Irrational Value Spreads: Fluctuating Market Efficiency

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Abstract

Extending the work of (**?**), we propose a method to measure the level of market inefficiency. We find that the market is not uniformly efficient and goes through periods of higher inefficiency reverting to efficiency in-line with **?**'s adaptive market hypothesis. We find that the value spread—the ratio of expensive to cheap stock valuations—is cointegrated with market inefficiency. By demonstrating that wider value spreads align with greater market inefficiency, we can empirically support **?**'s intuition that high spreads during events such as the dot-com bubble and 2019 reflect mispricing rather than fundamental changes. This evidence strengthens the case that the value premium is still alive but has been masked by market inefficiency. We also find that the market's temporary inefficiencies have grown more persistent over time.

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1 Introduction

?'s efficient market hypothesis posits that markets incorportate all information rationally and instantaneously. While this is an extreme assumption, it is established that markets tend to some form of efficiency. In an efficient market, investors are compensated for taking on systematic risk, therefore, understanding the level of market efficiency can be used to asses whether market participants will be rewarded fairly for the risks they bear.

"Value" in classical factor investing refers to stocks with high book-to-market ratios as per **?**, typically considered undervalued or distressed. Its relationship with market efficiency is critical: if markets are efficient, an investor should be rewarded for holding a portfolio of under-valued stocks. However, when inefficiencies arise, value stocks may become mispriced, with the market willing to pay more for already 'expensive' stocks, and less for already 'cheap' stocks. In these periods, investors may consider that the definition of value has changed, the subjective measure of 'expensive' and 'cheap' has shifted. We can define a value spread as the ratio of the value of expensive stocks to the value of cheap stocks. In this paper we show that generally, periods of inflated value spreads are correlated with increasing inefficiency, therefore we suggest that abnormal value spreads are not reflective of a fundamental change in the market, but rather a mispricing.

Existing methods for measuring market efficiency typically focus on testing whether stock prices follow a random walk (**?**, **?**). However, these methods often fail to capture variations in the level of efficiency over time. A more dynamic approach is needed to reflect the market's changing behavior.

? uses unbiasedness regressions to examine price informativeness around FOMC meetings. This method visually shows us how prices adjust to information flow, in speed, and under or overreaction, using R^2s and βs over the unbiasedness regression window, which is useful in an approach for measuring efficiency.

Building on this, we introduce the $Beta\_SSE$ score, which measures market efficiency at a point in time. By applying unbiasedness regressions within a rolling window, and constructing the $Beta\_SSE$ representation, we create a time series that tracks the level of market efficiency.

Our findings show that:

- The market is not uniformly efficient. Instead, it experiences cycles of inefficiency followed by reversion to efficiency.

- Over the last decade, inefficiency has been increasing, suggesting that the market has become less adept at incorporating available information into prices on longer time-scales.
- Furthermore, this rising inefficiency is cointegrated with the value spread, indicating a strong link between the weakness of the value premium and market inefficiency.

2 Data

3 Methodology

For the purposes of this paper we look at monthly returns of the S&P 500 from January 1927 to October 2024, but the methodology can be applied to any window at any frequency. I show an example of daily returns around earnings versus non-earnings months in the Appendix to build the intuition of interpreting the unbiasedness regression results.

3.1 Unbiasedness Regressions

The unbiasedness regression is a simple OLS regression of the cumulative logarithmic return of the asset over a normalized window of time $\text{SP500}_{[0,T]}$, on a subset of the logarithmic cumulative returns, $\text{SP500}_{[0,t]}$, where $t \leq T$, and t is the number of months from the beginning of the window. [1]

$$\text{SP500}_{[0,T]} = \alpha_t + \beta_t \text{SP500}_{[0,t]} + \epsilon_t, \text{ where } 0 \leq t \leq T. \tag{1}$$

For an illustrative example, let $T = 36$, so that the window $[0, T]$ represents a 3-year period, and let us look at the returns of the SP500.

For $t = 1$, the regression is of the form:

$$\text{SP500}_{[0,36]} = \alpha + \beta \text{SP500}_{[0,1]} + \epsilon \tag{2}$$

From this regression we extract the coefficient β and the R^2 value. The regression is repeated for $t = 2, t = 3, ..., t = 36$, and the coefficients β as well as the R^2 are plotted against t.

When log prices form a random-walk with drift, as in efficient markets, $\beta_t = 1$ for all t. As per **?**, when $\beta_t = 1$, the partial return from 0 to t provides an "efficient" forecast of the return from t to T. 2, in the sense that no amplification or attenuation of the partial return can improve the residual variance of the forecast error. If $\beta_t < 1$, then the partial return is attenuated or partially reversed in the total return, which is often interpreted as a symptom of price noise, a temporary component in prices, or "overreaction" (**?**). Conversely, if $\beta_t > 1$, the partial return

[1] $\text{SP500}_{[0,1]}$ is the matrix of returns of the SP500 from January 1927 to February 1927, February 1927 to March 1927, . . . , August 2024 to September 2024, September 2024 to October 2024.
$\text{SP500}_{[0,2]}$ would be the matrix of returns from January 1927 to March 1927, February 1927 to April 1927, . . . August 2024 to October 2024, and so on for the entire dataset.

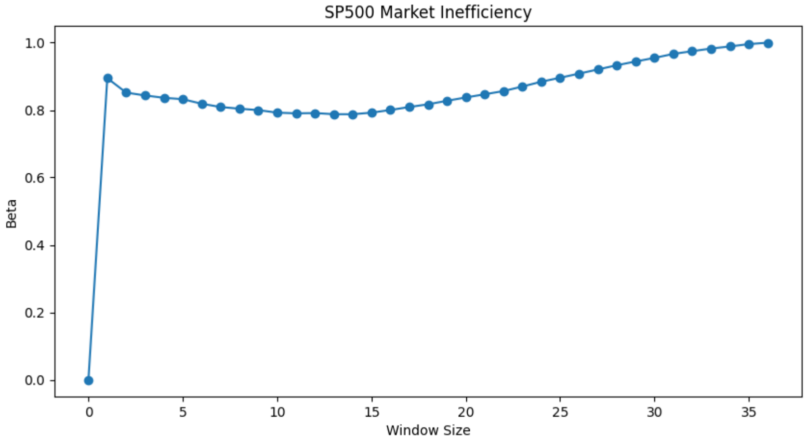

Figure 1: An example of an unbiasedness regression on the SP500 (1927 - 2024)

is amplified in the total return, suggesting "underreaction" or slow information processing.

In this paper we will be focusing on the β_t coefficient, but an explanation of the R^2 is provided in the Appendix for completeness. We now have the foundation to construct a score for market efficiency.

3.2 $Beta\_SSE$ Score

We have established the unbiasedness regression infrastructure, and that a $\beta_t < 1$ indicates overreaction, a $\beta_t > 1$ indicates underreaction, and $\beta_t = 1$ indicates efficiency. To construct a score that measures the level of market efficiency at a point in time, we have to create a representation of the β_t graph for a time period.

We define the $Beta\_SSE$ score as the sum of the squared differences between β_t and the horizontal line at $\beta = 1$, for all t in the window. Since a $\beta_t = 1$ indicates efficiency, the $Beta\_SSE$ score will be lower when the market is more efficient and larger when the market is less efficient, whether by under- or over-reaction.

To construct a timeseries of these scores, we run the unbiasedness regressions on a rolling window of five years. So $Beta\_SSE_t$ is constructed from the unbiasedness regressions of the SP500 where $t = 0$ in the regression are the month beginnings in the period $t - 5 \times 12$ to t. [2]

$$Beta\_SSE_t = \sum_{i=0}^{36} (\beta_{t,i} - 1)^2 \tag{3}$$

[2]Note that our unbiasedness regressions exted out to $t + 36$, so $Beta\_SSE_t$ score isn't tradeable at time t but at time $t + 36$. Consider the window [January 2000, January 2005], the observation starting at January 2005 requires the window of returns from January 2005 to January 2008 to make $\text{SP500}_{[0,36]}$.

Where $\beta_{t,i}$ is the β coefficient from the unbiasedness regression from a window ending on the date t, at the ith depth of the unbiasedness window.

3.3 The Value Spread

The value spread is the ratio of the book-to-market ratio of the portfolio of the most expensive 30% of stocks to the book-to-market ratio of the portfolio of the cheapest 30% of stocks, as per **?**. To construct this measure we use Kenneth French's data library (**?**). We get the annual Sum of BE / Sum of ME for the large-cap top 30% portfolio and bottom 30% portfolio directly from the library, and then calculate the value spread as the ratio of the two. The time series of the Value Spread can be found in Figure 2, recession periods are shaded in grey.

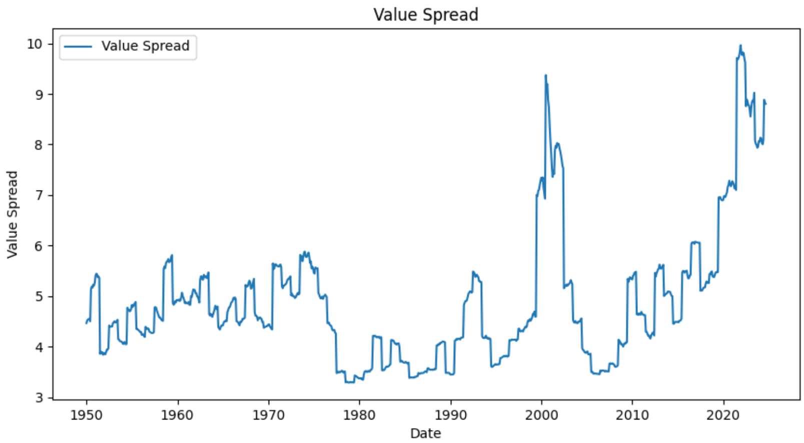

Figure 2: The value spread timeseries 1927 - 2024

4 Results

Using the $Beta\_SSE$ score, we construct a timeseries of the level of market efficiency as seen in Figure 3. We shade in grey the recession periods as defined by NBER based Recession Indicators (?)

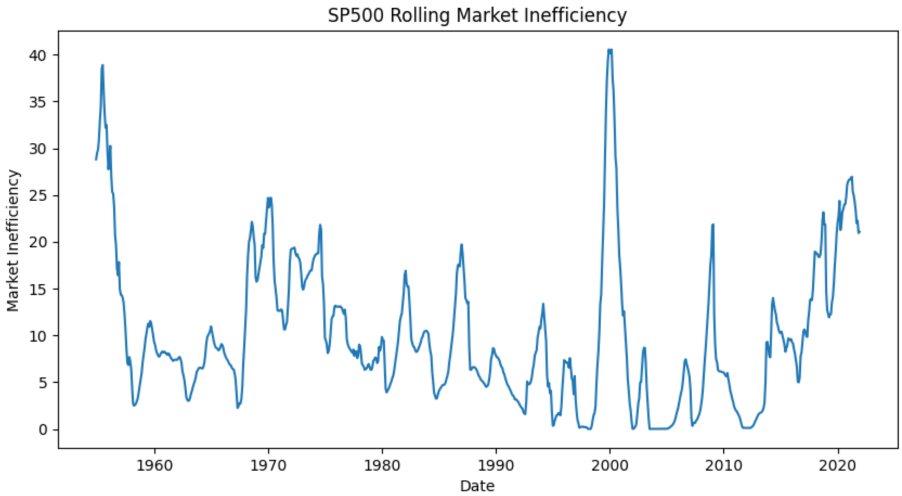

Figure 3: The $Beta\_SSE$ score timeseries of the SP500 (1950 - 2024)

We see the market is inefficient around the dot-com bubble, the 2008 financial crisis, and the COVID-19 pandemic, which is in line with our expectations. We can visually asses that the market is not uniformly efficient. Instead, it goes through periods of higher inefficiency followed by reversion to efficiency. We also see that the market has been increasingly inefficient in the last decade, which is in line with **?**.

Empirically we can test the mean reversion through an Augmented Dickey-Fuller test (**?**) on the $Beta\_SSE$ timeseries, which is significant at the 0.1% level.

Figure 4 shows the value spread and the $Beta\_SSE$ score timeseries. We see visually that the two are cointegrated, which we test with the Engle-Granger two-step cointegration test, which is significant at the 0.5% level.

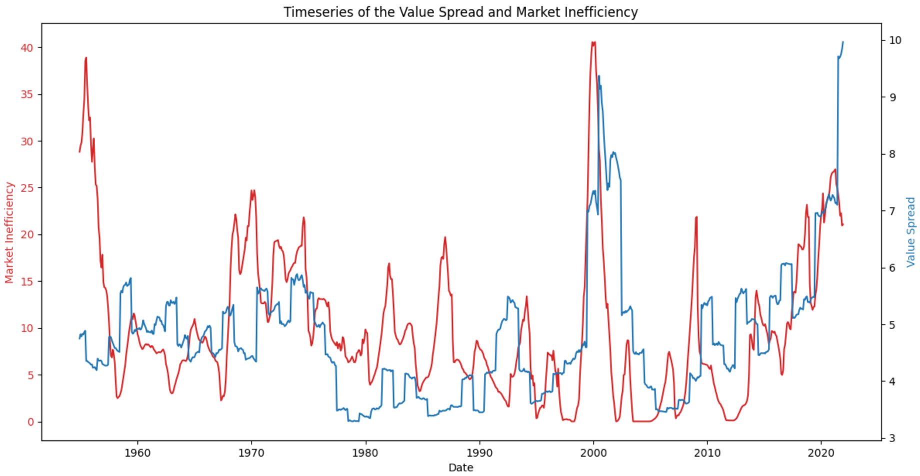

Figure 4: The value spread and $Beta\_SSE$ score timeseries of the SP500 (1950 - 2024)

5 Conclusion

6 Appendix