

### Confidence Interval for a Proportion

Confidence Interval =  $p \pm z \cdot (\sqrt{p(1-p)} / n)$

where: p: sample proportion z: the chosen z-value n: sample size

**A random sample of 500 apples was taken from, a large consignment and 60 were found to be bad. Obtain the 95%, 98% confidence limits for the percentage number of bad apple in the consignment.**

```
library(glue)
n<-500;
p<-(60/500)
SE <- sqrt(p*(1-p)/n)
z_star<- qnorm(1-(1 - 0.95)/2)
ME<-z_star*SE
glue("{p - ME}, {p + ME}")
```

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```

**A sample of 900 members has a mean 3.4 cms, and s.d. 2.61 cms. If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.**

$\bar{X} \pm z \cdot \sigma / \sqrt{n}$

```
library(glue)
n<-900;
sigma<-2.61
Xbar<-3.4
SE <- sigma/sqrt(n)
z_star<- qnorm(1-(1 - 0.95)/2)
ME<-z_star*SE
glue("{Xbar - ME}, {Xbar + ME}")
```

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n<-900;
sigma<-2.61
Xbar<-3.4
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```

solve

The mean muscular endurance score of a random sample of 60 subjects was found to be 145 with a s.d. of 40. Construct a 95% confidence interval for the true mean. Assume the sample size to be large enough for normal approximation.

### Confidence Interval for a Difference in Proportions

Confidence interval =  $(p_1 - p_2) \pm z \sqrt{p_1(1-p_1)/n_1 + p_2(1-p_2)/n_2}$

where:

$p_1, p_2$ : sample 1 proportion, sample 2 proportion

$z$ : the z-critical value based on the confidence level

$n_1, n_2$ : sample 1 size, sample 2 size

Write R code for the above formula and solve the following problem

A medical researcher conjectures that smoking can result in the wrinkled skin around the eyes. The researcher recruited **150 smokers** and **250 nonsmokers** to take part in an observational study and found that **95** of the **smokers** and **105** of the **nonsmokers** were seen to have prominent wrinkles around the eyes (based on a standardized wrinkle score administered by a person who did not know if the subject smoked or not). Find **CI** for the true difference that would exist between these two groups in the population.

### Confidence Interval for a Difference in Means

Then the confidence interval for the difference in the two population means is

$$\bar{x}_1 - \bar{x}_2 \pm (z \text{ critical value}) \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

$$S.E. (\bar{x}_1 - \bar{x}_2) = \sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)} = \sqrt{(s_1^2/n_1) + (s_2^2/n_2)}$$

Write R code for the above formula and solve the following problem

In a certain factory there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 ozs. with a standard deviation of 12 ozs. while the corresponding figures in a sample of 400 items from the other process are 124 and 14. Obtain the standard error of difference between the two sample means. Find the 99% confidence limits for the difference in the average weights of items produced by the two processes respectively.

Answer

$$\begin{aligned} |\bar{x}_1 - \bar{x}_2| \pm 2.58 \text{ S.E. } (\bar{x}_1 - \bar{x}_2) &= 4 \pm 2.58 \times 1.034 \\ &= 4 \pm 2.67 \text{ (approx.)} = 6.67 \text{ and } 1.33 \\ 1.33 < \mu_1 - \mu_2 < 6.67 \end{aligned}$$

#### **P-value from z score**

##### **Right-tailed test**

Suppose we want to find the p-value associated with a z-score of 2.02 in a right-tailed hypothesis test.

$$P = P(Z > 2.02) = 0.0217$$

`pnorm(q=2.02, lower.tail=FALSE)`

[1] 0.02169169

##### **Left-tailed test**

Suppose we want to find the p-value associated with a z-score of -0.77 in a left-tailed hypothesis test.

`pnorm(q=-0.77, lower.tail=TRUE)`

[1] 0.2206499

##### **Two-tailed test**

Suppose we want to find the p-value associated with a z-score of 2.83 in a two-tailed hypothesis test.

$$P = P(|Z| > 2.83) = 2P(Z < -2.83) = 0.0046 \quad 2 * \text{pnorm}(q=2.83, \text{lower.tail}=\text{FALSE})$$

[1] 0.0046548

##### **T test (two tailed)**

$$P = P(|T| > 2.06) \approx 0.06 \quad 2 * \text{pt}(q=2.06, 14, \text{lower.tail}=\text{FALSE})$$

[1] 0.05849421

\* here 14 is degrees of freedom

Similarly you can do for one tailed test (t)