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Exercise 3.1

a)

$\boxed{K(p \lor q) \land K\neg p}$

Apply AND-Rule:

$\boxed{\begin{array}{c} K(p \lor q) \land K\neg p, \\ K(p \lor q),\ K\neg p \end{array}}$

Apply Axiom T:

$\boxed{\begin{array}{c} K(p \lor q) \land K\neg p, \\ K(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p \end{array}}$

Apply OR-Rule:

$\boxed{\begin{array}{c} K(p \lor q) \land K\neg p, \\ K(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ p \end{array}}$ $\boxed{\begin{array}{c} K(p \lor q) \land K\neg p, \\ K(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ q \end{array}}$

$\perp$-Rule:

$\boxed{\begin{array}{c} K(p \lor q) \land K\neg p, \\ K(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ p, \\ \perp \end{array}}$ $\boxed{\begin{array}{c} K(p \lor q) \land K\neg p, \\ K(p \lor q),\ K\neg p, \\ p \lor q,\ \neg p, \\ q \end{array}}$

No more rules can be applied and there is an open premodel left, thus the formula is satisfiable. Kripke Model: $M = (W, R, V)$ with $W = \{w_1\}$, $R(K) = \{(w_1, w_1)\}$, $V(p) = \{\}$ and $V(q) = \{w_1\}$. Pointed S5 model (M, w) with M from above and $w = \{\neg p, q\}$.

b)

$\boxed{\neg (K(p \land q) \to Kp)}$

Apply NotImpl-Rule:

$\boxed{\begin{array}{c} \neg (K(p \land q) \to Kp), \\ (K(p \land q) \land \neg Kp) \end{array}}$

Apply AND-Rule:

$$\boxed{\begin{array}{c} \neg\left(K\left(p \wedge q\right) \rightarrow Kp\right), \\ \left(K\left(p \wedge q\right) \wedge \neg Kp\right), \\ K\left(p \wedge q\right), \neg Kp \end{array}}$$

Apply Axiom T:

$$\boxed{\begin{array}{c} \neg\left(K\left(p \wedge q\right) \rightarrow Kp\right), \\ \left(K\left(p \wedge q\right) \wedge \neg Kp\right), \\ K\left(p \wedge q\right), \neg Kp, \\ p \wedge q \end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c} \neg\left(K\left(p \wedge q\right) \rightarrow Kp\right), \\ \left(K\left(p \wedge q\right) \wedge \neg Kp\right), \\ K\left(p \wedge q\right), \neg Kp, \\ p \wedge q, \\ p, q \end{array}}$$

Duality:

$$\boxed{\begin{array}{c} \neg\left(K\left(p \wedge q\right) \rightarrow Kp\right), \\ \left(K\left(p \wedge q\right) \wedge \neg Kp\right), \\ K\left(p \wedge q\right), \neg Kp, \\ p \wedge q, \\ p, q, \\ \neg(\neg\hat{K}\neg p) \end{array}}$$

$$\boxed{\begin{array}{c} \neg\left(K\left(p \wedge q\right) \rightarrow Kp\right), \\ \left(K\left(p \wedge q\right) \wedge \neg Kp\right), \\ K\left(p \wedge q\right), \neg Kp, \\ p \wedge q, \\ p, q, \\ \hat{K}\neg p \end{array}}$$

<l>-Rule

$$\begin{array}{c} \boxed{\begin{array}{c} \neg\left(K\left(p \wedge q\right) \rightarrow Kp\right), \\ \left(K\left(p \wedge q\right) \wedge \neg Kp\right), \\ K\left(p \wedge q\right), \neg Kp, \\ p \wedge q, \\ p, q, \\ \hat{K}\neg p \end{array}} \\ \updownarrow \\ \boxed{\neg p} \end{array}$$

c)

$$\boxed{(K_a p \lor K_a \neg p) \land K_b (K_a p \lor K_a \neg p)}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c}(K_a p \lor K_a \neg p) \land K_b (K_a p \lor K_a \neg p),\\ (K_a p \lor K_a \neg p),\ K_b (K_a p \lor K_a \neg p)\end{array}}$$

d)

$$\boxed{\neg (O (Op \to p) \to (OOp \to Op))}$$

Apply NotImpl-Rule:

$$\boxed{\begin{array}{c}\neg (O (Op \to p) \to (OOp \to Op)),\\ (O (Op \to p) \land \neg (OOp \to Op))\end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c}\neg (O (Op \to p) \to (OOp \to Op)),\\ O (Op \to p) \land \neg (OOp \to Op),\\ O (Op \to p),\ \neg (OOp \to Op)\end{array}}$$

Apply NotImpl-Rule:

$$\boxed{\begin{array}{c}\neg (O (Op \to p) \to (OOp \to Op)),\\ O (Op \to p) \land \neg (OOp \to Op),\\ O (Op \to p),\ \neg (OOp \to Op),\\ OOp \land \neg Op\end{array}}$$

Apply AND-Rule:

$$\boxed{\begin{array}{c}\neg (O (Op \to p) \to (OOp \to Op)),\\ O (Op \to p) \land \neg (OOp \to Op),\\ O (Op \to p),\ \neg (OOp \to Op),\\ OOp \land \neg Op,\\ OOp,\ Op\end{array}}$$

e)

$$\boxed{\neg (OKp \to Op)}$$

Exercise 3.2

ϕ	$M, w_2 \models K_1\phi$	$M, w_2 \models K_2\phi$	$M, w_2 \models C\phi$	$M, w_2 \models D\phi$	$M, w_1 \models C\phi$	$M, w_1 \models D\phi$
p	☐	☒	☐	☒	☐	☐
q	☒	☐	☐	☒	☐	☒
$p \wedge q$	☐	☐	☐	☒	☐	☐
$p \vee q$	☒	☒	☒	☒	☒	☒