Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Instructions:

1. Print your name and student number above AND on the MC answer sheet. A test or MC answer sheet without a name and student number won't be marked. A page (except the front page) without a name at the top right, where shown, won't be marked.
2. Use only a pencil when filling in the MC answer sheet for the multiple choice questions. Circle the correct answers on your question paper first and only when you are certain of your answer fill in the MC answer sheet. Only the answers found on the MC answer sheet will be used when marking the multiple choice questions.
3. Check that you have all 5 PAGES before beginning the exam.
4. Pace yourself – you have ~50 minutes.
5. Use the blank spaces on exam pages for rough work. No scrap paper is permitted.
6. Simple non-programmable calculators (not cell phones or tablets) are allowed – but you can leave your answers as accurate and complete numerical expressions where calculations are required but need not waste time evaluating them.
7. If you have a cell phone or any electronic device (other than a pacemaker) with you – be sure it is turned off now, and stored in a safe place away from your desk.
8. Hand in BOTH this exam booklet AND the MC answer sheet. Taking an exam booklet from the exam room will result in an automatic grade of 'F' for this course.
9. This test is worth 17.5% of your final mark.

I have read, understood, and will comply with all of the above instructions:

______________________________ ______________

sign your full name here date

Formulas are at the end.

Choose the best answer in each of the following multiple-choice questions. 1 mark each

1. For a float data type the hidden bit has a value
a. 2^0 b. 2^1 c. 2^{-23} d. 2^{127} e. 2^{255}

2. The computer representation of a floating-point number is specified by the following three parts
a. overflow, exponent, mantissa b. sign bit, decimal point, mantissa c. underflow, sign bit, mantissa
d. hidden bit, exponent, machine epsilon e. none of the above is correct

3. For a binary number with m bits in the mantissa and n bits in the exponent, which of the following is true?
a. m determines epsilon; n determines overflow b. n determines epsilon; m determines overflow
c. n determines underflow; m determines overflow d. m determines the underflow; n determines overflow
e. m and n have nothing to do with epsilon and overflow

4. The *mantissa* of a float data type in memory consists of bits representing bit values in the range
a. 2^0 to 2^{-23} b. 2^{-0} to 2^{-23} c. 2^{-0} to 2^{-255} d. 2^{-1} to 2^{-23} e. 2^{-128} to 2^{+127}

5. For the `float` number stored in memory with a bias of 126, its base 10 value is
a. between 0.25 and 0.5 b. between 0.5 and 1.0 c. between 1.0 and 2.0 d. between 2 and 4
e. between 1.0 and 10.0

6. When 0.6 is stored as a float, its mantissa, including the hidden bit, has the decimal value (base 10)
a. 0.3 b. 0.6 c. 1.2 d. 2.4 e. none of the previous

7. The fractional error in any float number caused by mantissa roundoff is approximately
a. 2^0 b. 2^{-7} c. 2^{-23} d. 2^{-38} e. 2^{-127}

8. The value of the decimal fraction 11.125_{10} in binary is
a. 1011.010 b. 10.10110 c. 10.11101 d. 1011.001 e. 1011.110

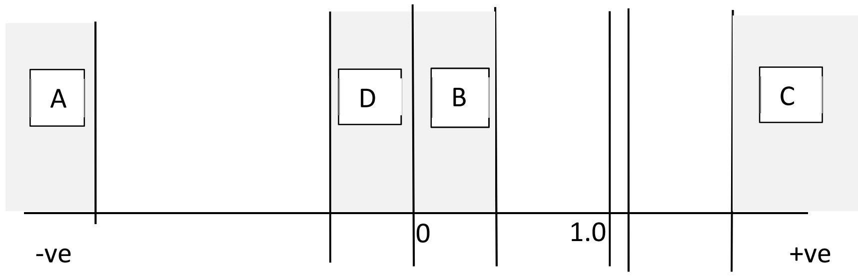
9. The next float number above $1+2^{-23}$ is
a. $2+2^{-23}$ b. $1+2^{-22}$ c. $1+2^{-7}$ d. 2^{-22} e. $1+2^{-24}$

10. The term fractional or relative error can be defined as
a. (approx. value – exact value)*exact value b. approx. value/exact value
c. exact value/(approx. value - exact value) d. approx. value – exact value/approx. value
e. none of the previous

11. Underflow for a floating point number means
a. it is smaller than epsilon b. it lies between 1.0 and (1.0 + epsilon) c. it is just larger than epsilon
d. it is too large to be registered in memory e. it is too small to be registered in memory

12. Which of the following is correct for the hexadecimal numbers composed from the bit field of a `float` number in memory?
a. the sign bit is combined with the first 3 bits of the exponent
b. the sign bit is combined with the first 3 bits of the mantissa
c. the hidden bit is combined with the first 3 bits of the exponent
d. the hidden bit is combined with the first 3 bits of the mantissa
e. the hidden bit and the sign bit are not stored in memory

Example Test 1 W18 CST8233 **NAME**



13. In the above graph of floating-point numbers what is the region A?
a. negative underflow b. positive underflow c. negative overflow d. positive overflow e. epsilon

14. In the above graph of floating-point numbers what is the region B?
a. negative underflow b. positive underflow c. negative overflow d. positive overflow e. epsilon

15. What is a difference between a Maclaurin and a Taylor series near the point x = a?
a. Maclaurin uses powers of x; Taylor uses powers of a
b. Maclaurin uses powers of a; Taylor uses powers of x
c. Maclaurin uses derivatives at x = 0; Taylor uses derivatives at x = a
d. Taylor uses derivatives at x = 0; Maclaurin uses derivatives at x = a
e. wrong, there is no difference between a Maclaurin and Taylor series at that point

16. Which of the following functions is NOT a good candidate for a Maclaurin series?
a. 1/x b. sin(x).cos(x) c. e^{2x} d. $e^x\sin(x)$ e. wrong, they are all good candidates

17. What are the first two terms of the Maclaurin series of $\frac{e^x}{2}$?
a. $\frac{1}{2} + e^x$ b. $1 + \frac{e^x}{2}$ c. $0.5 + \frac{x}{2}$ d. $\frac{1}{2} + x$ e. none of the previous

18. To improve the accuracy of a Maclaurin series approximation to a function you could
a. Increase to number of terms b. decrease the number of terms c. evaluate it farther from x = 0
d. increase the derivatives at x = 0 e. wrong, you can't improve the accuracy

19. In the following Maclaurin series for sin(x) near x = 0:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

what is the first order derivative at x = 0?
a. 0 b. 1 c. x d. -1 e. wrong, it doesn't have a first order derivative at x = 0.

20. A Maclauren series approximation to a function y = f(x) requires:
a. it has finite derivatives at x = 0 b. it has finite derivatives at x=1 c. it has finite derivatives at y = 0
d. it has infinite derivatives at y = 0 e. it has infinite derivatives at x = 0

21. A truncation error occurs when
a). a computer program terminates b) a double is rounded to a *float* c) the program casts a float to an int
d) a number is rounded in the computer e) an infinite mathematical series is terminated

22. What is true about the Maclaurin series representation of the sine and cosine functions?
a. sine series is more accurate near x = 0 but cosine series is more accurate near x = 1
b. sine series is more accurate near x = 1 but cosine series is more accurate near x = 0
c. cosine series has only odd powers of x and sine series has only even powers of x
d. sine series has only odd powers of x and cosine series has only even powers of x
e. a Maclaurin series cannot be used to represent either sine or cosine

23. What is the value of 6!/4! ?
a. 6 b. 12 c. 24 d. 30 e. 120

Short Questions

Show all your working to get partial marks

24. [10 marks] Show the derivation of the hexadecimal field in memory that is used to represent the `float` decimal number -0.5.

25. [10 marks]

25a [5 marks]

Derive the Maclaurin series for the first 3 non-zero terms of the function $\frac{1}{2}e^{\frac{x}{2}}$

25b. [5 marks]

Write down a numerical expression for the estimated % fractional error in your series from 25a at x = 1 when the series contains only 2 non-zero terms

Formulas

The general formula for Maclaurin Series:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0).x^k}{k!}$$

Maclaurin Series of Particular Functions:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad \text{for all } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

Derivatives:

$$\frac{d(\sin(x))}{dx} = \cos(x); \frac{d(\cos(x))}{dx} = -\sin(x); \frac{d(e^{ax})}{dx} = ae^{ax} \qquad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$