Contents

1 Question 1

Binomial likelihood, the posterior with a conjugate beta prior is:

$$p(\theta|y) = \frac{1}{B(\overline{\alpha}, \overline{\beta})} \theta^{\overline{\alpha}-1} (1-\theta)^{\overline{\beta}-1} \tag{1}$$

with

$$\begin{aligned} \overline{\alpha} &= \alpha_0 + y \\ \overline{\beta} &= \beta_0 + n - y \end{aligned} \tag{2}$$

The beta prior can be specified as:

$$\equiv \text{ binomial experiment with } (\alpha_0 - 1) \text{ successes in } (\alpha_0 + \beta_0 - 2) \tag{3}$$

2 Question 2

Check PPD for binomial likelihood on p. 151. We should take into account sampling variability of $\hat{\theta}$

3 Question 3

Contour probability: posterior evidence of H_0 with HPD interval. Defined as:

$$P\left[p(\theta|\boldsymbol{y}) > p\left(\theta_0|\boldsymbol{y}\right)\right] \equiv (1 - p_B) \tag{4}$$

p_B is computed from the smallest HPD interval containing θ_0.

Beta (α_0, β_0) prior is equivalent to a binomial experiment with $\alpha_0 - 1$ successes in $(\alpha_0 + \beta_0 - 2)$ experiments.

The non-informative beta prior has $\alpha_0 = 1, \beta_0 = 1$ and is equal the uniform prior on $[0, 1]$.

4 Question 4

Popular priors for BGLIM are normal proper priors with large variance. Gelman et al. however suggest Cauchy density with center 0 and scale parameter 2.5 for standardized continuous covariates.