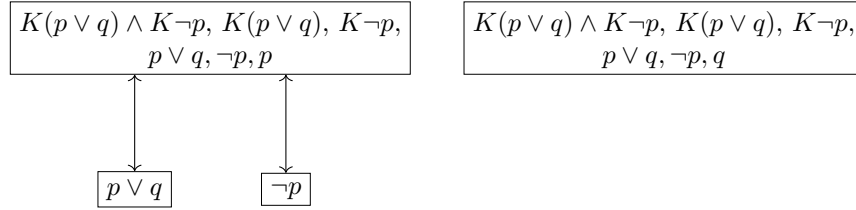


### Exercise 3.1

a)



This yields the model  $M = (W, R, V)$  with  $W = \{w_1\}$ ,  $R(K) = \{(w_1, w_1)\}$ ,  $V(p) = \{w_1\}$  and  $V(q) = \{w_1\}$ .

### Exercise 3.1 Variation

a)

$K(p \vee q) \wedge K\neg p$

Apply AND-Rule:

$K(p \vee q) \wedge K\neg p,$   
 $K(p \vee q), K\neg p$

Apply Axiom T:

$K(p \vee q) \wedge K\neg p,$   
 $K(p \vee q), K\neg p,$   
 $p \vee q, \neg p$

Apply OR-Rule:

$K(p \vee q) \wedge K\neg p,$   
 $K(p \vee q), K\neg p,$   
 $p \vee q, \neg p,$   
 $p$

$K(p \vee q) \wedge K\neg p,$   
 $K(p \vee q), K\neg p,$   
 $p \vee q, \neg p,$   
 $q$

$\perp$ -Rule:

$K(p \vee q) \wedge K\neg p,$   
 $K(p \vee q), K\neg p,$   
 $p \vee q, \neg p,$   
 $p,$   
 $\perp$

$K(p \vee q) \wedge K\neg p,$   
 $K(p \vee q), K\neg p,$   
 $p \vee q, \neg p,$   
 $q$

No more rules can be applied and there is an open premodel left, thus the formula is satisfiable. Kripke Model:  $M = (W, R, V)$  with  $W = \{w_1\}$ ,  $R(K) = \{(w_1, w_1)\}$

$\{(w_1, w_1)\}$ ,  $V(p) = \{\}$  and  $V(q) = \{w_1\}$ . Pointed S5 model  $(M, w)$  with  $M$  from above and  $w = \{\neg p, q\}$ .

b)

$$\boxed{\neg (K (p \wedge q) \rightarrow Kp)}$$

Apply NotImpl-Rule:

$$\boxed{\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp) \end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{l} \neg (K (p \wedge q) \rightarrow Kp), \\ (K (p \wedge q) \wedge \neg Kp), \\ K (p \wedge q), \neg Kp \end{array}}$$

### Exercise 3.2

$\phi$	$M, w_2 \models$ $K_1\phi$	$M, w_2 \models$ $K_2\phi$	$M, w_2 \models$ $C\phi$	$M, w_2 \models$ $D\phi$	$M, w_1 \models$ $C\phi$	$M, w_1 \models$ $D\phi$
$p$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$q$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
$p \wedge q$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$p \vee q$	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>