Greedy Color Completion

- Given
  - Intervals $I = \{ I_1 \sqsubset \ldots \sqsubset I_{2k} \}$,
  - $\omega(I) \leqslant k$,
  - $c' : \text{prefix}_\sqsubset I(I_k) \to [k]$ be a proper k-coloring on $\text{prefix}_\sqsubset I(I_k)$

- Task is to assign color to next k intervals

- Observation
  - Let $I = \{I_1 \sqsubset \ldots \sqsubset I_m\}$ be a multiset of unit intervals. If $\omega(I) < m$, then the extremal intervals are disjoint, i.e., $I_1 < I_m$.

Greedy Color Completion

- Algorithm
  - Let J = { $I_l, I_{l+1}, \dots I_k$ }, intervals that intersects I_k
  - Number of elements in J = k-(l-1)
  - Coloring of J requires k-(l-1) colors.
  - Now color next l-1 intervals with the other (l-1) colors, i.e. infix $\sqsubset I(I_{k+1}, I_{k+l-1})$
  - Now set $c(I_i) = c(I_{i-k})$ for $i \in \{k + l, \dots, 2k\}$ because by observation 1 intervals I_i and I_{i-k} do not intersect.

- Hence we colored 2k intervals from given k coloring.

Modulo Color Completion

- Given
  - n Intervals
  - $\omega(I) \leqslant k$,
  - Coloring of starting K intervals
  - K intervals assumes proper coloring

- Task is to assign color to every interval

- Algorithm
  - Apply Greedy Color Completion Algorithm on next K intervals recursively till no intervals are left.