Multi-Agent Systems, Winter Semester 2018/2019, Exercise 5
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Exercise 5.1

a)

$K(\phi \wedge \neg K\phi) \models_{N,K} K\phi \wedge K\neg K\phi \models_T K\phi \wedge \neg K\phi \models \bot$
An agent cannot know know ϕ while not knowing ϕ.
Rather, $[!\phi \wedge \neg K\phi]K\phi \wedge K\phi$ holds.

b)

$K_jK_i\phi \models K_jK_i\phi \wedge (K_i\phi \rightarrow_T \phi) \models_N K_jK_i\phi \wedge K_j(K_i\phi \rightarrow_T \phi) \models_K K_j\phi$
An agent knows all logical implications of his knowledge, including the use of axioms.

c)

$O(\phi \wedge \psi) \models O(\phi \wedge \psi) \wedge (\phi \wedge \psi \rightarrow \psi) \models_N O(\phi \wedge \psi) \wedge O(\phi \wedge \psi \rightarrow \psi) \models_K O(\psi)$
An agent is obliged to all generalizations of his obligations.

d)

$P\phi \wedge \neg P\psi \models P\phi \models_{N,K} P(\phi \vee \psi)$
If only ϕ is permitted while ψ is not, $\phi \vee \psi$ is permitted (weakening rule). This is a counterexample to the given statement.

e)

$\models_T O(\phi) \rightarrow \phi \models_N O(O(\phi) \rightarrow \phi)$

Exercise 5.2

p is true iff the passenger is alive. q is true iff the pedestrian is alive.

a)

$\Diamond(Kp \wedge K\neg q) \wedge \Diamond(\neg Kp \wedge \neg K\neg p \wedge Kq)$

b)

$O(\neg Kp \wedge \neg K\neg p \wedge Kq)$

c)

$O(\neg K\neg p \wedge \neg K\neg q)$

d)

$O(\neg Kp \wedge \neg K\neg p \wedge Kq) \models_{N,K} O(\neg K\neg p \wedge Kq) \models_{D,N,K} O(\neg K\neg p \wedge \neg K\neg q)$