Reflection Transformation for 3D Mirrors in Computer Graphics

A B. Tech Project Report Submitted
in Partial Fulfillment of the Requirements
for the Degree of

Bachelor of Technology

by

Rajendhar Reddy Nalabolu
(150101040)

Aman Agarwal
(150101005)

under the guidance of

Dr Pinaki Mitra

to the

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
GUWAHATI - 781039, ASSAM

CERTIFICATE

*This is to certify that the work contained in this thesis entitled "**Reflection Transformation for 3D Mirrors in Computer Graphics**" is a bonafide work of **Rajendhar Reddy Nalabolu (Roll No. 150101040)**, **Aman Agarwal(Roll No. 150101005)**, carried out in the Department of Computer Science and Engineering, Indian Institute of Technology Guwahati under my supervision and that it has not been submitted elsewhere for a degree.*

Supervisor: **Dr Pinaki Mitra**

Assistant Professor,

November, 2018 — Department of Computer Science & Engineering,

Guwahati. — Indian Institute of Technology Guwahati, Assam.

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Chapter 1

Introduction

Computer graphics are pictures and films created on computer's screen using data. Usually, the term refers to computer-generated image data created with the help of specialized graphical hardware and software. It is a vast area of computer science which has recently developed and has lots of scope of work. It is the visual representations of data displayed on a monitor of a computer. Computer-aided design relies on computer graphics for modelling and visualizing objects etc. Transformation in the field of computer graphics is an instrumental tool to picture proper objects into 2 dimensional or 3 dimensional space. An object that is drawn in either 2D or 3D comprises of infinitely many parts of different shapes and sizes. Combining these parts would comprise of the object that is desired on the screen. Transformation helps in translating, resizing or rotating a particular shape in a particular manner. This is extremely important so that the pieces can allign perfectly and mix within each other to get the desired object. Transformation means changing some graphics into other by applying a set of rules. Transformations can be of various types such as translation, scaling, rotation, reflection, etc.

1.1 Organization of The Report

This chapter provides a background for the topics covered in this report.

We already introduced computer graphics and the significance of transformations in computer graphics. Further in this chapter we will describe our problem statement. In chapter 2 we discussed about the prior works on this topic and the papers and journals we went through to get an understanding of this project. Then in chapter 3 we propose an algorithm to get an image of an object through a plane, i.e. we consider a plane as a mirror and get the image of an object through that mirror. And finally we conclude with our future goals and references that helped us in making this project a reality.

1.2 Problem Statement

Prediction of the change in image of an object through a plane mirror when it is rotated by a small angle.

Chapter 2

Review of Prior Works

In this chapter we review 3D Rotational transformation about an arbitrary axis and image of a point with respect to arbitrary plane in computer graphics.

2.1 Review of 3D Rotational Transformation

Transformation equations for rotation by an angle θ about z-axis of point (x, y, z) is:

$x' = x\ cos\theta$ - $y\ sin\theta$

$y' = x\ sin\theta + y\ cos\theta$

$z' = z$

Matrix notation of the above transformation equations can be expressed as follows:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

General notation of the above matrix is $P' = R_z(\theta)$*P here $P' = (x', y', z')$ is the transformed point and P=(x, y, z) original point.

Similarly for rotation about x-axis we can replace z $\rightarrow$ x $\rightarrow$ y $\rightarrow$ z and we can get transformation matrix about x-axis, similarly for y-axis we can get transformation matrix. For rotation about arbitrary axis represented by direction vector $V = (v_x, v_y, v_z)$ where $v_x^2 + v_y^2 + v_z^2 = 1$. For transformation we align arbitrary axis with one of the co-ordinate axis we can do this by applying following steps:

i) We rotate vector V about x-axis by an angle α such that it can be aligned in xz-plane.

ii) Then we rotate about y-axis by an angle β such that it can be aligned along z-axis.

iii) Now we rotate point P about z-axis by an angle θ and finally we inverse steps i and ii. (here angles α, β can be obtained from vector V).

For small angle rotation we can replace $sin\theta$ with θ and $cos\theta$ with 1 as θ is very small in the final transformation matrix.

2.2 Review of image of point in 3D

Reflection is 2D is image with respect to axis where as in case of 3D it is image with respect to plane.Transformation matrix for reflection about $X - Y$ plane is:

$$T_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly we can get reflection matrix for $Y - Z$ and $Z - X$ planes.

For reflection about an arbitrary plane which passes through points $P_1(x_0, y_0, z_0), P_2(x_1, y_1, z_1)$ and $P_3(x_2, y_2, z_2)$,which are noncollinear. For image we will align arbitrary plane with one of planes $X - Y$,$Y - Z$,$Z - X$. We can do this by following steps:

i) First translate the given arbitrary plane such that it passes through origin. We can achieve this with the help of known point P_1.

ii) We perform rotational transformations such that vector normal to the plane coincide with one of the axis then the plane coincides with one of $X-Y$,$Y-Z$,$Z-X$ planes.

iii) Now we can reflect point through that plane and we can get image of that point.

iv) Lastly we perform inverse transformations of steps ii and i respectively in same order.

2.3 Review of Reflected Ray for Rotation of mirror

If we rotate plane mirror about an arbitrary axis by an angle θ then reflected ray gets deviated by an angle of 2θ this can be know from laws of reflection shown in figure 2.1 .

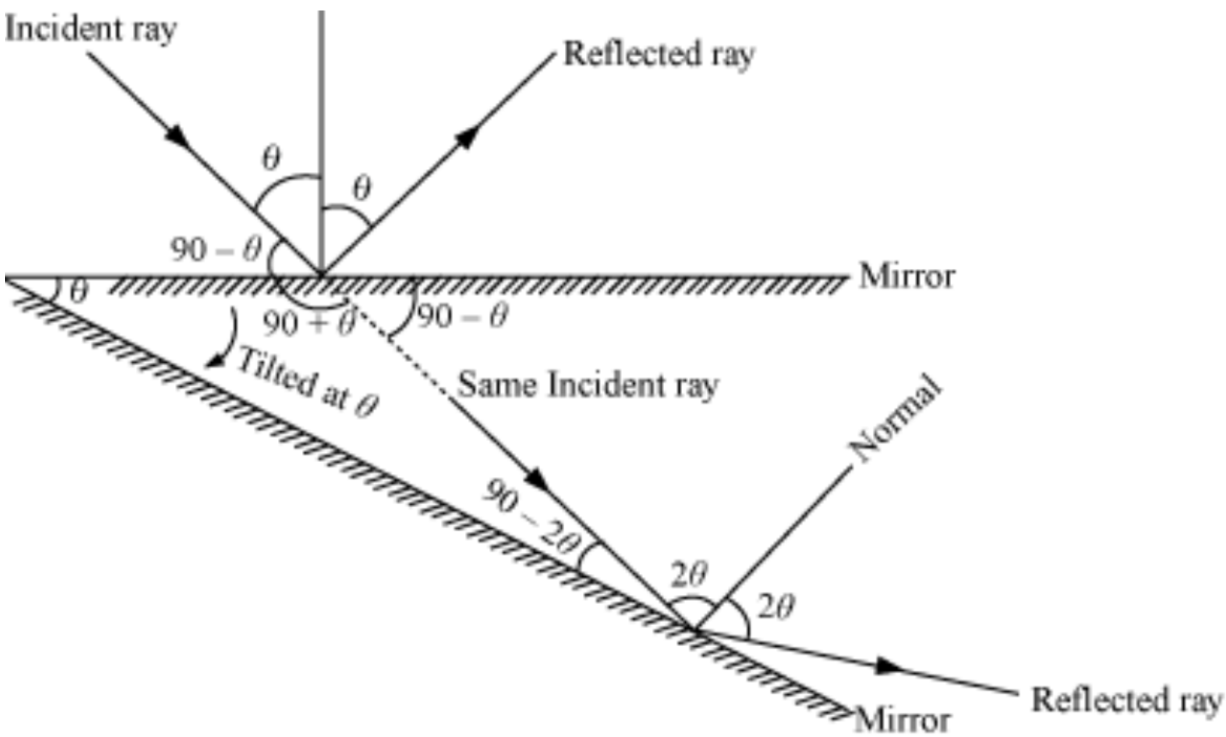

Fig. 2.1 Deviation in reflected ray.

2.4 Conclusion

In this chapter we explained about rotation and reflection matrices based on these We can use the above methods to find the image of the object through an arbitrary plane and the use rotational transformations to trace the image when an arbitrary plane is rotated as image rotates by twice the angle of rotation. We can also extend the same for small angles of rotation.

Chapter 3

Proposed Approach

Let us consider arbitrary mirror plane M which is denoted by distance d from view point to M and orientation which is angle it makes with x-axis α measured in anti clockwise direction and with z-axis ϕ measured in clockwise direction. We consider object as a line which passes through two arbitrary points.

3.1 Reflection Transformation

Consider view point as origin of the co ordinate system then the plane passes through point $P_0 = (0, 0,$d). Now to find an image of object through the plane M, we can use sequence of transformations which are explained in steps below:

i) First perform translation of the plane M such that it passes through origin this can be done by translating point P_0 so that it passes through origin

ii) Now perform rotation about x-axis by an angle α and then perform rotation about z-axis by an angle ϕ so that plane M coincides with $Z - X$ plane.

iii) Then reflect the points of object about this plane using reflection transformation.

iv) Finally perform inverse transformation of steps ii and i.

So the final transformation matrix denoted by T_{ref} can be written as follows:

$$T_{ref} = T(p_0)R_x(\alpha)R_z(\phi)T_{zx}R_z^{-1}(\phi)R_x^{-1}(\alpha)T^{-1}(p_0)$$

3.2 Mirror Rotation

From the laws of reflection we know that if the mirror is rotated by an angle θ then image is rotated by an angle 2θ using this by using rotational transformation about an arbitrary axis we can obtain new image with out actually finding the image from Reflection Transformation.

We already know that rotation about an arbitrary axis represented by direction vector $V = (v_x, v_y, v_z)$ where $v_x^2 + v_y^2 + v_z^2 = 1$ by an angle θ.

Rotational matrix for the above vector can be written as:

$$R(\theta) = R_x^{-1}(\alpha)R_y(\beta)_{-1}R_z(\theta)R_y(\beta)R_x(\alpha) =$$

$$\begin{bmatrix} v_x^2\text{+cos}\theta(1 - v_x^2) & v_x v_y(\text{1-cos}\theta)\text{-v}_z\text{sin}\theta & v_x v_z(\text{1-cos}\theta)\text{+v}_y\text{sin}\theta & 0 \\ v_x v_y(\text{1-cos}\theta)\text{+v}_z\text{sin}\theta & v_y^2\text{+cos}\theta(1 - v_y^2) & v_y v_z(\text{1-cos}\theta)\text{-v}_x\text{sin}\theta & 0 \\ v_x v_z(\text{1-cos}\theta)\text{-v}_y\text{sin}\theta & v_y v_z(\text{1-cos}\theta)\text{+v}_x\text{sin}\theta & v_z^2\text{+cos}\theta(1 - v_z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

But as the reflected ray is rotated twice the angle of rotation so the reflection can be obtained by replacing θ by 2θ.

$$R(2\theta) = R_x^{-1}(\alpha)R_y(\beta)_{-1}R_z(2\theta)R_y(\beta)R_x(\alpha) =$$

$$\begin{bmatrix} v_x^2\text{+cos}2\theta(1 - v_x^2) & v_x v_y(\text{1-cos}2\theta)\text{-v}_z\text{sin}2\theta & v_x v_z(\text{1-cos}2\theta)\text{+v}_y\text{sin}2\theta & 0 \\ v_x v_y(\text{1-cos}2\theta)\text{+v}_z\text{sin}2\theta & v_y^2\text{+cos}2\theta(1 - v_y^2) & v_y v_z(\text{1-cos}2\theta)\text{-v}_x\text{sin}2\theta & 0 \\ v_x v_z(\text{1-cos}2\theta)\text{-v}_y\text{sin}2\theta & v_y v_z(\text{1-cos}2\theta)\text{+v}_x\text{sin}2\theta & v_z^2\text{+cos}2\theta(1 - v_z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.3 For small angle Rotation

From the above transformation matrix for small angle $\theta \to 0$ then $cos\theta \to 1$ and $1-cos\theta \to 0$ also $sin\theta \to \theta$ and also $sin2\theta = 2sin\theta$, $cos2\theta = 2cos^2\theta - 1$ for small angle $sin2\theta \to 2\theta$ and

$cos2\theta \rightarrow 1$.

After applying these approximations of small angle the approximated rotational matrix becomes

$$R_{app}(\theta) = \begin{bmatrix} 1 & \text{-}v_z 2\theta & v_y 2\theta & 0 \\ v_z 2\theta & 1 & \text{-}v_x 2\theta & 0 \\ \text{-}v_y 2\theta & v_x 2\theta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.4 Conclusion

In this chapter we came across transformation matrix for rotation and reflection about an arbitrary axis and used laws of reflection to obtain the new image after rotation of mirror then applied approximation for small angle rotation to get final reflection transformation matrix.

Chapter 4

Results

From the transformation matrices obtained from chapter 3 we implemented section 3.1 to find image of a line through an arbitrary plane. We used midpoint line drawing algorithm to draw lines in computer graphics. We implemented this in Dev-C++ and output for image of line about the mirror is shown in figure 4.1.

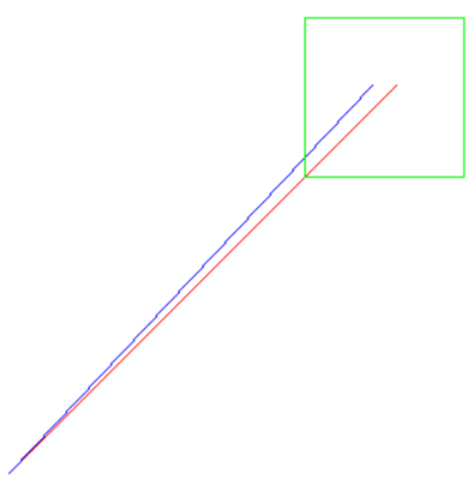

Fig. 4.1 Reflection of line about mirror.

In the figure 4.1 plane is drawn using green color, input line is denoted in red and output line is denoted in blue color respectivey.

Chapter 5

References

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