

Task 3B : Theme & Rulebook Questionnaire

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Question No.	Max. Marks	Marks Scored
Q1	5	
Q2	10	
Q3	5	
Q4	5	
Q5	10	
Q6	5	
Q7	10	
Q8	15	
Q9	5	
Q10	5	
Q11	10	
Q12	5	
Q13	10	
Total	100	

Q1. Briefly describe your experience in building the Lunar Scout bike.

A1.

Task-0 to task-3 till now, has been a journey never imagined before. Learned many new things whether it be Lagrangian way of dealing problems, control systems, 3D-modelling and control algorithms. Failed low many times, with no light of hope, but the inner motivation we had helped us push this much that our consistency, hard work, repeated efforts, dedication, and most importantly team work all combined gave us the much needed efforts and this will continue till the end. As we head into the next phase, we are more determined than ever. We know there will be more challenges, but we're ready to tackle them and keep making our Lunar Scout bike even cooler

Q2. In task 1, you were introduced to LQR controller design for a simple pendulum and asked to do mathematical modelling and LQR controller design for Rotary Inverted Pendulum. In that, you were asked to derive the equations, linearize around the equilibrium point and find the A & B matrix using the Jacobian function.

In this question, you have to choose the states for your Lunar Scout bike that you are going to design. Model the system using Euler-Lagrangian Mechanics that you learned in task 1. Linearize the system using jacobians around the equilibrium points representing your physical system. Use mathematical expressions for derivations and proper diagrams where necessary.

A2.

Q3. Which is the most optimal controller between PID and LQR. Justify your answer.

A3.

Optimality of a controller depends on the characteristics of the system to be controlled i.e Whether the system is Simple or Complex, Linear or non Linear, Single variate or multivariate etc. In our case, we needed to handle a complex system of self-balancing bike based on rotary inverted pendulum which is considered as an multivariate, non-linear, complex system handling which requires great deal of mathematical analysis, considering which we think LQR is much optimal controller than PID for this given system. Also PID not involves



any kind of mathematical analysis and random tuning works only for a given case, when situation changes tuning variables need to be re-tuned which again proves to be a hectic task.

Q4. What is the significance of finding Controllability and Observability of a system in state space approach?

A4.

Controllability— A system is represented in the form of state matrix in state space approach. State represents the controlled variables and their derivatives (change rate). Controlling a system means being able to control these states. For the same we use controllability matrix in deciding if the system is fully controllable or not. An uncontrollable system cannot be controlled how much we apply LQR on it. If some states are uncontrollable, we can devise some approach to deal with them.

Observability— In simple observability means being able to read input by seeing corresponding output. This is the function sensors perform in a system, they help us see output using which we get an idea about the related input or internal mechanism governing the concerned output. In technical terms, observability is determining current input state from output state only. Thus we use an observability matrix to check whether the above purpose can be accomplished or not i.e. system is fully observable or not

Q5. Briefly explain your opinion on having the centre of mass of the bike low or high. Use diagrams/calculations/examples to support your argument.

A5.

Centre of mass of the bike should be as low as possible in order to keep the bike stable (or in stable equilibrium). This would prevent the bike from tipping over. While designing the bike this could be achieved by placing the heavier components closer to the ground so that the centre of mass of overall body comes down.



Q6. In what cases will the run time be considered as the maximum time ($T_{max} = 300$ seconds) according to the scoring formula and theme rules?

A6.

According to the scoring formula the run time will be considered as the maximum time if:

- i) The bike's 5 seconds buzzer beep is done without reaching the SL location.
- ii) The 5 seconds buzzer beep is done without covering all the CS

Q7. Explain what you have understood from the Theme play in your own words.

A7. The theme run starts with a 1 seconds buzzer beep produced by the bike. The bike can choose any of the path and maximum time allotted for the run is 5 minutes. The bike will be controlled wirelessly through a remote control. The bike should visit each CS and should halt there for at least 3 seconds then only it will be considered valid visit. Once all the CS has been visited by the bike, it should go to any of the 3 SL and should give the 5 seconds buzzer beep and this will be considered as indication for 'Run completion' and the time for the run completion is noted. If the bike's 5 seconds buzzer beep is done without reaching the SL and/or is done without covering all the CS, then the maximum runtime (300 seconds) will be considered as the runtime for the bike. An indication is given by the LED and buzzer at each CS visit according to the instructions of the 'Theme play'.

Q8. What will be the SCORE in the following situation:

Given	Run	Configuration:
Start	Location	S1
Colony	Sites: 1, 2, 3, 5	
Obstacle: O1, O3		

In the given run there are four Colony Sites (CS). The bike started its journey from S1. It halted near the first CS, which has the north pole of the magnet facing the track. It indicated green LED and buzzer, then started its traversal.

- Now it has reached the second CS which has no magnet in it. It doesn't indicate any of the light and started its traversal.
- The bike crossed one obstacle and then indicated the first CS again with red led and buzzer with proper halt, then it continued forward.



- Now it reaches its final CS which is having a south pole facing towards the track and indicating green LED and buzzer beep while passing by the CS, but without halting near the CS.
- Bike then goes to the S2 position, stops and beeps the buzzer for 5 seconds. By this time 150 seconds have passed, from the start time. The bike did not have any MI/PP/HP during the run.

A8. Score=1100(if design bonus=100).

Q9. What is Parallel and Perpendicular axis theorem? Is it required for mathematical modelling? Justify your answer with respect to the lunar scout bike.

A9.

The parallel axis theorem states that the moment of inertia of a rigid body about any axis through the centre of mass is equal to the sum of the moment of inertia about the parallel axis and the product of the mass of the body and the square of the distance between the two axes. Mathematically, this can be expressed as $I = I_{cm} + md^2$, where I is the moment of inertia about the parallel axis, I_{cm} is the moment of inertia about the centre of mass axis, m is the mass of the body, and d is the distance between the two axes. The perpendicular axis theorem states that the moment of inertia of a planar body about an axis perpendicular to the plane of the body is equal to the sum of the moments of inertia about two perpendicular axes, and I_x and I_y are the moments of inertia about the two perpendicular axes in the plane of the body. These theorems are useful in physics and engineering for calculating the moments of inertia of complex objects, which is important for understanding how they will respond to rotational motion and external forces. They are used in the analysis and design of mechanical systems, such as in the design of machinery, vehicles, and structures.

The Parallel Axis Theorem and Perpendicular Axis Theorem are both important concepts in physics, particularly in the context of rotational motion and the calculation of moments of inertia. These theorems are not specific to mathematical modelling alone but are essential tools in



physics and engineering for analysing the motion and behaviour of objects undergoing rotational motion.

Now, in the context of the lunar scout bike, understanding these theorems would be crucial for engineers and physicists involved in designing and analysing the bike's performance. The moment of inertia is a key parameter when studying the rotational motion of objects, and these theorems provide a means to calculate it for various configurations.

For instance, if the lunar scout bike has irregular shapes or components with different masses distributed away from the axis of rotation, the Parallel Axis Theorem would be useful in calculating the total moment of inertia. Additionally, if the bike is a planar object, the Perpendicular Axis Theorem could simplify the calculation of the moment of inertia about an axis perpendicular to the plane.

In conclusion, while not explicitly required for all mathematical modelling tasks, these theorems are fundamental tools in physics and engineering, and their application becomes particularly relevant when dealing with rotational dynamics, such as in the case of the lunar scout bike or any rotating object.

Q10. How will you check whether the system is stable or not in a state-space approach?

A10.

In a state-space approach, stability analysis is crucial to determine whether a system is stable or not. Stability is a key property that ensures a system's response remains bounded and does not diverge over time. There are different methods to check the stability of a system in the state-space representation, and the most common approaches include:

1. Eigenvalue Analysis:

- One widely used method is to analyse the eigenvalues of the system matrix (A matrix) in the state-space representation. The system is stable if all the eigenvalues of matrix A have negative real parts. Mathematically, if all



eigenvalues λ_i satisfy $\text{Re}(\lambda_i) < 0$, then the system is stable. This condition ensures that the system response decays over time.

2. Lyapunov Stability Analysis:

- Lyapunov stability analysis involves the use of Lyapunov functions to determine stability. A Lyapunov function is a scalar function of the system state that, when its derivative is analyzed, provides information about the system's stability. The Lyapunov stability criterion states that if there exists a continuously differentiable function $V(x)$ such that its derivative along the system trajectories is negative definite, then the system is stable.

3. BIBO (Bounded Input, Bounded Output) Stability:

- BIBO stability is concerned with the system's response to bounded input signals. A system is BIBO stable if, for any bounded input signal, the output remains bounded. This can be analyzed using the system's transfer function or state-space representation.

4. Direct Stability Analysis:

- In some cases, direct stability analysis of the state-space equations may be performed. This involves analyzing the system dynamics and looking for characteristics that indicate stability, such as damping ratios and natural frequencies.

Here's a general procedure to check stability in a state-space approach:

- Write the system in state-space form: $\dot{x} = Ax + Bu$
- Analyze the eigenvalues of matrix A .
- If all eigenvalues have negative real parts, the system is stable.
- Alternatively, use Lyapunov functions or other stability analysis methods for additional confirmation.

It's important to note that stability analysis is context-dependent, and the specific approach may vary based on the characteristics of the system and the requirements of the application. Additionally, stability analysis is often performed using software tools that can compute eigenvalues or Lyapunov functions numerically.



Q11. What will be happening in the following situation:

The bike wrongly indicated the LED colour for a colony site while in halt. When starting to move it crosses the dotted line and hits the colony sites and falls. So Manual intervention has taken place. How many penalties will be imposed and what are they?

A11. There are 4 penalties while these movement happen

1. Wrong Indication(WI) .
2. Path Penalty(PP) .
3. Hit Penalty(HP) .
4. Manual Intervention(MI) .

Q12. How many different type sensors does a 6-axis Inertial Measurement Unit (IMU) have? Explain what physical quantities they measure exactly?

A12.

A 6-axis Inertial Measurement Unit (IMU) typically integrates sensors to measure specific physical quantities related to motion and orientation. The term "6-axis" indicates that the IMU is capable of measuring motion along six degrees of freedom. These axes are generally categorized into translational (linear) and rotational (angular) motion. The sensors commonly found in a 6-axis IMU include:

1. Accelerometer (3 axes for translational motion):

- Measures acceleration along three mutually perpendicular axes (usually denoted as X, Y, and Z). The accelerometer provides information about changes in velocity or linear acceleration. It is sensitive to dynamic forces, including gravity, and can be used to determine the orientation of the device.

2. Gyroscope (3 axes for rotational motion):

- Measures angular rate or rotational velocity around three axes (X, Y, and Z). Gyroscopes are essential for capturing the rate at which the device is rotating. They help in tracking changes in orientation and are particularly useful for stabilizing and maintaining orientation in applications like drones, robotics, and navigation systems.



Together, the accelerometer and gyroscope in a 6-axis IMU enable the device to capture motion in all six degrees of freedom. However, it's important to note that while an IMU with accelerometers and gyroscopes can determine changes in velocity and orientation, it may not be able to directly measure absolute position. Additional sensors or methods, such as magnetometers or external positioning systems, may be required for applications that need absolute position information.

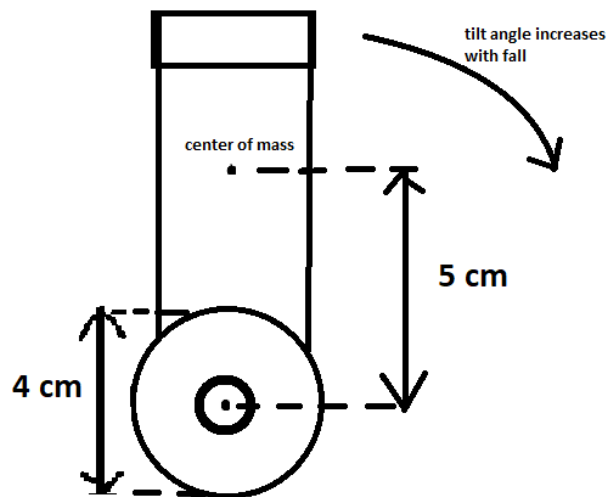
In summary, a 6-axis IMU typically includes:

- Accelerometer (3 axes):
 - Measures linear acceleration along X, Y, and Z axes.
- Gyroscope (3 axes):
 - Measures angular rate or rotational velocity around X, Y, and Z axes.

These sensors collectively provide valuable data for applications such as inertial navigation, motion tracking, gesture recognition, robotics, and other systems where understanding changes in motion and orientation is critical.

Q13. Consider a wheeled inverted pendulum having a vertical body balanced using a DC geared motor. The center of mass of the body is at 5 cm height from the axle of the wheel and motor shaft. The diameter of the wheel is 4 cm.

- What is the torque required from the DC geared motor(max RPM = 300), to be able to balance this body of total mass 1.5 kg(consider wheel & motor massless), with max correctable angular tilt as ± 5 degrees. (Mention steps for your calculation)



A13. The gravitational torque

To calculate the torque required from the DC geared motor to balance the inverted pendulum, we can use the principles of rotational dynamics and control theory.

1. Gravitational Torque Calculation:

Gravitational Torque, $\tau_{\text{gravity}} = \text{Force} \times \text{Distance}$

The force exerted by gravity on the pendulum is the weight of the pendulum's mass, calculated as:

$$F_{\text{gravity}} = m \times g$$

Where:

- $m = \text{mass of the pendulum} = 1.5 \text{ kg}$
- $g = \text{acceleration due to gravity} = 9.81 \text{ m/s}^2$

$$F_{\text{gravity}} = 1.5 \text{ kg} \times 9.81 \text{ m/s}^2 = 14.715 \text{ N}$$

The distance from the pivot point to the center of mass of the pendulum is 5 cm (0.05 m).

Now, we can calculate the gravitational torque:

$$\tau_{\text{gravity}} = F_{\text{gravity}} \times \text{Distance} = 14.715 \text{ N} \times 0.05 \text{ m} = 0.73575 \text{ Nm}$$

2. Torque for Angular Acceleration:

The maximum angular tilt allowed is ± 5 degrees. We'll consider $\theta = 5$ degrees $= 0.08727$ radians.

We convert the maximum RPM of the motor to angular velocity:



$$\omega_{\max} = (2\pi \times \text{max RPM}) / 60$$

$$\omega_{\max} = (2\pi \times 300) / 60 = 31.415 \text{ rad/s}$$

Now, we can calculate the maximum angular acceleration:

$$\alpha_{\max} = \omega_{\max}^2 \times \tan(\theta) = (31.415 \text{ rad/s})^2 \times \tan(0.08727)$$

$$\alpha_{\max} \approx 10.347 \text{ rad/s}^2$$

The moment of inertia of the pendulum can be approximated as the moment of inertia of a slender rod rotating about one end:

$$I = (1/3) \times m \times L^2$$

Where $L = 0.05 \text{ m}$ (distance from axle to center of mass)

$$I = (1/3) \times 1.5 \text{ kg} \times (0.05 \text{ m})^2 = 0.00125 \text{ kg}\cdot\text{m}^2$$

Now, we can calculate the torque required for acceleration:

$$\tau_{\text{acceleration}} = I \times \alpha \approx 0.00125 \text{ kg}\cdot\text{m}^2 \times 10.347 \text{ rad/s}^2 \approx 0.012934 \text{ Nm}$$

Total Torque Required:

$$\text{Total Torque Required} = \tau_{\text{gravity}} + \tau_{\text{acceleration}}$$

$$\text{Total Torque Required} \approx 0.73575 \text{ Nm} + 0.012934 \text{ Nm} \approx 0.7487 \text{ Nm}$$

So, the torque required from the DC geared motor to balance the body is approximately 0.7487 Nm.

