18.2 Continuous univariate distributions

Table 18.1 Beta density: Beta(α, β)

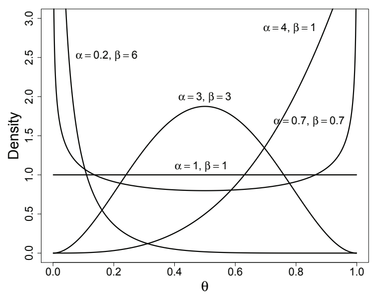
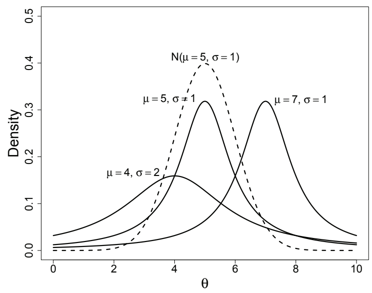
Model	Examples
$p(\theta) =$ $\frac{1}{B(\alpha,\beta)}\ \theta^{\alpha-1}(1-\theta)^{\beta-1}$ with $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ `Condition:` $\alpha > 0, \beta > 0$ `Range:` [0,1] `Parameters:` α, β: shape	
Moments	**Program commands**
`mean:` $\frac{\alpha}{(\alpha+\beta)}$	`R: dbeta(theta,alpha,beta)`
`mode:` $\frac{\alpha-1}{(\alpha+\beta-2)}$	`WB/JAGS: theta ~ dbeta(alpha,beta)`
`variance:` $\frac{\alpha\,\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	`SAS: theta ~ beta(alpha,beta)`

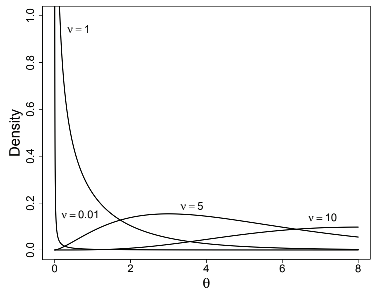
Table 18.2 Cauchy distribution: Cauchy(μ, σ)

Model	Examples
$p(\theta) = \frac{1}{\pi}\left(\frac{\sigma}{\sigma^2+(\theta-\mu)^2}\right)$ `Condition:` $\sigma > 0$ `Range:` $(-\infty, \infty)$ `Parameters:` μ: location, σ: scale	

Moments		Program commands	
`mean:`	-	`R:`	`dcauchy(theta,mu,sigma)`
`mode:`	μ	`WB/JAGS:`	-
`variance:`	-	`SAS:`	`theta ~ cauchy(mu,sigma)`

Note:
Cauchy distribution is a special case of location-scale t-distribution:
Cauchy$(\mu, \sigma) = \mathrm{t}(1, \mu, \sigma)$.

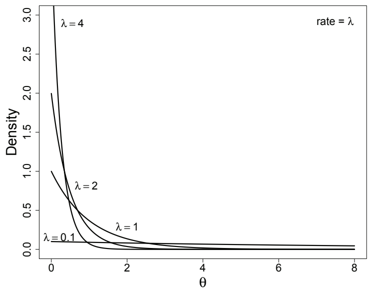
Table 18.3 Chi-squared density: $\chi^2(\nu)$

Model	Examples
$p(\theta) = \frac{1}{\Gamma(\nu/2)2^{\nu/2}}\theta^{(\nu/2)-1}\mathrm{e}^{-\theta/2}$ `Condition:` $\nu > 0$ `Range:` $\nu = 2 : [0, \infty)$ otherwise : $(0, \infty)$ `Parameters:` ν: degrees of freedom	

Moments		Program commands	
`mean:`	nu	`R:`	`dchisq(theta,nu)`
`mode:`	$\nu - 2$ $(\nu \geq 2)$, otherwise $-$	`WB/JAGS:`	`theta ~ dchisqr(nu)`
`variance:`	2ν	`SAS:`	`theta ~ chisq(nu)`

Note:
Chi-squared is a special case of a gamma distribution:
$\chi^2(\nu) = \text{Gamma}(\alpha = \nu/2, \beta = 1/2)$ (rate).
JAGS offers a non-central χ^2-distribution:
'theta $\sim$ dnchisqr(nu,delta)', $\delta > 0$ non-centrality parameter.
JAGS offers an F-distribution (ratio of 2 independent χ^2s):
'theta $\sim$ df(nu1, nu2)', with nu1, nu2 = dfs of numerator and denominator, resp.

Table 18.4 Exponential density: Exp(λ)

Model	Examples
`rate:` $p(\theta) = \lambda e^{-\lambda\theta}$ `Condition:` $\lambda > 0$ `Range:` $[0, \infty)$ `Parameters:` λ: rate	
Moments	**Program commands**

`rate:` λ
`mean:` $\frac{1}{\lambda}$
`mode:` 0
`variance:` $\frac{1}{\lambda^2}$
	`R:` `dexp(theta,lambda)`
	`WB/JAGS:` `theta ~ dexp(lambda)`
	`SAS:` `theta ~ expon(iscale=lambda)`
	`(scale)` `theta ~ expon(scale=ilambda)`

Note: Exponential is special case of gamma distribution: Exp(λ)= Gamma($\alpha = 1, \lambda$).	

Table 18.5 Gamma density: Gamma(α, β)

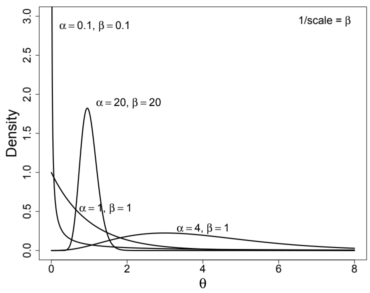
Model

`rate:`
$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\, \theta^{(\alpha-1)}\, \mathrm{e}^{-\beta\theta}$

`Condition:` $\alpha > 0, \beta > 0$
`Range:` $\alpha = 1 : (0, \infty)$
otherwise : $[0, \infty)$

`Parameters:`
α: shape, β: rate

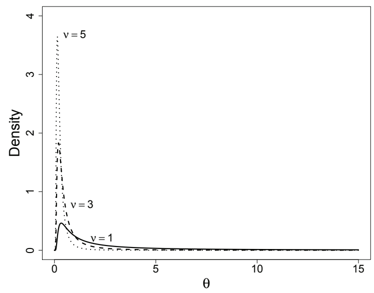
Examples



Moments		Program commands	
`rate:`	β		
`mean:`	$\frac{\alpha}{\beta}$	`R:`	`dgamma(theta,alpha,rate=beta)`
		`(scale)`	`dgamma(theta,alpha,scale=ibeta)`
`mode:`	$\frac{\alpha-1}{\beta}$ $(\alpha \geq 1)$	`WB/JAGS:`	`theta ~ dgamma(alpha,beta)`
`variance:`	$\frac{\alpha}{\beta^2}$	`SAS:`	`theta ~ gamma(alpha,iscale=beta)`
		`(scale)`	`theta ~ gamma(alpha,scale=ibeta)`

Note:
WB and JAGS offer a generalized gamma distribution *GenGamma*:
$\theta \sim \text{GenGamma}(\alpha, \beta^*, \lambda) \Leftrightarrow \theta^{1/\lambda} \sim \text{Gamma}(\alpha, \beta)$, with $\beta^* = \beta^{1/\lambda}$.
WB/JAGS command: 'theta $\sim$ dgen.gamma(alpha,beta,lambda)'.

Table 18.6 Inverse chi-squared density: $\mathrm{Inv}-\chi^2(\nu)$

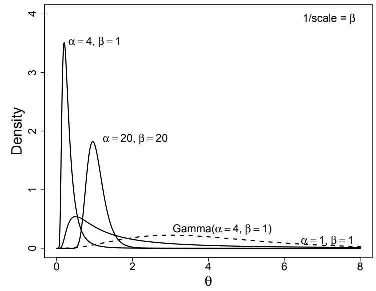
Model	Examples
$p(\theta) = \frac{1}{\Gamma(\nu/2)2^{\nu/2}}\theta^{-(\nu/2+1)}\mathrm{e}^{-1/(2\theta)}$ `Condition:` $\nu > 0$ `Range:` $(0, \infty)$ `Parameters:` ν: degrees of freedom	

Moments		Program commands	
`mean:`	$\frac{1}{\nu-2}$ $(\nu > 2)$	`R:`	`dchisq(1/theta,nu)/theta^2`
`mode:`	$\frac{1}{\nu+2}$	`WB/JAGS:`	`theta <- 1/itheta;` `itheta ~ dchisqr(nu)`
`variance:`	$\frac{2}{(\nu-2)^2(\nu-4)}$ $(\nu > 4)$	`SAS:`	`theta ~  ichisq(nu)`

Note:
Inverse χ^2 is a special case of the inverse gamma-distribution: (rate).
$\mathrm{Inv}-\chi^2(\nu) = \mathrm{IG}(\alpha = \nu/2, \beta = 1/2)$ (rate).
Inverse χ^2 is a special case of the scaled inverse χ^2-distribution with $\nu\, s^2 = 1$.

Table 18.7 Inverse gamma density: IG(α, β)

Model	Examples

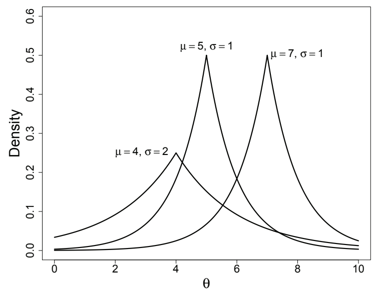
`rate:`
$p(\theta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \theta^{-(\alpha+1)} \mathrm{e}^{-\beta/\theta}$
 	
`Condition:` $\alpha > 0, \beta > 0$
`Range:` $(0, \infty)$
`Parameters:`
α: shape, β: rate
	

Moments	Program commands
`rate:` β	
`mean:` $\frac{\beta}{(\alpha-1)}$	`R:` `dgamma(1/theta,alpha,rate=beta)/theta^2` `(scale)` `dgamma(1/theta,alpha,scale=beta)/theta^2`
`mode:` $\frac{\beta}{(\alpha+1)}$	`WB/JAGS:` `theta <- 1/itheta;` `itheta ~ dgamma(alpha,beta)`
`variance:` $\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$	`SAS:` `theta ~ igamma(alpha,iscale=beta)` `(scale)` `theta ~ igamma(alpha,scale=ibeta)`

Note:
$\theta \sim \mathrm{IG}(\alpha, \beta) \Leftrightarrow 1/\theta \sim \mathrm{Gamma}(\alpha, \beta)$.

Table 18.8 Laplace density: Laplace(μ, σ)

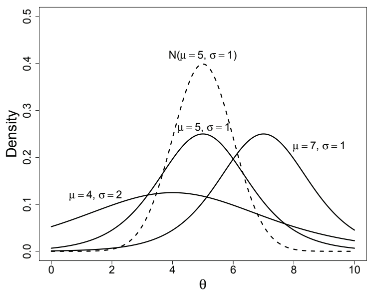
Model	Examples

`scale:`
$p(\theta) = \frac{1}{2\sigma}\,\mathrm{e}^{-(\theta-\mu)/\sigma}$
 	
`Condition:` $\sigma > 0$
`Range:` $(-\infty, \infty)$
`Parameters:`
μ: location, σ: scale
	

Moments	Program commands
`scale:` σ	
`mean:` μ	`R:` `dlaplace(theta,mu,sigma)`
`mode:` μ	`WB/JAGS:` `-` `(rate)` `theta ~ ddexp(isigma)`
`variance:` $2\sigma^2$	`SAS:` `theta ~ laplace(mu,scale=sigma)` `(rate)` `theta ~ laplace(mu,iscale=isigma)`

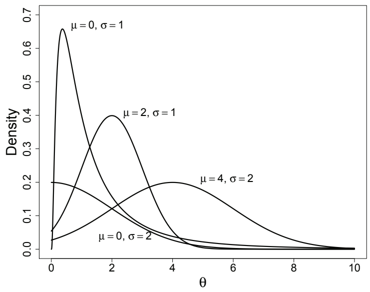
Note:
Laplace distribution is also called *double exponential distribution.*
R function dlaplace is available from R package 'VGAM'.

Table 18.9 Logistic distribution: Logistic(μ, σ)

Model	Examples
$p(\theta) =$ $$\exp\left(-\frac{\theta-\mu}{\sigma}\right)\ \left[\sigma\ \exp\left(-\frac{\theta-\mu}{\sigma}\right)\right]^2$$ `Condition:` $\sigma > 0$ `Range:` $(-\infty, \infty)$ `Parameters:` μ: location, σ: scale	

Moments		Program commands	
`mean:`	μ	`R:`	`dlogis(theta,mu,sigma)`
`mode:`	μ	`WB/JAGS:`	`theta ~ dlogis(mu,isigma)` (rate)
`variance:`	$\frac{\pi^2\sigma^2}{3}$	`SAS:`	`theta ~ logistic(mu,sigma)`

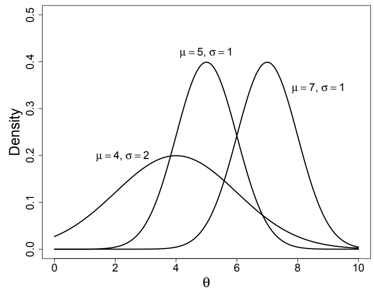
Table 18.10 Lognormal distribution: $\mathrm{LN}(\mu, \sigma^2)$

Model	Examples
$p(\theta) = \frac{1}{\theta\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log(\theta)-\mu)^2}{2\sigma^2}\right)$ `Condition:` $\sigma > 0$ `Range:` $(0, \infty)$ `Parameters:` μ: location, σ: scale	

Moments	Program commands
`mean:` $\exp(\mu + \sigma^2)$	`R: dlnorm(theta,mu,sigma)`
`mode:` $\exp(\mu - \sigma^2)$	`WB/JAGS: theta ~ dlnorm(mu,isigma2)`

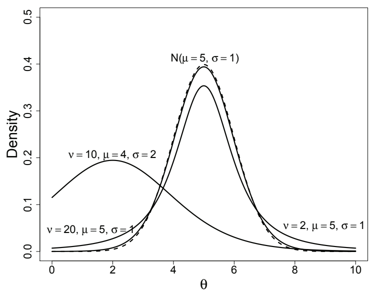
`variance:` $\exp(2(\mu + \sigma^2)) - \exp(2\mu + \sigma^2)$	`SAS: theta ~ lognormal(mu,sd=sigma)`
	`theta ~ lognormal(mu,var=sigma2)`
	`theta ~ lognormal(mu,prec=isigma2)`

Table 18.11 Normal distribution: $\mathrm{N}(\mu, \sigma^2)$

Model	Examples
$p(\theta) =$ $\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\theta-\mu)^2}{2\sigma^2}\right)$ `Condition:` $\sigma > 0$ `Range:` $(-\infty, \infty)$ `Parameters:` μ: location, σ: scale	
Moments	**Program commands**
`mean:` μ	`R:` `dnorm(theta,mu,sigma)`
`mode:` μ	`WB/JAGS:` `theta ~ dnorm(mu,isigma2)`

`variance:` σ^2	`SAS:` `theta ~ normal(mu,sd=sigma)`
	`theta ~ normal(mu,var=sigma2)`
	`theta ~ normal(mu,prec=isigma2)`

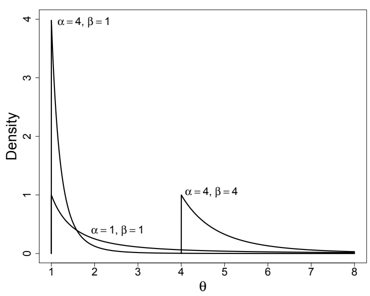
Table 18.12 Location-scale Student's t-distribution: $\mathrm{t}(\nu, \mu, \sigma)$

Model	Examples
$p(\theta) =$ $\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sigma\sqrt{\nu\pi}} \left(1 + \frac{(\theta-\mu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}$ `Condition:` $\sigma > 0, \nu > 0$ `Range:` $(-\infty, \infty)$ `Parameters:` μ: location, σ: scale ν: degrees of freedom	

Moments		Program commands	
`mean:`	μ (if $\nu > 1$)	`R:`	`dt(nu,(theta-mu)/sigma)/sigma`
`mode:`	μ	`WB/JAGS:`	`theta ~ dt(mu,isigma2,nu)`

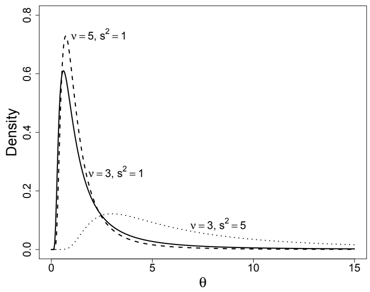
`variance:`	$\frac{\nu}{\nu-2}\sigma^2$ (if $\nu > 2$)	`SAS:`	`theta ~ t(mu,sd=sigma,nu)`
			 `theta ~ t(mu,var=sigma2,nu)`
			 `theta ~ t(mu,prec=isigma2,nu)`

Table 18.13 Pareto distribution: Pareto(α, β)

Model	Examples
$p(\theta) = \frac{\alpha}{\beta}\left(\frac{\beta}{\theta}\right)^{\alpha+1}$ `Condition:` $\alpha > 0, \beta > 0$ `Range:` (β, ∞) `Parameters:` α: shape, β: location	
Moments	**Program commands**
`mean:` $\frac{\alpha\beta}{\alpha-1}$ (if $\alpha > 1$)	`R: dpareto(theta,beta,alpha)`
`mode:` β	`WB/JAGS: theta ~ dpareto(alpha,beta)`
`variance:` $\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$ (if $\alpha > 2$)	`SAS: theta ~ pareto(alpha,beta)`

Note:
R function dpareto is available from R package 'VGAM'.

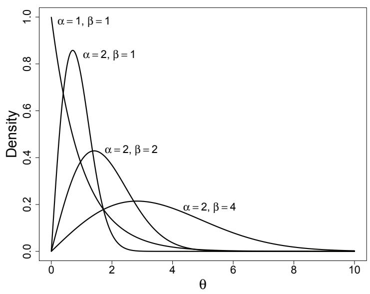
Table 18.14 Scaled inverse chi-squared density: $\mathrm{Inv}-\chi^2(\nu, s^2)$

Model	Examples
$p(\theta) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)} s^\nu \theta^{-(\nu/2+1)} \mathrm{e}^{-\nu s^2/(2\theta)}$ `Condition:` $\nu > 0, s > 0$ `Range:` $(0, \infty)$ `Parameters:` ν: degrees of freedom, s^2: scale	

Moments		Program commands	
`mean:`	$\frac{\nu}{\nu-2}s^2 \;\; (\nu > 2)$	`R:`	`dchisq(nu*s^2/theta,nu)nu* s^2/theta^2`
`mode:`	$\frac{\nu}{\nu+2}s^2$	`WB/JAGS:`	`theta <- nu*s^2/itheta; itheta ~ dchisqr(nu)`
`variance:`	$\frac{2\nu^2}{(\nu-2)^2(\nu-4)}s^4 \;\; (\nu > 4)$	`SAS:`	`theta ~ sichisq(nu,s)`

Note:
Scaled inverse chi-squared is a special case of the inverse gamma distribution:
$\mathrm{Inv}-\chi^2(\nu, s^2) = \mathrm{IG}(\alpha = \nu/2, \beta = \nu\, s^2/2)$ (rate).

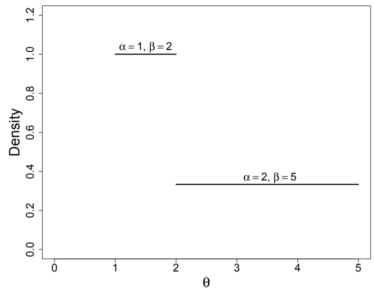
Table 18.15 Weibull distribution: Weibull(α, β)

Model	Examples
$p(\theta) =$ $\frac{\alpha}{\beta} \left(\frac{\theta}{\beta}\right)^{(\alpha-1)} \exp\left(-(\theta/\beta)^{\alpha}\right)$ `Condition:` $\alpha > 0, \beta > 0$ `Range:` $\alpha = 1 : [0, \infty)$ otherwise : $(0, \infty)$ `Parameters:` α: shape, β: scale	

Moments		Program commands	
`mean:`	$\beta\Gamma(1 + 1/\alpha)$	`R:`	`dweibull(theta,alpha,beta)`
`mode:`	$\beta(1 - 1/\alpha)^{1/\alpha}$ (if $\alpha > 1$)	`WB/JAGS:`	`theta ~ dweib(alpha,ibeta)`
`variance:`	$\beta^2 \left[\Gamma(1 + 2/\alpha) - \Gamma^2(1 + 2/\alpha)\right]$	`SAS:`	`theta ~ weibull(0,alpha,beta)`

Note:
SAS: more general Weibull distribution with additional $\mu > 0$ = lower limit of range: 'weibull(mu,alpha,beta)', with θ/β in Weibull distribution replaced by $(\theta - \mu)/\beta$.

Table 18.16 Uniform distribution: $U(\alpha, \beta)$

Model	Examples
$p(\theta) = \frac{1}{\beta-\alpha}$ `Condition:` $\beta > \alpha$ `Range:` $[\alpha, \beta]$ `Parameters:` α: lower limit, β: upper limit	

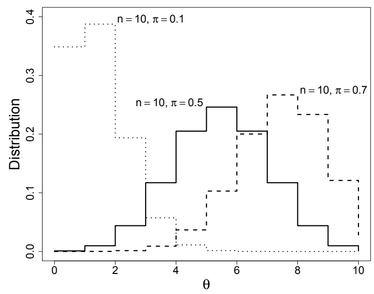
Moments	Program commands
`mean:` $\frac{\alpha+\beta}{2}$	`R: dunif(theta,alpha,beta)`
`mode:` -	`WB/JAGS: theta ~ dunif(alpha,beta)`
`variance:` $\frac{(\beta-\alpha)^2}{12}$	`SAS: theta ~ uniform(alpha,beta)`

Note:
Uniform is a special case of the beta distribution: U(0,1) = Beta(1,1).

18.3 Discrete univariate distributions

Table 18.17 Binomial distribution: $\text{Bin}(n, \pi)$

Model	Examples

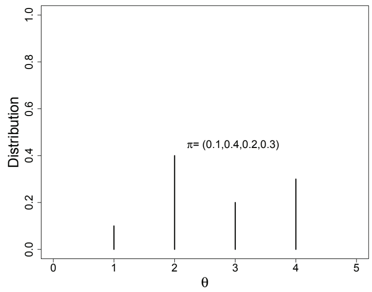
$p(\theta) = \binom{n}{\theta}\pi^{\theta}(1-\pi)^{n-\theta}$
 	
`Conditions:`
$n = 0, 1, 2, \ldots$
$0 \leq \pi \leq 1$
`Range:` $\theta \in \{0, 1, \ldots, n\}$
`Parameters:`
n: sample size
π: probability of success
	

Moments		Program commands	
`mean:`	$n\pi$	`R:`	`dbinom(theta,n,pi)`
`mode:`	$\lfloor(n+1)\pi\rfloor$	`WB/JAGS:`	`theta ~ dbin(pi,n)`
`variance:`	$n\pi(1-\pi)$	`SAS:`	`theta ~ binomial(n,pi)`

Note:
$\lfloor(n+1)\pi\rfloor$ = greatest integer in value.
Special case: *Bernoulli distribution* = Bern(π) = Bin(1,π).
Commands Bernoulli dist: R: dbern(pi), WB: dbern(pi), SAS: binary(pi).

Table 18.18 Categorical distribution: Cat(π)

Model	Examples

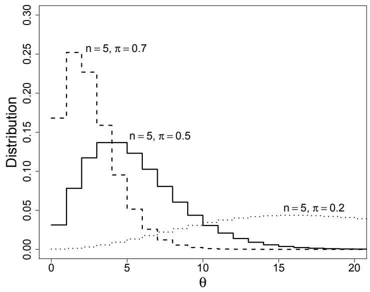
$p(\theta) = \pi_\theta$
 	
`Conditions:`
$\pi_\theta > 0$, $\sum \pi_\theta = 1$
`Range:` $\theta \in \{0, 1, \ldots, n\}$
`Parameters:`
π_θ: class probabilities
	

Moments		Program commands	
`mean:`	–	`R:`	`dmultinom(theta,size=1,pi)`
`mode:`	–	`WB/JAGS:`	`theta ~ dcat(pi)`
`variance:`	–	`SAS:`	`theta ~ multinom(pi)`

Note:
Categorical is a special case of the multinomial distribution with $n = 1$.
JAGS only requires that π_θ is positive, they must not add up to 1.

Table 18.19 Negative binomial distribution: NB(n, π)

Model	Examples

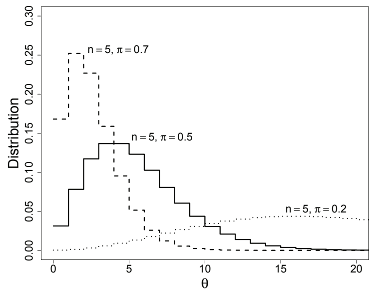
$p(\theta) = \binom{\theta+n-1}{\theta}\pi^n(1-\pi)^\theta$
 	
`Conditions:`
$n = 0, 1, 2, \ldots$
$0 \leq \pi \leq 1$
`Range:` $\theta \in \{0, 1, \ldots, n\}$
`Parameters:`
n: number of successes
π: probability of success
	

Moments		Program commands	
`mean:`	round $\left(\frac{n(1-\pi)}{\pi}\right)$	`R:`	`dnegbin(theta,n,pi)`
`mode:`	round $\left(\frac{(n-1)(1-\pi)}{\pi}\right)$	`WB/JAGS:`	`theta ~ dnegbin(pi,n)`
`variance:`	$\frac{n(1-\pi)}{\pi^2}$	`SAS:`	`theta ~ negbin(n,pi)`

Note:
Special case: *Geometric distribution*: geom(p)=NB(1,π).

We have seen alternative parametrizations of the negative binomial distribution in the book:
Expression (3.15): $\pi = \beta/(1+\beta)$ and $n = \alpha$ a real value.
Expression (6.19): $\pi = 1/(1+\kappa\lambda)$ and $n = 1/\kappa$ a real value.

Table 18.20 Poisson distribution: Poisson(λ)

Model	Examples
$p(\theta) = \frac{\lambda^\theta}{\theta!}\exp(-\lambda)$ `Condition:` $\lambda \geq 0$ `Range:` $\theta \in \{0, 1, \ldots, n\}$ `Parameters:` λ: average number of counts	

Moments		Program commands	
`mean:`	λ	`R:`	`dpois(theta,lambda)`
`mode:`	round(λ)	`WB/JAGS:`	`theta ~ dpois(lambda)`
`variance:`	λ	`SAS:`	`theta ~ poisson(lambda)`

18.4 Multivariate distributions

Table 18.21 Dirichlet distribution: Dirichlet($\boldsymbol{\alpha}$)

Model	Program commands
$p(\boldsymbol{\theta}) = \frac{\Gamma(\sum_{j=1}^{J}\alpha_j)}{\prod_{j=1}^{J}\Gamma(\alpha_j)} \prod_{j=1}^{J}\theta_j^{\alpha_j-1}$	R: `ddirichlet(vtheta,valpha)`
`Condition:` $\alpha_j > 0$ $(j = 1, \ldots, J)$	WB/JAGS: `vtheta[] ~ ddirich(valpha[])`
`Range:` $\theta_j > 0,\ \sum_{j=1}^{J}\theta_j = 1$	SAS: `vtheta ~ dirich(valpha)`
`Parameters:` α_j: probabilities	

Moments	
`mean:` $\alpha_j / \sum_{j=1}^{J}\alpha_j$	`mode:` $(\alpha_j - 1)/\sum_{j=1}^{J}\alpha_j$
`variances:` $\frac{\alpha_j(\sum_m \alpha_m - \alpha_j)}{(\sum_m \alpha_m)^2(\sum_m \alpha_m - \alpha_j)}$	`covariances:` $-\frac{\alpha_j\alpha_k}{(\sum_m \alpha_m)^2(\sum_m \alpha_m + 1)}$

Table 18.22 Inverse Wishart distribution: IW(R, k)

Model	Program commands
$p(\Sigma) =$ $c\,\det(\mathrm{R})^{k/2}\det(\Sigma)^{-(k+p+1)/2}\exp\left[-\frac{1}{2}\mathrm{tr}\left(\Sigma^{-1}\mathrm{R}\right)\right]$ with $c^{-1} = 2^{kp/2}\pi^{p(p-1)/4}\prod_{j=1}^{p}\Gamma\left(\frac{k+1-j}{2}\right)$	R: `diwish(Sigma, k, Rinv)` (Rinv=R^{-1} in MCMCpack)
`Condition:` R pos definite, $k > 0$	WB/JAGS: -
`Range:` Σ symmetric	SAS: `Sigma ~ iwishart(k,R)`
`Parameters:` k: degrees of freedom & R: inverse of cov matrix	

Moments	
`mean:` $\mathrm{R}/(k-p-1)$ (if $k > p+1$)	`mode:` $\mathrm{R}/(k+p+1)$

Table 18.23 Multinomial distribution: Mult$(k, \boldsymbol{\pi})$

Model	Program commands
$p(\boldsymbol{\theta}) = \frac{n!}{\theta_1!\theta_2!\ldots\theta_k!} \prod_{j=1}^{k} \pi_j^{\theta_j}$,	R: `dmultinom(theta,size=n,prob=vpi)`
`Condition:` $\sum_{j=1}^{k} \pi_j = 1$	WB/JAGS: `vtheta[] ~ dmulti(pi[],n)`
`Range:` $\theta_j \in \{0, \ldots, n\}$ with $\sum_{j=1}^{k} \theta_j = n$	SAS: `vtheta ~ multinom(vpi)`
`Parameters:` π_j: probabilities	
Moments	
`mean:` $n \cdot \boldsymbol{\pi}$	
`variances:` $n\pi_j(1 - \pi_j)$	`covariances:` $-n\pi_j\pi_k$

Table 18.24 Multivariate Normal distribution: $\mathrm{N}_p(\boldsymbol{\mu}, \Sigma)$

Model	Program commands
$p(\boldsymbol{\theta}) = \frac{1}{(2\pi)^{p/2}\det(\Sigma)^{1/2}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\mu})^T \Sigma^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu})\right]$	R: `mvrnorm(vtheta,vmu,S)` (MASS)
`Condition:` Σ positive definite	WB/JAGS: `vtheta[] ~ dmnorm(vmu[],S[,])`
`Range:` $-\infty < \theta_j < \infty$	SAS: `vtheta ~ mvn(vmu,S)`
`Parameters:` $\boldsymbol{\mu}$: mean vector & Σ: $p \times p$ covariance matrix	
Moments	
`mean:` $\boldsymbol{\mu}$	`mode:` $\boldsymbol{\mu}$
`variances:` Σ_{jj}	`covariances:` Σ_{jk}

Table 18.25 Multivariate Student's t-distribution: $\mathrm{T}_\nu(\boldsymbol{\mu}, \Sigma)$

Model	Program commands
$p(\boldsymbol{\theta}) =$ $c\,\det(\Sigma)^{-1/2}\left[1+\frac{1}{\nu}(\boldsymbol{\theta}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu})\right]^{-(\nu+p)/2}$ with $c = \frac{\Gamma[(\nu+p)/2]}{\Gamma(\nu/2)(k\pi)^{p/2}}$	`R: -`
`Condition:` Σ positive definite, $\nu > 0$	`WB/JAGS: vtheta[] ~ dmt(vmu[],S[,],nu)`
`Range:` $-\infty < \theta_j < \infty$	`SAS: -`
`Parameters:` $\boldsymbol{\mu}$: mean vector Σ: $p \times p$ covariance matrix ν: degrees of freedom	

Moments			
`mean:`	$\boldsymbol{\mu}$ (if $\nu > 1$)	`mode:`	$\boldsymbol{\mu}$
`variances:`	$\frac{\nu}{\nu-2}\Sigma_{jj}$ (if $\nu > 2$)	`covariances:`	$\frac{\nu}{\nu-2}\Sigma_{jk}$ (if $\nu > 2$)

Table 18.26 Wishart distribution: Wishart(R, k)

Model	Program commands
$p(\Sigma) =$ $c\,\det(\mathrm{R})^{-k/2}\det(\Sigma)^{(k-p-1)/2}\exp\left[-\frac{1}{2}\mathrm{tr}\,(\mathrm{R}^{-1}\Sigma)\right]$ with $c^{-1} = 2^{kp/2}\pi^{p(p-1)/4}\prod_{j=1}^{p}\Gamma\left(\frac{k+1-j}{2}\right)$	`R: dwish(Sigma, k, Rinv)` (Rinv=R^{-1} in MCMCpack)
`Condition:` R pos definite, $k > 0$	`WB/JAGS: Sigma[,] ~ dwish(R[,],k)`
`Range:` Σ symmetric	`SAS: -`
`Parameters:` k: degrees of freedom & R: covariance matrix	

Moments	
`mean:` $k\mathrm{R}$	`mode:` $(k-p-1)\mathrm{R}$ (if $k > p+1$)
`variances:` $\mathrm{var}(\Sigma_{ij}) = k(r_{ij}^2 + r_{ii}r_{jj})$	`covariances:` $\mathrm{cov}(\Sigma_{ij}, \Sigma_{kl}) = k(r_{ik}r_{jl} + r_{il}r_{jk})$

Note:
WinBUGS uses an alternative expression of the Wishart distribution: In the above expression R is replaced by R^{-1} and hence represents a covariance matrix in WinBUGS.