Interval Graph

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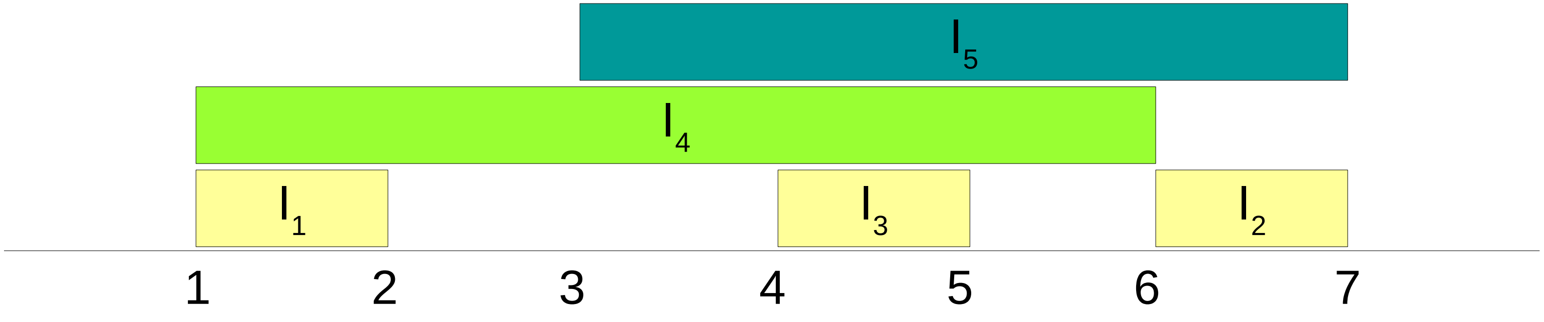
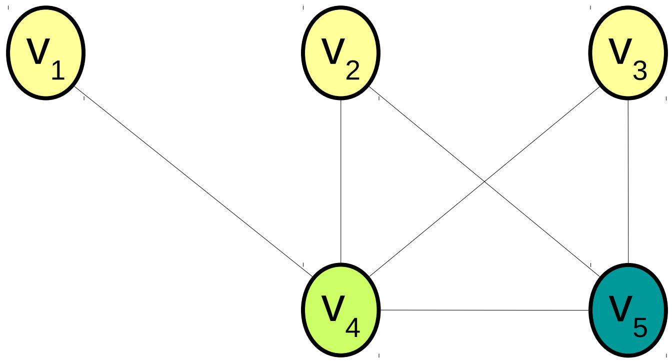
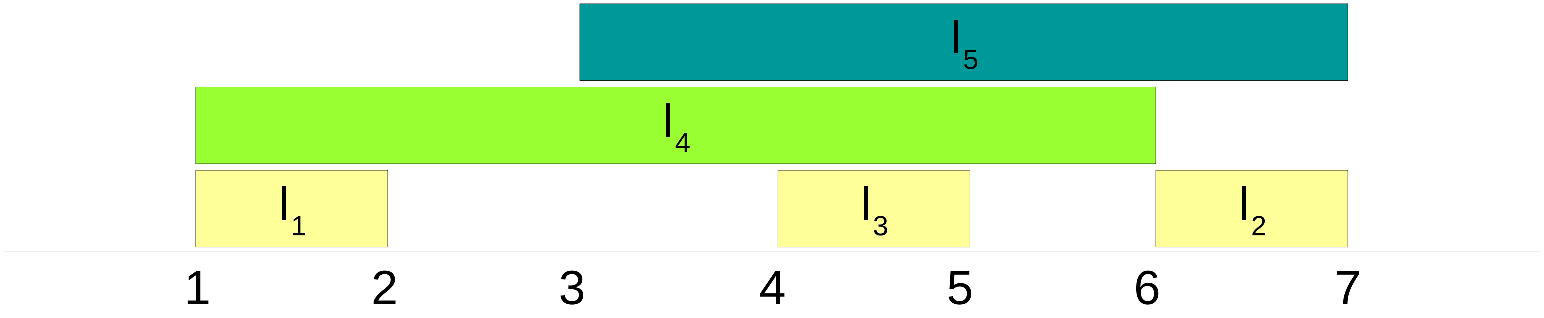


Interval Graph





Online Interval Graph Coloring

In the online setting, there is an order

$[\ I_1,\ I_2,\ I_3, \dots\dots\dots\dots, I_n\ ]$

and intervals appear in this order

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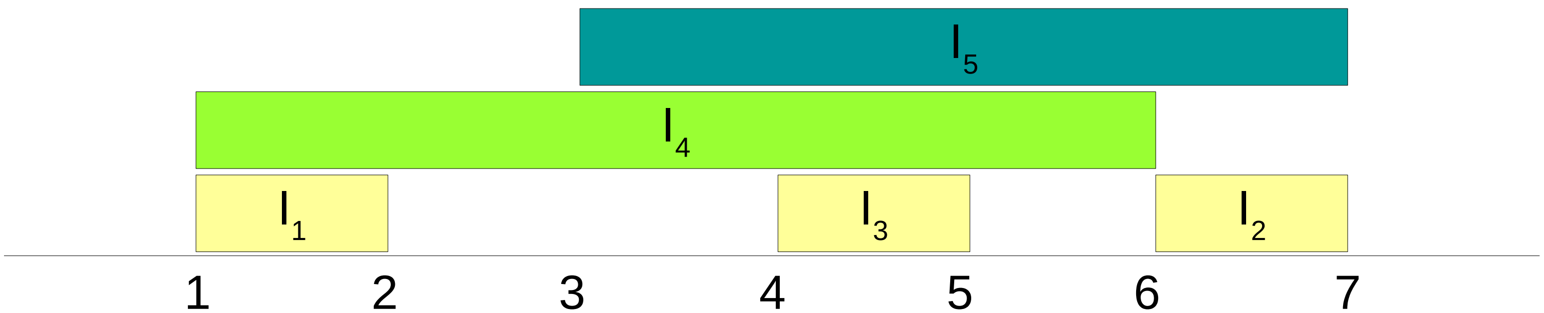
Order : $[\ I_1, I_2, I_3, I_4, I_5\ ]$



Online Interval Graph Coloring

In the online setting, there is an order

$[I_1, I_2, I_3, \ldots\ldots\ldots, I_n]$

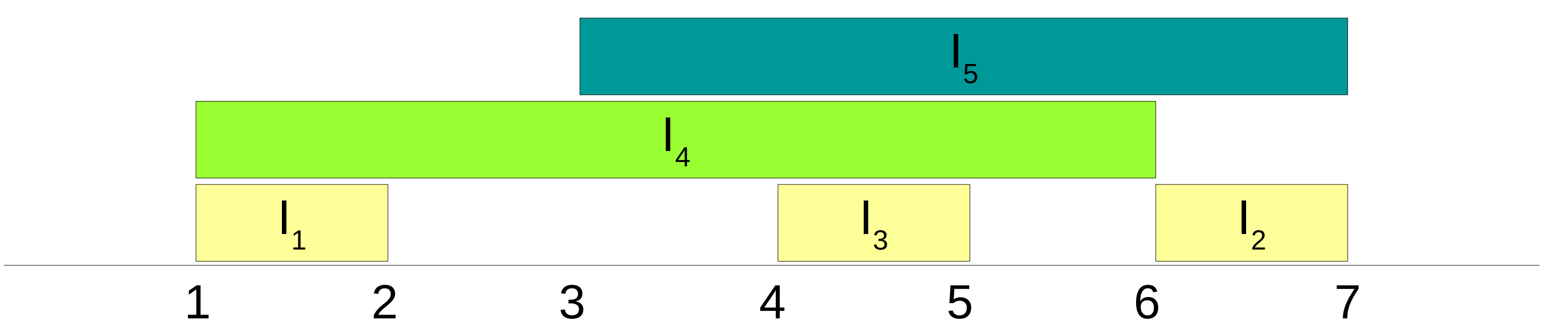
and intervals appear in this order

A color is assigned to the interval I_i before the appearance of interval I_{i+1}

Goal is to use as few colors as possible without recoloring any of the intervals from

$I_1, I_2, I_3, \ldots\ldots\ldots\ldots\ldots I_{i-1}$

Order : $[I_1, I_2, I_3, I_4, I_5]$



Online Interval Graph Coloring

Kierstead and Trotter presented a 3-competitive algorithm [KT]

Kierstead and Trotter also proved that their result is optimum

[KT]: Henry A Kierstead and William T Trotter, An extermal problem in recursive combinatorics, Congressus Numerantium, 1981

Overview of KT-algorithm

color assigned to vertex v is a tuple (p(v), o(v))

p(v) is called position or level
o(v) is called offset

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Overview of KT-algorithm

color assigned to vertex v is a tuple (p(v), o(v))
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Key property is that for every edge {u, v}
(p(u), o(u)) is different from (p(v), o(v))

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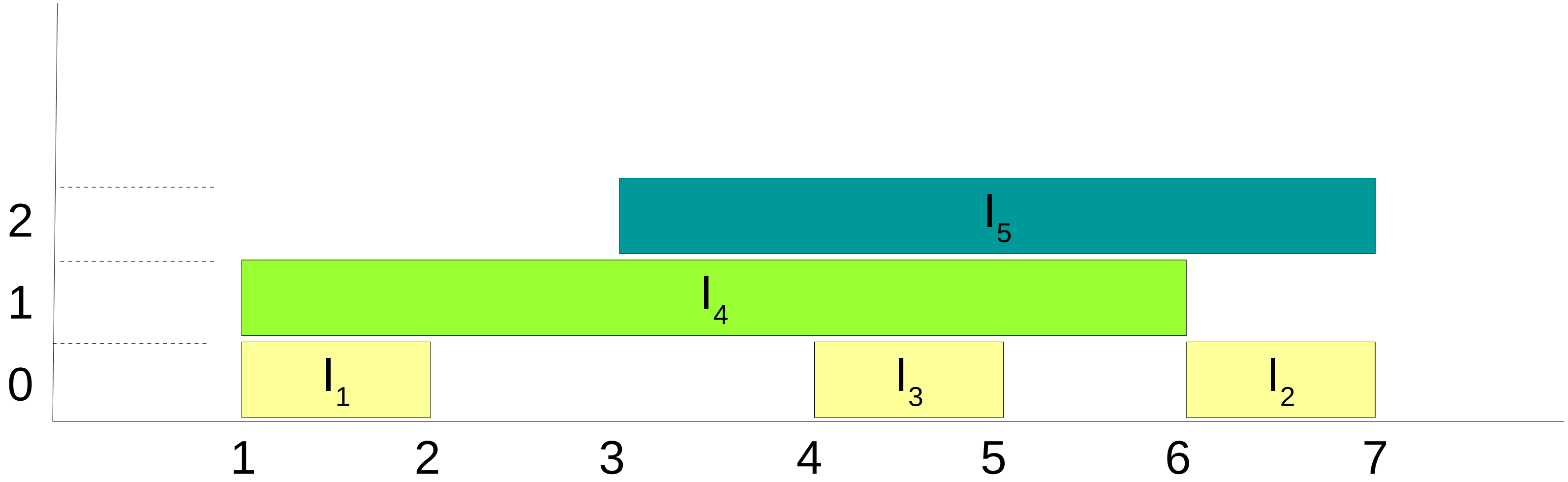
Overview of KT-algorithm

Imagine X-Y plane

Intervals from X-axis
Levels are integer points on Y-axis



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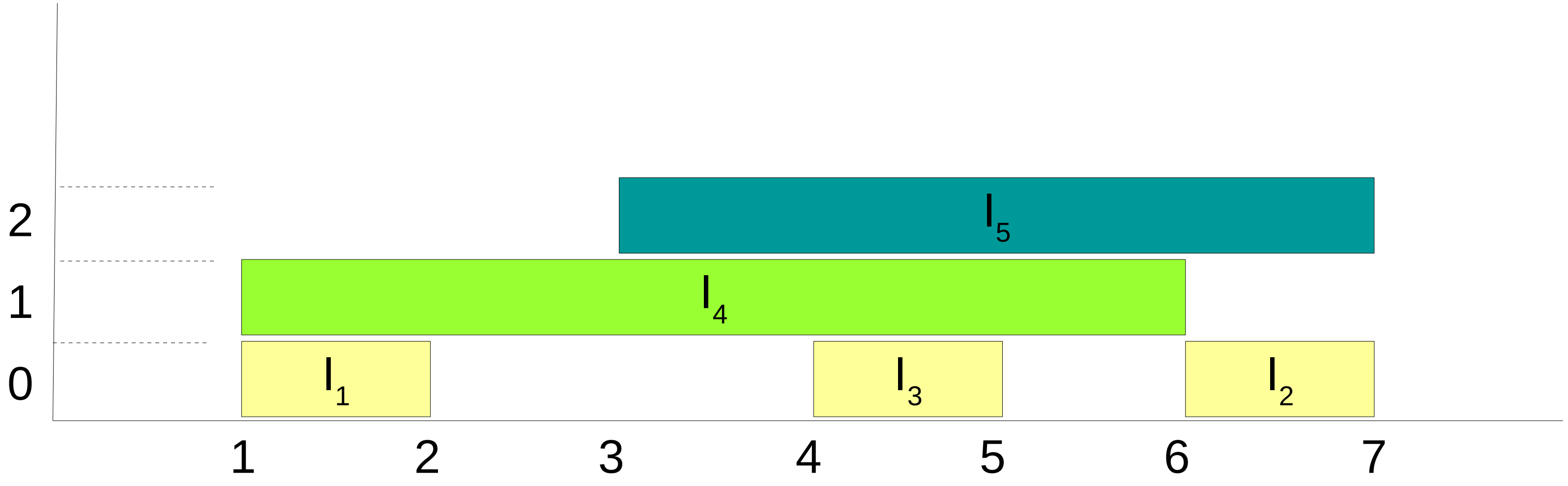

How to compute p(v) and o(v)?

Overview of KT-algorithm

Computing p(v)

I_i is the interval appearing and v_i is the corresponding vertex in the interval graph G

Overview of KT-algorithm

Computing p(v)

$\mathbf{G_r(v_i)}$ is the **induced subgraph** of G on

$$\mathbf{\{v_j | v_j \in V(G), j < i, p(v_j) \le r, (v_i, v_j) \in E(G)\}}$$

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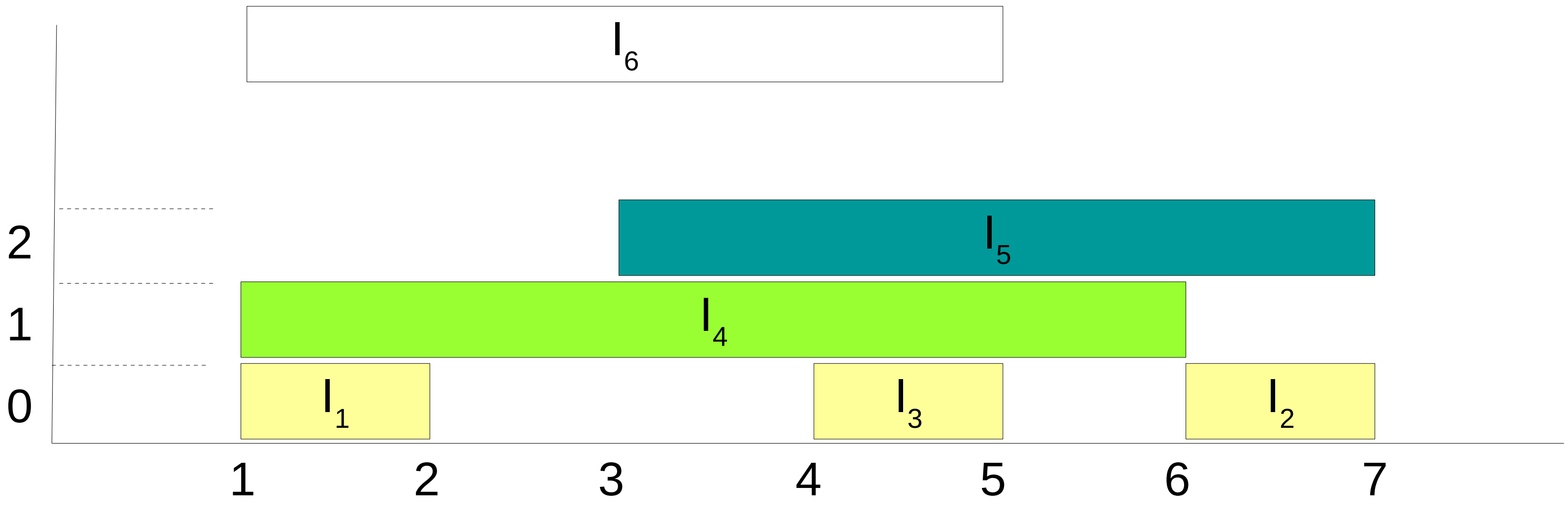

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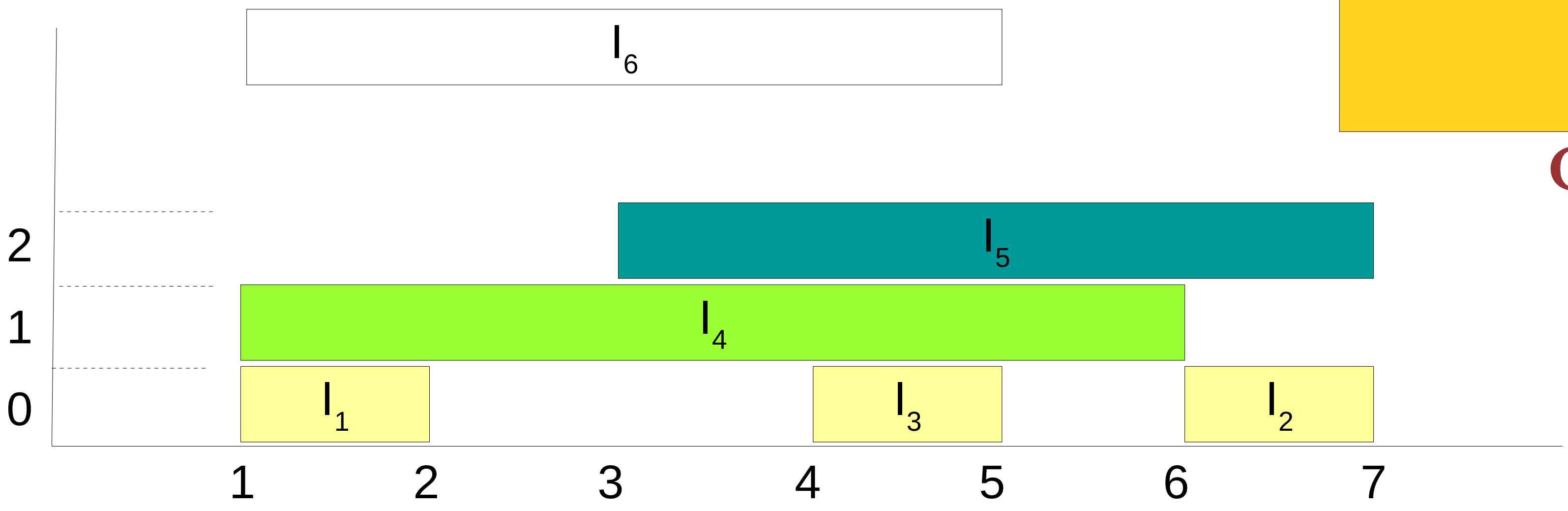

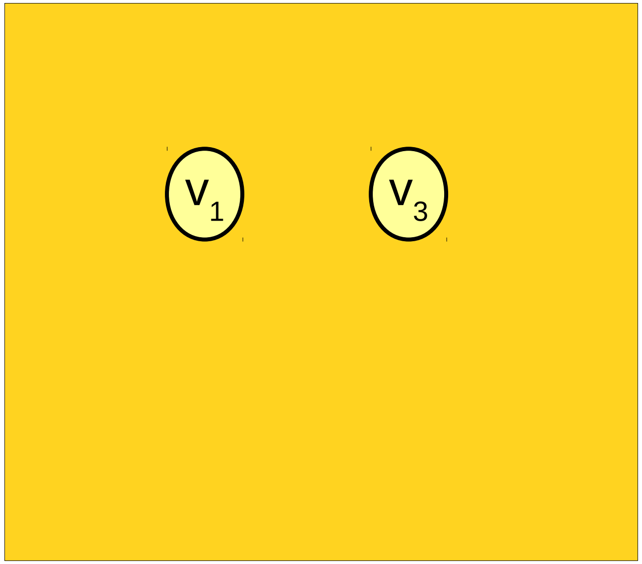

$\mathbf{G_0(v_6)}$

Overview of KT-algorithm

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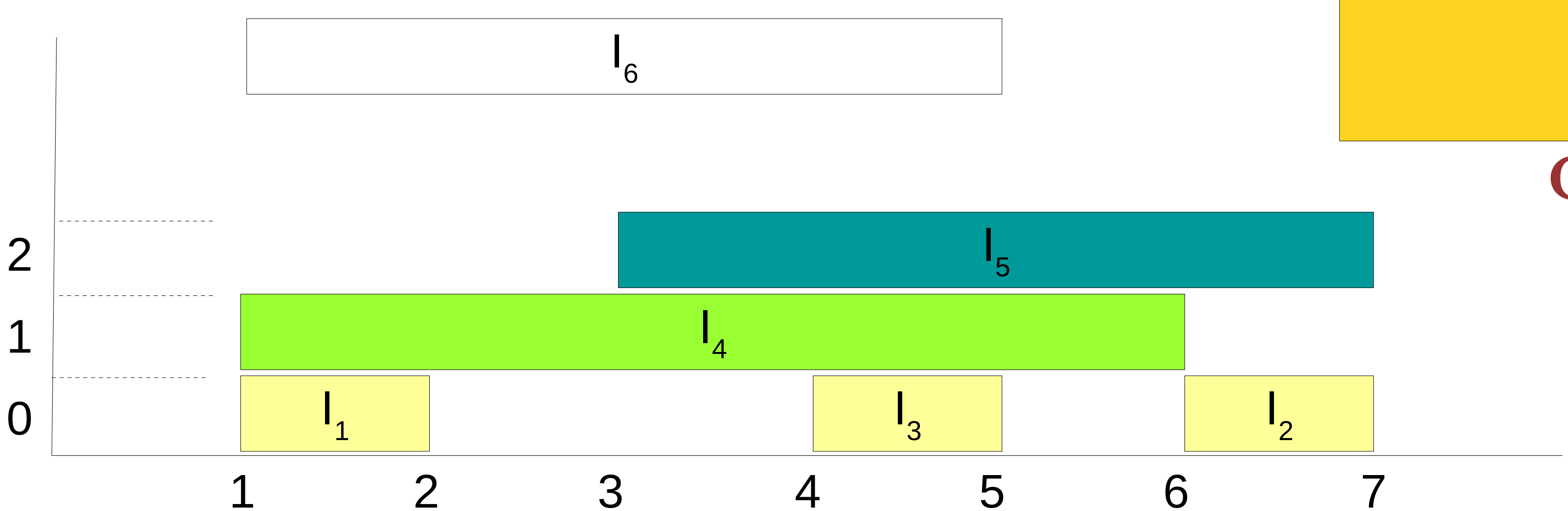
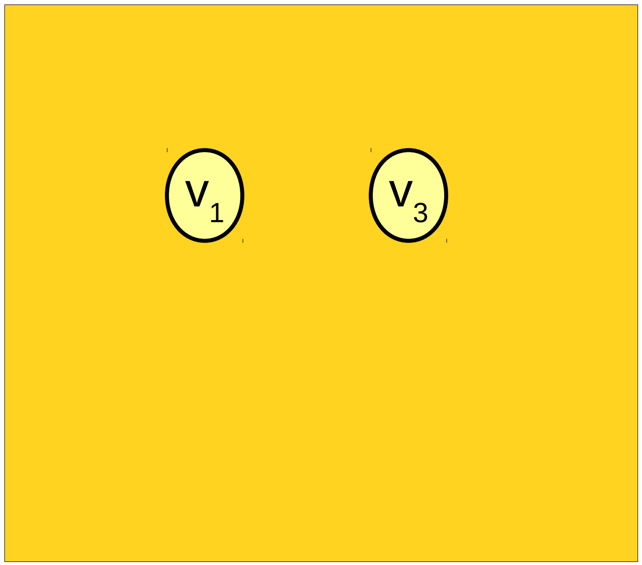
$$\mathbf{p(v_i)} = \min\{\mathbf{r} | \omega(\mathbf{G_r(v_i)}) \leq \mathbf{r}\}$$





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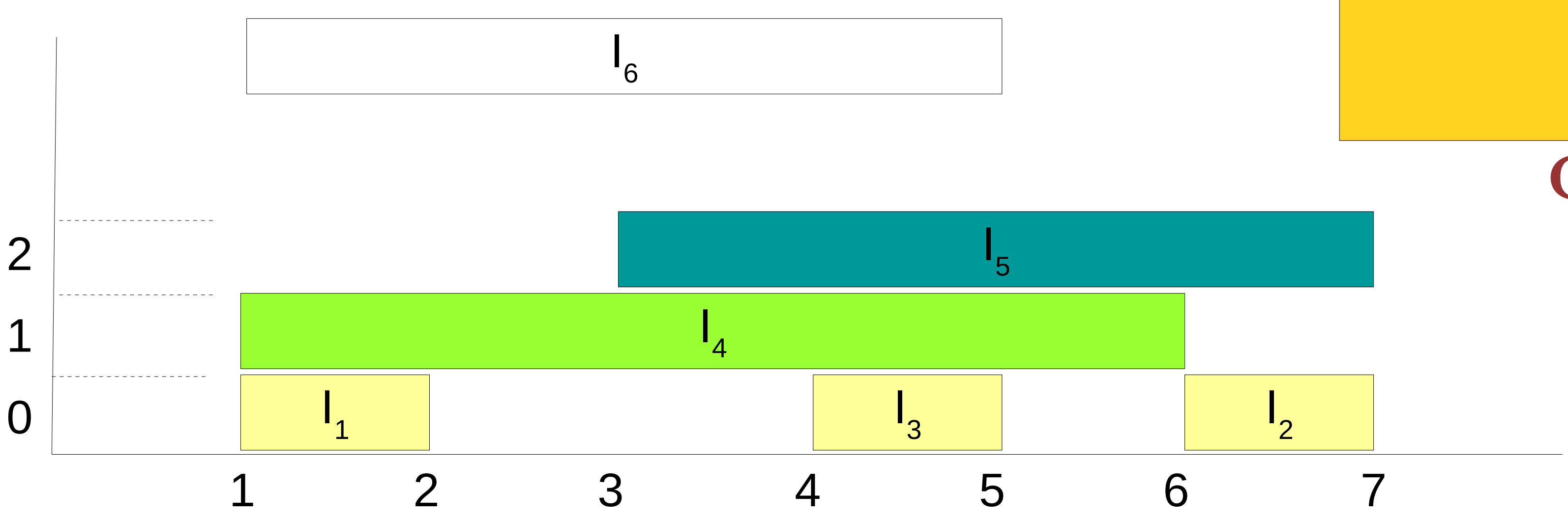
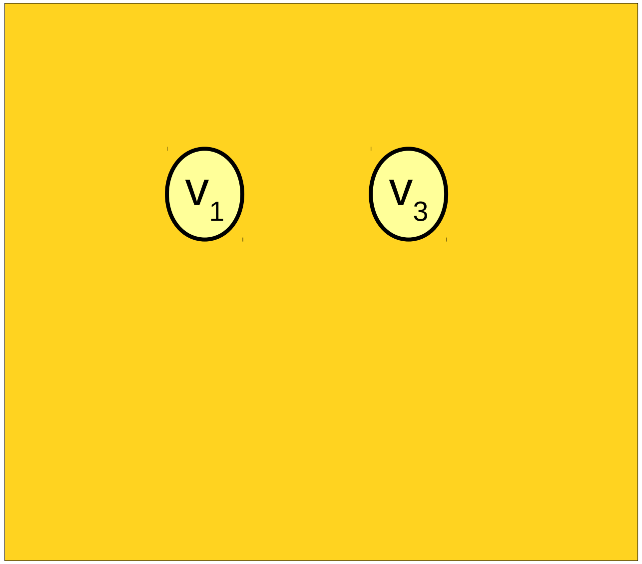
Maximum clique size





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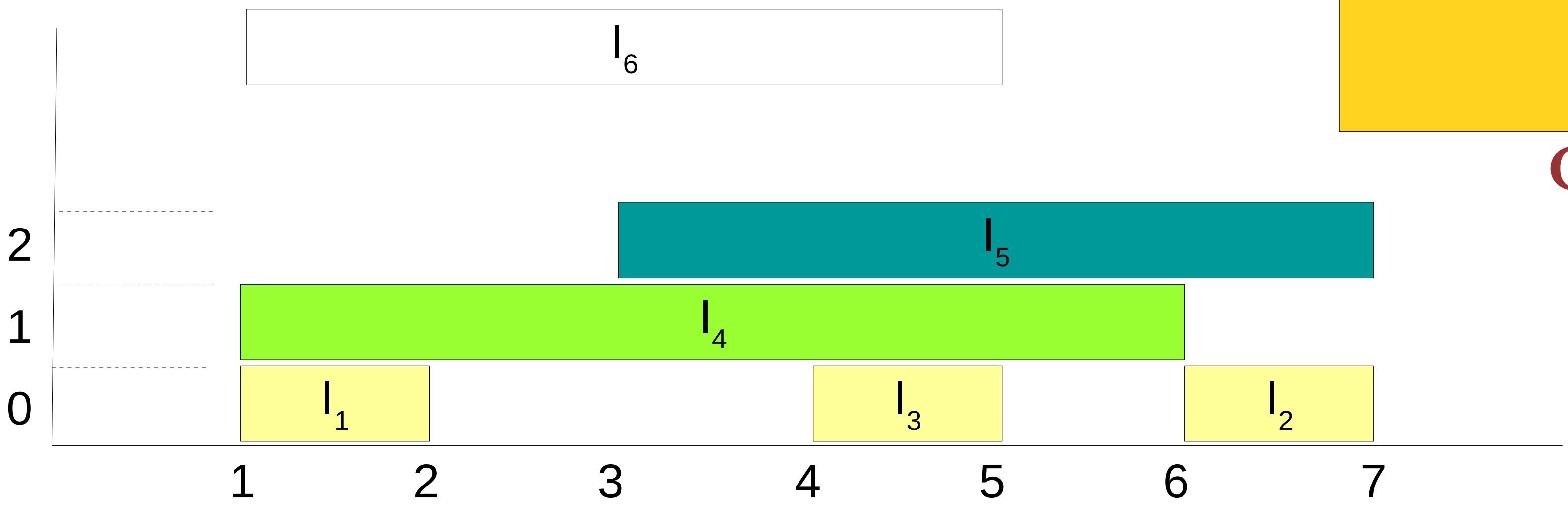
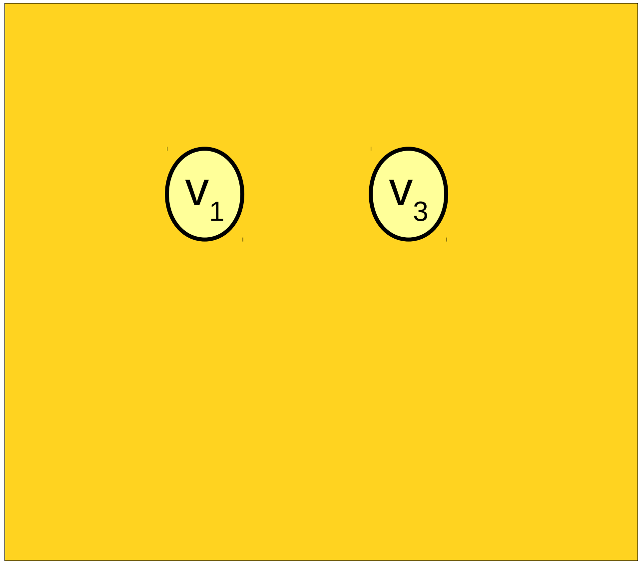
$$\mathbf{p(v_i)} = \min\{\mathbf{r} | \omega(\mathbf{G_r(v_i)} \leq \mathbf{r}\}$$

$$\omega(\mathbf{G_0(v_6)}) = \mathbf{1}$$





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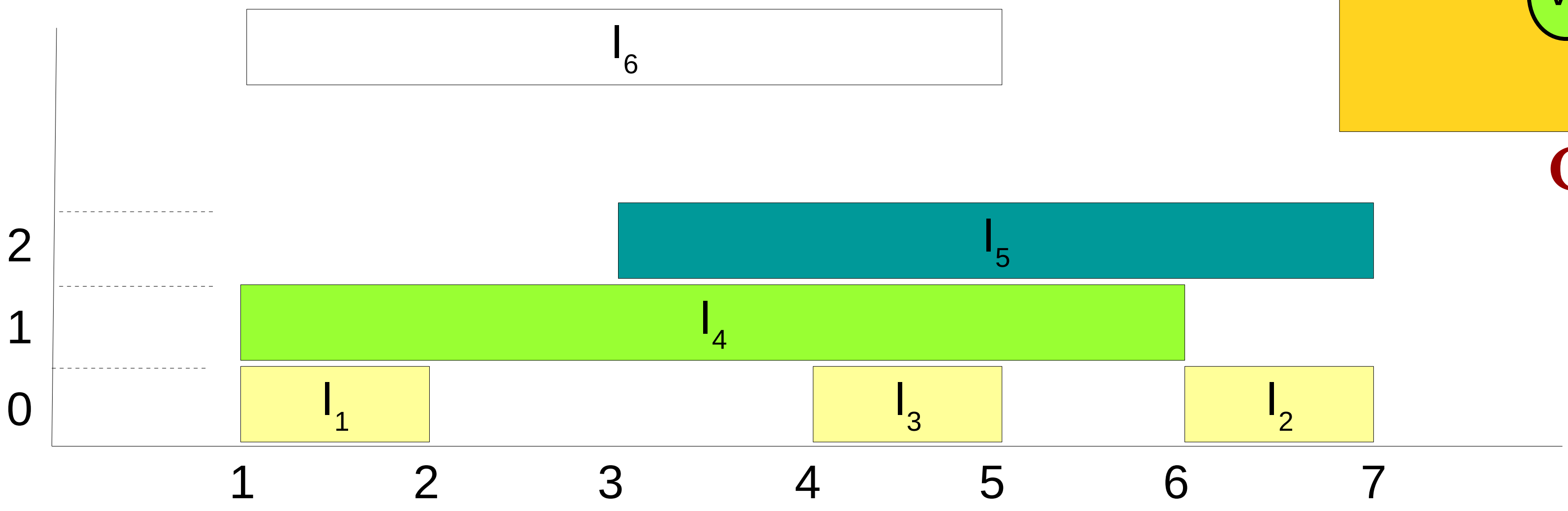
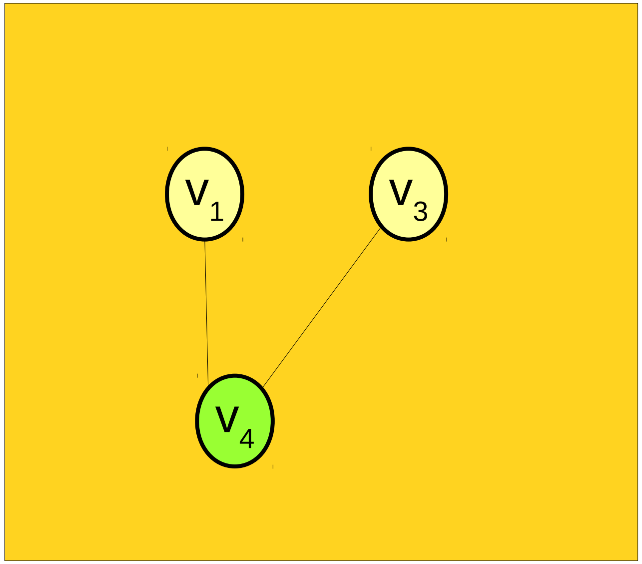
$\mathbf{G_r(v_i)}$ is the induced subgraph of G on $\{\mathbf{v_j} | \mathbf{v_j} \in \mathbf{V(G)}, \mathbf{j} < \mathbf{i}, \mathbf{p(v_j)} \le \mathbf{r}, (\mathbf{v_i}, \mathbf{v_j}) \in \mathbf{E(G)}\}$

$$\mathbf{p(v_i)} = \min\{\mathbf{r} | \omega(\mathbf{G_r(v_i)}) \le \mathbf{r}\}$$

$$\omega(\mathbf{G_1(v_6)}) = \mathbf{2}$$





$\mathbf{G_1(v_6)}$

Overview of KT-algorithm

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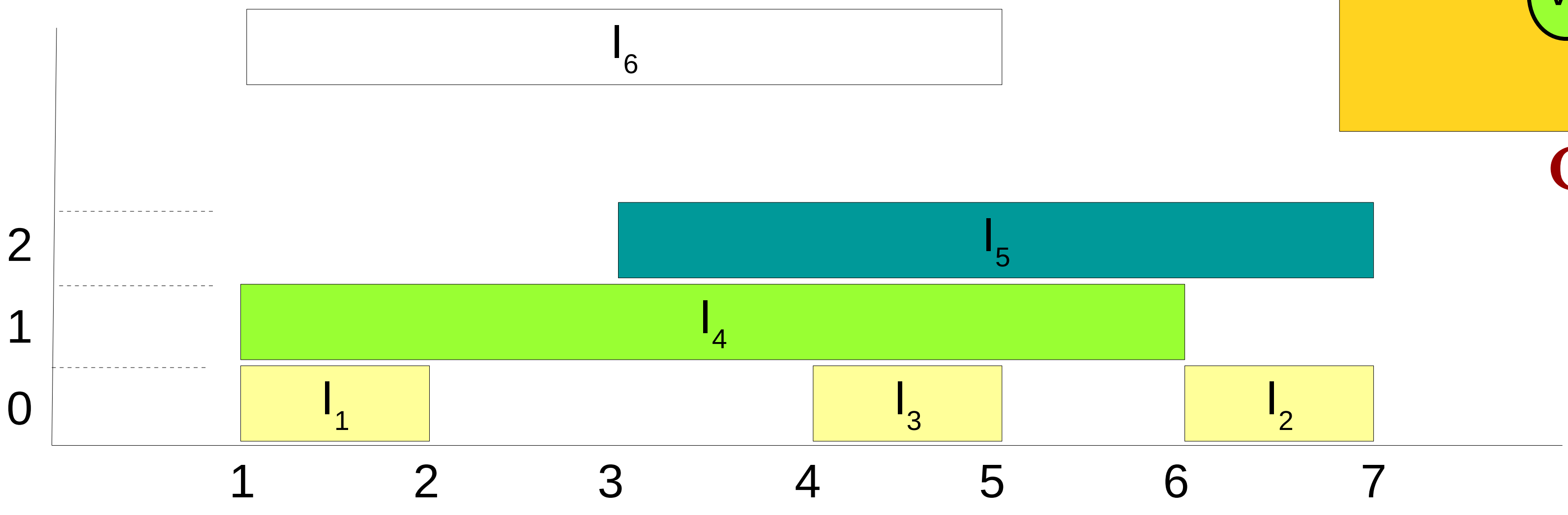
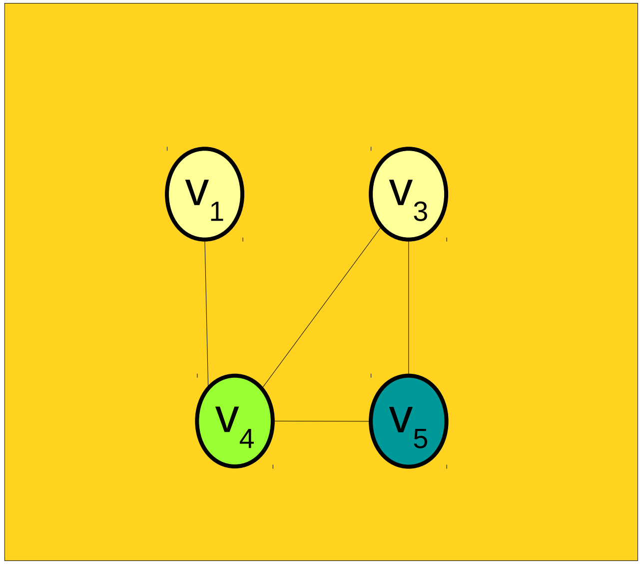
$$\mathbf{p(v_i)} = \min\{\mathbf{r} | \omega(\mathbf{G_r(v_i)}) \leq \mathbf{r}\}$$

$$\omega(\mathbf{G_2(v_6)}) = \mathbf{3}$$





$\mathbf{G_2(v_6)}$

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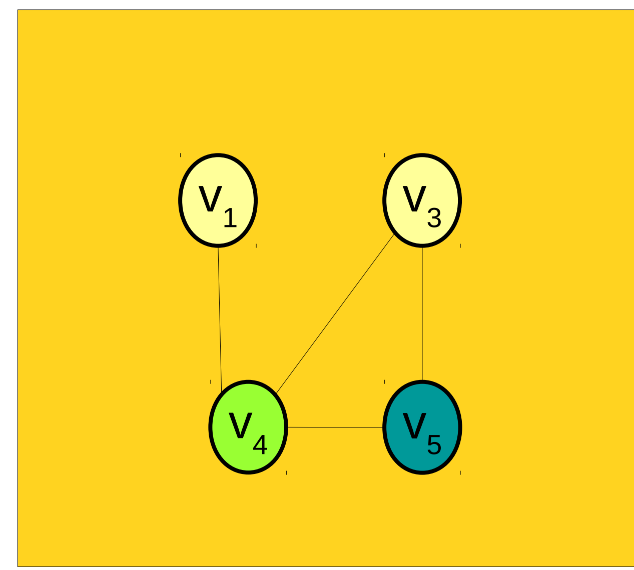
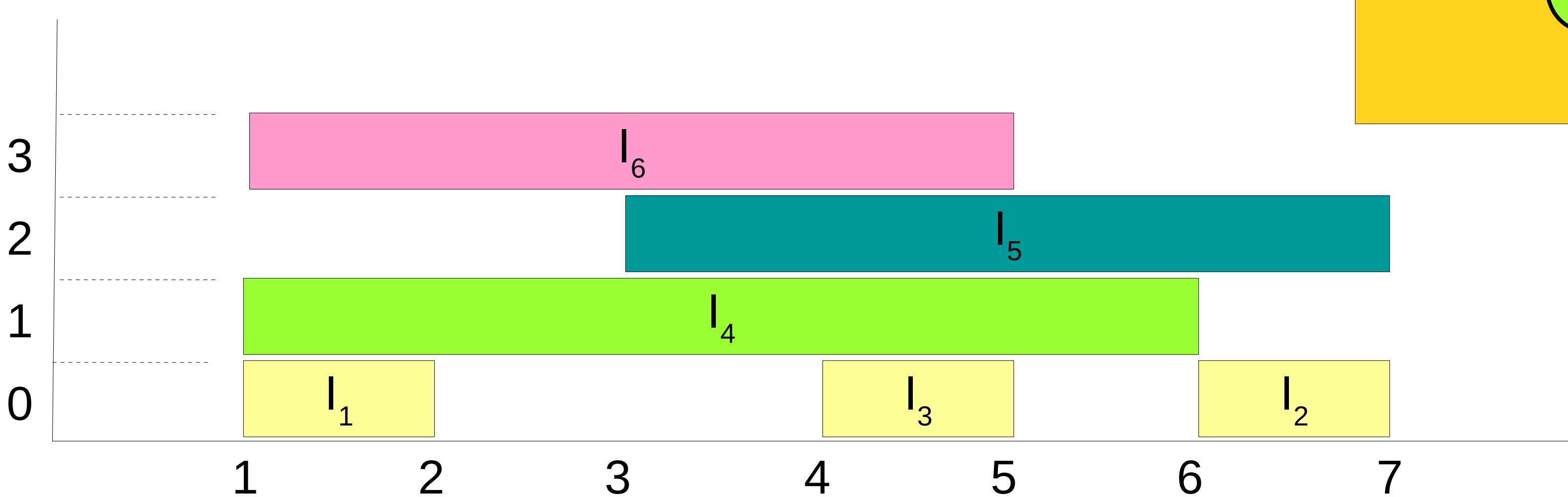
$$\omega(\mathbf{G_3(v_6)}) = \mathbf{3}$$



$\mathbf{G_3(v_6)}$



Overview of KT-algorithm

Computing p(v)

Compute : $\mathbf{G_r(v)}$ for $\mathbf{r \geq 0}$

Assign : $\mathbf{p(v) = \min\{r | \omega(G_r(v)) \leq r\}}$

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Properties satisfied by p(v)

P1: $\mathbf{p(v)} \leq \omega - \mathbf{1}$

P2: $\{\mathbf{v} | \mathbf{p(v)} = \mathbf{0}\}$ is an independent set

P3: Induced subgraph on $\{\mathbf{v_j} | \mathbf{p(v_j)} = \mathbf{i}\}$ has max degree 2

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Computing o(v)

Vertex v has at most 2 neighbors such that their level is p(v).
Compute the smallest value in the set {1,2,3} different from the offset of these neighbors and set it to o(v)