Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Answer - Let the probability of not meeting comm. be P(E).
calculate the z-score
Given : µ= 45 , time = 50 Minutes
Z-Score = (time – mean time)/std dev => (50-45)/8 = 0.625
probability from table = 0.7324
P(E) = 1 – 0.7324 = 0.2676 (B)

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer – (A) False
Explanation - most of the values(age) lies between –SD to +SD because it's normally distributed. Therefore more values must be less than 44(age).
(B) True.

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In [1]: import scipy.stats as stats

In [11]: x = 30
         mean = 38
         sd = 6
         #probability
         p = stats.norm.cdf(x= x, loc = mean, scale = sd)
         print(p)
         print(p*400)
         round(p*400)
         #probability is approx equal to 36.

         0.09121121972586788
         36.484487890347154

Out[11]: 36
```

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer – X1 and X2 both are identical and independent normal distributions. X1 + X2 results in $N(\mu1+\mu2, \sigma^2+ \sigma^2)$, prameters wont change in 2X1.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Answer - to get symmetry about mean = (1-0.99)/2 = 0.005

z-score is -2.57.

To find the a,b values = 20x(-2.57) +- 100. It gives (48.6,151.44) answer is D.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer –

(A) Add up the profits as they are normal dist.
    Annual_profit $\sim N(5+7, 3^2 + 4^2) \Rightarrow N(12, 5^2)$
    Rupee Range = [99008103.48, 980991896.52]
(B) 5th percentile is 143 million RS (approx.)
(C) Division 1 will have larger prob. For making losses.

```
In [11]: mean = 12
         std = 5
         p = 0.95
         #million
         mean = mean*(10**6)*45
         std = std*(10**6)*45
         stats.norm.interval(alpha = p, loc = mean, scale = std)
         stats.scoreatpercentile([99008103.48, 980991896.52],5)
         division1 = stats.norm.cdf(0,5,3)
         division2 = stats.norm.cdf(0,7,4)
         print(division1, division2)
         #div1 has more prob. of facing losses.

         0.0477903522728147 0.040059156863817086
```