Set 1 - Exercise 1

Exercise: Given the vectors $\mathbf{R}_1 = \boldsymbol{a}_x + 2\boldsymbol{a}_y + 3\boldsymbol{a}_z$, $\mathbf{R}_2 = 3\boldsymbol{a}_x + 2\boldsymbol{a}_y + \boldsymbol{a}_z$. Find a) the dot product $\mathbf{R}_1 \sqcup \mathbf{R}_2$, b) the projection of $\mathbf{R}_1$ on $\mathbf{R}_2$, c) the angle between $\mathbf{R}_1$ and $\mathbf{R}_2$. Write a MATLAB program to verify your answer.

$$\vec{R}_1 = (1, 2, 3)$$
$$\vec{R}_2 = (3, 2, 1)$$

a)
$$A = \vec{R}_1 \circ \vec{R}_2$$
$$= 1(3) + 2(2) + 3(1)$$
$$= 10$$

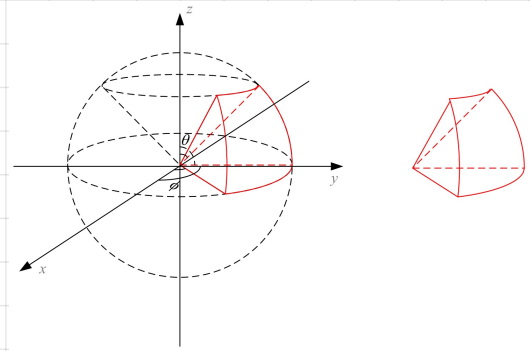
b)
$$\vec{B} = \text{proj}_{\vec{R}_1} \vec{R}_2$$
$$= \vec{a}_{R2} \left(\vec{R}_1 \cdot \vec{a}_{R2} \right)$$
$$= \vec{a}_{R2} \left(1\left(\frac{3}{\sqrt{14}}\right) + 2\left(\frac{2}{\sqrt{14}}\right) + 3\left(\frac{1}{\sqrt{14}}\right) \right)$$
$$= \vec{a}_{R2} \left(\frac{10}{\sqrt{14}}\right)$$
$$= \frac{10}{\sqrt{14}} \left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right)$$
$$= \frac{30}{14}, \frac{20}{14}, \frac{10}{14}$$
$$= \left(\frac{15}{7}, \frac{10}{7}, \frac{5}{7} \right)$$

$$\vec{a}_{R2} = \frac{\vec{R}_2}{|\vec{R}_2|}$$
$$= \frac{1}{\sqrt{3^2 + 2^2 + 1^2}} \vec{R}_2$$
$$= \frac{1}{\sqrt{14}} \vec{R}_2$$
$$= \frac{1}{\sqrt{14}} (3, 2, 1)$$
$$= \left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right)$$

c)
$$\theta_{R12} = ?$$
$$R_1 \cdot a_{R2} = |R_1||a_{R2}| \cos\theta_{R12}$$
$$\frac{10}{\sqrt{14}} = \left(\sqrt{1^2 + 2^2 + 3^2}\right)(1) \cos\theta_{R12}$$
$$\frac{10}{\sqrt{14}} = \sqrt{14} \cos\theta_{R12}$$
$$\frac{10}{14} = \cos\theta_{R12}$$
$$\theta_{R12} = \cos^{-1}\left(\frac{5}{7}\right)$$
$$= 0.7752 \text{ rad}$$

Set 2 - Exercise 1

Exercise: The surfaces $r = 0$ and $r = 2$, $\phi = 45^\circ$, $\phi = 90^\circ$, $\theta = 45^\circ$ and $\theta = 90^\circ$ define a closed surface. Find the enclosed volume and the area of the closed surface S. Write a MATLAB program to verify your answer.



- $0 \le r \le 2$
- $\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$
- $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

a) $V = ?$

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$V = \int_{\phi=\pi/4}^{\pi/2} \int_{\theta=\pi/4}^{\pi/2} \int_{r=0}^{2} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_{r=0}^{2} r^2 \, dr \times \int_{\theta=\pi/4}^{\pi/2} \sin\theta \, d\theta \times \int_{\phi=\pi/4}^{\pi/2} d\phi$$

$$= \frac{1}{3} r^3 \Big|_0^2 \times -\cos\theta \Big|_{\pi/4}^{\pi/2} \times \phi \Big|_{\pi/4}^{\pi/2}$$

$$= 8/3 \times \sqrt{2}/2 \times \pi/4$$

$$= \frac{8\sqrt{2}\pi}{3(2)(4)}$$

$$V = \frac{\sqrt{2}}{8}\pi$$

$$= 1.48 \text{ m}^3$$

b) $S = ?$

- S_1 : outer spherical surface @ $r = 2$m

$$dS = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 2^2 \sin\theta \, d\theta \, d\phi$$

$$= 4 \sin\theta \, d\theta \, d\phi$$

$$= 4 \int_{\pi/4}^{\pi/2} \sin\theta \, d\theta \int_{\pi/4}^{\pi/2} d\phi$$

$$= 4 \times -\cos\theta \Big|_{\pi/4}^{\pi/2} \times \phi \Big|_{\pi/4}^{\pi/2}$$

$$= \cancel{4} \times \sqrt{2}/2 \times \pi/\cancel{4}$$

$$= \frac{\sqrt{2}}{2}\pi$$

$$S_1 = 2.22 \text{ m}^2$$

- S_2 : inner spherical surface @ $r = 0$m

$\because r = 0$, $r^2 = 0$

$\therefore S_2 = 0 \text{ m}^2$

- S_3 & S_4 : surfaces at constant ϕ of $\phi = \pi/4, \pi/2$

$$dS = r \, dr \, d\theta$$

$$= \int_0^2 r \, dr \int_{\pi/4}^{\pi/2} d\theta$$

$$= \frac{1}{2} r^2 \Big|_0^2 \times \theta \Big|_{\pi/4}^{\pi/2}$$

$$= 2 \times \pi/4$$

$$= \pi/2$$

$$S_3 = S_4 = 1.57 \text{ m}^2$$

- S_5 & S_6 : surfaces @ constant θ of $\theta = \pi/4, \pi/2$

$$dS = r \sin\theta \, dr \, d\phi$$

$$= \sin\theta \times r \, dr \times d\phi$$

$$= \sin\theta \times \frac{1}{2} r^2 \Big|_0^2 \times \phi \Big|_{\pi/4}^{\pi/2}$$

$$= \sin\theta \times 2 \times \pi/4$$

$$S_5 = \sin(\pi/4) \times 2 \times \pi/4$$

$$= \frac{\sqrt{2}}{\cancel{2}} \times \cancel{2} \times \frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{4}\pi$$

$$= 1.11 \text{ m}^2$$

$$S_6 = \sin\left(\frac{\pi}{2}\right) \times 2 \times \pi/4$$

$$= 1 \times 2 \times \pi/4$$

$$= \frac{1}{2}\pi$$

$$= 1.57 \text{ m}^2$$

$$S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$

$$= \frac{\sqrt{2}}{2}\pi + 0 + \pi/2 + \pi/2 + \frac{\sqrt{2}}{4}\pi + \frac{1}{2}\pi$$

$$= 2.22 + 0 + 1.57 + 1.57 + 1.11 + 1.57$$

$$S = 8.04 \text{ m}^2$$