

$$e \rightarrow 7, d \rightarrow 23$$

$$PK \rightarrow (e, n), EK \rightarrow (d, n)$$

$$\text{Message, } M = C^d \pmod n$$

$C \rightarrow \text{cipher}$

$$C = M^e \pmod n$$

$M \rightarrow \text{HIDE}$

7834

$$C \rightarrow 7^7 \div 55 \rightarrow 28$$

$$8^7 \div 55 \rightarrow 2$$

$$3^7 \div 55 \rightarrow 42$$

$$4^7 \div 55 \rightarrow 49$$

$$(28^{23} \pmod n)$$

$$(28^3 \times 28^5 \times 28^5 \times 28^5 \times 28^5) \pmod{55}$$

$$(7 \times 43 \times 43 \times 43 \times 43) \div 55 \rightarrow 7$$

Miller Rabin

Private key = p, q

Public = n

$$C = P^2 \pmod n$$

$$p \pmod 4 = 3$$

$$q \pmod 4 = 3$$

$$\left. \begin{array}{l} p \pmod 4 = 3 \\ q \pmod 4 = 3 \end{array} \right\} p, q \equiv 3 \pmod 4 = 3$$

\rightarrow don't proceed if doesn't satisfy

$$\therefore p = 11, q = 7$$

$$n = p \times q = 77$$

$$C = P^2 \pmod n$$

$$= 45^2 \pmod{77} = 23$$

Find a, b decryption

$$axp + b \times q = 1$$

3210

 $f(17) = \dots$
 $g(17) = \dots$

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 $h(2) =$ $m =$ $P_0 =$

$$h(P_0) = \frac{3210}{1} = 3210$$

$$h^2(P_0) = 3210$$

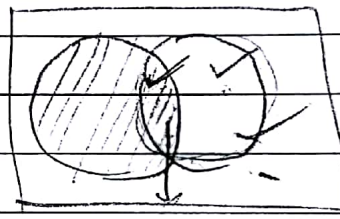
$$h^3(P_0) = h(3210) = \frac{3210}{2} = 1605$$

$$h^4(P_0) =$$

$$3210$$

$$n = 1000$$

3210



gcd(11, 7)

q	x_1	x_2	x	s_1	s_2	s	t_1	t_2	t
1	11	7	4	1	0	1	0	1	-1
1	7	4	-3	0	1	-1	1	-1	2
1	4	3	1	1	-1	2	-1	2	-3
3	3	1	0	-1	2	-7	2	-3	11
	1	0		2	-7		-3	11	
				\downarrow			\downarrow		
				a			b		

$$m_p = c^{p-1/4} \mod p = 1$$

$$m_q = c^{q-1/4} \mod q = 4$$

$$R_1 = (a \times m_q \times p + b \times m_p \times q) \mod n = 67$$

$$R_2 = n - R_1 = 10$$

$$R_3 = (a \times m_q \times p - b \times m_p \times q) \mod n = 32$$

$$R_4 = 45$$

Apply hash fn. to all 4 to get 4 hash values.

If any hash value is equal to the hash value send by the sender then that is the plain text.

Q. $p = 23, q = 7$

$p = 24$

$23 \bmod 4 = 3 \checkmark$

$7 \bmod 4 = 3 \checkmark$

$n = p \times q = 161$

$C = 24^2 \bmod 161 = 93$

Decryption

$\text{gcd}(23, 7)$

q	x_1	x_2	x	t_1	t_2	t	s_1	s_2	s
-----	-------	-------	-----	-------	-------	-----	-------	-------	-----

3	23	7	2	0	1	-3	1	0	1
---	----	---	---	---	---	----	---	---	---

3	7	2	1	1	-3	10	0	1	-3
---	---	---	---	---	----	----	---	---	----

2	2	1	0	-3	10	-23	1	-3	7
---	---	---	---	----	----	-----	---	----	---

\downarrow 0

\downarrow 10 -23

\downarrow -3 7

\downarrow 6

\downarrow a

$m_p = 93^{24/4} \bmod p = -1$

$m_q = 93^{24/4} \bmod q = 4$

$R_1 = (-3 \times 4 \times 23 + 10 \times 1 \times 7) \bmod 161 = 116$

$R_2 = 45$

$R_3 = 137$

$R_4 = 24$

Elgamal Cryptosystem

Alice \rightarrow Bob

Public (e_1, e_2, p)

Private (d)

key generation

$$p = 11$$

$$e_1 = 2 \in \mathbb{Z}_p^*$$

$$d = 5 \in \mathbb{Z}_p^* \quad \text{given} \quad 1 \leq d \leq p-2$$

$$\mathbb{Z}_p^* = \{1, 2, 3, \dots, 10\}$$

compute e_2

$$e_2 = e_1^d \bmod p$$

$$= 2^5 \bmod 11 = 10$$

Public $(2, 10, 11)$

Encryption

$$x = 4, \quad m = 7 \quad \text{given}$$

$$c_1 = e_1^x \bmod p$$

$$c_2 = (m \times e_2^x) \bmod p$$

} 2 ciphertexts are produced

$$c_1 = 2^4 \bmod 11 = 5$$

$$c_2 = 7 \times 10^4 \bmod 11 = 7$$

Decryption

$$m = c_2 (c_1^d)^{-1} \bmod p$$

$$= c_2 c_1^{-d} \bmod p$$

$$= c_2 c_1^{p-1-d} \bmod p$$

$$\{a^{-1} \bmod p = a^{p-2} \bmod p\}$$

$$m = 7 \times 5^{11-1-5} \bmod 11 = 7$$

Q. $p = 17, d = 5, e_1 = 6, m = 13, x = 10$

$$6 \in \mathbb{Z}_p^* \checkmark, 5 \in \mathbb{Z}_p^* \checkmark, 1 \leq 5 \leq 1$$

$$e_2 = 6^5 \bmod 17 = 7$$

$$c_1 = 6^{10} \bmod 17 = 15$$

$$c_2 = 13 \times 7^{10} \bmod 17 = 9$$

2

$$m = 9 \times 15^{17-1-5} \text{ mod } 17$$

$$= \underline{13}$$

Properties of Hash Function

- 1) Preimage Resistance - given a hash value ~~has~~ should not be able to get original image from hash value
- 2) Second Preimage Resistance - given a specific image M_1 we should not be able to find another message M_2 such that $\text{hash}(M_1) = \text{hash}(M_2)$
- 3) Collision Resistance - given any 2 msg. M_1 & M_2 we ~~can~~ ^{shouldn't} find a relation such that $\text{hash}(M_1) = \text{hash}(M_2)$

Random Oracle Model

- 1) when a new msg. of any length is given to an oracle it creates a msg. digest randomly having 0's & 1's
- 2) when a msg. is given & digest exists, oracle simply gives the msg digest
- 3) No formula was used to create the digest

SHA - 512

- the msg. should be less than 2^{128} bits otherwise this method fails
- msg. should be in hexadecimal Eg → 'a' = 37 = 61
- convert that into binary & calculate the length
- write the length in hexadecimal

$$\boxed{\text{Msg}} + \boxed{\text{Padding}} + \boxed{\text{Length}} \quad \left. \begin{array}{l} \uparrow \text{in hexa} \quad \uparrow 128 \text{ bits} \end{array} \right\} \text{multiple of } 1024$$

$$(-\text{Msg} - 128) \bmod 1024 = \text{Padding Bits}$$

↓
1 followed by 0's

16 × 64 bit

~~64 × 16~~ words is input

→ w_0, w_1, \dots, w_{15}

1 word = 16 bits

64 bit words = 1024 bits

But 80 rounds are there

Rot shift

→ circular right

so words are to be expanded

→ shift length left

$$\Rightarrow w_{16} = w_{15}$$

$$\text{Rot Shift}_{1-8-7}(x) \rightarrow \text{Circular}_{\text{right}}^{\text{shift}}(x) \oplus \text{Circular Shift}_8(x) \oplus \text{Shift left}_7(x)$$

$w_{i-16}, w_{i-15},$

- Take 1st 8 prime nos. & take square roots of them, convert the 16 decimal digits to hexa to get initial digest

- Take 1st 80 prime nos. & take cube roots of them, convert the 16 decimal digits to hexa to get other digest

Majority for creation of A + Conditional for creation of E

Majority

3 inputs of 4 bits is given

→ The 1st 3 bits (MSB to LSB) of all the 3 inputs are taken

→ if no. of 1's > 1 then majority bit is 1

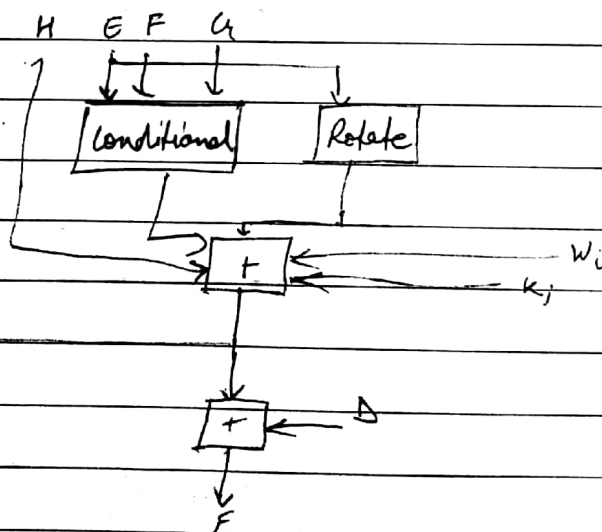
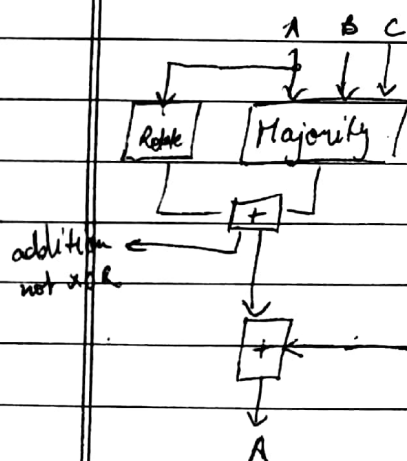
→ if ————— ≤ 1 ————— 0

→ then the 4th bit (LSB) of all 3 inputs are taken + calculated the majority

$$\text{Rotate}(A) = \text{Rotate}_{16}(A) \oplus \text{Rotate}_{24}(A) \oplus \text{Rotate}_{28}(A)$$

$\begin{matrix} \text{FOR} & & \text{FOR} & & \text{FOR} \\ \text{1011} & & \text{0100} & & \text{1100} \end{matrix}$

Conditional if value of E = 1 → conditional val. = val. of F
if ————— = 0 → ————— = ————— G_1



After 70th round the result generated is added to the initial Digest.