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Statistics

Random Variable :-

If the numeric value assumed by variable is result of some chance factor i.e we cannot predict the value in advance, then the variable is called R.V/ chance variable/ stochastic variable

These R.V are denoted capital letters

Ex. X, Y, Z etc.

* R.V are of 2 types

(i) Discrete R.V

A discrete R.V is one which assumes isolated values.

Ex- If X is R.V holding no. of heads in 4 tosses of a coin, it assume it holds the values (0, 1, 2, 3, 4)

(ii) Continuous R.V

A R.V which assumes any value within an interval i.e all values in a continuous scale,

Ex- The weights in kg in a group of individual

Discrete probability Distribution

let the R.V (X) assumes values $x_1, x_2, x_3 \ldots x_n$ with the corresponding probabilities $(P_1, P_2, P_3 \ldots P_n)$ such that

$\Sigma p_i = 1$ & $p_i \geq 0$ is called as DPD for R.V (X)

and it tells us how the total prob. of i is distributed over several values of R.V

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NOTE: If X_1, X_2 are 2 R.V then cX_1, $c_1X_1 + c_2X_2$, X_1X_2 are all R.V

Distribution func^n

let X be a R.V define for all real (x) by $\boxed{F(x) = P(X \le x)}$ is called distribution func^n of R.V (X)

The distributive func^n F is also called commutative distribution func^n,

$$F_X(x) = P(X \le x)$$

Properties of Distribution func^n

If F is a D.F of R.V (X) and $(a < b)$ then,

(i) $P(a < X \le b) = F(b) - F(a)$

(ii) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$

(iii) $F(\infty) = \lim_{x \to \infty} F(x) = 1$

Mean, Variance & Standard Deviation

Mean $(\mu) = \frac{\Sigma p_i x_i}{\Sigma x_i} = \Sigma p_i x_i$

$\sigma^2 = \Sigma p_i (x_i - \mu)^2$

$S.D = \sqrt{\sigma^2} \neq \sigma$

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Ques A R.V (X) has following prob. distribution funcⁿ

X:	0	1	2	3	4	5	6	7
$P(X=x_i)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) find k

(ii) $P(X<6)$

(iii) $P(X \geq 6)$

(iv) $P(3<X\leq 6)$

(v) min val of 'x' $\geq P(X\leq x) > 1/2$

Solⁿ Here sum of all prob is 1 which implies on addition

$(0+k+2k \text{ ---- } 7k^2+k=1)$

$$\boxed{k = 1/10}$$

(ii) $P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$k^2+5k = 1/100 + 8/10 = 81/100$

or

$P(X<6) = 1-P(X\geq 6)$

$= 1-[P(X=6)+P(X=7)]$

$= 1-[9k^2+k]$

$= 81/100$

(iii) $P(X\geq 6) = P(X=6)+P(X=7)$

$= 9k^2+k$

$= 9/100 + 1/10$

$= 19/100$

(iv) $P(X=4)+P(X=5)+P(X=6)$

$3k+k^2+2k^2 = 3k+3k^2 = 33/100$

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(v) $P(X \le 0) = 0$

$P(X \le 1) = 0 + K$

$P(X \le 2) = 0 + K + 2K = 3K = 3/10$

$P(X \le 3) = 5K = 5/10 = 1/2$

$P(X \le 4) = 1/2 + 3K = 1/2 + 3/10 > 1/2$

Moments

μ_r Moments about mean (central moments) for individual as well as frequency data

Moments about origin (— μ_r) (ν_r)

Moments about arbitary number (μ_r')

★ Relationships b/w μ_R, μ_R' & ν_r

★ Theoritical frequency distribution.

Generally f d are formed from the experimental or observed data.

there are 3 types of probability distribution

(i) Binomial distribution } the R.V is discrete.

(ii) Poisson distribution }

(iii) Normal distribution → Here R.V is continuous.

Binomial Distribution

Let there be 'n' independent trials in an experiment.

Let the R.V X denote the no. of successes in these 'n' trials.

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Let 'p' be the probability of success and 'q' be the prob of failure such that $p+q=1$, then binomial distribution speaks about r success in n trial given by the formula:-

$$\boxed{P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}}$$

Note
- In a binomial distribution n is finite that mean the total no. of experiment.
- Each trial has only two possible outcomes success & failure
- All trials are independent & finally p & q are fixed & constant.

Mean & variance of B.D :-

* Mean of distribution $\mu = n \cdot p$
* Variance $(\sigma^2) = n \cdot p \cdot q$
* S.D $(\sigma) = +\sqrt{n \cdot p \cdot q}$

Recurrence formula for the prob. of binomial distribution

W.K.T $p(r) = {}^{n}C_{r}\, p^{r} q^{n-r} = \frac{n!}{(n-r)!\, r!} p^{r} q^{n-r}$ — (1)

$p(r+1) = {}^{n}C_{r+1}\, p^{r+1} q^{n-r-1} = \frac{n!}{(n-r-1)!\,(r+1)!} p^{r+1} \cdot q^{n-r-1}$ — (2)

$$\frac{p(r+1)}{p(r)} = \frac{(n-r)!}{(n-r-1)!} \cdot \frac{r!}{(r+1)!} \cdot \frac{p}{q} = \frac{(n-r)\,\cancel{(n-r-1)!}}{\cancel{(n-r-1)!}} \cdot \frac{\cancel{r!}}{(r+1)\,\cancel{(r-1)!}} \cdot \frac{p}{q}$$

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$$= \left(\frac{n-r}{r+1}\right)\left(\frac{p}{q}\right) \quad -(2)$$

$$\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} \cdot \frac{p}{q}$$

$$\Rightarrow \boxed{P(r+1) = \left(\frac{n-r}{r+1}\right)\left(\frac{p}{q}\right) \cdot P(r)}$$

Ques Comment upon the following statement for B.D
mean = 6, $\sigma^2 = 9$

Sol^n mean $\Rightarrow$ $np = 6$ —(1)
variance $npq = 9$ —(2)

$$q = 9/6 = 1.5$$

Since q is prob. of failure which cannot excess, the given statement is false

Ques A dice is toss thrice. A success is getting '1' or '6'
Find the mean & variance of the no. of success

$n = 3$, $p = 2/6 = 1/3$

$q = 1 - p = 1 - 1/3 = 2/3$

mean $= np = 3 \times 1/3 = 1$

variance $= npq = 3 \times 1/3 \times 2/3$
$= 2/3$

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Ques If 10% of bulbs produced by a machine are defective. Determine the probability that out of 10 bulbs are chosen at random.

(i) 1 bulb

(ii) no bulb (iii) atmost 2 bulbs will be defected

Sol^n let X be a R.V choosing the no. of defective bulbs.

here, $n = 10$, $p = 10/100 = 1/10$, $q = 1 - 1/10 = 9/10$

(i) $P(X=1) = {}^{10}C_1 (1/10)^1 (9/10)^9$

$= 0.3874$

(ii) $P(X=0) = {}^{10}C_0 (1/10)^0 (9/10)^9$

$= 0.3486$

(iii) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.9297$

Ques A binomial variate (x) satisfies the relation $9P(X=4) = P(X=2)$ when $n=6$. Find the value of the parameter p and $P(X=1)$

Sol^n

$9[{}^6C_4 p^4 q^2] = {}^6C_2 p^2 q^4$

$9\,{}^6C_4 p^2 = {}^6C_2 q^2$

$9p^2/q^2 = \frac{6!}{4! \times 2!} \times \frac{4! \times 2!}{6!}$

$p/q = 1/3 \Rightarrow \boxed{p = q/3}$

$p + q = 1$

$q/3 + q = 1$

$4q/3 = 1$

$\boxed{q = 3/4}$

$p = 1 - 3/4 = 1/4$, $\boxed{p = 1/4}$

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$$P(x=1) = {}^6C_1 (1/4)^1 (3/4)^5$$
$$= 6 \times 1/4 \times 3/4 \times 3/4 \times 3/4 \times 3/4 \times 3/4$$
$$= 6 \times 1/4 \times 0.237$$

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Ques A student is given 8 true-false exam with 8 ques. If he corrects atleast 7 ques, he passes the exam. Find the probability that he'll pass
Given :- he guesses all the questions.

(Ans = 0.351)

Soln $p = 1/2$, $q = 1/2$, $n = 8$

$$P(x \geq 7) = P(x=7) + P(x=8)$$
$$= {}^8C_7 (1/2)^7 (1/2)^1 + {}^8C_8 (1/2)^8 (1/2)^0$$
$$= 8 \times (1/2)^8 + 1 \times (1/2)^8$$
$$= 8 \times 3.90 \times 10^{-3} + 3.90 \times 10^{-3}$$
$$= 0.0312 + 3.90 \times 10^{-3}$$
$$= 0.351$$

Note The 2 parameters of the binomial distribution are n & p

Curve Fitting

⇒ Fitting of Binomial Distribution

The general form of the fit is
$$N(q+p)^n$$
where, $(q+p)^n = P(x=r) = {}^nC_r p^r q^{n-r}$

⇒ $N(q+p)^n = N \cdot {}^nC_r p^r \cdot q^{n-r}$
where, N is Σf and 'n' is no. of trials.
p and q are probabilities of success & failure resp.

x	f	$f_i x_i$
x_1	f_1	$f_1 x_1$
x_2	f_2	$f_2 x_3$
x_3	f_3	$f_3 x_3$
⋮	⋮	⋮
x_n	f_n	$f_n x_n$
	$\Sigma f_i = N$	$\Sigma f_i x_i$

Mean $\mu = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = np$

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Steps :-

(i) Let the values of x and f are given. Create a column fx. Calculate mean $\mu = \frac{\Sigma f_i x_i}{\Sigma f_i}$ and equate it to np, where n is the no. of trials. Upon simplifying, we get p, $q = 1-p$

The theoritical binomial distribution formula is

$N(q+p)^n$ where, $N = \Sigma f_i$

Ques Fit a binomial distribution of the following frequency

X	0	1	2	3	4
f	30	62	46	10	2

Soln

X	f	$f_i x_i$
0	30	0
1	62	62
2	46	92
3	10	30
4	2	8
	$N = \Sigma f_i = 150$	$\Sigma f_i x_i = 192$

$$\therefore \mu = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{192}{150}$$

$$= 1.28$$

from the data, $n = 4$

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$\therefore$ mean $\mu = np$

$\Rightarrow 1.28 = 4 \times p$

$\boxed{p = 0.32}$

$q = 1 - 0.32$

$\boxed{q = 0.68}$

$N = 150$

$\therefore$ the binomial distribution is $N(q+p)^n$

$= 150({}^nC_r\, p^r\, q^{n-r})$

$= 150[{}^4C_r\,(0.32)^r\,(0.68)^{4-r}]$

(i) for $r=0 \Rightarrow 150 \times {}^4C_0 \times (0.32)^0 \times (0.68)^4 = 32.07$

(ii) for $r=1 \Rightarrow 150 \times {}^4C_1 \times (0.32)^1 \times (0.68)^3 = 60.37$

(iii) for $r=2 \Rightarrow 150 \times {}^4C_2 \times (0.32)^2 \times (0.68)^2 = 42.61$

(iv) for $r=3 \Rightarrow 150 \times {}^4C_3 \times (0.32)^3 \times (0.68) = 13.36$

(v) for $r=4 \Rightarrow 150 \times {}^4C_4 \times (0.32)^4 \times (0.68)^0 = 1.57$

Ques If the probability of hiting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit atleast one?

Soln

$n = 10$

$p = 10/100 = 1/10$

$q = 1 - 1/10 = 9/10$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - {}^{10}C_0\,(1/10)^0\,(9/10)^{10}$

$= 1 - (9/10)^{10}$

$= 0.6513$

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Que. Out of 800 families with 4 children each, how many families would be expected to have

(i) 2 boys & 2 girls

(ii) atleast 1 boy

(iii) no girl

(iv) atmost 2 girls

assume equal probabilities for boys and girls.

Sol^n

$n = 4$

$p = 1/2$

$q = 1/2$

(i) $P(X=2) = {}^4C_2 (1/2)^2 (1/2)^2$

$= \frac{4 \times 3}{2} \times (1/2)^4$

$= 3 \times (1/2)^3$

$= 0.375$

No. of families $= 800 \times 0.375$

$= 300$

(ii) $P(X \geq 1) = 1 - P(X=0)$

$= 1 - {}^4C_0 (1/2)^0 (1/2)^4$

$= 1 - (1/2)^4$

$= 1 - 0.0625$

$= 0.9375$

No. of families $= 800 \times 0.9375$

$= 750$

(iii) $P(X=0) = {}^4C_0 (1/2)^0 (1/2)^4$

$= 0.0625$

No. of families $= 800 \times 0.0625$

$= 50$

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(iv) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$= {}^4C_2 (1/2)^2 (1/2)^2 + {}^4C_3 (1/2)^3 (1/2) + {}^4C_4 (1/2)^4$

$= (1/2)^4 + 4 \times (1/2)^4 + 6 (1/2)^4$

$= 0.6875$

No. of families $= 800 \times 0.6875$

$= 550$

Ques Six dices are thrown 729 times. How many times do you expect atleast 3 dice to show a 5 or 6? (233)

Soln

Soln Assuming that on the average one telephone no. out of 15 called b/w 2 to 3 pm on week days is busy. What is the probability that if 6 randomly selected no. are called

(i) not more than 3 (0.999)

(ii) atleast 3 (0.005)

of them will be busy?

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Poisson Distribution

It is a special case which can be considered as an approximation to the binomial distribution

as $x \to \infty$, $p \to 0$

such that np always remain same

Note :- P.D is used when n is very large and p is small

The P.D formula is given by -

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!} \quad \text{where } \lambda = np, \quad r = 0,1,2 \ldots$$

from the above formula, λ is the parameter of the distribution

Note In poisson distribution, mean and variance are same

mean = variance = $\lambda = np$

WKT, sum of all the properties should be 1

Consider $P(0) + P(1) + P(2) + P(3) + \ldots$

$$= e^{-\lambda} + \frac{e^{-\lambda}\lambda}{1} + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!} + \ldots$$

$$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots\right] = e^{-\lambda} \cdot e^{\lambda} = 1$$

Recursion relation of the probabilities of P.D

We know that,

$$P(X=r) = \frac{e^{-\lambda} \cdot \lambda^{r}}{r!} \quad \text{--- (1)}$$

$$P(X=r+1) = \frac{e^{-\lambda} \cdot \lambda^{r+1}}{(r+1)!} \quad \text{--- (2)}$$

(2)/(1) $\quad P(r+1) = \left(\frac{\lambda}{r+1}\right) P(r)$

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Note Application of poission distribution include arrival rate of patient in a hospital, telephone calls, demand rate of certain computer spare parts, etc.

Ques If the variance of P.D is 2, find the probs. for $r = 1, 2, 3, 4, \ldots$ Also find $P(X \geq 4)$.

Soln

Given :- $\lambda = 2$

consider, $P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = e^{-\lambda} = e^{-2} = 0.1353$

WKT,

$$P(r+1) = \frac{\lambda}{r+1} P(r) \rightarrow \text{(Recursion formula)}$$

$\therefore$ if $r = 0$, $P(1) = \frac{\lambda}{1} \cdot P(0)$

$$P(1) = 2P(0) = 0.2706$$

$$P(2) = \frac{\lambda}{2} P(1)$$

$$= \frac{2}{2} P(1) = P(1) = 0.2706$$

$$P(3) = \frac{\lambda}{3} P(2)$$

$$= 2/3 \cdot P(2) = 0.1804$$

$$P(4) = 0.0902$$

$$P(X \geq 4) = 1 - P(X < 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$
$$= 0.1431$$

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Ques Assume that the probability of an individual coal miner being killed in a mine accident during an year is 1/2400. Use poission distribution to calculate the probability that in a mine employing 200 miners. There will be atleast one fatal accident in a year.

Sol^n Given $p = 1/2400$ and $n = 200$ $\Rightarrow \lambda = np = 200/2400$

$$\boxed{\lambda = 0.083}$$

$$P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} = \frac{e^{-0.083} (0.083)^r}{r!}$$

$$P(X \geq 1) = 1 - P(X=0)$$
$$= 1 - [e^{-0.083}]$$
$$= 0.08$$

Ques Suppose that a book of 600 pages contain 40 printing mistakes assume that these errors are randomly distributed throughout the book, and (r) = no. of errors per page has a poission distribution. What is the prob that 10 pages selected at random will be free from errors.

Sol^n

$$n = 10, \quad p = 40/600 = 1/15$$
$$\lambda = np$$
$$= 10 \times 1/15$$
$$\boxed{\lambda = 0.67}$$

$$P(X=0) = \frac{e^{-0.67} (0.67)^0}{0!}$$
$$= e^{-0.67}$$
$$= 0.51$$

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Calculation of theoritical frequency

The formula for theoritical frequencies is

$$\frac{N e^{-\lambda} \lambda^{r}}{r!} \quad i.e, \quad N \cdot P(X=r)$$

Ex - Fit a P.D to the following data and calculate theoritical distribution values.

x:	0	1	2	3	4
f:	122	60	15	2	1

$\lambda = \frac{\Sigma f_i x_i}{\Sigma f_i} = 0.5$

x	Observed frequencies f	Theoritical frequencies $(200)\, e^{-0.5} \frac{(0.5)^r}{r!} = \frac{(121.306)(0.5)^r}{r!}$
0	122	121.306
1	60	60.653
2	15	15.163
3	2	2.527
4	1	0.315
$\Sigma x_i = 10$	$\Sigma f_i = 200$	

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Normal Distribution

The N.D is a continuous dist. whose general eqⁿ for the dist. is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \quad -(1)$$

Where the variable x consumes all the values b/w $-\infty$ to ∞

σ, μ are the parameters of the distribution

The random variable x which follows N.D is called normal variate represented by $x : N(\mu, \sigma^2)$

Properties of normal distribution

eqⁿ (1) is called the probability density funcⁿ which follows the following properties

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) The N.D is symmetric about its mean

(iv). The mean, median, mode of the N.D coincides.

Standard form of normal distribution

If X is a normal variate with the mean (μ) and S.D (σ), we define a variable $(Z) = \frac{x-\mu}{\sigma}$ which has the N.D with $\mu = 0$ & $\sigma = 1$

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The R.V (z) is called standard normal random variable whose P.D.F is given by

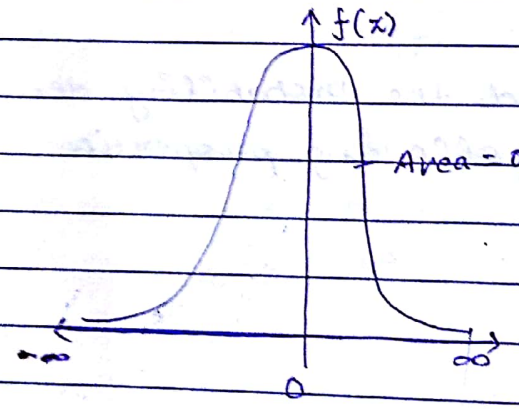
$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-1/2 (z)^2}$$

which is free from any parameter.

$$P(z_1 \le z \le z_2) = \int_{z_1}^{z_2} f(z)\, dz$$

Area under normal curve.

By taking $z = \left(\frac{x-\mu}{\sigma}\right)$, a standard normal curve is formed the total area under this curve is 1



The area under curve is divided into 2 equal parts by $z = 0$. The area b/w the ordinate $z = 0$ & any other ordinate can be noted from standard normal table. It should be noted that mean of the N.D is zero.

Ques A sample of 100 dry battery cells tested to find the length of life produced the following results $\bar{x} = 12$ hrs, $\sigma = 3$ hrs.

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Assuming the data to be normally distributed: What % of the battery cells are expected to have life.

(i) more than 15 hrs.

(ii) less than 6 hrs

(iii) b/w 10 - 14 hrs.

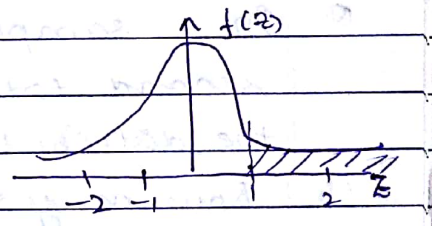
Sol^n let x denotes the length of the life of dry battery cells.

let $z = \frac{x-\mu}{\sigma} \Rightarrow \frac{x-12}{3}$

(i) more than 15 hrs

$x > 15$

when $x = 15$, $z = \frac{15-12}{3} = 1$



$\therefore P(x > 15) = P(z > 1)$

$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.3413$

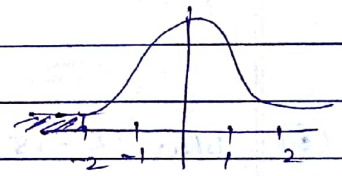
$= 0.1587$

$\therefore$ 15.87% have their life more than 15 hrs.

(ii) less than 6 hrs

$x < 6$

when $x = 6$, $z = \frac{6-12}{3} = -2$



$\therefore P(x < 6) = P(z < -2)$

$\Rightarrow \int_{-\infty}^{-2} f(z)\,dz = \int_{2}^{\infty} f(z)\,dz$

$= 0.5 - P(0 < z < 2)$

$= 0.5 - 0.4772$

$= 0.0228$

$\therefore$ 2.28% have their life less than 6 hrs

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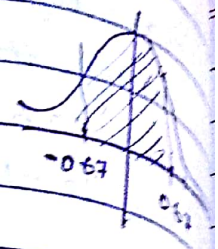
(iii) b/w 10 to 14 hrs.

$10 < x < 14$

When $x = 10$, $z = \frac{10-12}{3} = -2/3$

When $x = 14$, $z = \frac{14-12}{3} = 2/3$



$P(10 < x < 14) = 2\,P(0 < z < 0.67)$

$= 2 \times [0.2485]$

$= 0.497$

49.7 %

Q. In a sample of 1000 cases, the mean of second test is 14 & S.D is 2.5. Assuming the distribution to be normal. Find out

(i) how many students score b/w 12 and 15

(ii) how many score above 18?

(iii) how many score below 8.

Sol^n Let x denotes the

Let $z = \frac{x-\mu}{\sigma} = \frac{x-14}{2.5}$

(i) b/w 12 - 15

$12 < x < 15$

When $x = 12$, $z = \frac{12-14}{2.5} = \frac{-2}{2.5} = -0.8$

$x = 15$, $z = \frac{15-14}{2.5} = \frac{1}{2.5} = 0.4$.

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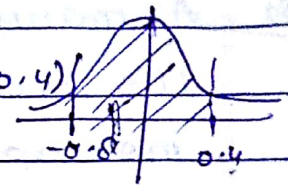
$P(12 < x < 15) = P(-0.8 < z < 0.4)$

$= P(0 < z < 0.8) + P(0 < z < 0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$



443.5 % ⇒ 444 students score b/w 12 - 15

(ii) above 18

$x > 18$

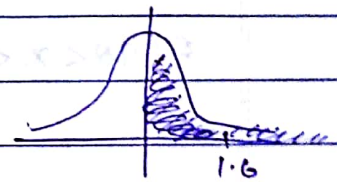
when $x = 18$, $z = \frac{18-14}{2.5} = \frac{40}{2.5} = 1.6$

$P(x > 18) = P(z > 1.6)$

$= 0.5 - P(0 < z < 1.6)$

$= 0.5 - 0.4452$

$= 0.0548$



5.48 % = 55 students score above 18

(iii) below 8

$x < 8$

when $x = 8$, $z = \frac{8-14}{2.5} = \frac{-6}{2.5} = -2.4$

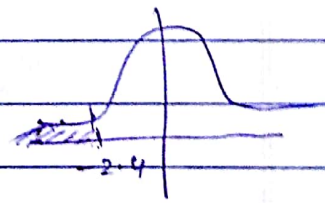
$P(x < 8) = P(z < -2.4)$

$= 0.5 - P(0 < z < -2.4)$

$= 0.5 - 0.4918$

$= 0.0082$

$= 0.2$



9 students score below 8

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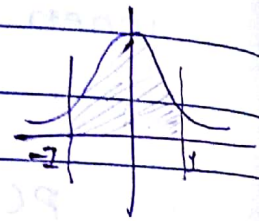
Ques. A manufacturer knows from his experience that the resistance of the resistors he produces is normal with mean = 100 & S.D = 2.
What % of resistors will have resistance in b/w 98 - 102 Ω

Sol^n

$\mu = 100, \sigma = 2$

When $x = 98 \Rightarrow z = \frac{98-100}{2} = -2/2 = -1$

$x = 102 \Rightarrow z = \frac{102-100}{2} = +2/2 = 1$



$P(98 < x < 102) = 2P(0 < z < 1)$
$= 2 \times 0.3413$
$= 0.6826$

68.26 % of resistors will have resistance in b/w 98 - 102

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★ Moment about arbitary pt. (A) [Raw moments]

Series :-

$$\mu_r' = \frac{\Sigma (x_i - A)^r}{n}$$

freq. Distribution data

$$\mu_r' = \frac{\Sigma f_i (x_i - A)^r}{N}, \quad N = \Sigma f_i$$

Note :- $\mu_0' = 1$

$$\mu_1' = \frac{1}{N} \Sigma f_i (x_i - A)^1$$

$$= \frac{1}{N} \Sigma f_i x_i - \frac{1}{N} \Sigma f_i A$$

$$= \bar{x} - \frac{A}{N} \cdot N$$

$$= \bar{x} - A$$

Sometimes in order to simplify the computations if there is a common factor 'h' in the values of $\bar{x} - A$ we define a new variable

$$u = \frac{x - A}{h}$$

In this case, $\mu_r' = \frac{1}{N} \left(\Sigma f_i u_i^r \right) h^r$

★ Moments about origin (ν_r)

A series data $\nu_r = \frac{\Sigma x_i^r}{n}$

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for frequency data

$$\mu_r' = \frac{1}{N} \sum f_i x_i^r$$

* Relation b/w μ_r and μ_r'

$$\mu_1 = 0$$
$$\mu_2 = \mu_2' - \mu_1'^2$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

* Relation b/w ν_r and μ_r

$$\nu_1 = \bar{x}$$
$$\nu_2 = \mu_2 + \bar{x}^2$$
$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$
$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Karl Pearson (β & γ coefficients)

The following 4 coefficients based on the first 4 moments of a freq. distribution about its mean is given by

$$\beta_1 = \mu_3^2 / \mu_2^3$$

$$\beta_2 = \mu^4 / \mu_2^2$$

$$\gamma_1 = +\sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

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Ques The first 3 moments of the distribution about the no. 2 of a variable are 1, 16, −40. Show that the mean is 3, variance is 15 and μ_3 is −86

Sol^n Given values are moments about arbitary no. 2

$\therefore \mu_1' = 1, \mu_2' = 16, \mu_3' = -40$

$\mu_1' = \bar{x} - A$

$1 = \bar{x} - 2$

$\boxed{\bar{x} = 3}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 16 - 1$

$\boxed{\mu_2 = 15}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= -40 - 3 \times 16 \times 1 + 2 \times 1^3$

$= -40 - 48 + 2$

$\boxed{\mu_3 = -86}$ Proved

Ques Calculate the variance and third central moment from the following data.

x_i	0	1	2	3	4	5	6	7	8
f_i	1	9	26	59	72	52	29	7	1

Sol^n

x_i	f_i	$u = \frac{x - A}{h}$ [h=1, A=4]	fu	fu^2	fu^3
0	1	−4	−4	16	−64
1	9	−3	−27	81	−243
2	26	−2	−52	104	−208
3	59	−1	−59	59	−59
4	72	0	0	0	0
5	52	1	52	52	52
6	29	2	58	116	232
7	7	3	21	63	159
8	1	4	4	16	64
	$N = 256$		-7	$= 507$	$\Sigma fu^3 = -37$

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$$\mu_r' = \frac{1}{N}\left(\Sigma f_i^* u_i^{*r}\right) h^r$$

$$\therefore \mu_1' = \frac{1}{N}\left(\Sigma f_i^* u_i^*\right) h^1$$

$$= \frac{-7}{256} = -0.02734$$

$$\mu_2' = \frac{1}{N}\left(\Sigma (f_i^* u_i^{*2})\right) h^2$$

$$= \frac{507}{256}$$

$$= 1.9805$$

$$\mu_3' = -0.1445$$

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$= 1.9805 - (7.47 \times 10^{-4})$

$= 1.9805 - 0.000747$

$= 1.9797$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= -0.1445 - 3 \times 1.9797 \times (-0.02734) + 2 \times (-0.02734)^3$

$= -0.1445 + 5.9392 \times 0.02734 + 2 \times (-0.02734)^3$

$= -0.1445 + 0.1623 - 2 \times (0.02734)^3$

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Ques 4 The first 4 moments of the distribution about the value of a variable are $-1.5, 17, -30$ and 108. Find the moment about mean, origin and β_1 and β_2

Skewness

If the curve of the distribution is not symmetrical it is called skewness which means lack of symmetry in the distribution.

* Moments coefficients of skewness

It is given by the formula $\frac{\mu_3}{\sqrt{\mu_2^3}}$

Ques The first 3 central moments of a distribution are $0, 15$ & -31. Find the moment coefficient of skewness

Sol^n

$$= \frac{-31}{\sqrt{15^3}}$$

$$= \frac{-31}{58.09}$$

$$= 0.533$$

Ques Calculate moment coff. of skewness from the following data

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

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Sol^n

x	f	$u = \frac{x-A}{h}$ $[h=1, A=4]$	fu	fu^2	fu^3
0	1	-4	-4	16	-64
1	8	-3	-24	72	-216
2	28	-2	-56	112	-224
3	56	-1	-56	56	-56
4	70	0	0	0	0
5	56	1	56	56	56
6	28	2	56	112	224
7	8	3	24	72	216
8	1	4	4	16	64
	$\Sigma N = 256$		$fu = 0$	$fu^2 = 512$	0

$$\mu_r' = \frac{1}{N}\left(\Sigma f_i^0 u_i^{0r}\right) h^r$$

$$\therefore \mu_1' = \frac{1}{N}\left(\Sigma f_i^0 u_i^0\right) h^1$$

$$= \frac{1}{256}(0)(1)$$

$$= 0$$

$$\therefore \mu_2' = \frac{1}{N}\left(\Sigma\left(f_i^0 u_i^{02}\right)\right) h^2$$

$$= \frac{1}{256}(512)$$

$$= 2$$

$$\mu_3' = \frac{1}{N}\left(\Sigma\left(f_i^0 u_i^{03}\right)\right) h^3$$

$$= \frac{1}{256}(0) = 0$$

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$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$= 2 - 0 = 2$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 0 - 3 \times 2 \times 0 + 2 \times 0$

$= 0$

$$\text{skewness} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{0}{\sqrt{2^3}}$$

$$= 0$$

Probability Density Func^n

If 'x' is a continuous random variable then func^n $f(x)$ is said to be prob. density func^n if it is defined as

(i) $f(x) \geq 0, \; -\infty \leq x \leq \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) $P(a < x < b) = \int_a^b f(x)\,dx$

Cummulative Distribution func^n

A cummulative D.F for the continuous R.V is define as

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

NOTE (i) $F(-\infty) = 0$

(ii) $F(\infty) = 1$

(iii) $P(a < x \leq b) = P(a \leq x \leq b) = P(a \leq x < b) = P(a < x < b) = F(b) - F(a)$

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Ques A.R.V 'X' has the distribution func^n x is $f(x) = 6x(1-x)$, $0 \le x \le 1$. Check whether $f(x)$ is P.D.F or not.

Sol^n If $\int_0^1 f(x)\,dx = 1$ then $f(x)$ is PDF

$$\Rightarrow \int_0^1 6x(1-x)\,dx = 1$$

$$\int_0^1 (6x - 6x^2)\,dx = 1$$

$$\left[\frac{\cancel{6}^{3}x^2}{\cancel{2}} - \frac{\cancel{6}^{2}x^3}{\cancel{3}}\right]_0^1 = 1$$

$$\left[3x^2 - 2x^3\right]_0^1 = 1$$

$$(3-2) = 1$$

$$1 = 1$$

Ques Let X be continuous R.V with PDF

$$f(x) = \begin{cases} Kx & 0 \le x < 1 \\ K & 1 \le x < 2 \\ -Kx + 3K & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find K, (ii) $P(X<2)$

Sol^n $\int_0^3 f(x)\,dx = 1 \Rightarrow \int_0^1 Kx\,dx + \int_1^2 K\,dx + \int_2^3 (-Kx + 3K)\,dx =$

$$\left[\frac{Kx^2}{2}\right]_0^1 + [Kx]_1^2 + \left[-\frac{Kx^2}{2} + 3Kx\right]_2^3 = 1$$

$$\frac{K}{2} + 2K - K + \left[\left(-\frac{9K}{2} + 9K\right) - \left(-\frac{4K}{2} + 6K\right)\right] = 1$$

$$\frac{3K}{2} + \left[\frac{9K}{2} + \frac{4K}{2} - 12K\right] = 1$$

$$\frac{4K}{2} = 1 \Rightarrow 2K = 1$$

$$\boxed{K = 1/2}$$

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(ii) $P(X<2) = 0 + \int_0^1 Kx\,dx + \int_1^2 K\,dx$

$= \left[\frac{Kx^2}{2}\right]_0^1 + \left[Kx\right]_1^2$

$= \frac{K}{2} - 0 + 2K - K$

$= \frac{K}{2} + K$

$= 3K/2 = 3 \times 1/4 = 3/4$

Moment generating func^n

In theoritical developments an indirect method for computing moments is used. The method depends on finding moment generating func^n In case of R.V X, the moment generating func^n is defined as

$$M(t) = \int_a^b e^{tx} f(x)\,dx$$

Note

- The limits of a and b can be from $-\infty$ to $+\infty$
- Absolute value of $\left|\frac{d^r}{dt^r} M(t)\right|_{t=0} = \mu_r$

  where μ_r is r^{th} moment about origin
- Moment generating func^n ~~has~~ be defined for

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if $z = x - m$, where m is the mean of distribution then the rth moment about z will give r^{th} moment ~~of~~ x about m

Therefore, we write

$$M_{x-m}(t) = \int_a^b e^{t(x-m)} f(x)\,dx = e^{-mt}\int_a^b e^{tx} f(x)\,dx = e^{-mt} M_x(t)$$

- If x is discrete,

then

$$M_x(t) = \sum e^{tx} \cdot p$$

where p is the prob for the R.V $(X = x)$

Ques Find the MGF of the exponential distribution

$$f(x) = \frac{1}{c} e^{-x/c} \; ; \; 0 \le x \le \infty, \; c > 0$$

Also find μ & σ.

Sol^n

Here the variable is continuous R.V

hence the MGF is given by

$$M_x(t) = \int_0^\infty e^{tx} \cdot f(x)\,dx = \int_0^\infty e^{tx} \cdot \frac{1}{c} e^{-x/c}\,dx$$

$$= \frac{1}{c}\int_0^\infty e^{(t - 1/c)x}\,dx$$

$$= 1/c \int_0^\infty e^{-(-t+1/c)x}\,dx$$

$$= 1/c \left[\frac{e^{-(-t+1/c)x}}{-(-t+1/c)} \right]_0^\infty$$

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$$= \frac{1}{c}\left(\frac{1}{-t+1/c}\right)^{0}$$

$$= \frac{1}{c} \cdot \frac{1}{\frac{1-ct}{c}} = \frac{1}{1-ct}$$

$$M_x(t) = \frac{1}{1-ct} = (1-ct)^{-1}$$

$$M_x(t) = 1 + ct + (ct)^2 + (ct)^3 + \cdots$$

$$\text{Mean} = \mu_1 = \left[\frac{d(M_x(t))}{dt}\right]_{t=0} = \left[c + 2c\,(ct) + \cdots\right]_{t=0}$$

$$\text{Mean} = c$$

$$\mu_2 = 2c^2$$

$$\text{Variance } \mu_2 = \mu_2 - \mu_1^2 = 2c^2 - c^2 = c^2$$

$$S.D = \sqrt{\sigma^2} = \sqrt{c^2} = c$$

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