

# Seminar Nr.1, Euler's Functions; Counting, Outcomes, Events

## Theory Review

**Euler's Gamma Function:**  $\Gamma : (0, \infty) \rightarrow (0, \infty), \Gamma(a) = \int_0^{\infty} x^{a-1} e^{-x} dx.$

1.  $\Gamma(1) = 1;$
2.  $\Gamma(a+1) = a\Gamma(a), \forall a > 0;$
3.  $\Gamma(n+1) = n!, \forall n \in \mathbb{N};$
4.  $\Gamma\left(\frac{1}{2}\right) = \sqrt{2} \int_0^{\infty} e^{-\frac{t^2}{2}} dt = \int_{\mathbb{R}} e^{-t^2} dt = \sqrt{\pi}.$

**Euler's Beta Function:**  $\beta : (0, \infty) \times (0, \infty) \rightarrow (0, \infty), \beta(a, b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx.$

1.  $\beta(a, 1) = \frac{1}{a}, \forall a > 0;$
2.  $\beta(a, b) = \beta(b, a), \forall a, b > 0;$
3.  $\beta(a, b) = \frac{a-1}{b} \beta(a-1, b+1), \forall a > 1, b > 0;$
4.  $\beta(a, b) = \frac{b-1}{a+b-1} \beta(a, b-1) = \frac{a-1}{a+b-1} \beta(a-1, b), \forall a > 1, b > 1;$
5.  $\beta(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \forall a > 0, b > 0.$

**Arrangements:**  $A_n^k = \frac{n!}{(n-k)!};$

**Permutations:**  $P_n = A_n^n = n!;$

**Combinations:**  $C_n^k = \frac{A_n^k}{P_k} = \frac{n!}{k!(n-k)!}.$

**De Morgan's laws:**

$$\overline{\bigcup_{i \in I} A_i} = \bigcap_{i \in I} \overline{A_i} \quad \text{and} \quad \overline{\bigcap_{i \in I} A_i} = \bigcup_{i \in I} \overline{A_i}.$$

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1. In how many ways can 10 students be seated in a classroom with

- a) 15 chairs?
- b) 10 chairs?

2. Find the number of possible outcomes for the following events:

- a) three dice are rolled;
- b) two letters and three digits are randomly selected.

3. A firm offers a choice of 10 free software packages to buyers of their new home computer. There are 25 packages available, five of which are computer games, and three of which are anti-virus programs.

- a) How many selections are possible?
- b) How many selections are possible, if exactly three computer games are selected?
- c) How many selections are possible, if exactly three computer games and exactly two anti-virus programs are selected?

4. A person buys  $n$  lottery tickets. For  $i = \overline{1, n}$ , let  $A_i$  denote the event: the  $i^{th}$  ticket is a winning one. Express the following events in terms of  $A_1, \dots, A_n$ .

- a) A: all tickets are winning;

- b) B: all tickets are losing;
- c) C: at least one is winning;
- d) D: exactly one is winning;
- e) E: exactly two are winning;
- f) F: at least two are winning;
- g) G: at most two are winning.

5. Three shooters aim at a target. For  $i = \overline{1, 3}$ , let  $A_i$  denote the event: the  $i^{th}$  shooter hits the target. Express the following events in terms of  $A_1, A_2$  and  $A_3$ .

- a) A: the target is hit;
- b) B: the target is not hit;
- c) C: the target is hit exactly three times;
- d) D: the target is hit exactly once;
- e) E: the target is hit exactly twice.