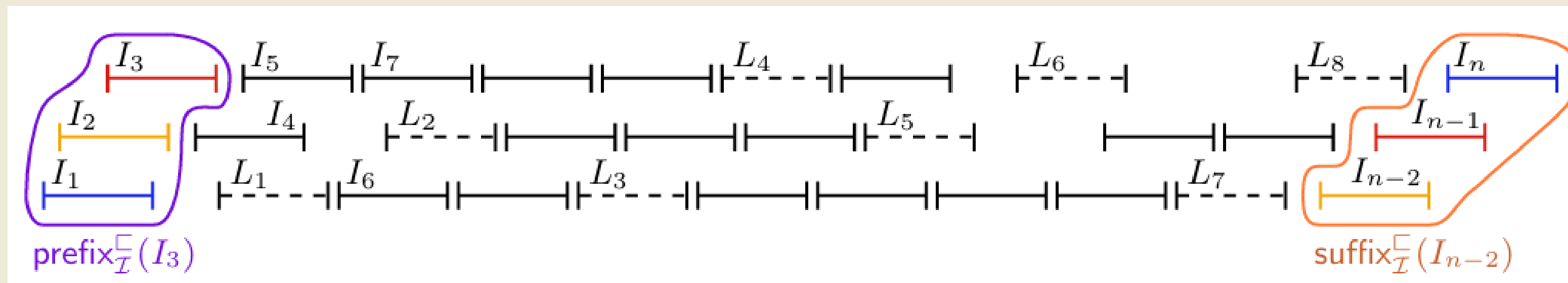




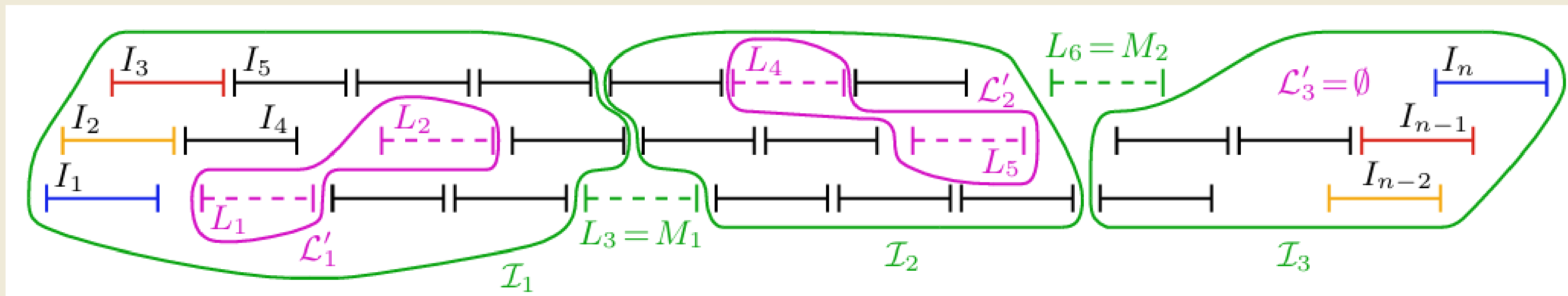
# Unit Precoloring Extension Problem

- Given
  - $I = \{ I_1 \sqsubset \dots \sqsubset I_m \}$ ,  $m > 2k$
  - Proper  $k$ -coloring  $c'$  on prefix  $\sqsubset I(k) \rightarrow$  the first  $K$  intervals
  - Proper  $k$ -coloring  $c''$  on suffix  $\sqsubset I(m-k+1) \rightarrow$  the last  $K$  intervals
- Assumption
  - We can fit between the prefix and the suffix additional  $k^2 - 1$  mutually disjoint unit intervals without increasing the chromatic number of  $I$
  - Multiset of unit intervals  $I$  is connected as a graph
  - Because of these assumption it is not NP-hard
- Task is to assign color to every interval while retaining  $c'$  and  $c''$

- Introduce  $L = \{ L_1 < L_2 < \dots < L_{k^2-1} \}$
- Divide  $L$  such that
  - $L = L_1 \cup \{M_1\} \cup L_2 \cup \dots \cup \{M_{k-1}\} \cup L_k$ , where
    - $M_i = L_{ik}$  for  $i \in [k-1]$
    - $|L_i| = k - 1$
- $\text{prefix} \sqsubset I(l_k) \sqsubset L_1 < \{M_1\} < L_2 < \dots < \{M_{k-1}\} < L_k \sqsubset \text{suffix} \sqsubset I(l_{n-k+1})$



- Now, partition  $I$  into  $k$  parts:  $I = I_1 \cup I_2 \cup \dots \cup I_k$ , such that
  - $\text{prefix} \sqsubset I(I_k) \subseteq I_1 \sqsubset \{M_1\} \sqsubset I_2 \sqsubset \dots \sqsubset \{M_{k-1}\} \sqsubset I_k \supseteq \text{suffix} \sqsubset I(I_{n-k+1})$
- Now, we add few  $L_i$ 's in  $I_i$ 's such that it is multiple of  $K$ , i.e.
  - $J_i = I_i \cup L_i'$
- $J_1, J_2, \dots, J_k$  satisfies
  - $|J_i| = k p_i$ ,  $p_i$  is a natural number
  - $\text{prefix} \sqsubset I(I_k) \subseteq J_1 \sqsubset \{M_1\} \sqsubset J_2 \sqsubset \dots \sqsubset \{M_{k-1}\} \sqsubset J_k \supseteq \text{suffix} \sqsubset I(I_{n-k+1})$



- Now we have  $J_1, J_2, \dots, J_k$ 
  - Apply Modulo Color Completion algorithm for  $J_i$  using suffix of  $J_{i-1}$
  - If we don't use  $M_{i-1}$  then
    - prefix of  $J_i$  will copy suffix of  $J_{i-1}$  as it is
  - If we use  $M_{i-1}$  then
    - it shifts all colors in the permutation down by one
  - Assumption
    - At each stage  $i$ , the algorithm has already matched the permutation given by  $c''$  on the last  $i-1$  positions.
    - So after applying Modulo Color Completion, it will match on the last  $i$  positions.

