Exercise 12.1

a)

- *Symmetry*
  The sum operator is commutative and the permutations of N can be rearranged in a way for i and j to substitute each other. $i \notin C_i(o) \quad \forall o$.
  $j \in S$: $\mu_i(S) = v(S \cup i) - v(S) = v((S \setminus j \cup i) \cup j) - v(S \setminus j \cup i) = \mu_j(S \setminus j \cup i)$
  $j \notin S$: $\mu_i(S) = v(S \cup i) - v(S) = v(S \cup j) - v(S) = \mu_j(S)$
- *Dummy Player*
  $\mu_i(S \cup j) = v(S \cup j \cup i) - v(S \cup j) = (v(S \cup i) + v(\{j\})) - (v(S) + v(\{j\}) = v(S \cup i) - v(S) = \mu_i(S)$
- *Additivity*
  $(v_1 + v_2)(S \cup i) - (v_1 + v_2)(S) = v_1(S \cup i) - v_1(S) + v_2(S \cup i) - v_2(S)$

b)

$$
\begin{aligned}
\sum_{i \in N} \Psi_i(N, v) =& \sum_{i \in N} \frac{1}{N!} \sum_{o \in \Pi(N)} \mu_i(C_i(o)) \\
=& \frac{1}{N!} \sum_{o \in \Pi(N)} \sum_{i \in N} (v(C_i(o) \cup i) - v(C_i(o))) \\
=& \frac{1}{N!} \sum_{o \in \Pi(N)} (v(N) - v(C_{o_{N-1}}(o)) + v(C_{o_{N-1}}(o)) - \cdots - v(\emptyset)) \\
=& \frac{|\Pi(N)|}{N!} v(N) \\
=& v(N)
\end{aligned}
$$

Exercise 12.2

See `taxi.py`