Gdansk University of Technology, dr hab. J. Guthmuller

Monte Carlo Integration:

The goal of the exercise is to calculate the integral of a function $f(x)$:

$$I \equiv \int_a^b f(x)dx$$

In the Monte Carlo method the integral is obtained from

$$\int_a^b f(x)dx = (b-a)\langle f \rangle$$

where $\langle f \rangle$ is the average value of the function in the interval $[a,b]$.

Using a list of *N* random numbers uniformly distributed between *a* and *b* the function average can be estimated from

$$\langle f \rangle_N = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

Thus, the Monte Carlo estimate of the integral is

$$\int_a^b f(x)dx \approx (b-a)\langle f \rangle_N$$

The standard deviation σ_N of probability theory is defined by

$$\sigma_N = \sqrt{\frac{\frac{1}{N}\sum_{i=1}^{N} f(x_i)^2 - \left(\frac{1}{N}\sum_{i=1}^{N} f(x_i)\right)^2}{N-1}}$$

Meaning that

$$\int_a^b f(x)dx \approx (b-a)\left(\langle f \rangle_N \pm \sigma_N\right)$$

with 68.3% confidence.

Exercise:

- Calculate $I = \int_0^1 e^x dx$ and σ_N using *N*=10, 100, 1000,…,10 000 000 random numbers. Compare with the exact value.

- Using *Importance sampling* the integral *I* can be written as

$$I = \int_0^{\frac{3}{2}} \frac{e^{-1+\sqrt{1+2y}}}{\sqrt{1+2y}}dy$$

Calculate I and σ_N using *N*=10, 100, 1000,…,10 000 000 random numbers. Compare with the exact value.

- For $I = \int_0^1 e^x dx$ calculate 10 000 integral estimates with *N*= 100 and *N*= 1000. Plot the distribution of integral values for $1.5 \le I \le 1.9$. For this, count the number of integral values obtained in each interval of size 0.002.