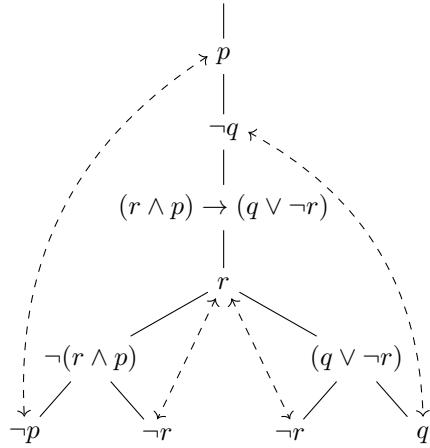
$$\neg((\neg p \wedge q \wedge r) \vee (p \wedge q) \vee (\neg p \wedge \neg r) \vee \neg q)$$



Since its negation is unsatisfiable,  $(\neg p \wedge q \wedge r) \vee (p \wedge q) \vee (\neg p \wedge \neg r) \vee \neg q$  is a tautology.

**c)**

$$p \wedge \neg q \wedge (r \wedge p) \rightarrow (q \vee \neg r) \wedge r$$



$p \wedge \neg q \wedge (r \wedge p) \rightarrow (q \vee \neg r) \wedge r$  is unsatisfiable, therefore  
 $\{p \wedge \neg q, (r \wedge p) \rightarrow (q \vee \neg r)\} \models \neg r$

### Exercise 1.3

Detailed transformations for the first statement, with  $C$  meaning that Cyclope is guilty and  $\neg C$  that Cyclope is not guilty. Other abbreviations analogous.

$$\begin{aligned}
 & (\neg C \wedge E \wedge \neg A) \vee (C \wedge \neg(E \wedge \neg A)) \\
 \equiv & (\neg C \wedge E \wedge \neg A) \vee (C \wedge (\neg E \vee A)) \\
 \equiv & (\neg C \vee (C \wedge (\neg E \vee A))) \wedge (E \vee (C \wedge (\neg E \vee A))) \wedge (\neg A \vee (C \wedge (\neg E \vee A))) \\
 \equiv & (\neg C \vee C) \wedge (\neg C \vee \neg E \vee A) \wedge (E \vee C) \wedge (E \vee \neg E \vee A) \wedge (\neg A \vee C) \wedge (\neg A \vee \neg E \vee A) \\
 \equiv & (\neg C \vee \neg E \vee A) \wedge (E \vee C) \wedge (\neg A \vee C)
 \end{aligned}$$

This leads to the following set of formulae (applying the steps above to all statements):

$$\{\neg C, \neg E, A\} \quad (1.1)$$

$$\{C, E\} \quad (1.2)$$

$$\{C, \neg A\} \quad (1.3)$$

$$\{\neg E, \neg G, \neg L\} \quad (2.1)$$

$$\{E, G\} \quad (2.2)$$

$$\{E, L\} \quad (2.3)$$

$$\{\neg A, \neg Sh, L\} \quad (3.1)$$

$$\{A, Sh\} \quad (3.2)$$

$$\{A, \neg L\} \quad (3.3)$$

$$\{\neg G, \neg W, P\} \quad (4.1)$$

$$\{G, W\} \quad (4.2)$$

$$\{G, \neg P\} \quad (4.3)$$

$$\{\neg I, E, St\} \quad (5.1)$$

$$\{I, \neg E\} \quad (5.2)$$

$$\{I, \neg St\} \quad (5.3)$$

$$\{\neg L, \neg I, W\} \quad (6.1)$$

$$\{L, I\} \quad (6.2)$$

$$\{L, \neg W\} \quad (6.3)$$

$$\{\neg M, \neg P, C\} \quad (7.1)$$

$$\{M, P\} \quad (7.2)$$

$$\{M, \neg C\} \quad (7.3)$$

$$\{\neg P, \neg E, \neg G\} \quad (8.1)$$

$$\{P, E\} \quad (8.2)$$

$$\{P, G\} \quad (8.3)$$

$$\{\neg Sh, \neg C, I\} \quad (9.1)$$

$$\{Sh, C\} \quad (9.2)$$

$$\{Sh, \neg I\} \quad (9.3)$$

$$\{\neg St, \neg G, M\} \quad (10.1)$$

$$\{St, G\} \quad (10.2)$$

$$\{St, \neg M\} \quad (10.3)$$

$$\{\neg V, \neg M, Sh\} \quad (11.1)$$

$$\{V, M\} \quad (11.2)$$

$$\{V, \neg Sh\} \quad (11.3)$$

$$\{\neg W, \neg St, \neg V\} \quad (12.1)$$

$$\{W, St\} \quad (12.2)$$

$$\{W, V\} \quad (12.3)$$

Now apply resolution:

$$(1.2) + (8.1) : \{C, \neg P, \neg G\} \quad (13)$$

$$(4.3) + (13) : \{C, \neg P\} \quad (14)$$

$$(7.2) + (14) : \{C, M\} \quad (15)$$

$$(7.3) + (15) : \{M\} \quad (16)$$

$$(10.3) + (16) : \{St\} \quad (17)$$

$$(5.3) + (17) : \{I\} \quad (18)$$

$$(9.3) + (18) : \{Sh\} \quad (19)$$

$$(11.1) + (19) : \{V\} \quad (20)$$

$$(12.1) + (20) : \quad \{\neg W, \neg St\} \quad (21)$$

$$(17) + (21) : \quad \{\neg W\} \quad (22)$$

$$(6.1) + (22) : \quad \{\neg L, \neg I\} \quad (23)$$

$$(18) + (23) : \quad \{\neg L\} \quad (24)$$

$$(2.1) + (24) : \quad \{E\} \quad (25)$$

$$(4.2) + (22) : \quad \{G\} \quad (26)$$

$$(8.1) + (25) : \quad \{\neg P, \neg G\} \quad (27)$$

$$(26) + (27) : \quad \{\neg P\} \quad (28)$$

$$(3.1) + (19) : \quad \{\neg A, L\} \quad (29)$$

$$(24) + (29) : \quad \{\neg A\} \quad (30)$$

$$(1.1) + (25) : \quad \{\neg C, A\} \quad (31)$$

$$(30) + (31) : \quad \{\neg C\} \quad (32)$$

This leads to the following set of guilty people: Emma Frost, Gambit, Iceberg, Malicia, Shadowcat, Storm, and Vega