ANALYSIS OF RLC CIRCUIT

BY

AMAN CHHAJED

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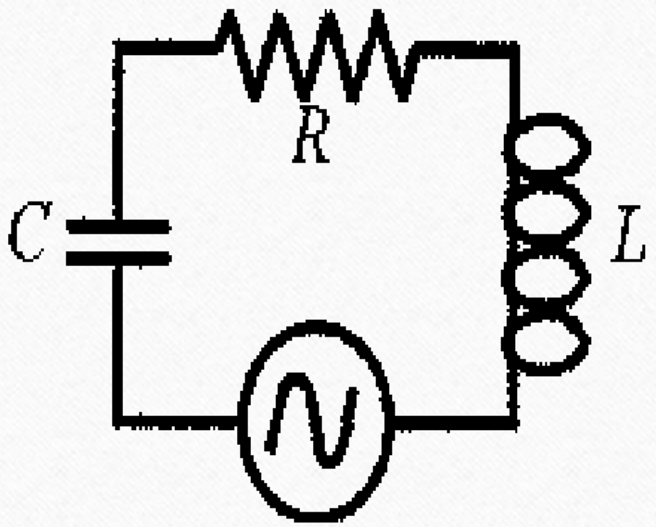
Basic RLC Circuit

L= Inductance
R=Resistance
C=Capacitance
V = Voltage
q=Charge
dq/qt=I



FARADAY'S LAW

Voltage drop across the inductor $= V_L$

$$V_L = L\frac{di}{dt}$$

OHM'S LAW

Voltage drop across the Resistor $= V_R$

$$V_R = RI$$

COULOMB'S LAW

The Voltage drop across the Capacitor=

$$V_C = \frac{1}{c}\, q$$

KIRCHHOFF'S VOLTAGE LAW

$$V_L + V_R + V_C = V$$

2^{nd} order differential equation:

- For the case where the source is an unchanging voltage, differentiating with respect to t and dividing by L leads to the second order differential equation:

$$I'' + \frac{R}{L} I' + \frac{1}{LC} I = 0$$

The differential equation

- Substituting V = IR in the previous equation we get

$$V'' + \frac{R}{L}V' + \frac{1}{LC}V = 0$$

The characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Eigenvalues of the characteristic equation:

$$S = -\frac{R}{2L} \mp \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Three Cases

Overdamped $\dfrac{R^2}{L^2} - \dfrac{4}{LC} > 0$

Critically damped $\dfrac{R^2}{L^2} - \dfrac{4}{LC} = 0$

Underdamped $\dfrac{R^2}{L^2} - \dfrac{4}{LC} < 0$

Three Cases

Over damped $R > \sqrt{\frac{4L}{C}}$

Critically damped $R = \sqrt{\frac{4L}{C}}$

Under damped $R < \sqrt{\frac{4L}{C}}$

Illustration

$$V'' + \frac{R}{L}V' + \frac{1}{LC}V = 0$$

V(0) = 3
V’(0) = 0

L = 0.003 C = 1e-10

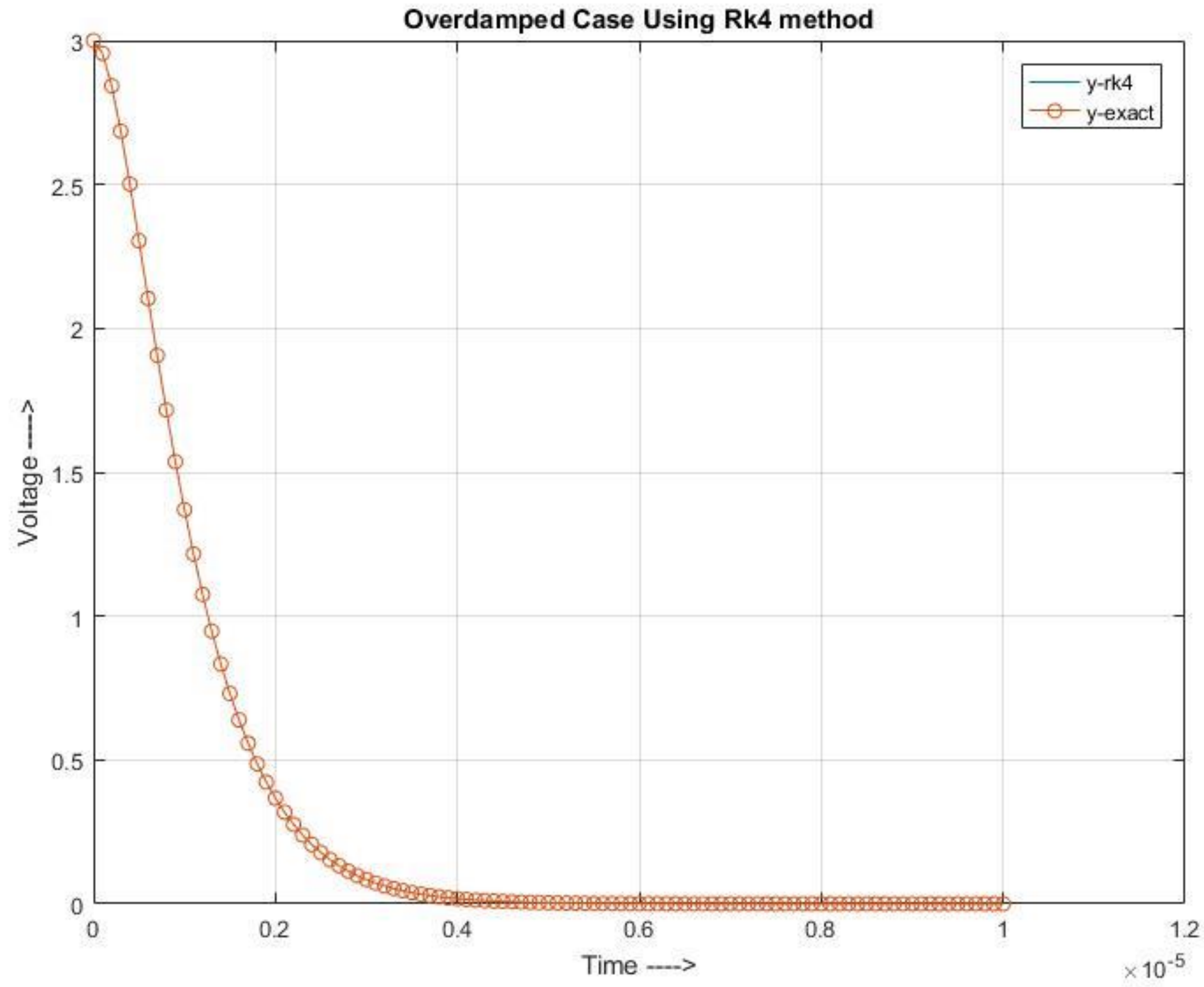
For critically damped case,

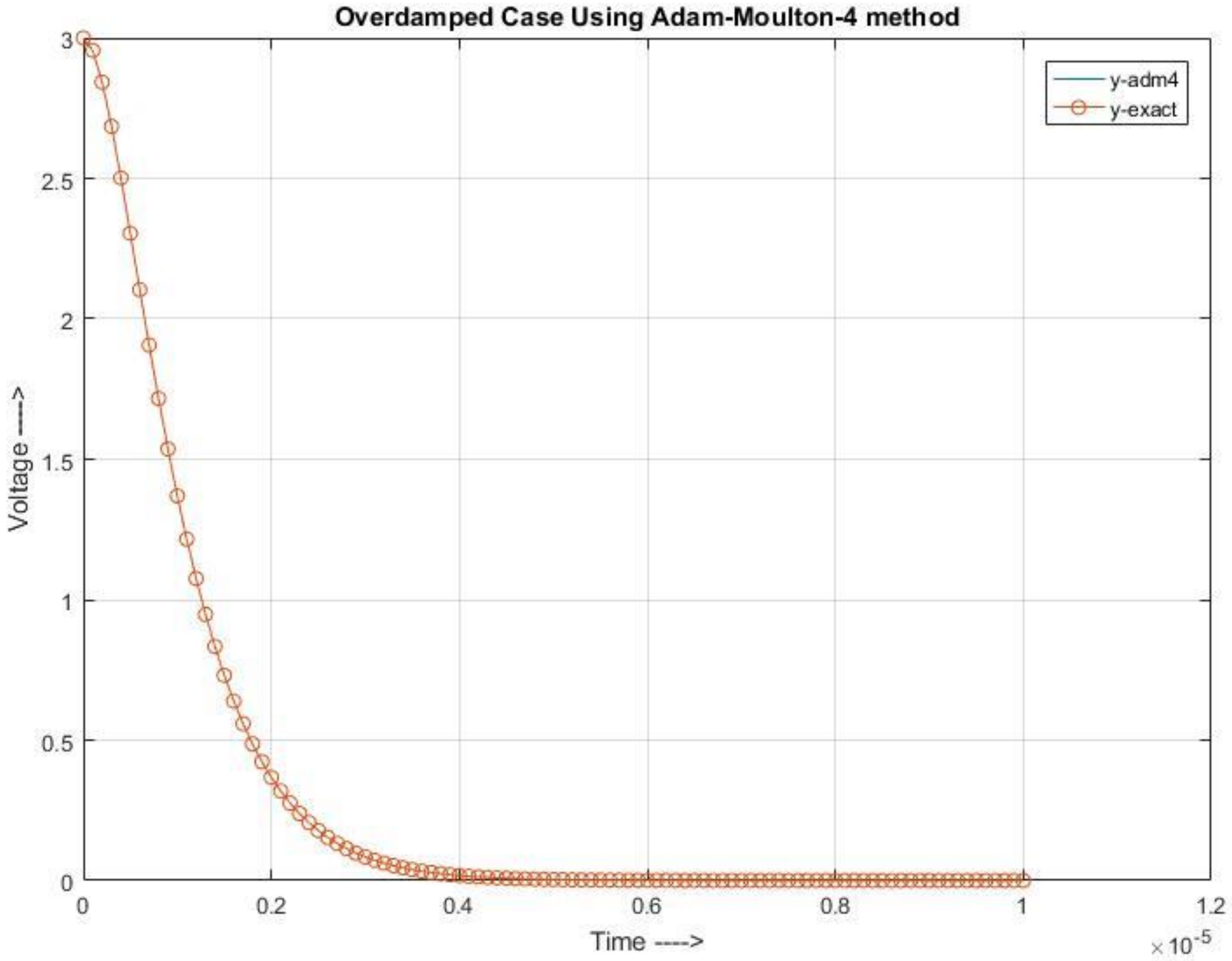
$$\frac{R^2}{L^2} - \frac{4}{LC} = 0$$

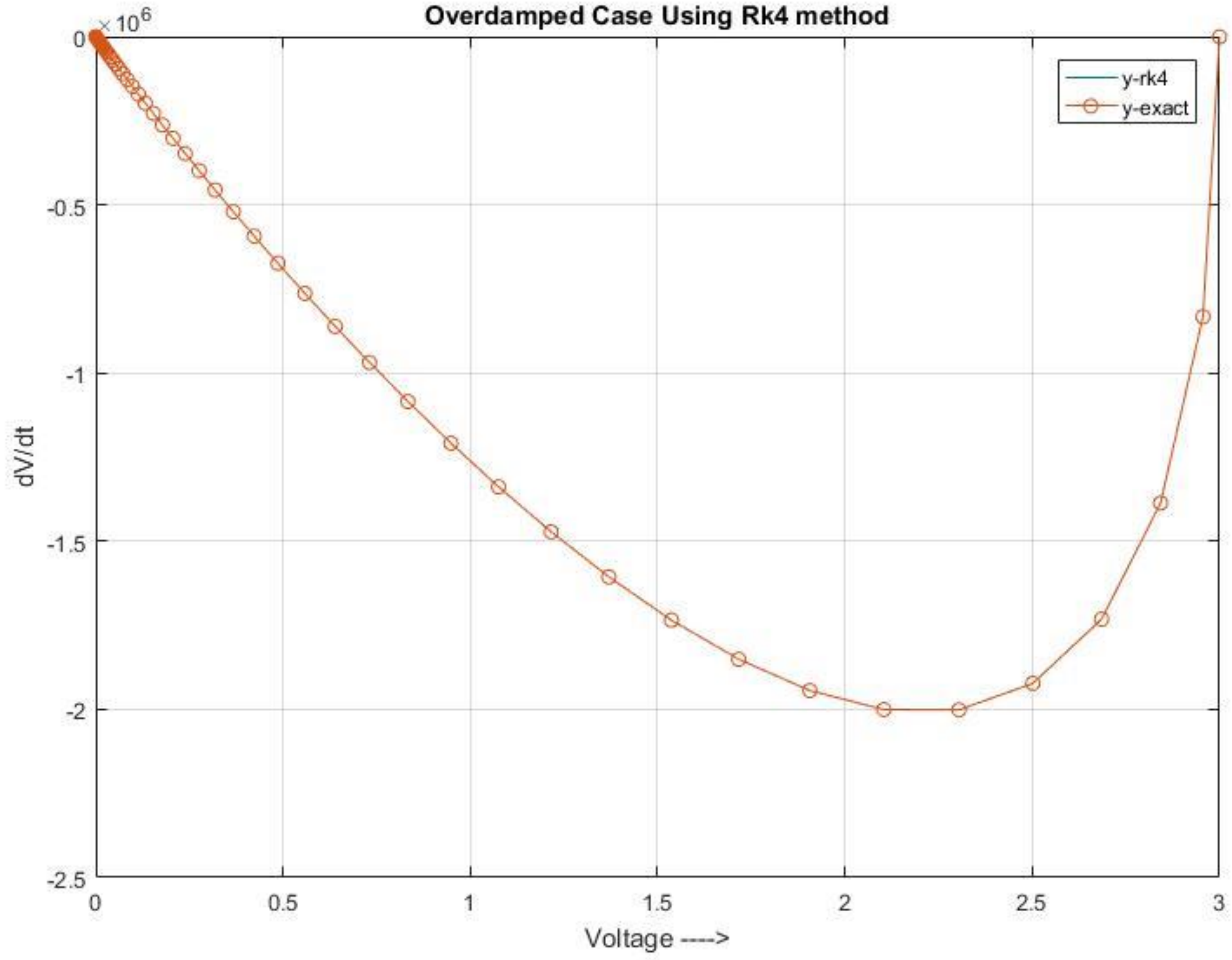
$$R = \sqrt{\frac{4(0.003)}{(1*10^{-10})}} = 10954.45$$

Overdamped Case

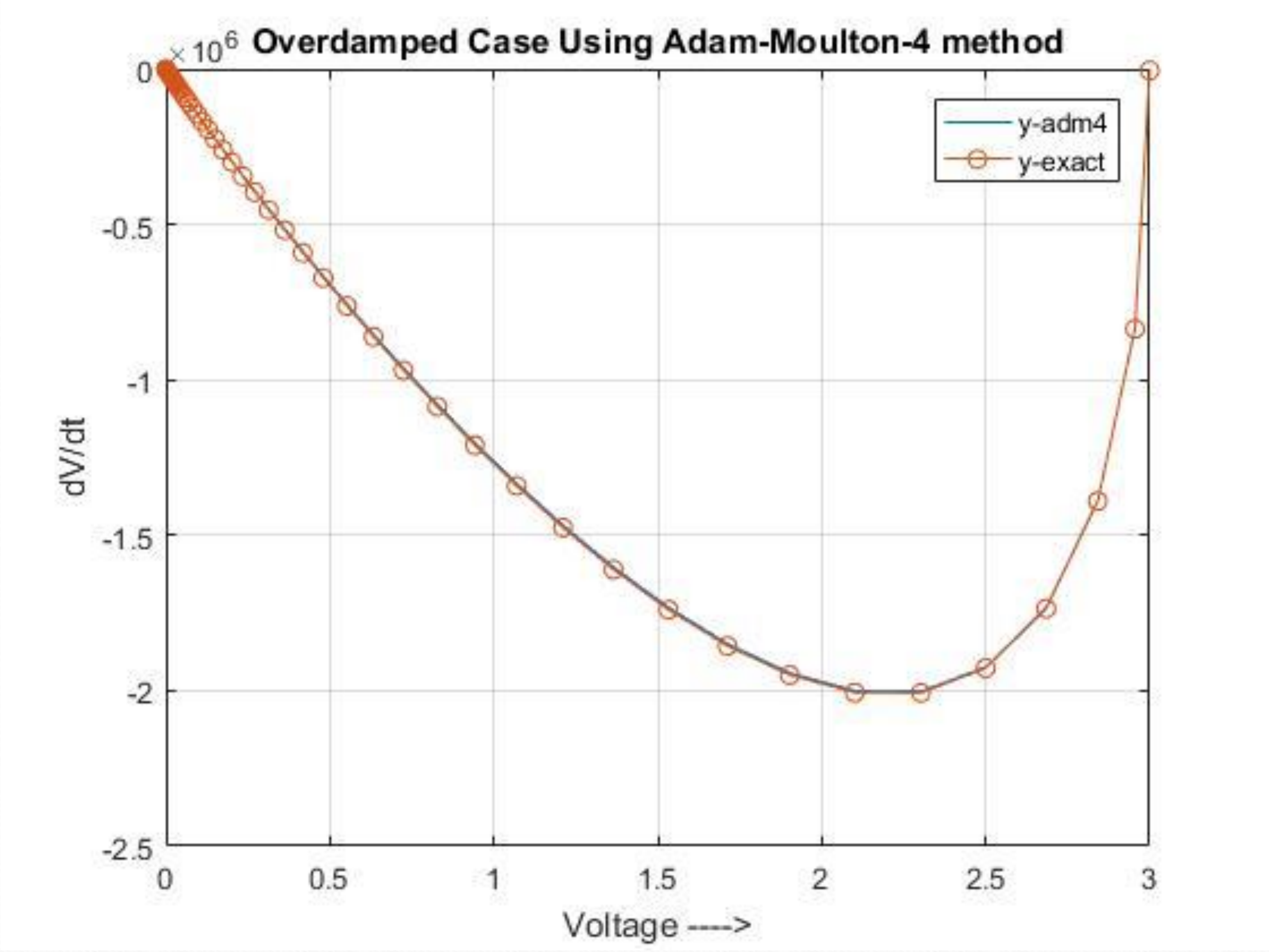
- R = 11000 (>10954.45)
- 2 real roots of the characteristic equation.
- S_1 = -1.6667e+6
- S_2 = -2e+6
- The voltage is not oscillatory and decays to equilibrium.
- Energy loss in resistor is faster than energy transferred.


Overdamped Case Using Rk4 method
y-rk4
y-exact
Voltage ----->
Time ---->
× 10^-5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2


Overdamped Case Using Adam-Moulton-4 method
y-adm4
y-exact
Voltage ---->
Time ---->
$\times 10^{-5}$
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2



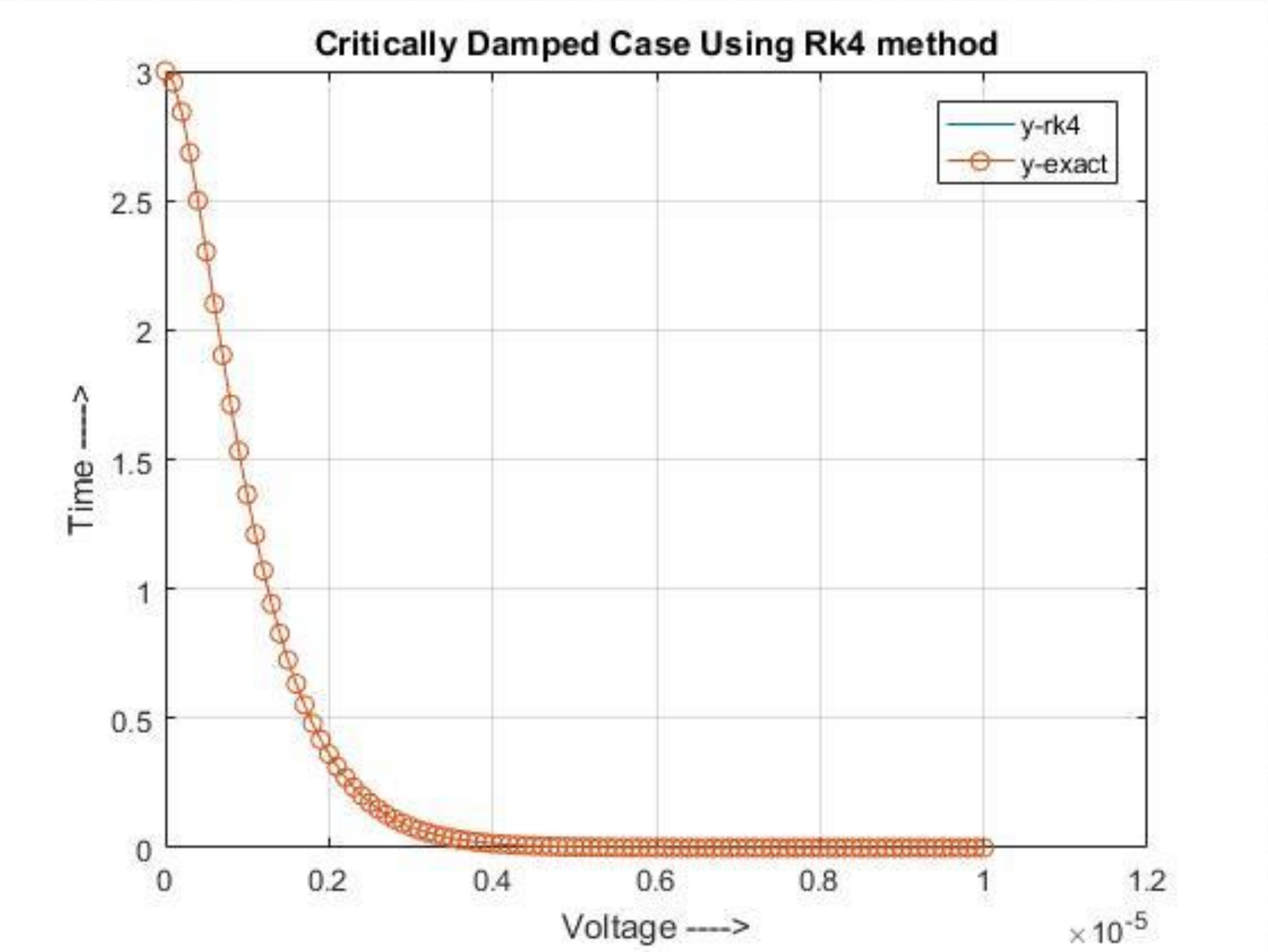
Overdamped Case Using Rk4 method
$\times 10^6$
0
-0.5
-1
-1.5
-2
-2.5
dV/dt
0
0.5
1
1.5
2
2.5
3
Voltage ---->
y-rk4
y-exact

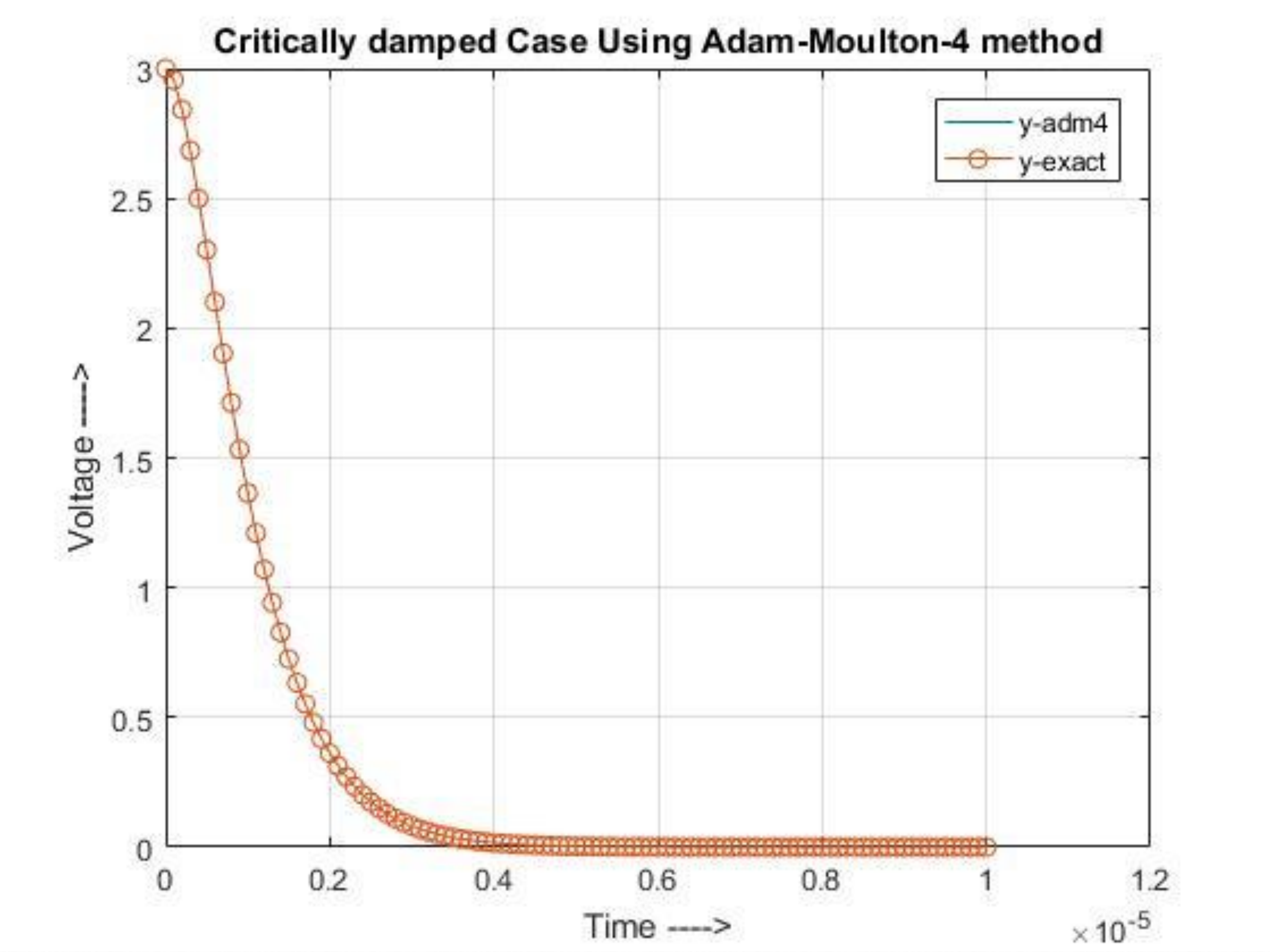


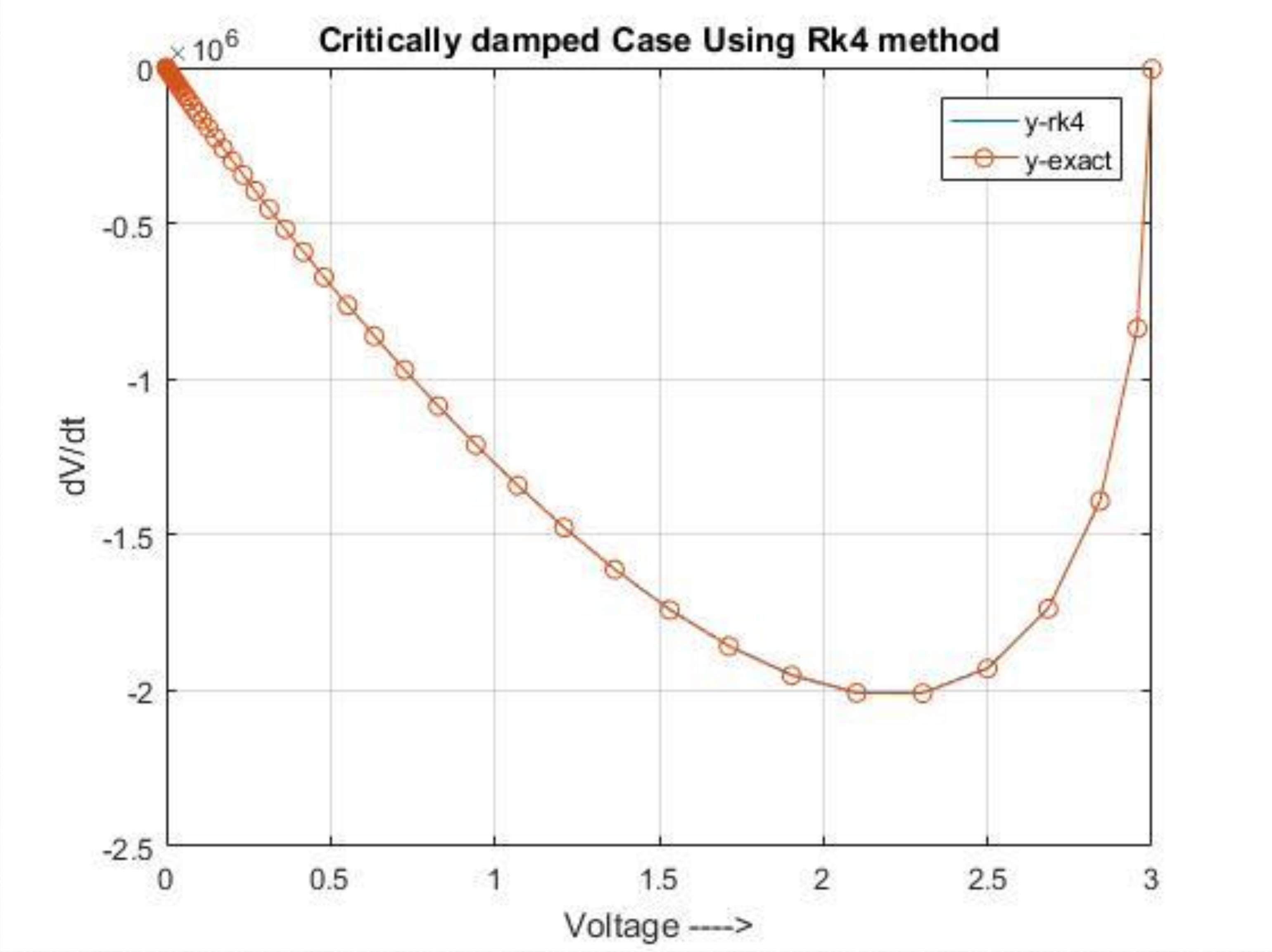
Overdamped Case Using Adam-Moulton-4 method
× 10^6
0
-0.5
-1
-1.5
-2
-2.5
dV/dt
0
0.5
1
1.5
2
2.5
3
Voltage ---->
y-adm4
y-exact

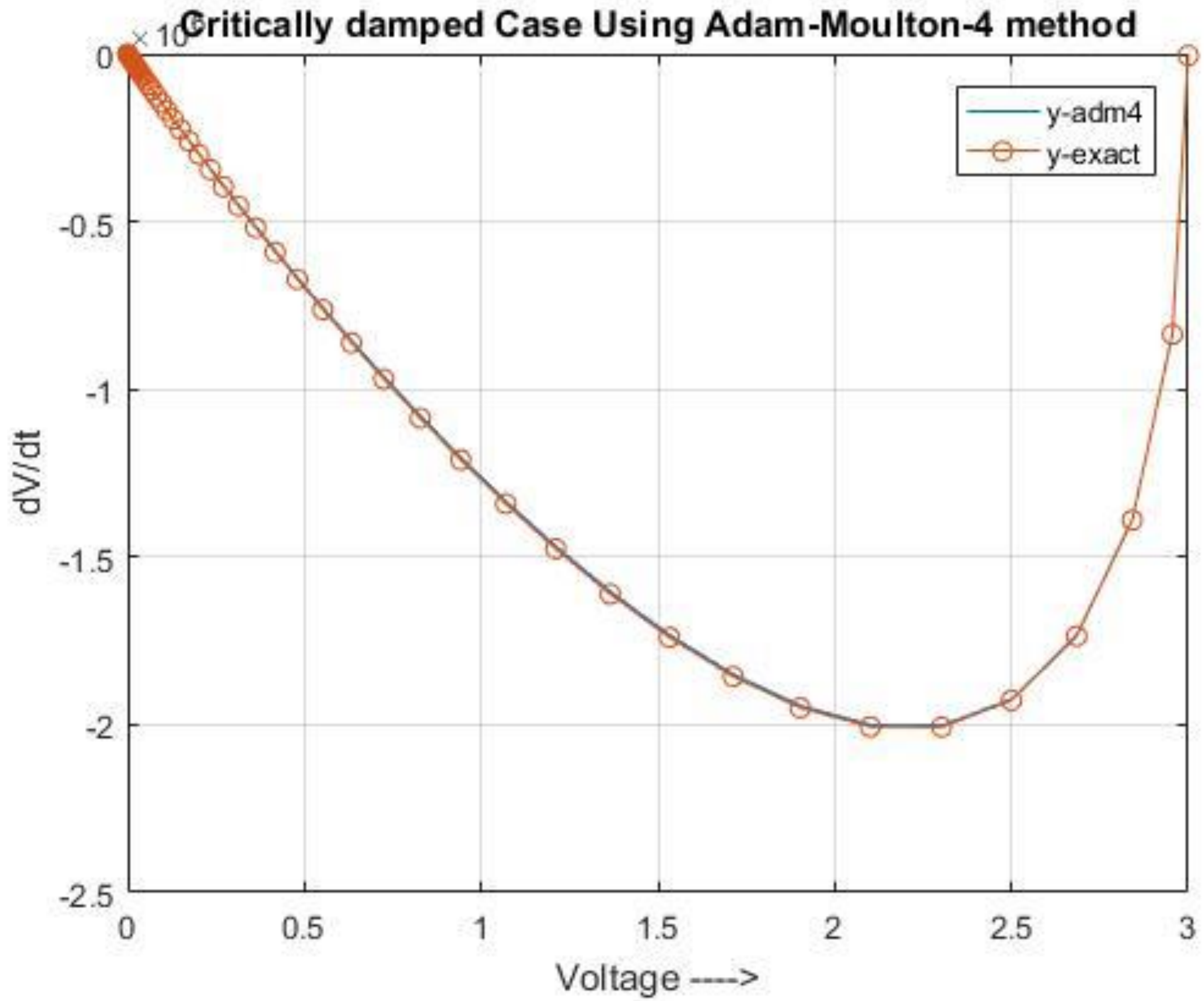
Critically damped Case

- R = 10954.45
- 2 equal real roots of the characteristic equation.
- S = -1.8257e+6
- The voltage returns to equilibrium in the minimum time and there is just enough damping to prevent oscillation.


Critically Damped Case Using Rk4 method
y-rk4
y-exact
Time ---->
Voltage ---->
×10^-5
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2

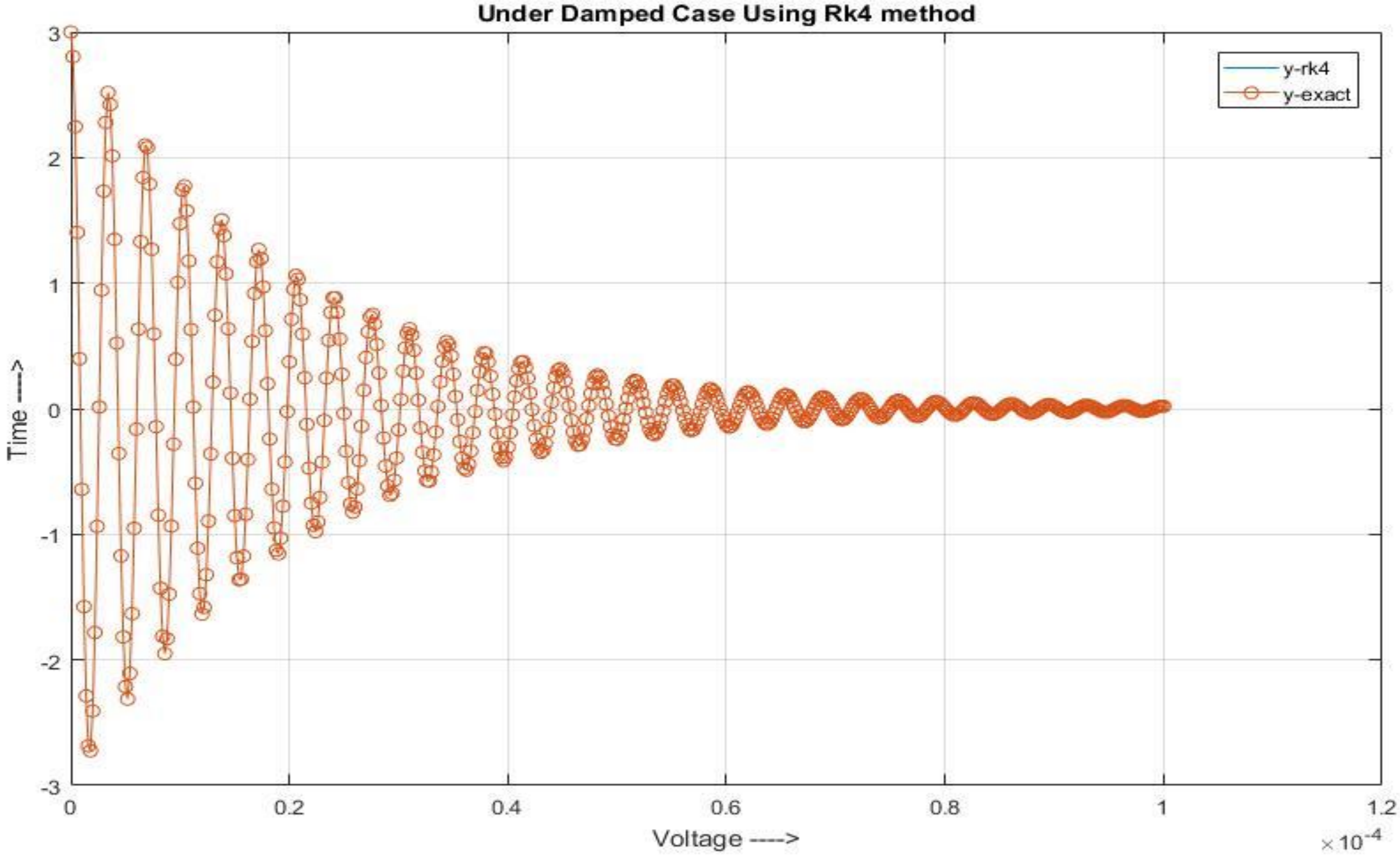

Critically damped Case Using Adam-Moulton-4 method
y-adm4
y-exact
Voltage ---->
Time ---->
×10^-5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2

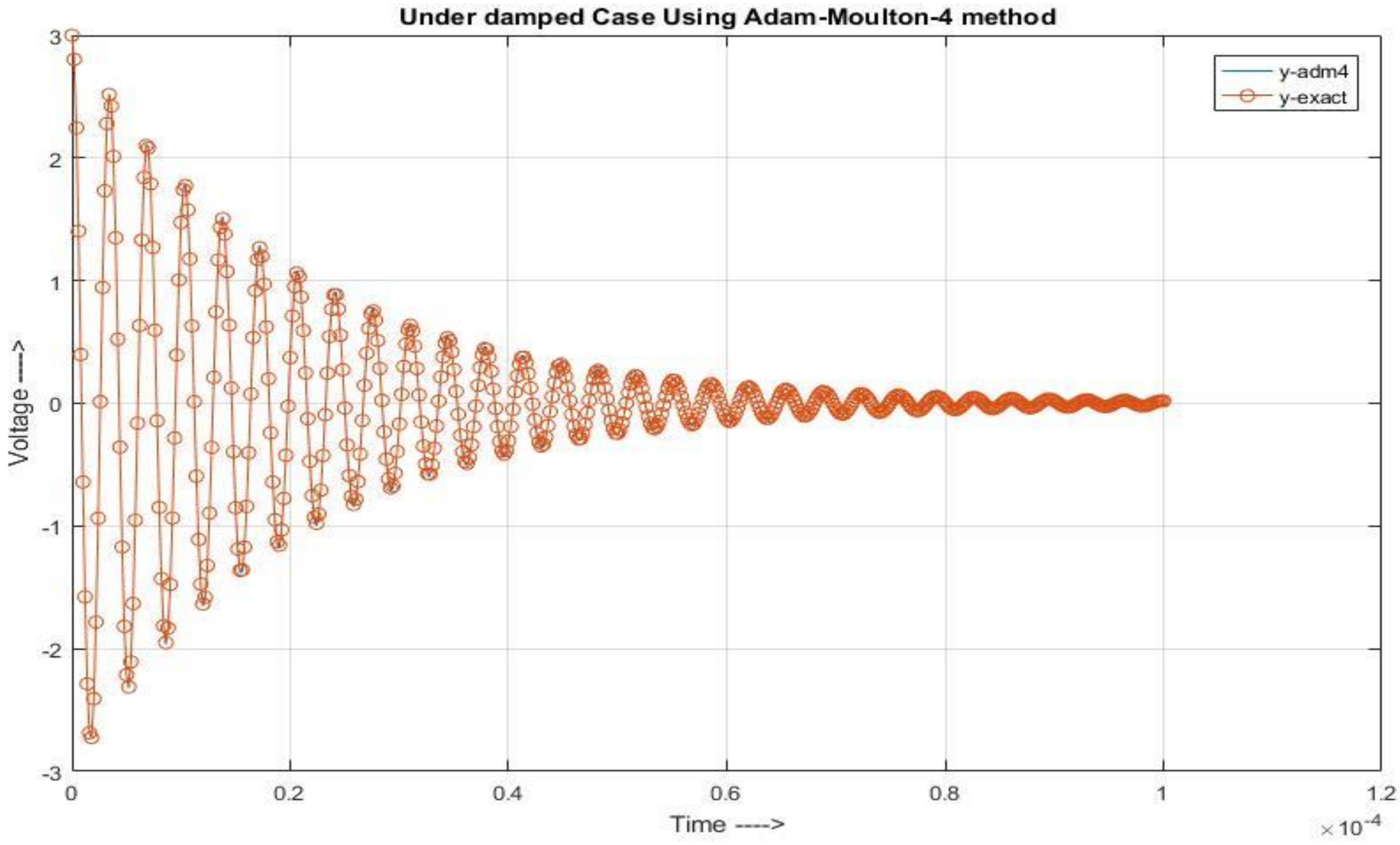

Critically damped Case Using Rk4 method
$\times 10^6$
0
-0.5
-1
-1.5
-2
-2.5
dV/dt
0
0.5
1
1.5
2
2.5
3
Voltage ---->
y-rk4
y-exact


Critically damped Case Using Adam-Moulton-4 method
× 10
y-adm4
y-exact
dV/dt
Voltage ---->
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
3

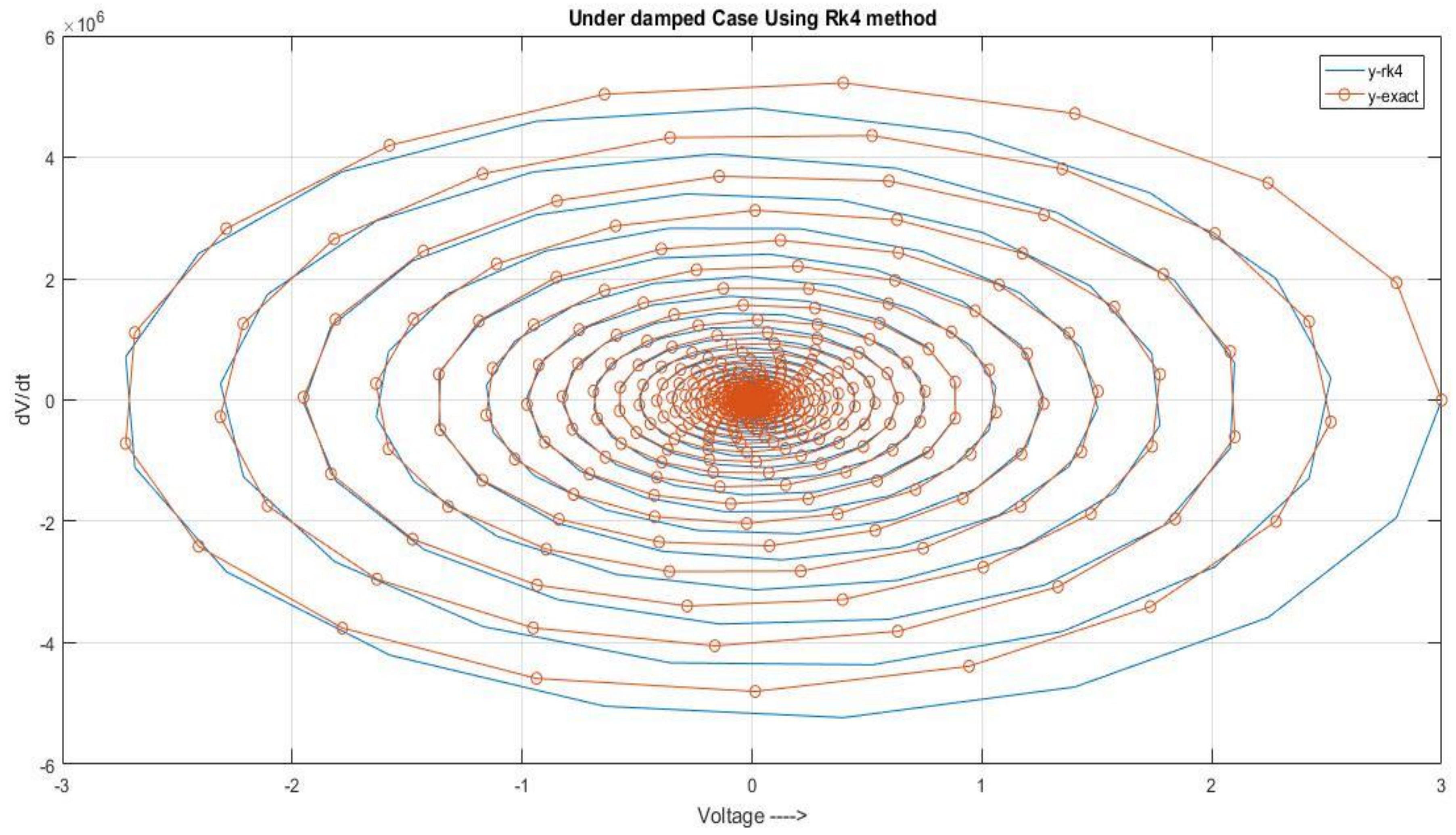
Under damped Case

- R = 300 (<10954.45)
- 2 imaginary roots of the characteristic equation.
- S_1 = -5e+4 + i1.825e+6
- S_2 = -5e+4 – i1.825e+6
- The voltage is oscillatory, amplitude decreases exponentially and finally it decays to equilibrium.

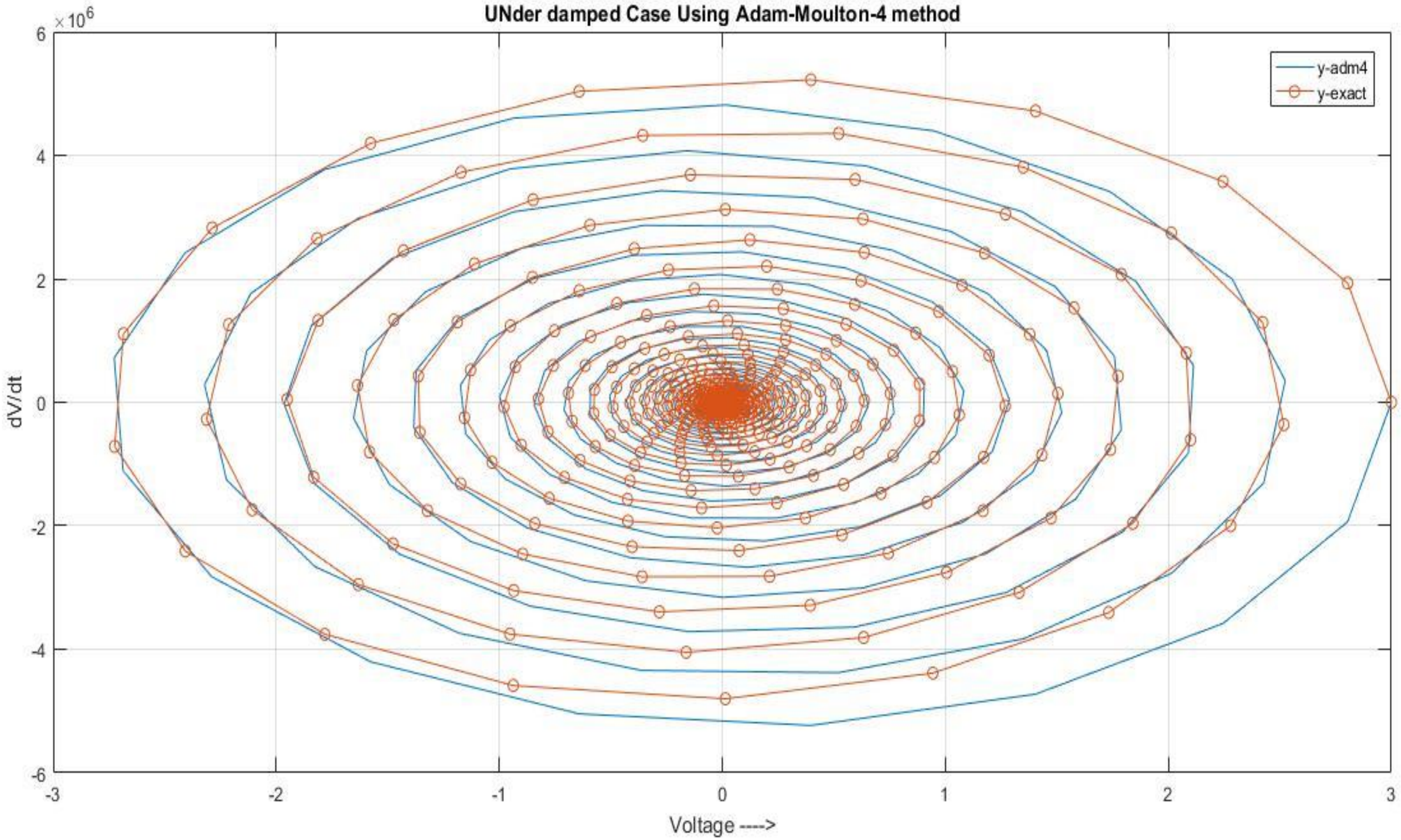

Under Damped Case Using Rk4 method
y-rk4
y-exact
Time ---->
Voltage ---->
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
×10^-4



Under damped Case Using Adam-Moulton-4 method
y-adm4
y-exact
Voltage ---->
Time ---->
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
× 10^-4



Under damped Case Using Rk4 method
×10^6
y-rk4
y-exact
dV/dt
Voltage ---->
6
4
2
0
-2
-4
-6
-3
-2
-1
0
1
2
3


UNder damped Case Using Adam-Moulton-4 method
× 10^6
6
4
2
0
-2
-4
-6
dV/dt
-3
-2
-1
0
1
2
3
Voltage ---->
y-adm4
y-exact