A Methodology for Measuring Market Inefficiency, and Its Relationship to the Value Spread *

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Abstract

Extending the work of (Boguth et al., 2023), we propose a method to measure the level of market inefficiency using unbiasedness regressions. We find that the market is not uniformly efficient and goes through periods of higher inefficiency reverting to efficiency in-line with Lo (2004)'s adaptive market hypothesis. We find that the value spread—the ratio of expensive to cheap stock P/B—is cointegrated with market inefficiency. By demonstrating that wider value spreads align with greater market inefficiency, we can empirically support Asness (2024)'s intuition that high spreads during events such as the dot-com bubble and Covid-19 could reflect mispricing rather than fundamental changes. This evidence strengthens the case that the value premium is still alive but has been masked by market inefficiency. We also find that the market's temporary inefficiencies have grown more persistent over time.

*I thank Professor Charles Martineau. Code and data supporting this analysis is available at: https://github.com/Aman-Rana-02/Irrational_Value_Spreads

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1 Introduction

Fama (1965)'s efficient market hypothesis propsed that markets incorporate all information rationally and instantaneously. While this is an extreme assumption, it is established that markets tend to some form of efficiency. In an efficient market, investors are compensated for taking on systematic risk, therefore, understanding the level of market efficiency can be used to asses whether market participants will be rewarded fairly for the risks they bear.

"Value" in classical factor investing refers to stocks with low price-to-book ratios as per Fama and French (1993) being considered undervalued. Being exposed to the Value factor means going long on stocks that exhibit an undervalued nature, and short on stocks that exhibit an overvalued nature in order to capture the value premium; compensation for the risk of holding undervalued companies. Its relationship with market efficiency is critical: if markets are efficient, an investor should be rewarded for risk. However, when inefficiencies arise, value stocks may become mispriced, with the market willing to pay more for already 'expensive' stocks, and less for already 'cheap' stocks. In these periods, investors may consider that the definition of value has changed, the subjective measure of 'expensive' and 'cheap' has shifted. We can define a value spread as the ratio of the value of expensive stocks to the value of cheap stocks. In this paper we show that generally, periods of inflated value spreads are correlated with an increase in inefficiency, therefore we suggest that abnormal value spreads may not be reflective of a fundamental change in the market, but rather a mispricing.

Existing methods for measuring market efficiency typically focus on testing whether stock prices follow a random walk (Fama, 1965) (Lim and Brooks, 2010). However these methods to not represent the variation of efficiency over time. A more dynamic approach is needed to reflect the market's changing behavior.

Boguth et al. (2023) uses unbiasedness regressions to examine price informativeness around FOMC meetings. This method visually shows us how prices adjust to information flow, in speed, and under or overreaction, using R^2s and βs over the unbiasedness regression window, which is useful in an approach for measuring efficiency.

Our estimand is Market Inefficiency, which we define as the relative level of the markets ability to price in future information. Building on this, we introduce the $Beta\_SSE$ estimate, which measures market efficiency at a point in time. By applying unbiasedness regressions within a rolling window, and constructing the $Beta\_SSE$ representation, we create a time series that tracks the relative level of market efficiency.

Our findings show that:

- The market is not uniformly efficient. Instead, it experiences cycles of inefficiency followed by reversion to efficiency.
- Over the last decade, inefficiency has been increasing, suggesting that the market has become less adept at incorporating available information into prices on longer time-scales.
- This rising inefficiency is correlated with the widening value spread, indicating a strong link between the weakness of the value premium and market inefficiency.

The rest of the paper is structured as follows: Section 2 describes the data used in this analysis. Section 3 outlines the methodology used to measure market inefficiency. Section 4 presents the results of our analysis. Section 5 concludes and discusses the implications of our findings. Section 6 provides additional details on the methodology.

2 Data

We use Python (Van Rossum and Drake, 2009) to analyze S&P500 price data from Yahoo Finance (Finance, 2024) and the Fama-French 3-factor data library (French, 2024). Yahoo finance provides daily price data of the S&P500 which we resample to monthly returns for the period 1950 - 2024. We get monthly log returns by taking the difference of the log of the adjusted close price of the S&P500 for each month.

2.1 Cumulative Log Returns

The unbiasedness regression require windows of cumulative returns. For any month t, we get every forward month's returns up to T months. By taking expanding window sums of the columns of log returns, we get a cumulative log returns matrix.

2.2 The Value Spread

The value spread is the ratio of the price-to-book of the portfolio of the portfolio of the most expensive 30% of stocks to the price-to-book of the portfolio of the cheapest 30% of stocks, as per Fama and French (1993). To construct this measure we use Kenneth French's data library (French, 2024). Using French's 3x2 sort on price-to-book and market equity, we get the monthly market value weighted average of the price-to-book of the portfolio of the 30% most expensive large-cap stocks and the portfolio of the 30% cheapest large-cap stocks. The value spread is the ratio of these two portfolios. It represents how much more expensive the expensive stocks are compared to the cheap stocks. Figure 1.

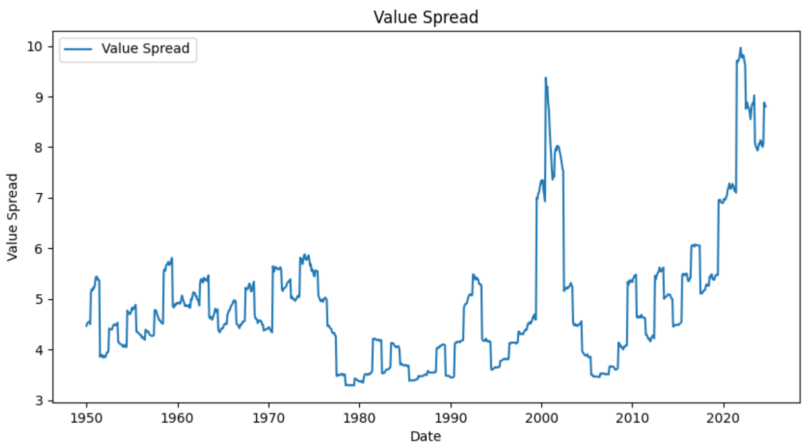

Figure 1: The value spread timeseries 1950 - 2024

3 Measurement of Market Efficiency

In this section, we will introduce the $Beta\_SSE$ score, which is a measure of market efficiency based on the unbiasedness regressions framework.

3.1 Unbiasedness Regressions

The unbiasedness regression is a simple OLS regression of the cumulative logarithmic return of the asset over a normalized window of time $\text{SP500}_{[0,T]}$, on a subset of the logarithmic cumulative returns, $\text{SP500}_{[0,t]}$, where $t \leq T$, and t is the number of months from the beginning of the window. [1]

$$\text{SP500}_{[0,T]} = \alpha_t + \beta_t \text{SP500}_{[0,t]} + \epsilon_t, \text{ where } 0 \leq t \leq T. \tag{1}$$

For an illustrative example, let $T = 36$, so that the window $[0, T]$ represents a 3-year period, and let us look at the returns of the SP500.

For $t = 1$, the regression is of the form:

$$\text{SP500}_{[0,36]} = \alpha + \beta \text{SP500}_{[0,1]} + \epsilon \tag{2}$$

From this regression we extract the coefficient β and the R^2 value. The regression is repeated for $t = 2, t = 3, ..., t = 36$, and the coefficients β as well as the R^2 are plotted against t.

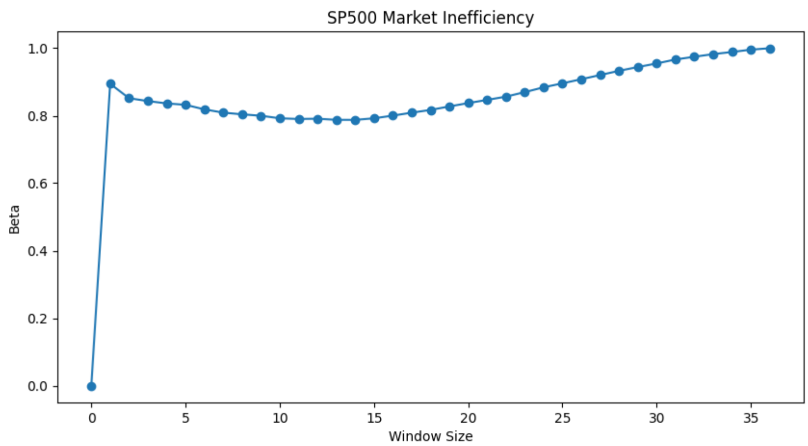

Figure 2: An example of an unbiasedness regression on the SP500 (1950 - 2024)

[1] $\text{SP500}_{[0,1]}$ is the matrix of returns of the SP500 from January 1950 to February 1950, February 1950 to March 1950, . . . , August 2024 to September 2024, September 2024 to October 2024.
$\text{SP500}_{[0,2]}$ would be the matrix of cumulative returns from January 1950 to March 1950, February 1950 to April 1950, . . . August 2024 to October 2024, and so on for the entire dataset.

$\beta_t = 1$ for all t when log prices move efficiently, following a random walk. When $\beta_t = 1$ the partial returns from 0 to t provide a forecast of the return from t to T (Mincer and Zarnowitz, 1969) that is as efficient as possible and do not need to be amplified or dampened. If $\beta_t < 1$, the partial return is dampened in the total returns, there fore there was a temporary component in prices which decays representing an "overreaction" (Barclay and Hendershott, 2003). Conversely, if $\beta_t > 1$, the partial return is amplified in the total return, suggesting the market has improperly processed the information and has underreacted.

In this paper we will be focusing on the β_t coefficient, but an explanation of the R^2 is provided in the Appendix for completeness. We now have the foundation to construct a score for market efficiency.

3.2 $Beta\_SSE$ Score

We have established the unbiasedness regression infrastructure, and that a $\beta_t < 1$ indicates overreaction, a $\beta_t > 1$ indicates underreaction, and $\beta_t = 1$ indicates efficiency. To construct a score that measures the level of market efficiency at a point in time, we have to create a representation of the β_t graph for a time period.

We define the $Beta\_SSE$ score as the sum of the squared differences between β_t and the horizontal line at $\beta = 1$, for all t in the window. Since a $\beta_t = 1$ indicates efficiency, the $Beta\_SSE$ score will be lower when the market is more efficient and larger when the market is less efficient, whether by under- or over-reaction.

To construct a timeseries of these scores, we run the unbiasedness regressions on a rolling window of five years. So $Beta\_SSE_t$ is constructed from the unbiasedness regressions of the SP500 where $t = 0$ in the regression are the month beginnings in the period $t - 5 \times 12$ to t. [2]

$$Beta\_SSE_t = \sum_{i=0}^{36} (\beta_{t,i} - 1)^2 \tag{3}$$

Where $\beta_{t,i}$ is the β coefficient from the unbiasedness regression from a window ending on the date t, at the ith depth of the unbiasedness window.

[2] Note that our unbiasedness regressions exted out to $t + 36$, so $Beta\_SSE_t$ score isn't tradeable at time t but at time $t + 36$. Consider the window [January 2000, January 2005], the observation starting at January 2005 requires the window of returns from January 2005 to January 2008 to make $\text{SP500}_{[0,36]}$.

4 Results

Using the $Beta\_SSE$ score, we construct a timeseries of the level of market efficiency as seen in Figure 3. We shade in grey the recession periods as defined by NBER based Recession Indicators (Federal Reserve Bank of St. Louis, 2024)

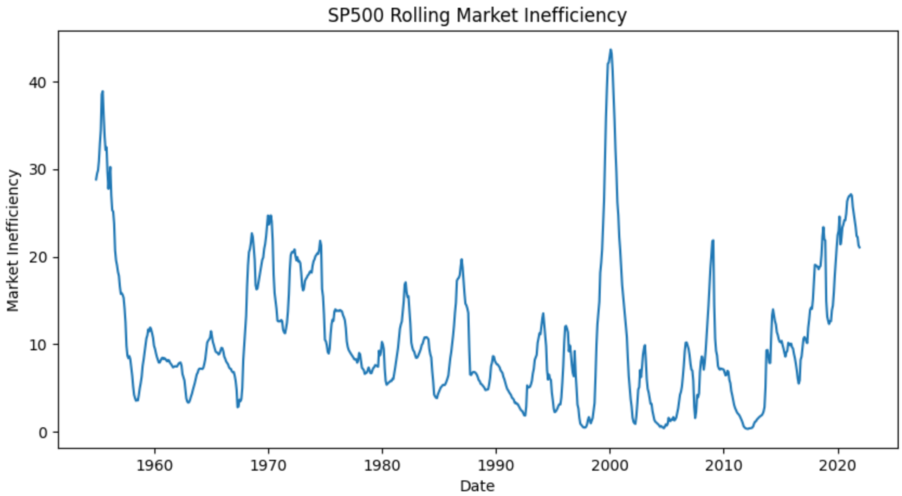

Figure 3: The $Beta\_SSE$ score timeseries of the SP500 (1950 - 2024) showing mean reversion and periods of inefficiency.

We see the market is inefficient around the dot-com bubble, the 2008 financial crisis, and the COVID-19 pandemic, which is in line with our expectations. We can visually asses that the market is not uniformly efficient. Instead, it goes through periods of higher inefficiency followed by reversion to efficiency. We also see that the market has been increasingly inefficient in the last decade, which is in line with Asness (2024).

Empirically we can test the mean reversion through an Augmented Dickey-Fuller test (Cheung and Lai, 1995) on the $Beta\_SSE$ timeseries, which is significant at the 0.1% level.

Figure 4 shows the value spread and the $Beta\_SSE$ score timeseries. We see visually that the two are cointegrated, which we test with the Engle-Granger two-step cointegration test, which is significant at the 0.1% level.

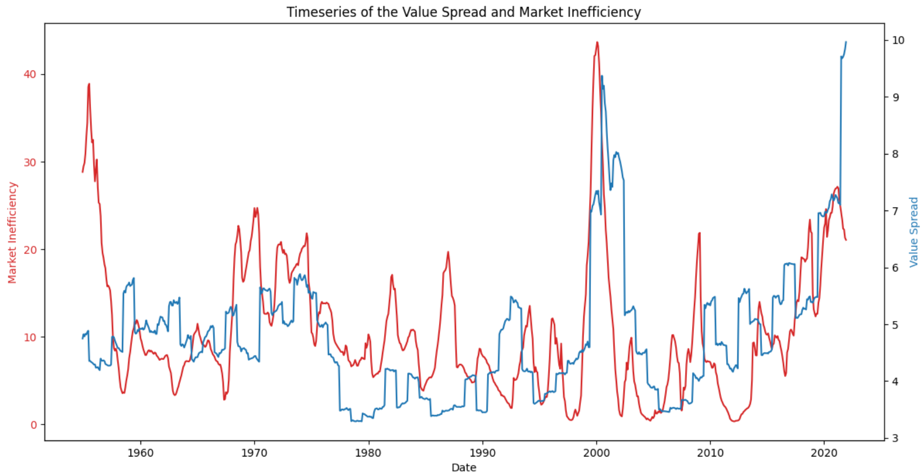

Figure 4: The value spread and market inefficiency move together, and have been increasing in tandem for the last decade.

5 Conclusion

In this paper, we introduced the $Beta\_SSE$ score, a measure of market inefficiency based on the unbiasedness regressions framework. We found that the market is not uniformly efficient, but instead goes through cycles of inefficiency followed by reversion to efficiency. Over the last decade, inefficiency has been increasing, suggesting that the market has become less adept at incorporating available information into prices on longer time-scales. This rising inefficiency is correlated with the widening value spread, indicating a strong link between the weakness of the value premium and market inefficiency. Causality between the two is not established, and can be investigated in a future work. However, this paper compliments the intuition presented in Asness (2024) that the value premium has been weakening over the last decade because of market inefficiency. A time series of market inefficiency is useful, since if market inefficiency can be forecasted it has implications for risk management. Portfolios looking to profit from the value premium may want to consider the prevailing level of market inefficiency when constructing their portfolios, adjusting their exposure during periods of inefficiency.

6 Appendix

6.1 Unbiasedness Regressions

In this Appendix we build on the intuition behind the unbiasedness regressions and the measurement of market inefficiency. We provide plots from simulated efficienct and inefficient markets, and show how the unbiasedness regressions can be used to measure the degree of inefficiency in the market.

6.1.1 Simulated Efficient Market

In this section we simulate an efficient market, where returns are normally distributed with a mean of 0 and a standard deviation of 0.1. They are independent random events.

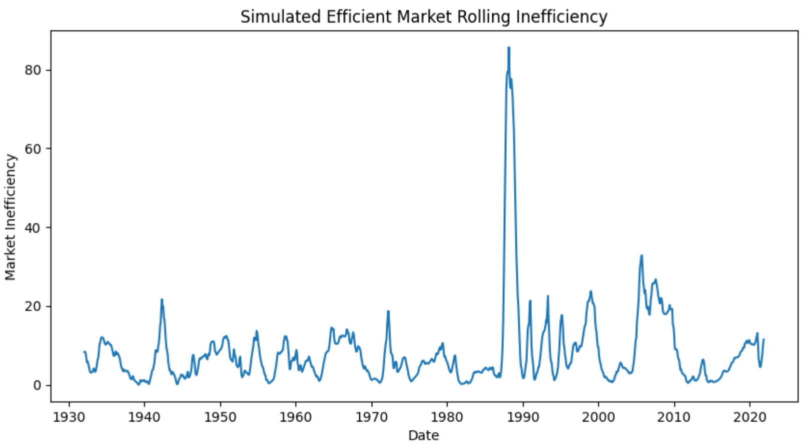

Figure 5: Simulated Efficient Market

In Figure 5 we see the rolling inefficiency of the simulated efficient market. Notice that the level remains low, with only small spikes in inefficiency. There is an outlier near the end of the period [TODO: why? just the instability of the rolling window methodology?]

6.1.2 Simulated Inefficient Market

In this section we simulate an inefficient market, where returns are an AR(1) process:

$$r_t = \phi r_{t-1} + \epsilon_t \tag{4}$$

where $\phi = 0.5$ and $\epsilon_t \sim N(0, 0.1)$. The resuling series of market inefficiency is shown in Figure 6. We see that there are much more frequent spikes of inefficiency, and the level of inefficiency is much higher than in the efficient market.

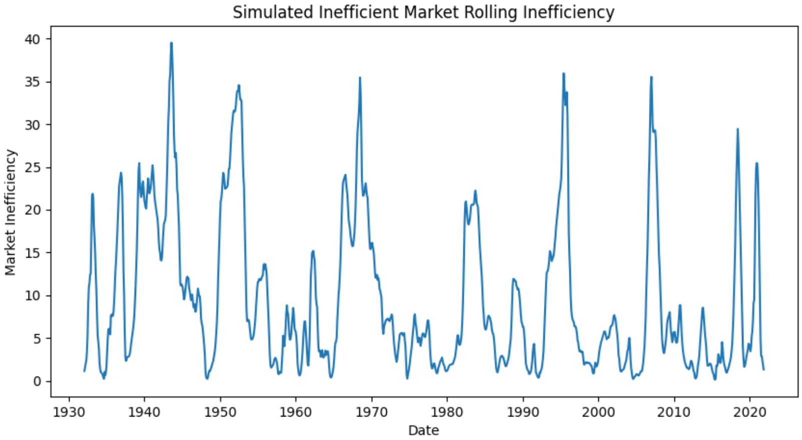

Figure 6: Simulated Inefficient Market

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