$U(t,x) =$

$$\frac{d^2 x}{dx^2}$$

$u(t,10) = 0$

$u(t,0) = 0$

$u(0,x) = e^{(-(x-5)^2)}$

$U(t,x) = X(x)T(t)$

$$\begin{cases} X''(x) + \lambda X(x) = 0 \\ T'(t) + \lambda T(t) = 0 \end{cases}$$

$T(t) = Ce^{(-\lambda t)}$

$X(x) = A\cos(\sqrt{\lambda}\, x) + B\sin(\sqrt{\lambda} \cdot x)$

$u(t,x) = Ce^{(-\lambda t)}\left[A\cos(\sqrt{\lambda}\, x) + B\sin(\sqrt{\lambda} \cdot x)\right]$

* Warunki brzegowe

$u(\overset{t}{t},\overset{x}{10}) = Ce^{(-\lambda t)}\left[\overset{\to 0}{A}\cos(10\cdot\sqrt{\lambda}) + B\sin(10\sqrt{\lambda})\right] = 0$

$u(\overset{t}{t},\overset{x}{0}) = Ce^{(-\lambda t)}\left[A\overset{\to 1}{\cos(0\cdot\sqrt{\lambda})} + B\overset{\to 0}{\sin(0\cdot\sqrt{\lambda})}\right] = 0$

$A\cdot T(t) = 0$

$A = 0$

$Ce^{(-\lambda t)} \cdot B\sin(10\sqrt{\lambda}) = 0$

$\sin 10\sqrt{\lambda} = 0$

$\sqrt{\lambda} \cdot 10 = k\pi$

$\sqrt{\lambda} = \frac{k\pi}{10}$

$\underline{\lambda} = \left(\frac{k\pi}{10}\right)^2$

$C_k = C\cdot B$

$u_k(t,x) = \sum_{k=0}^{\infty} C_k e^{(-\lambda t)}\left[\sin(\sqrt{\lambda_k}\cdot x)\right]$

* Warunek początkowy

$u(0,x) = \sum_{k=0}^{\infty} C_k e^{(-\lambda 0)} \sin(\sqrt{\lambda_k}\cdot x) = f(x)$

mnożymy $\sin(\sqrt{\lambda_m}\cdot x)$ i całkujemy $\int_0^{10}$

$\sum_{k=0}^{\infty} \int_0^{10} C_k \sin(\sqrt{\lambda_k}\, x)\sin(\sqrt{\lambda_m}\, x)dx = \int_0^{10} f(x)\sin(\sqrt{\lambda_m}\, x)dx$

$C_m = \frac{2}{10}\int_0^{10} f(x)\sin(\sqrt{\lambda_m}\cdot x)dx$

$C_m \to$ Najpierw liczę C_m a następnie liczę sumę ze wzoru $\sum_0^{\infty} C_m \exp(-\lambda_k t)\sin(\sqrt{\lambda_k}\cdot x)$