Verificare Formala

Aplicatii ale SMT solving

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Bazat pe suportul de curs: *Satisfiability Checking (Erika Ábrahám), RTWH Aachen*
Bazat pe cartile: (1) The calculus of computation - Z. Manna, A. Bradley; (2) Decision Procedures - An algorithmic Point of View - D. Kroening, O. Strichman

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Outline

Contents

Optimization

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More details at
`https://theory.stanford.edu/~nikolaj/programmingz3.html`.

Optimization (cont'd)

In Z3, the default optimization is lexicographic.
One can set up other types of optimization by adding (SMT-LIB version, but there exists also the corresponding commands for Python API):

- (set-option :opt.priority pareto)
- (set-option :opt.priority box)

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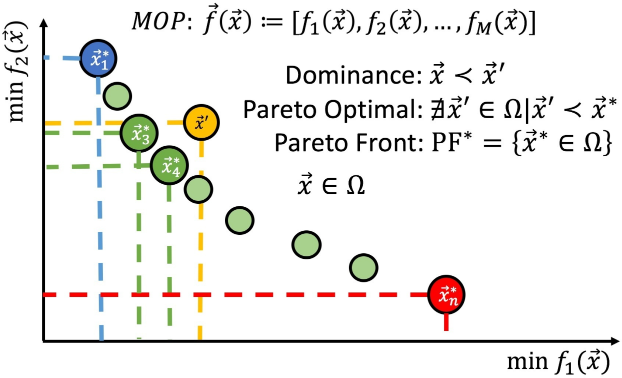
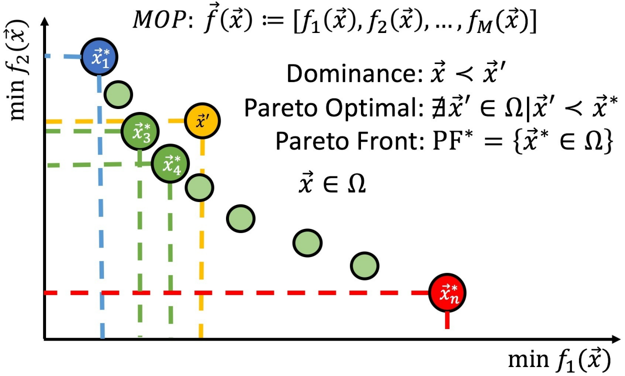
A set of feasible solutions are Pareto efficient/optimal when no further Pareto improvements can be made.

Pareto Optimality

Solutions along the line are all non-dominated solutions.



Solutions along the line are all non-dominated solutions.
Dominated solutions are inside the line as there is another solution on the line with at least one objective that is better.



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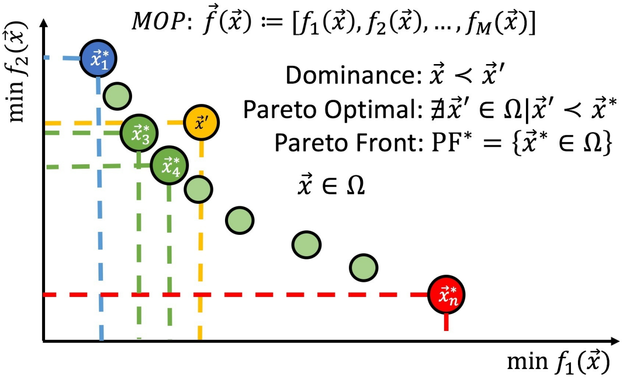

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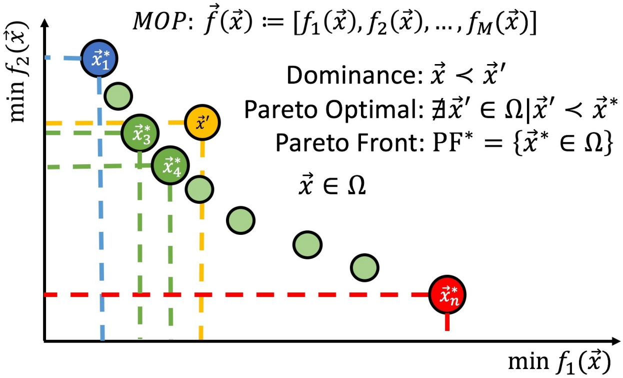

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Virtual Machine Placement Problem

Assume that we have three virtual machines (VMs) which require 100, 50 and 15 GB hard disk respectively. There are three servers with capabilities 100, 75 and 200 GB in that order. Find out a way to place VMs into servers in order to:

1. Minimize the operation cost (the servers have fixed daily costs 10, 5 and 20 USD respectively.)
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Solution. We first formalize the problem. Then we can translate it into the SMT-LIB format and use an SMT solver to find a solution.
Let x_{ij} denote that VM i is placed on the server j and y_j denote that server j is in use.
We need to express the followings.

Virtual Machine Placement Problem (cont'd)

1. Implicit constraints

2. Explicit constraints

3. Optimization functions

Virtual Machine Placement Problem (cont'd)

1. Implicit constraints
   - A VM is on exactly one server:

$$x_{i1} + x_{i2} + x_{i3} = 1, \quad \forall i, j = \overline{1,3}$$

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1. Implicit constraints
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   - A used server has at least a VM on it:

$$(x_{1j} = 1) \vee ... \vee (x_{3j} = 1) \Rightarrow (y_j = 1), j = \overline{1,3}$$

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2. Explicit constraints
   - Capacity constraints:
     $100x_{11} + 50x_{21} + 15x_{31} \leq 100y_1$
     $100x_{12} + 50x_{22} + 15x_{32} \leq 75y_2$
     $100x_{13} + 50x_{23} + 15x_{33} \leq 200y_3$
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   - $y_1 + y_2 + y_3$

Virtual Machine Placement Problem (cont'd)

There are various ways of encoding the variables of the problem, for example:

(Variant 1) as integers

(Variant 2) as real

(Variant 3) as bool

(Variant 4) using `assert-soft` constraints

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Variant 1. We declare each variable as integer, e.g.:

```
(declare-const x11 Int)
```

We also need to ensure that variables are 0/1, e.g.:

```
(assert (and (>= x11 0) (>= x12 0) (>= x13 0) (>= x21 0)...
(assert (and (<= y1 1) (<= y2 1) (<= y3 1)))
```

Another variant for encoding the 0/1 integers is:

```
(assert (or (>= x11 0) (<= x11 1)
(assert (or (>= x12 0) (<= x12 1)...
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Variant 1. Constraint of type 2 can be encoded in 2 ways. For example:

```
(assert (and (>= y1 x11) (>= y1 x21) (>= y1 x31)))
...
```

Or as:

```
(assert (implies (= y1 1) (or (= x11 1) (= x21 1) (= x31 1) ))
...
```

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There are various ways of encoding the variables of the problem, for example:

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Variant 2. We declare each variable as real. The constraints should be the same as for the integer encoding.

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Variant 3. We declare each variable as bool. The capability constraints require only integer/real variables, so we need to transform the bool variables into integer/real. This can be done by declaring a function as follows:

```
(define-fun bool_to_int ((b Bool)) Int (ite b 1 0) )
```

and cast the bool variables to int/real, e.g.

```
(assert (<= (+ (* 100 (bool_to_int x11)) ...  )  (...)))
```

Virtual Machine Placement Problem (cont'd)

There are various ways of encoding the variables of the problem, for example:

(Variant 1) as integers

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(Variant 4) using `assert-soft` constraints

Variant 4. Use `assert-soft` constraints (soft constraints) (see `https://rise4fun.com/Z3/tutorialcontent/optimization`). For our problem, soft constraints can be used to encode the optimization goals:

`(assert-soft (not y1) :id num_servers)` ...

`(assert-soft (not y1) :id costs :weight 10)` ...

The `assert-soft` command represents MaxSMT (maximize the number of constraints which can be satisfied) which tries to maximize the weighted sum of boolean expressions belonged to the same `id`. Since we are doing minimization, negation is needed to take advantage of MaxSMT support.