ALGONQUIN COLLEGE

NUMBER SYSTEMS

Binary Arithmetic
Addition and Subtraction

Convert binary 1101 to a decimal (unsigned).

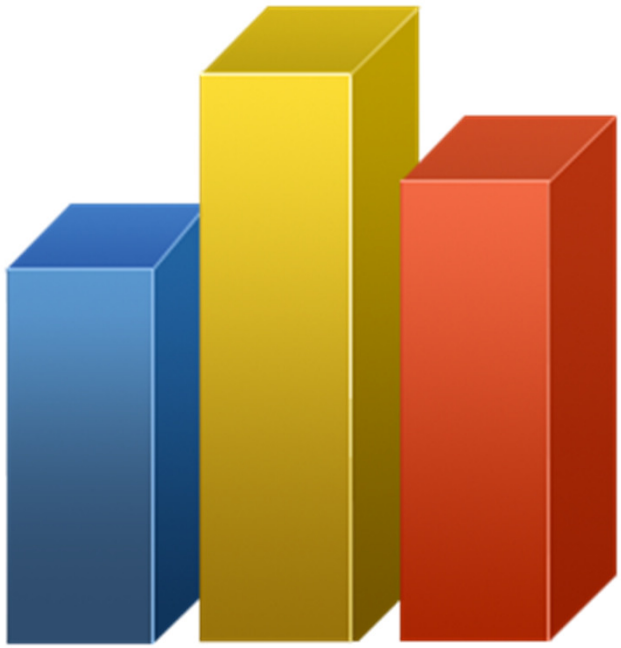
A 11

B 12

C 13

D 14

E 15



Binary - Decimal

1 1 0 1

$= 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$

= 8 + 4 + 0 + 1

= 13

Convert decimal number 5 to a 4-bit unsigned binary number

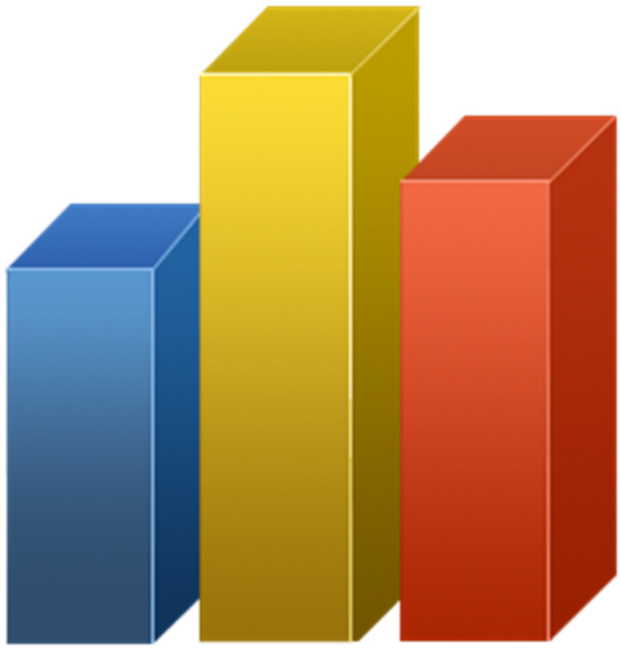
A 0100

B 0101

C 0110

D 0111

E 1001



Decimal - Binary

Multiples of 2 in 4-bit are….8 - 4 - 2 - 1

- How many multiples of 8 in 5? 0
- How many multiples of 4 in 5? 1
- 5-4 = 1
- How many multiples of 2 in 1? 0
- How many multiples of 1 in 1 ? 1

- So answer is 0101

Convert decimal 132 to 8-bit binary

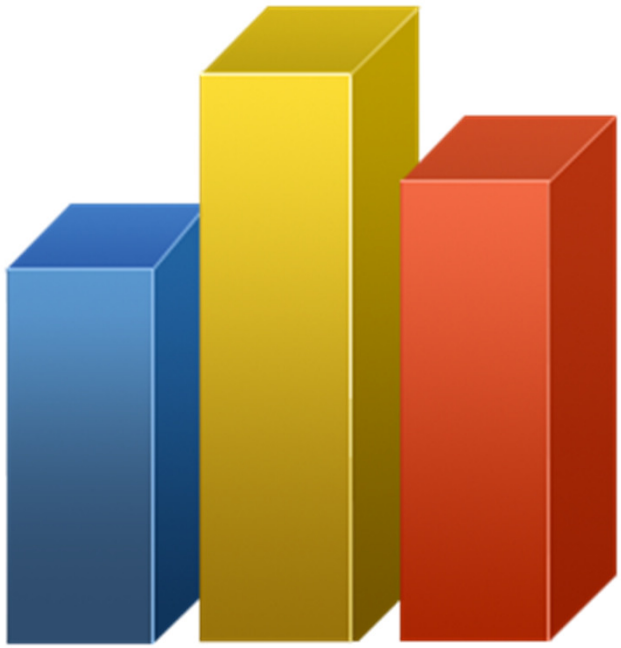
A 1000 0110

B 0100 0110

C 0100 0010

D 1000 0111

E 1000 0010



Decimal - Binary

Multiples of 2 in 8 bit - 128-64-32-16-8-4-2-1

- How many multiples of 128 in 132? 1 (132-128 = 4)
- How many multiples of 64 in 4? 0
- How many multiples of 32 in 4? 0
- How many multiples of 16 in 4? 0
- How many multiples of 8 in 4? 0
- How many multiples of 4 in 4? 1 (4-4 = 0)
- How many multiples of 2 in 0? 0
- How many multiples of 1 in 0 ? 0

- So answer is 1000 0100

Binary Addition

- Remember the binary system has only two symbols - 0 and 1
- if we add 0 + 0 we get 0
- if we add 0 + 1 we get 1
- if we add 1 + 0 we get 1
- if we add 1 + 1 we get “2” or 10 in binary (0 carry the 1)
- This is similar to 9 + 1 in decimal - we get 10 (0 carry the 1)

Binary Addition

- Example — Check in decimal
- 1100 — 12
  + 0011 — 3
- 1111 — 15

Binary Addition

- Example
- 0010 0010 0001 1100
  + 0001 1100 1010 0000
- 0

Binary Addition

- Example
- 0010 0010 0001 1100
  + 0001 1100 1010 0000
- 00

Binary Addition

- Example
- 0010 0010 0001 1100
  + 0001 1100 1010 0000
- 100

Binary Addition

- Example
- 0010 0010 0001 1100
  + 0001 1100 1010 0000
- 0011 1110 1011 1100

- Double check in decimal
- 8732 + 7328 = 16060

Binary Addition

- Example 2
- 0101 1110
  + 0100 1111
- 1

Binary Addition

- Example 2
- ${}^{1}$
- $0101\ 1110$
  $+\ 0100\ 1111$
- 01

Binary Addition

- Example 2
- 1 1
- 0101 1110
  + 0100 1111
- 101

Binary Addition

- Example 2
- 1 1 1 1 1
- 0101 1110
  + 0100 1111
- 1010 1101

- Double check in decimal
- 94 + 79 = 173

Binary Subtraction

- First we need to think about expression num1 - num2
- This is the same as num1 + (-num2)
- This is how we do binary subtraction - we take negative of second number and add it to first
- We know binary addition. So we just need to learn how to find negative of binary number

Finding the Negative of a Binary Number

- First we need to know that binary numbers use the very first digit of the number to indicate positive (0) or negative (1)
- So 0000 0001 is a positive number while 1000 0001 is a negative number
- This is based on first digit of the number - so that digit can no longer be used as part of the number.

One's Complement

- This is a special term which means that you take a binary number and basically change all it's 0 digits to 1's and all it's 1 digits to 0's.
- For example
- 0100 1100 0011 has a one's complement of 1011 0011 1100

Two's Complement

- This is a special term which means that you take the one's complement of a binary number and add 1 to it.
- For example
- 0100 1100 0011 has a one's complement of 1011 0011 1100 and two's complement of 1011 0011 1101_____

Back to Subtraction

- The negative of a binary number is it's two's complement
- So if we wanted to subtract using binary arithmetic decimal 1680 - 1219, we first convert both to binary
- 0110 1001 0000 - 0100 1100 0011
- We then find the one's complement of second number (1219) as 1011 0011 1100
- We then find the two's complement by adding 1 to get 1011 0011 1101

Example continued

- 0110 1001 0000 (which in decimal was 1680)
  +1011 0011 1101 (two's complement of 1219)
- 10001 1100 1101
- Notice the extra 1 on the beginning of the answer…we have to drop this
- So our answer is 0001 1100 1101 which indeed is decimal 461

Why do we learn this?

- Because this is how computer actually does the arithmetic we perform in our programs.

Summary

- We have seen how to do binary addition and
- binary subtraction by taking two's complement of second number (ie its negative) and adding it to the first number.