

Top-down Predictive Parsing

Article #16

Grammar

Original	Transformed
$E \rightarrow E + T \mid T$	$E \rightarrow TE'$
	$E' \rightarrow +TE' \mid \epsilon$
$T \rightarrow T * F \mid F$	$T \rightarrow FT'$
	$T' \rightarrow *FT' \mid \epsilon$
$F \rightarrow v \mid (E)$	$F \rightarrow v \mid (E)$

FIRST and FOLLOW sets

Nonterminal	FIRST	FOLLOW
E	{ v , (}	{) , \$ }
E'	{ + , ϵ }	{) , \$ }
T	{ v , (}	{ + ,) , \$ }
T'	{ * , ϵ }	{ + ,) , \$ }
F	{ v , (}	{ * , + ,) , \$ }

Parsing Table

Nonterminal	Input Symbol (token)					
	v	+	*	()	\$
E	TE'			TE'		
E'		+TE'			ϵ	ϵ
T	FT'			FT'		
T'		ϵ	*FT'		ϵ	ϵ
F	v			(E)		

Parsing Table

Nonterminal	Input Symbol (token)					
	v	+	*	()	\$
E	TE'			TE'		
E'		+TE'			ϵ	ϵ
T	FT'			FT'		
T'		ϵ	*FT'		ϵ	ϵ
F	v			(E)		

Parsing program (a + b) * c with Predictive Parser

Stack	Input	Table [A,a]
\$E	(v+v)*v\$	[E,(] => E->TE'
\$E'T	(v+v)*v\$	[T,(] => T->FT'
\$E'T'F	(v+v)*v\$	[F,(] => F->(E)
\$E'T')E((v+v)*v\$	Match and Remove (
\$E'T')E	v+v)*v\$	[E,v] => E->TE'
\$E'T')E'T	v+v)*v\$	[T,v] => T->FT'
\$E'T')E'T'F	v+v)*v\$	[F,v] => F->v
\$E'T')E'T'v	v+v)*v\$	Match and Remove v
\$E'T')E'T'	+v)*v\$	[T',+] => T'-> ϵ
\$E'T')E'	+v)*v\$	[E',+] => E'->+TE'
\$E'T')E'T+	+v)*v\$	Match and Remove +
\$E'T')E'T	v)*v\$	
\$E'T')E'T'F	v)*v\$	
\$E'T')E'T'v	v)*v\$	
\$E'T')E'T')*)v\$	
\$E'T')E')*)v\$	
\$E'T'))*)v\$	
\$E'T'	*)v\$	
\$E'T'F*	*)v\$	
\$E'T'F	v\$	
\$E'T'v	v\$	
\$E'T'	\$	
\$E'	\$	[E',]\$ => E'-> ϵ
\$	\$	Match and Stop

Parsing Table with Error Recovery Synchronization Entries

Nonterminal	Input Symbol (token)					
	v	+	*	()	\$
E	TE'			TE'	synch	synch
E'		+TE'			ϵ	ϵ
T	FT'	synch		FT'	synch	synch
T'		ϵ	*FT'		ϵ	ϵ
F	v	synch	synch	(E)	synch	synch

Parsing *if statement* grammar with predictive parser

<statement> ->

if <condition> then <statement>

| if <condition> then <statement> else <statement>

| <other statements>

<condition> -> boolean value

The following is an abstract presentation of the same grammar:

$S \rightarrow iCtS \mid iCtSeS \mid O$

$C \rightarrow b$

$O \rightarrow a$

This grammar is not an LL grammar because it contains a “left-factor”

Grammar

Original	Transformed
$S \rightarrow iCtS \mid iCtSeS \mid O$	$S \rightarrow iCtSS' \mid O$
	$S' \rightarrow eS \mid \epsilon$
$C \rightarrow b$	$C \rightarrow b$
$O \rightarrow a$	$O \rightarrow a$

FIRST and FOLLOW sets

Nonterminal	FIRST	FOLLOW
S	{ i , a }	{ e , \$ }
S'	{ e , ϵ }	{ e , \$ }
C	{ b }	{ t }
O	{ a }	{ e , \$ }

Parsing Table

Nonterminal	Input Symbol (token)					
	a	b	e	i	t	\$
S	O			iCtSS'		
S'			eS ϵ			ϵ
C		b				
O	a					

The grammar is ambiguous because the $[S', e]$ cell contains two entries.
A grammar whose parsing table has no multiple entries in a cell is said to be LL(1) grammar.

Another way to prove that the *if grammar* is ambiguous.

$S \rightarrow i C t S \mid i C t S e S \mid O$
 $C \rightarrow b$
 $O \rightarrow a$

Given the sentence:

if b then if b then a else a

or

i b t i b t a e a

The following two left-most derivations can be built to parse the sentence:

$S \Rightarrow i C t S \Rightarrow i b t S \Rightarrow i b t i C t S e S \Rightarrow i b t i b t S e S$
 $\Rightarrow i b t i b t O e S \Rightarrow i b t i b t a e S \Rightarrow i b t i b t a e O \Rightarrow i b t i b t a e a$
and

$S \Rightarrow i C t S e S \Rightarrow i b t S e S \Rightarrow i b t i C t S e S \Rightarrow i b t i b t S e S$
 $\Rightarrow i b t i b t O e S \Rightarrow i b t i b t a e S \Rightarrow i b t i b t a e O \Rightarrow i b t i b t a e a$

The grammar can be reworked to remove the ambiguity:

$S \rightarrow M \mid U$
 $M \rightarrow \text{if } C \text{ then } M \text{ else } M \mid O$
 $U \rightarrow \text{if } C \text{ then } S \mid \text{if } C \text{ then } M \text{ else } U$

but the grammar is still not an LL(1) grammar.

LL(1) Grammar Definition

A grammar is an LL(1) grammar if and only if whenever the grammar has a production $A \rightarrow \alpha \mid \beta$ with two distinctive options α and β the following conditions hold:

1. For no terminal a do both α and β have a in their FIRST sets.
2. At most one of α and β can derive the empty string.
3. If $\beta \Rightarrow \epsilon$, then α does not have in its FIRST set any terminal which is in FOLLOW(A).

Non- immediate left recursion

$S \rightarrow Aa \mid b$
 $A \rightarrow Ac \mid Sd \mid \epsilon$

Eliminating the recursion
Step 1.

$S \rightarrow Aa \mid b$
 $A \rightarrow Ac \mid Aad \mid bd \mid \epsilon$

Step 2.

$S \rightarrow Aa \mid b$
 $A \rightarrow bdA' \mid A'$
 $A' \rightarrow cA' \mid adA' \mid \epsilon$