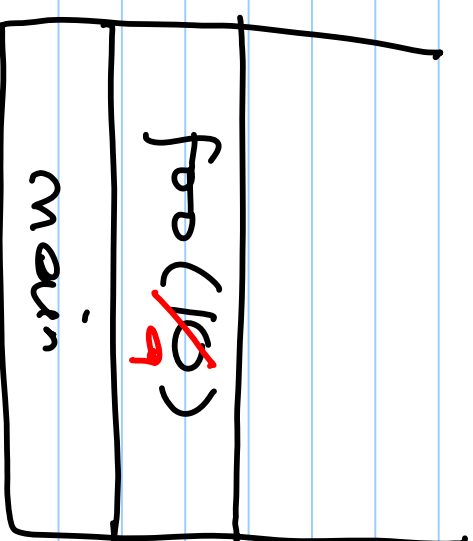
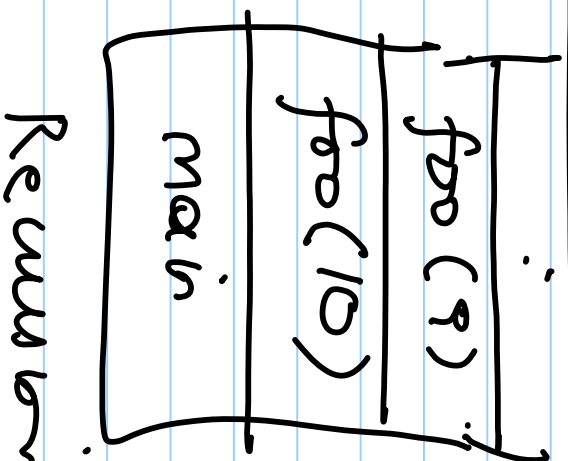


COL100 Lecture 2.8

Tail recursion: (last lecture, yay!)



```
int fact (int n)
{
    if (n == 0)
        return 1;
    else {
        return n * fact(n-1);
    }
}
```

Not tail recursive

// computes $accum * n!$ \rightarrow by computing $(accum * n) \times (n-1)!$
 $accum$ all

```
int fact_helper(int n, int accum)
{
    if (n == 0) {
        return accum;
    } else {
        return fact_helper(n-1, accum * n);
    }
}

int fact(int n)
{
    return fact_helper(n, 1);
}
```

10^{100}

```
int fact (int n)
```

```
{
```

```
    int accu = 1;
```

```
    while (n != 0)
```

recursive
call

```
        int new_n = n - 1;
        int new_accu = accu * n;
```

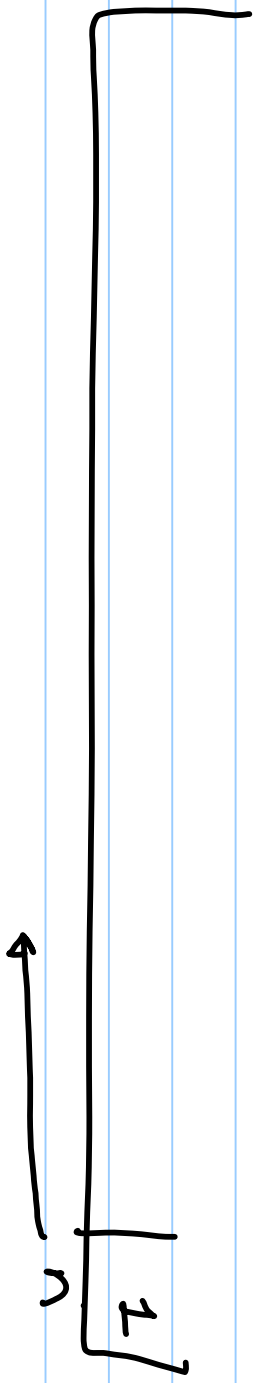
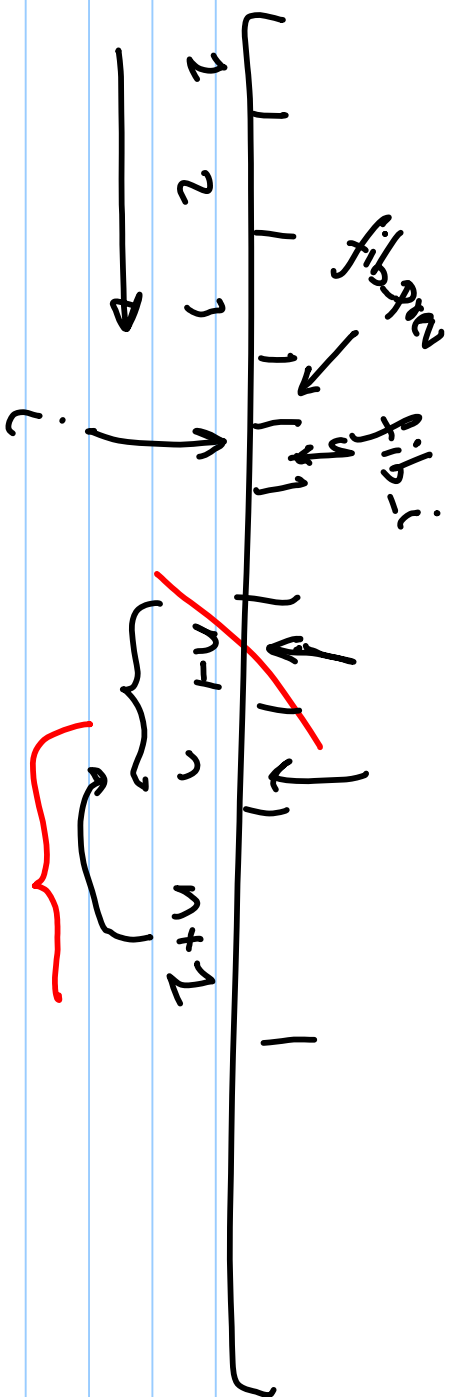
```
        n = new_n;
```

```
        accu = new_accu;
```

```
    }
```

base case: return accu;

```
}
```



```

    int fib_helper(int n, int i, int fib_i,
    {
        if (i == n) {
            return fib_i;
        } else {
            return fib_helper(n, i+1, fib_i +
            {
                fib_i);
            }
        }
    }
}

```

tail
recursion

```

    int fib(int n)
    {
        return fib_helper(n, 1, 1, 0);
    }
}

```

```
int fib(int n)
```

```
{  
    int i = 1;  
    int fib_i = 1;  
    int fib_prev = 0;
```

```
    while (i != n)
```

```
    {  
        int new_i = i + 1;  
        int new_fib_i = fib_i + fib_prev;  
        int new_fib_prev = fib_i;  
        i = new_i;  
        fib_i = new_fib_i;  
        fib_prev = new_fib_prev;  
    }  
    return fib_i;  
}
```

$i == N - n + 1$

`int`

`fib-helper (int n, int fib-i, int fib-prev)`

`if (n == 1) {`

`return fib-i;`

`} else {`

`return fib-helper (n-1, fib-i + fib-prev, fib-i);`

`}`

`int fib (int n)`

`{`

`return fib-helper (n, 1, 0);`

`}`


```
int fib (int n)
```

```
{  
    int fib-i = 1;
```

```
    int fib-prev = 0;
```

```
    while (n != 1)
```

```
{
```

```
    int new-n = n - 1;
```

```
    int
```

```
    int new-fib-i = fib-i + fib-prev;
```

```
    int
```

```
    new-fib-prev = fib-i;
```

```
    n = new-n;
```

```
    fib-i = new-fib-i;
```

```
}
```

```
    fib-prev = new-fib-prev;
```

```
    return fib-i;
```

```
}
```

Exhaustive Search

- Exploring every possible combination from a set of choices

General pseudo-code algorithm for exhaustive search

Choosing

- we iterate over "decisions". What are we iterating over here?
- What are the "choices" for each decision?

Exploring

- How can we "represent" the choice and the decision made so far

Un-Choosing

- How do we "un-modify" the choices in the previous step

Base case or Copy

Print All Binary (2):

0 0
0 1
1 0
1 1

Print All Binary (3):

0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1

decision?

characters
at its
position

choices? 0 or 1

repeated?

string So far

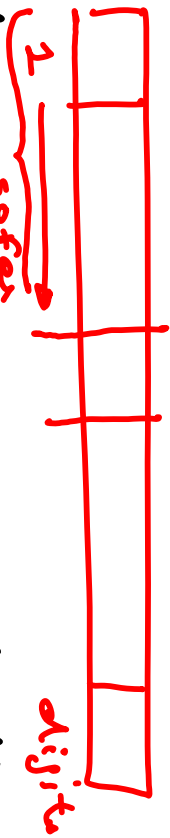
Unchoosing?

Create new string
for every call

Base case?

when all?

decisions have
been made
print string So far



void Print All Binary Helper (int digits, string sofar)

```
{
    if (digits == 0) {
```

```
        cout << sofar << endl;
```

```
    }
```

```
    else { Print All Binary Helper (digits-1, sofar + "0");
```

```
        Print All Binary Helper (digits-1, sofar + "1");
```

```
    }
```

```
}
```

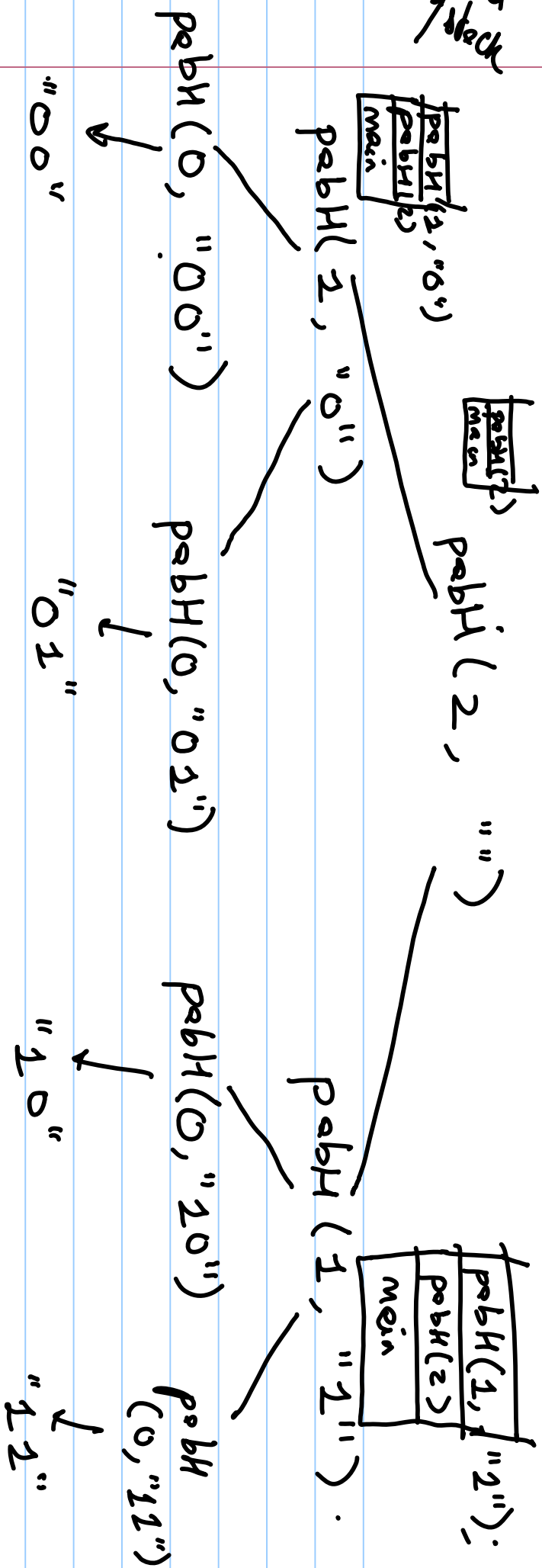
don't need sofar
be careful
pass-by-value

```
{
    void Print All Binary (int digits)
    {
        Print All Binary Helper (digits, "");
    }
}
```

empty

string

cell
↑
stack




call tree (tree of decisions)

post-
order

```
void print AllBinaryHelper ( int digits , string &sofar )  
{
```

```
    if ( digits == 0 ) {  
        cout << sofar << endl ;  
    } else {
```

```
        sofar = sofar + "0" ;  
        print AllBinaryHelper ( digits - 1 , sofar ) ;  
        sofar = sofar . substr ( 0 , sofar . length() - 1 ) ;  
        sofar = sofar + "1" ;  
        print AllBinaryHelper ( digits - 1 , sofar ) ;  
         sofar = sofar . substr ( 0 , sofar . length() - 1 ) ;
```

```
    }
```

