Multi-Agent Systems, Winter Semester 2018/2019, Exercise 2

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Exercise 2.1

a)

$$
\begin{array}{rl}
 & M, w_1 \models K_1(\neg l \wedge \hat{K}_2 l) \wedge [a](g \wedge K_1 l \wedge K_2 l) \\
1 & M, w_1 \models K_1(\neg l \wedge \hat{K}_2 l) \\
1.1 & \text{for every } u : \text{if } (w_1, u) \in R(1) \text{ then } M, u \models (lnotl \wedge \hat{K}_2 l) \\
1.1.1 & M, w_1 \models (\neg l \wedge \hat{K}_2 l) \\
1.1.1.1 & M, w_1 \models \neg l \quad ☺ \\
1.1.1.2 & M, w_1 \models \hat{K}_2 l \\
1.1.1.2.1 & \text{for some } u : (w_1, u) \in R(2) \text{ and } M, u \models l \\
1.1.1.2.1.2 & \text{we find } u : (w_1, w_1) \in R(2) \text{ and } M, u \models l \quad ☺ \\
2 & M, w_1 \models [a](g \wedge K_1 l \wedge K_2 l) \\
2.1 & \text{for every } u : \text{if } (w_1, u) \in R(a) \text{ then } M, u \models g \wedge K_1 l \wedge K_2 l \\
2.1.1 & M, w_3 \models g \wedge K_1 l \wedge K_2 l \\
2.1.1.1 & M, w_3 \models g \quad ☺ \\
2.1.1.2 & M, w_3 \models K_1 l \\
2.1.1.2.1 & \text{for every } u : \text{if } (w_3, u) \in R(1) \text{ then } M, u \models l \\
2.1.1.2.1.1 & M, w_3 \models l \quad ☺ \\
2.1.1.3 & M, w_3 \models K_2 l \\
2.1.1.3.1 & \text{for every } u : \text{if } (w_3, u) \in R(2) \text{ then } M, u \models l \\
2.1.1.3.1.1 & M, w_3 \models l \quad ☺
\end{array}
$$

b)

Agent 1 is standing inside a garage and knows both whether the garage door is open and whether the light inside the garage is on. Agent 2 is standing outside this garage and always knows whether the garage door is open. Only if this is the case, he also knows that the light is on. Opening the garage turns on the light if it was off before.

Exercise 2.2

a)

“ $\Longrightarrow$ ”:

T holds in our Kripke frame (W, R), i.e. $[I]x \implies x \quad \forall x \in W$. Since $V(\varphi) \subseteq W \quad \forall \varphi \in \mathcal{P}$ for all valuations V, $[I]\varphi \implies \varphi \quad \forall \varphi \in \mathcal{P}$.

“ $\Longleftarrow$ ”:

Given that $[I]\varphi \implies \varphi \quad \forall \varphi \in \mathcal{P}$ holds in all models, this is also true for the identity model with $\mathcal{P} = W$ and $V(u) = \{u\} \quad \forall u \in \mathcal{P}$.