15/1

PHYSICS

LASERS → Coherent Beam of light.
↓
Highly

Light
Amplification by
Stimulated
Emission of
Radiation.

The Redistribution of Intensity in the Region of Superimpose is called Interference.

To make interference to happen we need coherent Sources.

Crest on Crest.
Constructive interference → Intensity is more.
→ Crest on Trough
Destructive " → " " Less.

$$\boxed{y = A\sin(\omega t \pm \phi) \pm kx.}$$

$\omega = \frac{2\pi}{T}$

A is Amplitude ($I \propto A^2$)

$k = \frac{2\pi}{\lambda}$

we need to Study phase difference to Study Coherence.

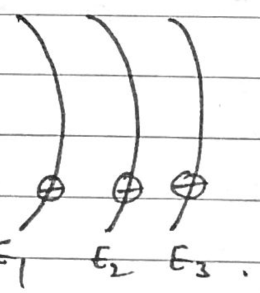
→ phase Diff: - Zero (In phase)
- 180° (out of phase)
- constant
- Random

→ For sources to be coherent the phase difference should be zero or 180°.

Two Tubelights can never interfere because Phase difference is constantly changing. So it's phase difference is Random.

$$\boxed{\text{phase diff} = \frac{2\pi}{\lambda}\cdot(\text{path diff.})}$$

— x — x — x —



$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} = 0.529\, n^2 A^\circ$$

n = Principle Quantum number.

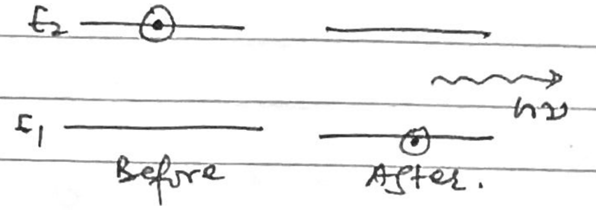
$$E_n = \frac{-m z^2 e^4}{8 \epsilon_0^2 n^2 h^2}$$

$$= \frac{-13.6}{n^2} \text{ ev}.$$

$$E = \left(\frac{V}{d}\right) \cdot \quad V = \frac{Q}{4\pi\epsilon_0 d}. \qquad P.E = (-e)\,V$$

$E = h\nu$

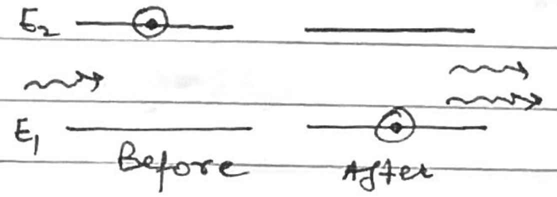
$= \dfrac{hc}{\lambda}$



1. → ABSORPTION

2. Spontaneous Emission

3. Stimulated Emission.



The no. of atoms present in any state is called it's population.

Q → Difference b/w Spontaneous & Stimulated Emission ?

→ Refer Book

Q → What are the characteristics of Laser Beam ?

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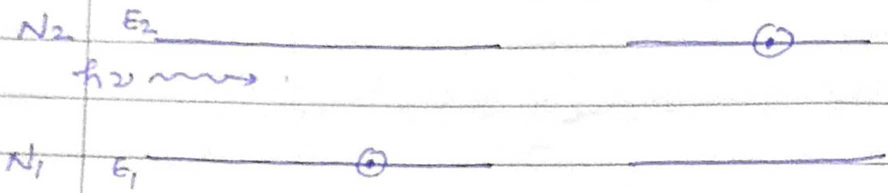
EINSTEIN'S COEFFICIENTS

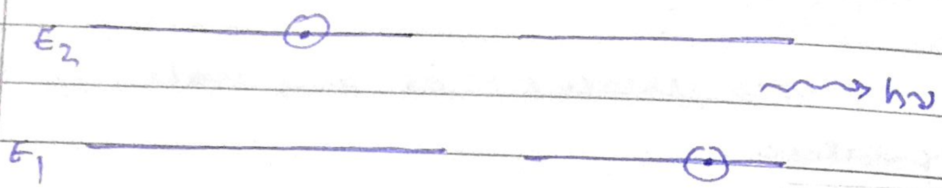
1.



STIMULATED ABSORPTION (ATTENUATION)

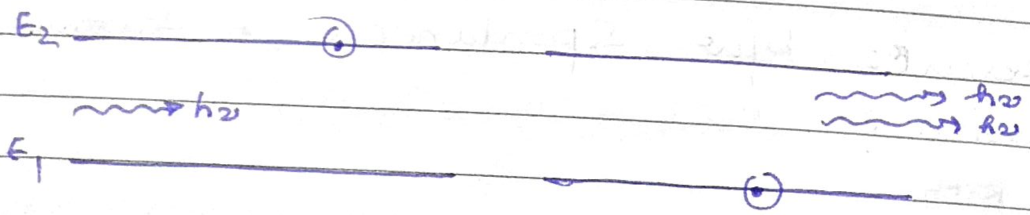
$$E_2 - E_1 = h\nu$$

2.



SPONTANEOUS EMISSION

3.



STIMULATED EMISSION (AMPLIFICATION)

For absorption

Now for 1. $P_{12} \propto u(\nu)$

Energy density

$\therefore P_{12} = B_{12}\, u(\nu)$

Einstein's co-efficient for stimulated Absorption

(a) Constant - Spontaneous Enmission

2. $P_{21} = A_{21}$

Einsteins coefficient of Spontaneous Emmision

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(b) variable – Stimulated Emission

3. $P_{21} \propto u(\nu)$

$\therefore \; P_{21} = \underbrace{B_{21}}_{\downarrow} u(\nu)$

Einstein's Coefficient for stimulated Emission.

Net probability $\rightarrow \quad P_{21} = A_{21} + B_{21} u(\nu)$

So Total probability for Absorption : $N_1 P_{12} = N_1 [B_{12} u(\nu)]$

Emission $\rightarrow \quad N_2 P_{21} = N_2 [A_{21} + B_{21} u(\nu)]$

$\Rightarrow$ At thermal Equilibrium.

$$N_1 P_{12} = N_2 P_{21}$$

$$N_1 [B_{12} u(\nu)] = N_2 [A_{21} + B_{21} u(\nu)]$$

$$[N_1 B_{12} - N_2 B_{21}] u(\nu) = N_2 A_{21}$$

$$\Rightarrow u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$= \frac{A_{21}}{\left(\frac{N_1}{N_2}\right) B_{12} - B_{21}}$$

$$= \frac{A_{21}}{B_{21}\left[\left(\frac{N_1}{N_2}\right)\frac{B_{12}}{B_{21}} - 1\right]}$$

Now at Thermal equilibrium Both the variable probabilities will be same i.e. $B_{12} = B_{21}$

$$\therefore u(\nu) = \frac{A_{21}}{B_{21}} \left[\frac{1}{\left(\frac{N_1}{N_2}\right) - 1} \right]$$

$$= \frac{A_{21}}{B_{21}} = \frac{A_{21}}{B_{21}} \frac{1}{\left[\left(\frac{N_1}{N_2}\right) - 1\right]}$$

Boltzman's law

$$\boxed{N_r = N_0 \, Exp\left(-\frac{E_r}{KT}\right)}$$

- $N_1 = N_0 \, Exp\left(\frac{-E_1}{KT}\right)$
- $N_2 = N_0 \, Exp\left(\frac{-E_2}{KT}\right)$

$$\frac{N_1}{N_2} = exp\left(\frac{E_2 - E_1}{KT}\right)$$

$$\frac{N_1}{N_2} = Exp\left(\frac{h\nu}{KT}\right)$$

Substuting.

$$\therefore u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left[Exp\left(\frac{h\nu}{KT}\right) - 1\right]}$$

Planck's Radiation Law

$$\frac{E_\nu \, d\nu}{(\nu \to \nu + d\nu)} = \frac{8\pi h\nu^3}{c^3} \left[\frac{d\nu}{Exp\left(\frac{h\nu}{KT}\right) - 1} \right]$$

If we remove $d\nu$

$N_2 \rightarrow$ Particles in Excited State
$N_1 \rightarrow$ " " Ground State.

$$E_\nu = \frac{8\pi h \nu^3}{c^3}\left[\frac{1}{\mathrm{Exp}\left(\frac{h\nu}{kT}\right)-1}\right]$$

If we compare these Equations

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}}$$

Now in terms of wavelength

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{d\lambda}{\mathrm{Exp}\left(\frac{hc}{\lambda kT}\right)-1}\right]$$

Final outcome is

$$\boxed{\frac{A_{21}}{B_{21}} \propto \nu^3}$$

"POPULATION INVERSION"

also called Inverted populated state or Negative Temprearature state.

↓

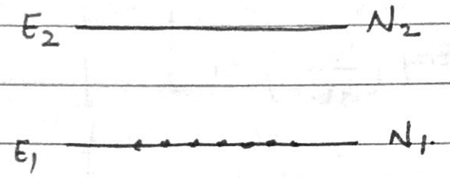
where Inverted populated State is reach is called Active System.

↓

Active Medium

↓

Active Material.



Here $N_1 > N_2$

But if there comes a state i.e. $\boxed{N_2 > N_1}$ then it is called populated Inverted state.

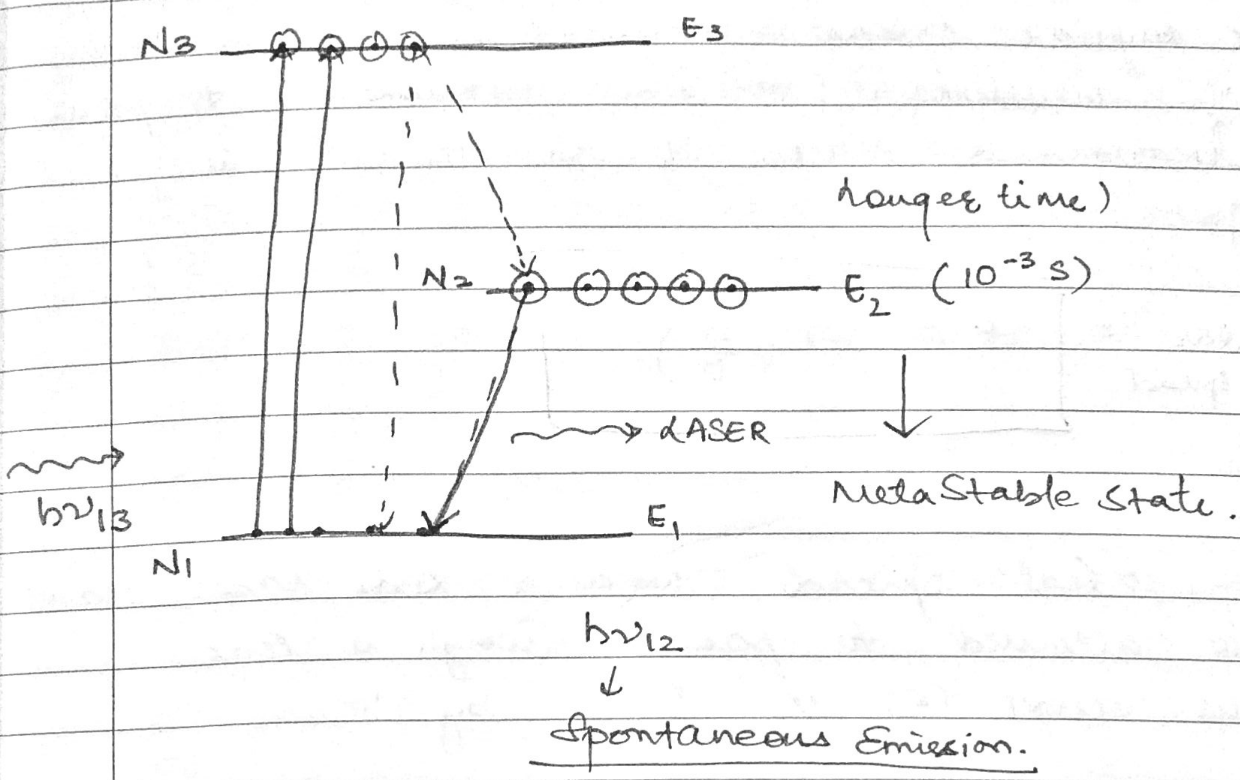
Population Inversion is done by ~~4 methods~~ Pumping

↓

Methods to Supply the Energy to do population Inversion.

PUMPING SOURCES

1. Optical pumping — RUBY LASER
2. Electrical Discharge — HE-NE LASER
3. Chemical Reaction — ~~SEMI CONDUCTOR~~ CO_2 LASER
4. Direct Conversion. — ~~CO_2 LASER~~, SEMICONDUCTOR LASER.


N3
E3
Longer time)
N2
E2 (10^-3 s)
LASER
Metastable state.
hν13
E1
N1
hν12
Spontaneous Emission.

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Tutorial

Some formulae

⇒ The Angular spread produced by a Laser Beam of wavelength (λ) when allowed to pass through a mirror of Diameter (D) is given by :-

$$\text{Angular spread} = \boxed{d\theta = 1.22\left(\frac{\lambda}{D}\right)}$$

⇒ The areal spread when a ~~laser~~ laser beam is allowed to pass through a lens of Focus length (F) is given by :-

$$\text{Areal Spread} = \boxed{dA = f(d\theta)^2}$$

⇒ The Radius of the Laser Beam focused at a particular point is given by :-

$$\text{radius} = \frac{\lambda_o f}{a}$$

where a = Beam Radius
f = focal length of given lens.

⇒ The area of the focused spot

$$\text{area} = \pi\left[\frac{\lambda_o f}{a}\right]^2$$

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Q Consider a 2mW laser Beam ($\lambda_0 = 6 \times 10^{-5}$ cm or 6000 Å)

Incident on an eye whose focal length is given by 2.5 cm If the pupil diameter $2a = 2$mm. then find the Intensity of the laser Beam.

Sol:

$$I = \frac{P}{A}$$

$P = 2$mW $= 2 \times 10^{-3}$ W

$a = 1$ mm $= 0.1$ cm

$\lambda_0 = 6 \times 10^{-5}$

$f = 2.5$ cm.

$$A = \pi \left[\frac{\lambda_0 f}{a} \right]^2$$

$$A = 3.14 \left[\frac{6 \times 10^{-5} \times 2.5}{0.1} \right]^2$$

$$A = 3.14 \times (150 \times 10^{-5})^2$$

$$A = 3.14 \times 22500 \times 10^{-10}$$

$$A = 3.14 \times 22500 \times 10^{-10}$$

$$A = 706.5 \times 10^{-9}$$

$$A = 7.06 \times 10^{-7}$$

$$I = \frac{2 \times 10^{-3}}{3.14 \times 22500 \times 10^{-10}}$$

$$I = \frac{2 \times 10^{7}}{3.14 \times 22500}$$

$$\boxed{I = 2.8 \times 10^{6} \ W/m^2}$$

Q 2

3 MW → Laser Beam

$2a = 1\,Cm$

$f = 5\,Cm.$

$\lambda_0 = 6 \times 10^{-5}\,Cm$

$$A = \pi \left[\frac{\lambda_0 f}{a} \right]^2$$

$\lambda_0 = 6 \times 10^{-5}\,Cm$

$= 6 \times 10^{-3}\,m$

$a = 0.5\,Cm. = 0.005\,m$

$f = 0.05\,m.$

$$A = 3.14 \left[\frac{6 \times 10^{-3} \times 0.05}{0.005} \right]^2$$

$$A = 3.14 \left[\frac{6 \times 10^{-3} \times 5 \times 10^{-2}}{5 \times 10^{-3}} \right]$$

$$A = 3.14 \left[\frac{6 \times 10^{-5}\,Cm \times 5}{0.5} \right]^2$$

$$A = 3.14 \left[\frac{\overset{6}{\cancel{30}} \times 10^{-5} \times 10.}{\cancel{5}} \right]^2$$

$$A = 3.14 \left[6 \times 10^{-4} \right]^2$$

$A = 3.14 \times 36 \times 10^{-8}$

$A = 113.04 \times 10^{-8}.$

$$I = \frac{P}{A}$$

Power is Energy per unit time!

$$I = \frac{3\times10^{6}}{113.04\times10^{-8}}$$

$$I = \frac{3\times10^{14}}{113.04}$$

$$I = 0.0265\times10^{14}$$

$$\boxed{I = 2.65\times10^{16}}\ W/M^2$$

Q2 Calculate the power per unit area delivered by a laser pulse of energy 4×10^{-3} Joule and pulse length 10^{-9} sec when the pulse is focused on a Target to a very small spot of radius 1.5×10^{-5} m.

Ans:

$$\text{power} = \frac{\text{Energy}}{\text{time}} = \frac{4\times10^{-3}}{10^{-9}} = 4\times10^{6}\ W.$$

$$I = \frac{P}{A} = \frac{4\times10^{6}}{3.14\times(1.5\times10^{-5})^2}$$

$$= \frac{4\times10^{6}}{3.14\times1.5\times1.5\times10^{-10}}$$

$$= \frac{4\times10^{6}}{7.065\times10^{-10}}$$

$$= 0.566\times10^{16}$$

$$= 5.6\times10^{15}\ W/M^2$$

TUSHY
18|1|18

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MAIN COMPONENTS OF A LASER

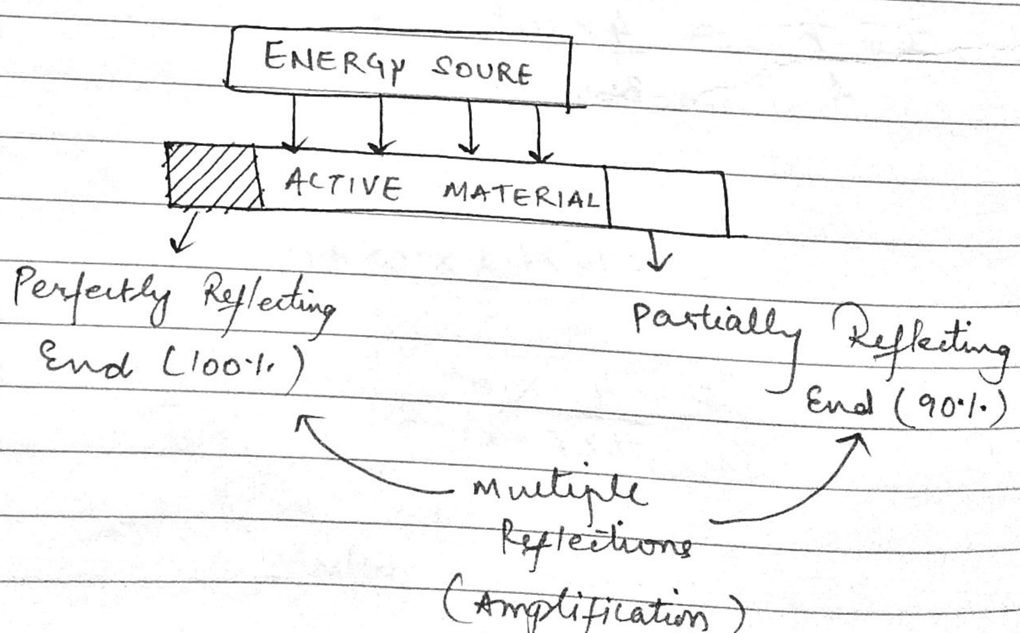
NECESSITIES

1. Active Material
   - Molecule
   - Atom
   - Ion

   Solid
   Liquid
   Gaseous.

2. Pumping Source.
   * Pop Inversion
   * ~~Metabst~~ Meta Stable state
   * Sp. Em
   * St. Em

3. Resonant cavity
   optical Resonator.



Losses → Attenuation

Gain → Amplification

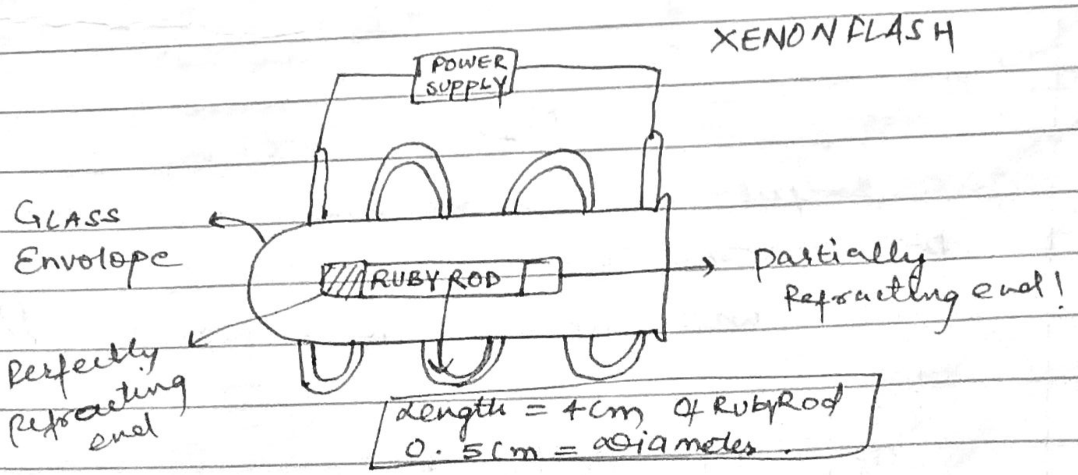
Steady state

1. Absorption
2. Transmission
3. Scattering

There may be loss Based on these 3 Reason. But gains are also there.
In the steady state the losses are compensated by the gains.

→ RUBY LASER (Al_2O_3, Cr_2O_3)
(Pink)

also called as → CRYSTAL LASER
THREE LEVEL LASER
PULSED.



Active material = Cr^{+3} ion.

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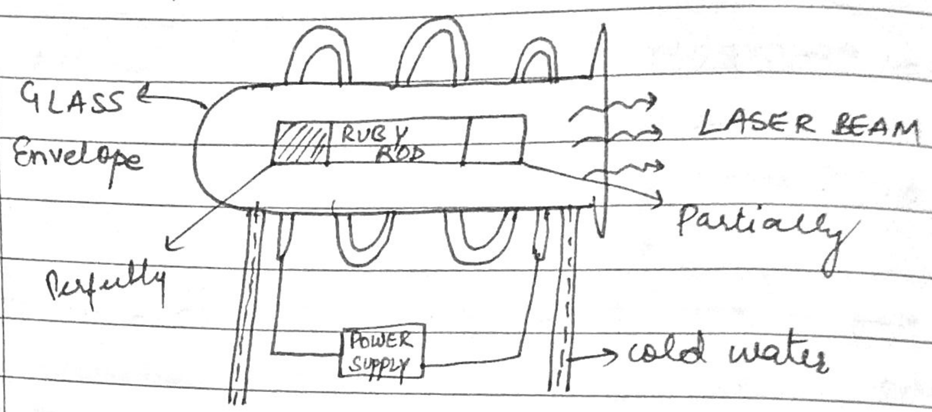
Drawing Diagram again

Active ~~Medium~~ material = Cr^{+3} ion

Active ~~material~~ medium = Ruby Laser



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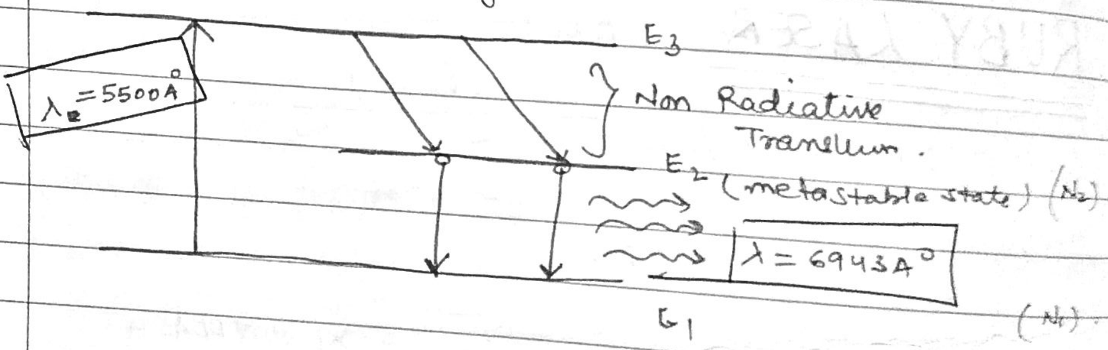
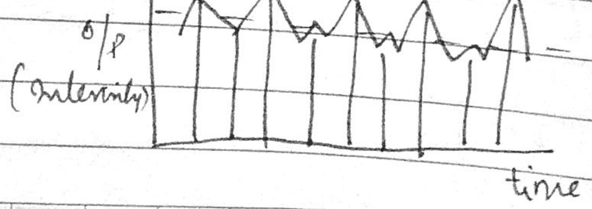
Energy level diagram.



$N_2 > N_1$ (P.I)

~~output will be always a~~ ~~pto~~ ~~pulsed~~ wave

output will always be a pulsed wave



Active material is always less in number.

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Applications

→ Drilling purpose

→ whenever you ~~see~~ want pulses.

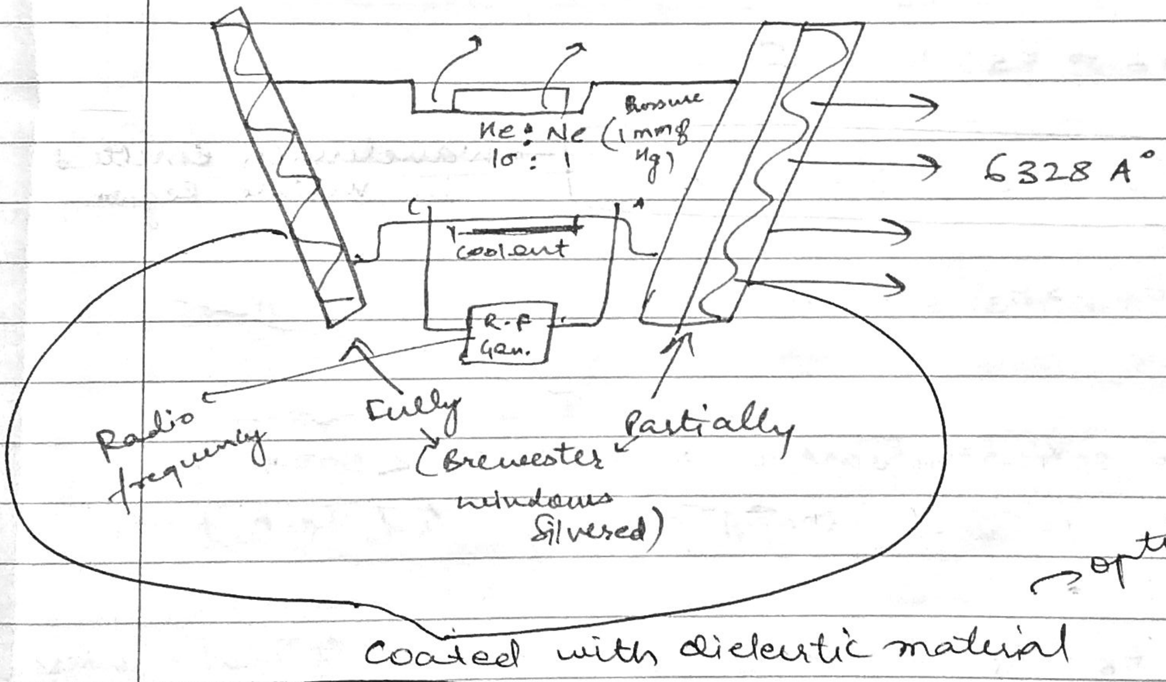
Disadvantages

→ Efficiency ~~is~~ is less than 10%.

→ Conversion Rate is very less. (Because of 3 level system).

→ Large amount of heat is used, so a coolent is to be provided.

→ "He - Ne Laser" → 4 level Laser.



optical ~~lenses~~ absorbers.

$\lambda < 5000 A°$ (absorbed.

Coated with dielectric material

→ 1st Gas Laser.

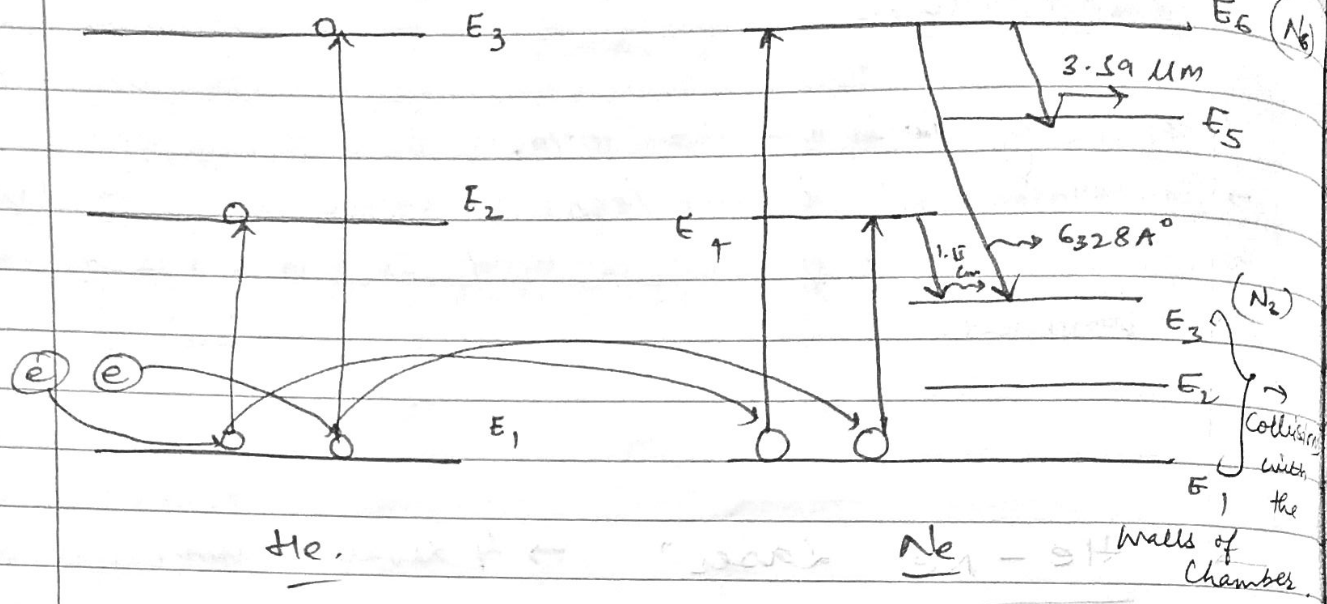
Active material → Ne atom.

→ Pumping is Radio - Frequency Generator. (Electric discharge)

→ He : Ne = 10 : 1 Ratio. (Ne is less since it is active material)

→ Length & diameter 80 : 10 cm.

Energy level Diagram



$E_6 \rightarrow E_5$ $\lambda = 3.39\,\mu m$

$\boxed{E_6 \rightarrow E_3 \quad \lambda = 6328 A^\circ}$ → wavelength emitted in visible region.

$E_4 \rightarrow E_3$ $\lambda = 1.15\,\mu m$

Due to optical absorber wavelengths < 5000A° gets absorbed and > 5000A° gets reflected back to the system!

⇒ $E_6 \rightarrow E_5$ & $E_4 \rightarrow E_3$ is reduced by optical absorbers.

→ $N_6 > N_3$ ~~should be~~ continously and for this the downward transition $E_3 \rightarrow E_2$ & $E_2 \rightarrow E_1$ should be fast.

and to take care of this we take the ratio of the tube (80 : 10 cm) (length : diameter)

→ Efficiency of He-Ne is less than 20%.

→ Primary Reasons

(i) You can not Restrict $E_6 - E_5$, $E_4 \rightarrow E_3$ & $E_1 \rightarrow E_4$.

Applications

(i) Bar Code Readers.

To over come ISSUES of He-Ne Laser we have CO_2 laser. & CO_2 Laser is the best Industrial Lasers.

~~Q what does the CD ROM contain~~

Q2 How the Groves of a CD are filled so that you can read & erase ~~the~~ & write the data again & again on a CD or a DVD?

Q2 what does a R/W CD should contain so that the Groves are created ~~on the~~ into lands of flat surface again & again?

Q2 what should your CD-writer do to create the lands again and again. (other than laser)?

23/1

HOLO → whole
GRAPHY → WRITING
(Graphine)

HOLOGRAPHY

By DENNIS GABOR

First hologram was produced by LEITH UPATNICKS

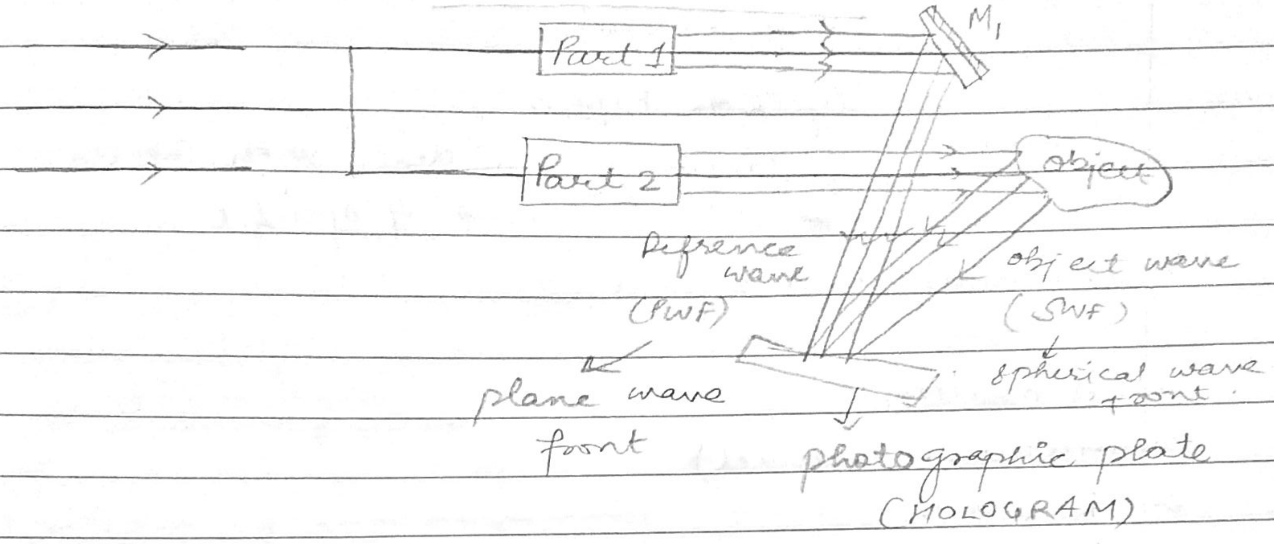
photography	Holography
1. Lens	1. no need of lens.
2. 2D Image of 3D object.	2. 3D Image of 3D object.
3. Intensity (Amplitude)	3. Intensity variations + phase variations.
	4. Complicated Interference pattern.
	5. optical code ↓ photographic plate.

to do all this we need Highly Co-herent Beam. (laser)

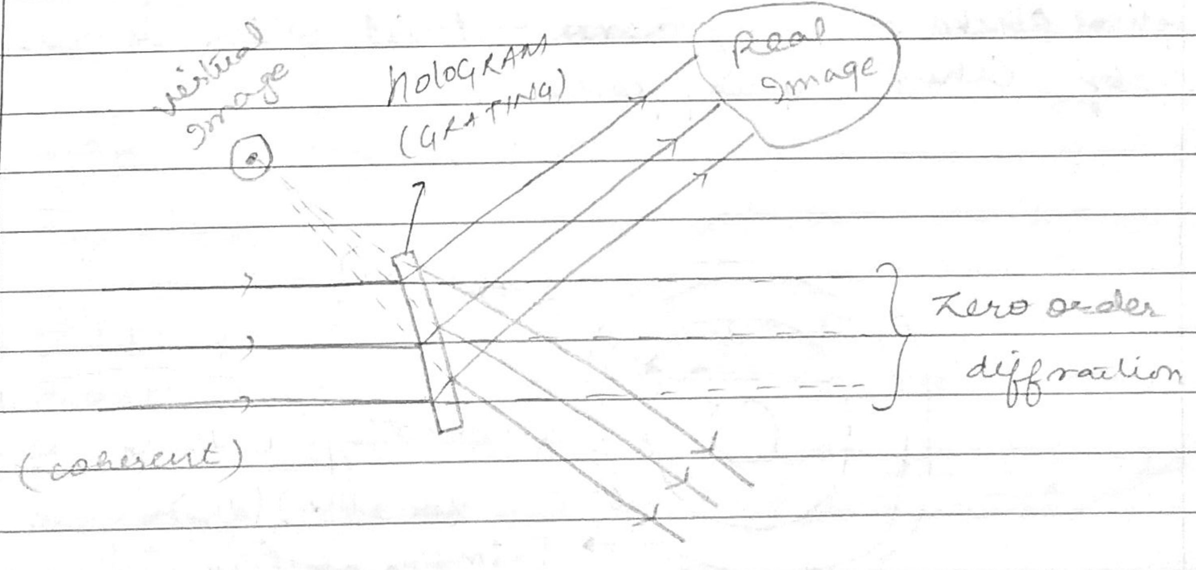
→ HOLOGRAM → Construction
→ Reconstruction,

→ Construction :-

(Recording



→ Re-construction



25/Jan.

'FIBRE OPTICS'

'OPTICAL FIBRE'

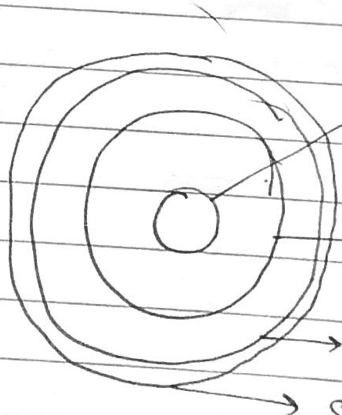
(Branch of physics which deals with communication system with the help of optics.)

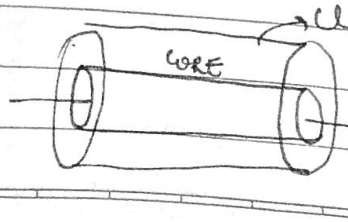
→ Communication System

1. Transmitter
2. medium (channel)
3. Reciever.

→ Total Internal Reflection is the phenomena used in optical fibre

→ optical fibre is a very very narrow ~~nano~~ material which is made up of either Glass / plastic.





For normal Cu wire → 48 Speeches Can be delivered simulta ansously
By an optical fibre Can deliver 15000 Speeches Simultaneously

Refractive Index of core = n_1
" " " Cladding = n_2

for TIR $\boxed{n_1 > n_2.}$

29 JAN

ADvantages

1. Multiplexing — Bandwidth ↑
2. Saftey is very high.
3. Bent / Flexible

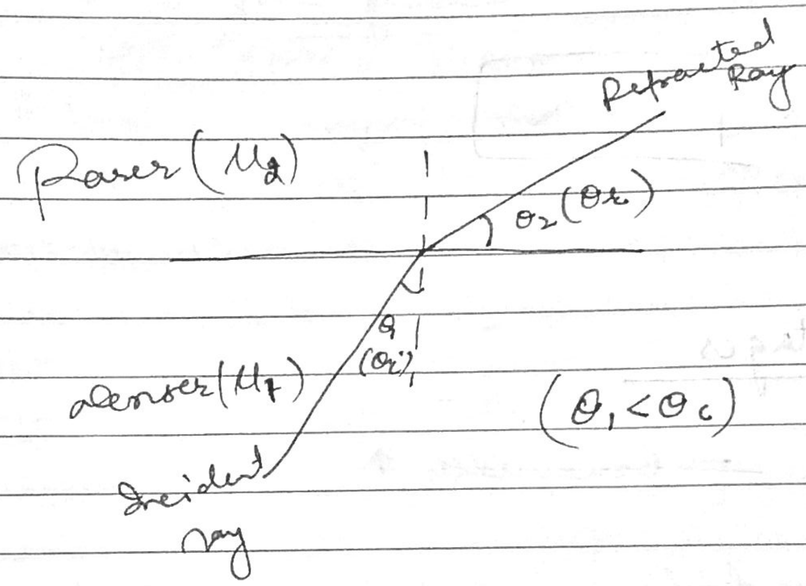
→ Main function of Cladding:

* Chipping (Total Insulation of light way. ~~Lithous~~ the light is to stay Inside & outside light Shouldn't Come in)
* Cladding should h've lower R. Index for TIR to happen.

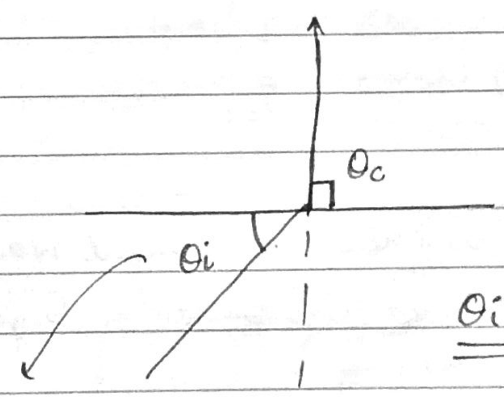
→ Total Internal Reflection

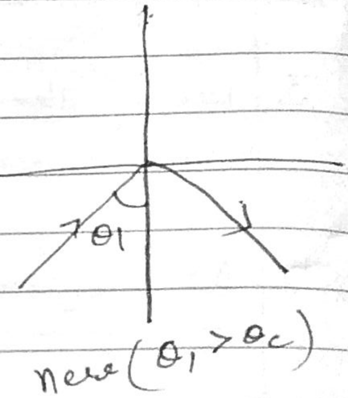
For a particular angle of ~~Reflection~~ Incidence where the angle of Refraction is 90° that particular angle is Called critical angle

Form Refraction (Denser to Rarer) :-



For TIR





$\theta_i = \theta_c$

That particular angle of incidence where ~~θ_i~~ Refraction angle is 90°.

Snell's Law

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$\mu_1 \sin\theta_c = \mu_2$

$\Rightarrow \sin\theta_c = \left(\frac{\mu_2}{\mu_1}\right) \Rightarrow \boxed{\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)}$

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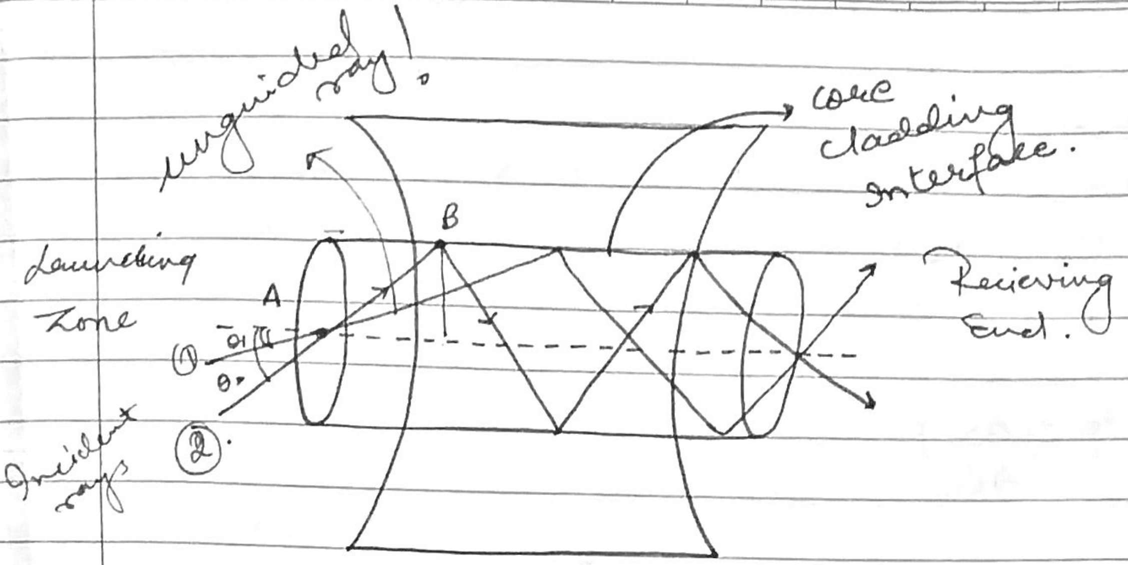

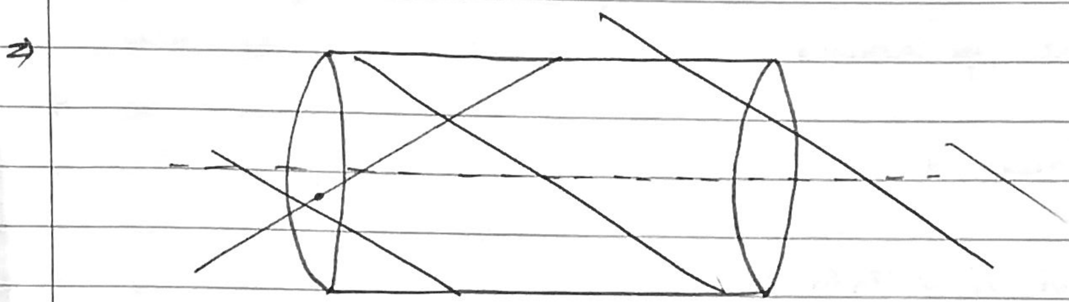

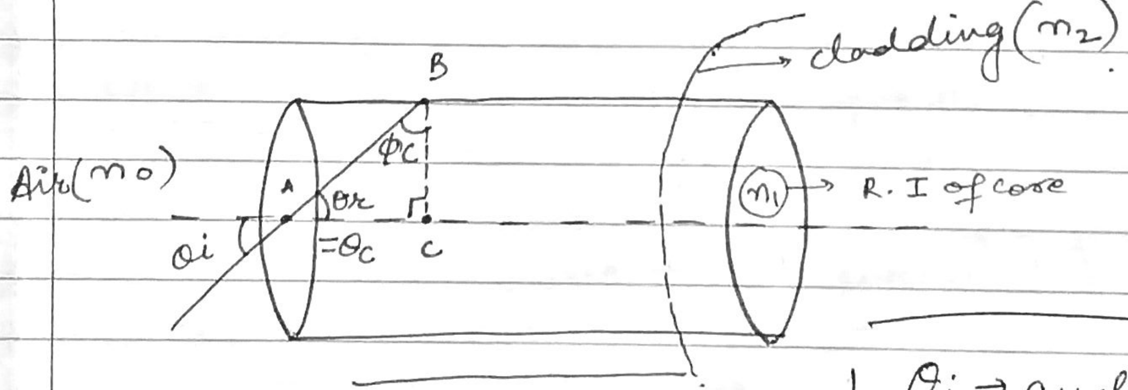

$\theta_i \rightarrow$ angle of Incidence
$\theta_r \rightarrow$ angle of refracting

$\theta_r + \phi_c + 90^\circ = 180^\circ$

$\boxed{\theta_r = 90^\circ - \phi_c}$

$\boxed{\theta_c = 90^\circ - \theta_r}$

ϕ_c = Critical angle

θ_c = Critical propagation angle (angle which is ~~to~~ responsible for the critical angle)

30 January θ_i = angle of incidence

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In ΔABC

$$\sin\phi_c = \left(\frac{AC}{AB}\right) = \left(\frac{n_2}{n_1}\right)$$

$$\cos\theta_c = \left(\frac{AC}{AB}\right)$$

→ NUMERICAL APERTURE (NA) (maximum angle of incidence)

* light gathering capability of an optical fibre.

Snell's law

$$n_0 \sin\theta_i = n_1 \sin\theta_r$$

$$\sin\theta_i = \left(\frac{n_1}{n_0}\right)\sin\theta_r$$

from ΔABC

$$\theta_r + \phi_c + 90^\circ = 180^\circ$$

$$\theta_r = 90^\circ - \phi_c.$$

Substituting the value.

$$\sin\theta_i = \left(\frac{n_1}{n_0}\right) \sin(90^\circ - \phi_c]$$

$$= \frac{n_1}{n_0}\cos\phi_c.$$

$\sin\phi_c$ will be defined when we will find θ_i which will be max for ϕ_c. thus $\theta_i = (\theta_i)_{max}$

Angle of incidence.

$$\boxed{\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

$$\sin\phi_c = \frac{n_2}{n_1}$$

So now from this value for $\cos\phi_c = \sqrt{1-\frac{n_2^2}{n_1^2}}$

$$= \frac{\sqrt{n_1^2-n_2^2}}{n_1}$$

$$\therefore \sin(\theta_i)_{max} = \left(\frac{\cancel{n_1}}{n_0}\right)\frac{\sqrt{n_1^2-n_2^2}}{\cancel{n_1}}$$

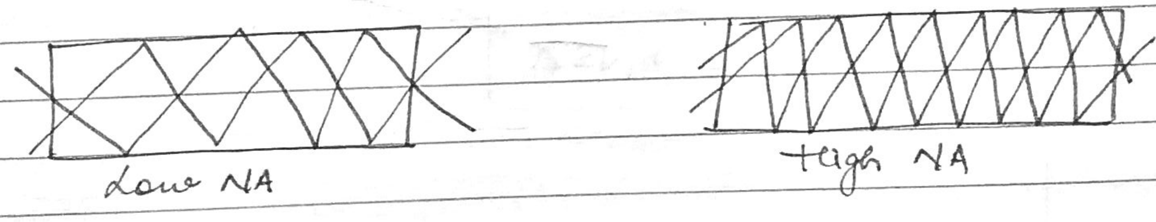
$$\sin(\theta_i)_{max} = \sqrt{n_1^2-n_2^2} \quad (\because n_0 = 1 \text{ (for air)})$$

$$\boxed{NA = \sin(\theta_i)_{max} = \sqrt{n_1^2-n_2^2}}$$

Numerical Aperture is defined as Maximum Angle of Incidence.

$NA \longrightarrow 0.13 - 0.50$.

→ Now consider two optical fibres



In an optical fibre with High Numerical Aperture (NA) no. of modes are more as compared to the one with Low NA.

Relative Refractive Index (Δ)
(fractional)

$$\boxed{\Delta = \frac{n_1 - n_2}{n_1}}$$

$(\Delta \rightarrow 0 - 0.01)$

$\rightarrow$ $NA = \sqrt{n_1^2 - n_2^2}$

$$(NA)^2 = n_1^2 - n_2^2$$
$$= (n_1 + n_2)(n_1 - n_2)$$
$$= 2n_1 \left(\frac{n_1 + n_2}{2}\right)\left(\frac{n_1 - n_2}{n_1}\right)$$
$$= 2n_1 \left(\frac{2n_1}{2}\right)\left(\frac{n_1 - n_2}{n_1}\right)$$

($\because$ Diff b/w n_1 & n_2 is very less & it will come as n_1 only
Let $n_1 = 1.556$
$n_2 = 1.55$)

$$= 2n_1^2 [\Delta]$$

$$\boxed{NA = n_1 \sqrt{2\Delta}}$$

POWER LOSSES.

P_i = Input power
P_o = Output power.

$$P_o = P_i e^{-\alpha L}$$

$\alpha \rightarrow$ Attenuation constant.
$L \rightarrow$ Length of fibre whichever we are considering!

$$\frac{P_o}{P_i} = e^{-\alpha L}$$

$$\ln \frac{P_o}{P_i} = -\alpha L$$

$$\Rightarrow \alpha L = \ln\left(\frac{P_i}{P_o}\right)$$

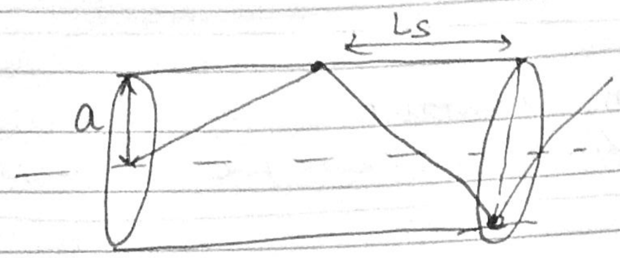
$$\alpha = \frac{1}{L} \ln\left(\frac{P_i}{P_o}\right)$$

For Base 10

$$\boxed{\alpha = \frac{10}{L} \log_{10}\left(\frac{P_i}{P_o}\right) \frac{dB}{Km}}$$

SKIP DISTANCE (L_s)

The distance b/w any two successive Reflections taking place inside the core is called SKIP Distance.



$$\rightarrow \quad L_s = 2a \left[\left(\frac{n_1}{n_0 \sin\theta_1} \right)^2 - 1 \right]^{1/2}$$

$$\rightarrow \quad N_r = \frac{L}{L_s}$$

(Total no. of Reflection)

length of the optical fibre

$$= \frac{L}{2a} \left\{ \frac{1}{\left[\left(\frac{n_1}{\sin\theta_i^\circ} \right)^2 - 1 \right]^{1/2}} \right.$$

$$\boxed{N_r = \frac{L}{2a} \frac{1}{\left[\left(\frac{n_1}{\sin\theta_i} \right)^2 - 1 \right]^{1/2}}}$$

V-Number

(normalised Frequency)

$$V = \frac{\pi d}{\lambda}[NA]$$

$$= \frac{\pi d}{\lambda}\left[\sqrt{n_1^2 - n_2^2}\right]$$

$$= \frac{\pi d}{\lambda}\left[n_1\sqrt{2\Delta}\right]$$

no. of modes $= \frac{V^2}{2}$ For single mode

$\frac{V^2}{4}$ For multimode.

1/Feb/2018

Types of fibres

Based on

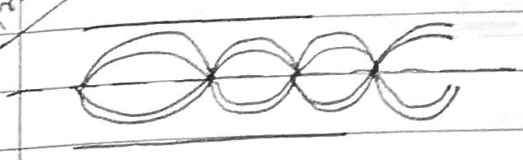
- Refractive Index profile
  - Step Index fibre
  - Graded Index fibre
- modes of propagation
  - Single mode Step Index fibre (SM SI / SIF / SMF)
  - Multimode Step Index fibre
  - multimode Graded Index fibre
- materials
  - Glass / Glass
  - Glass / Plastic
  - plastic cladded Glass (PCS)

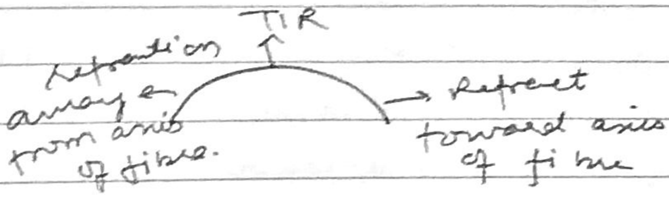
M.M Step Index	Graded Index
d core ⇒ 500 – 100 μm	d core = 50 μm
d clad ⇒ 100 μm	d clad = 70 μm
High dispersion	Low dispersion
meridonial ~~prog~~ propagation	Skew Helical
Band width is low	very High Bandwidth
NA is Large	
Longer Lifetime	
Cost of fibre is low.	Cost of fibre is Expensive (diff. to manufacture.)

→ Step has one uniform n_1, then comes down and hence has another uniform n_2.
In Graded, n_1 gradually decreases & becomes n_2.

$\alpha = 2$ in case of M.M.G.I.F

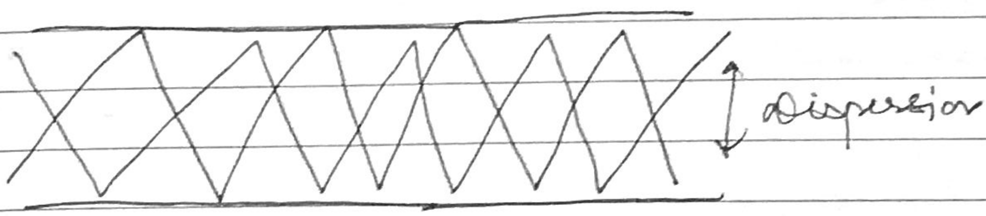
Graded





Skew helical / self periodic focusing

step:



Graded

$$\boxed{n(r) = n_1 \sqrt{1 - \left[2\Delta\left(\frac{r}{a}\right)\right]^{\alpha}}} \quad (r<a)$$

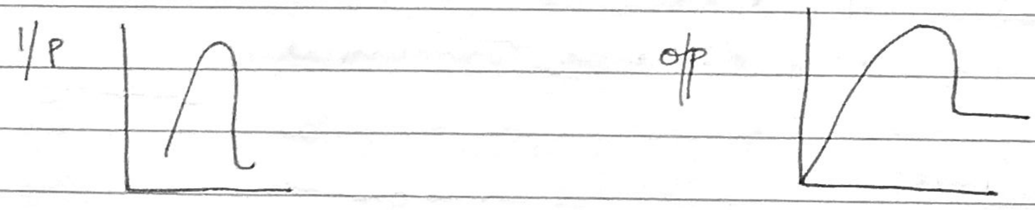
$$= n_2 \quad (r>a).$$

$a \rightarrow$ radius of core

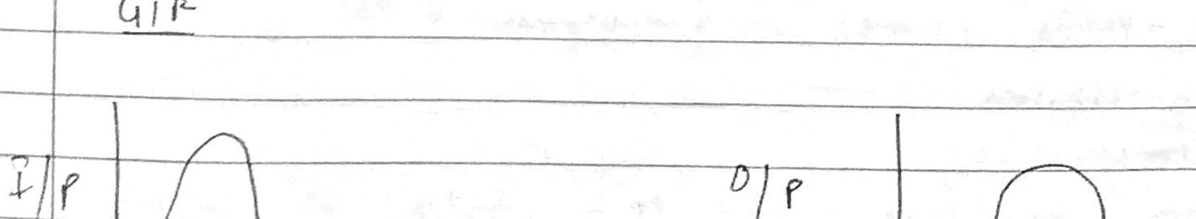
$r =$ radius of cladding

α is 2 since 2 reflections take place.

SIF



GIF



	SMSIF	MMSIF	MM GRIN
Core made of Cladding is made of silica lightly doped with phosphorus	Ge doped si		
	$d_{core} \rightarrow$ 18-12 μm	50-100 μm	50-200 μm
	$d_{clad} \rightarrow$ 125 μm	150-250 μm	125-400 μm
	$n(r) = n_1 (r<a)$ $= n_2 (r>a)$	Finite no. of modes signal degradation which leads to high attenuation Δ & NA high	$n(r) = n_1 \sqrt{1 - \{2\Delta (\frac{r}{a})\}}$ $(r>a)$ where $\alpha = 2$ $r = n_2 \; (r<a)$
	only zero order mode		$NA = n_1 \sqrt{2\Delta \left[1 - \left(\frac{r}{a}\right)^2\right]}$
	Δ & NA = small		

Rays making larger angles with axis traverse longer path but they travel in a region of lower refractive index & hence have higher speed of propagation.

Glass fibres

		or	or → Ge is added to n
Core	SiO_2	SiO_2	$GeO_2 SiO_2$
Cladding	$GeO_2 SiO_2$	$B_2O_3 SiO_2$	SiO_2

n_1 (silica) = 1.458 at λ = 850 nm.

Used in long distance communication

Plastic fibres

Core - polystyrene	(1.6)	PMMA	(1.49)
Clad - PMMA	(1.49)	copolymer	(1.40)

Polymethylon

Cost effective

Temp. sensitive upto 80°C, after 80°, copolymer may melt.

PCS.

Core - High purity quartz material (1.05)

Clad - Teflon.

Short distance Communication app.

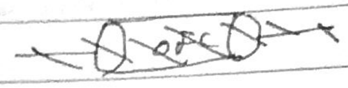
used in SIF only.

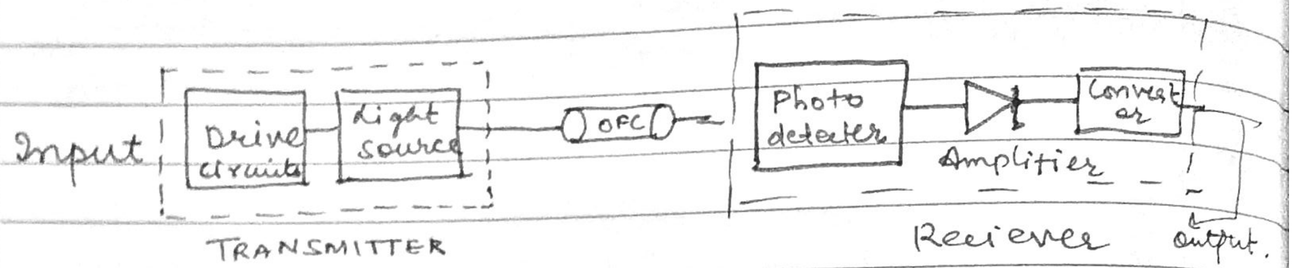
Less Expensive & have v. high attenuation.

5|2

"OPTICAL FIBRE COMMUNICATION"

1. Transmitter
2. channel – optical fibre cable (OFC)
3. Reciever





Non Electrical Signal → Transducer

↓

Electrical Signal

↓

These electrical signals In Transmitter are coupled with light source

- LEDs
- Laser diodes

↓

IR Region

850 nm

1300 nm

1500 nm

ANALOG MODULATION

↓

DIGITAL MODULATION

↓

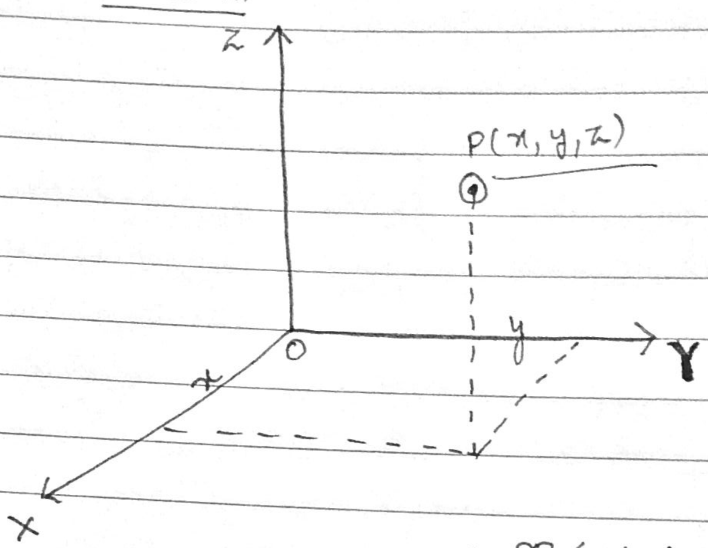
Signal Transmission to longer distance with Same power.

photo detectors detects the signal then it's amplified and then converted converts it back to it's original form of signal.

"CO-ORDINATE SYSTEMS"

1. Cartesian (x, y, z)
2. Cylindrical (ρ, ϕ, z)
3. Spherical (r, θ, ϕ)

Cartesian



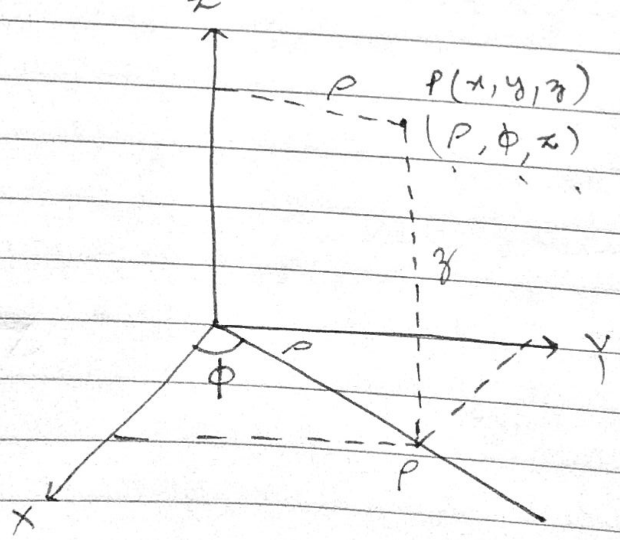
$-\infty \le x \le +\infty$

$-\infty \le y \le +\infty$

$-\infty \le z \le +\infty$

Cylindrical

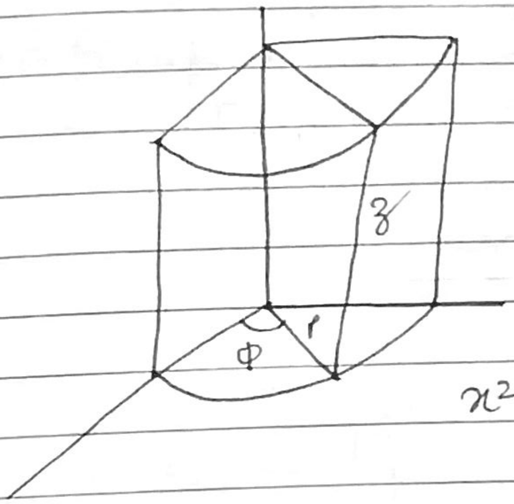
(ρ, ϕ, z)



$0 < \rho \le \infty$

$0 < \phi \le 2\pi$

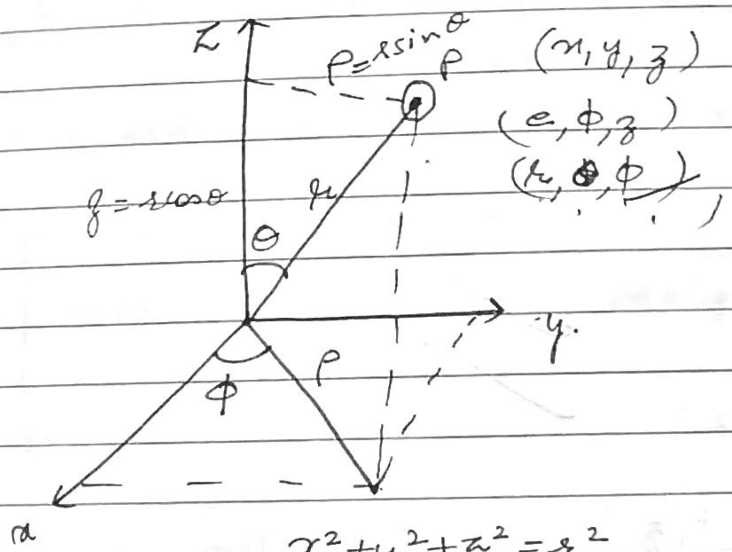
$-\infty \le z \le \infty$



$$x^2+y^2=\rho^2.$$

→ "SPHERICAL"

(r, θ, ϕ)



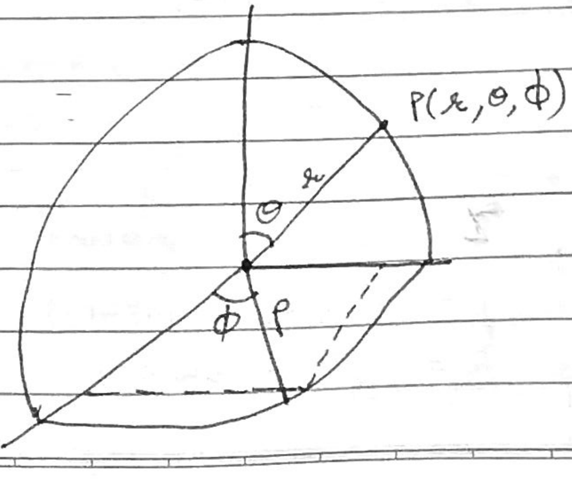
$$x^2+y^2+z^2=r^2$$

$$\rho^2+z^2=r^2.$$

$0 < r \leq \infty$

$0 < \theta \leq \pi$

$0 < \phi \leq 2\pi$



25/Jan.

'FIBRE OPTICS'

'OPTICAL FIBRE'

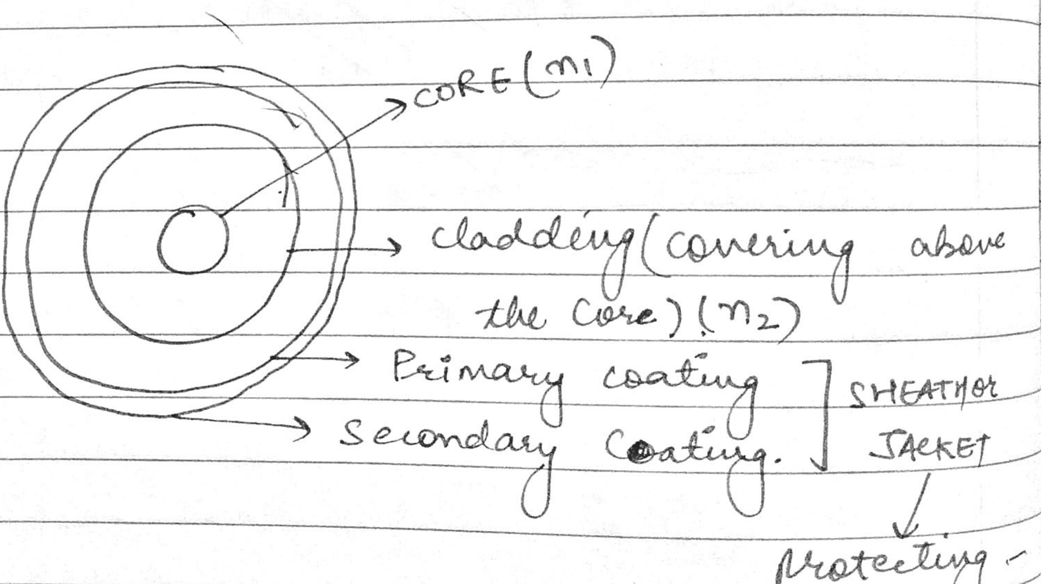
(Branch of physics which deals with communication system with the help of optics.)

→ Communication System

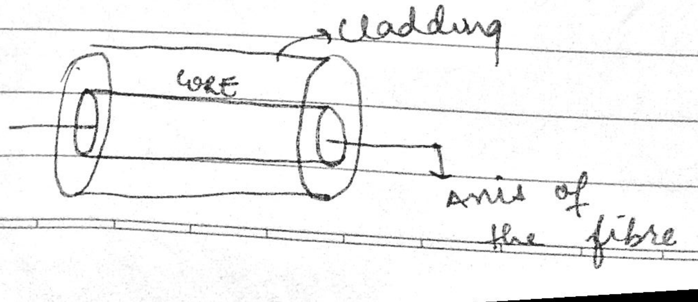
1. Transmitter
2. Medium (channel)
3. Reciever.

→ Total Internal Reflection is the phenomena used in optical fibre

→ optical fibre is a very very narrow ~~nano~~ material which is made up of either Glass / plastic.



— the core and the cladding.



For normal Cu wire → 48 Speeches can be delivered Simultaneously
By an optical fibre can deliver 15000 Speeches Simultaneously

Refractive Index of core $= n_1$
" " " Cladding $= n_2$

for TIR $\boxed{n_1 > n_2}$.

29 JAN

ADVantages

1. Multiplexing — Bandwidth ↑
2. Saftey is very high.
3. Bent | Flexible

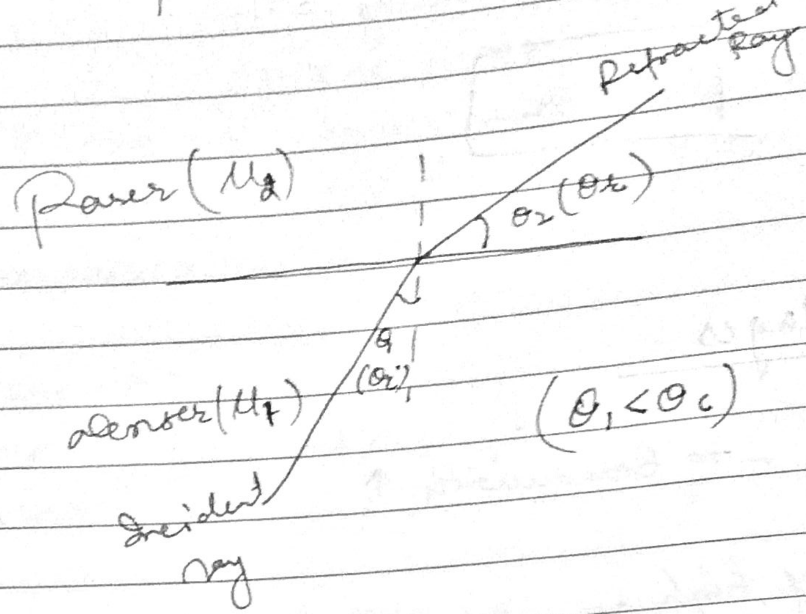
→ Main function of Cladding:

* Chipping (Total Insulation of Light way. ~~Without~~ the light is to stay Inside & outside Light shouldn't Come in)
* Cladding should h've lower R. Index for TIR to happen.

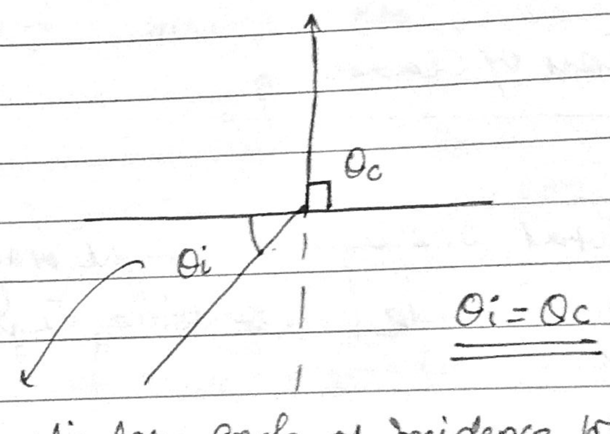
→ Total Internal Reflection

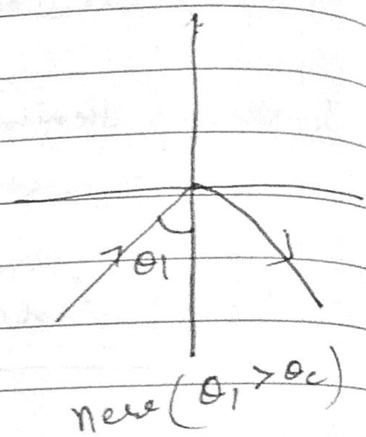
For a particular angle of ~~Reflection~~ Incidence where the angle of Refraction is 90° that particular angle is called Critical angle

~~For~~ Refraction (Denser to Rarer) :-



For TIR





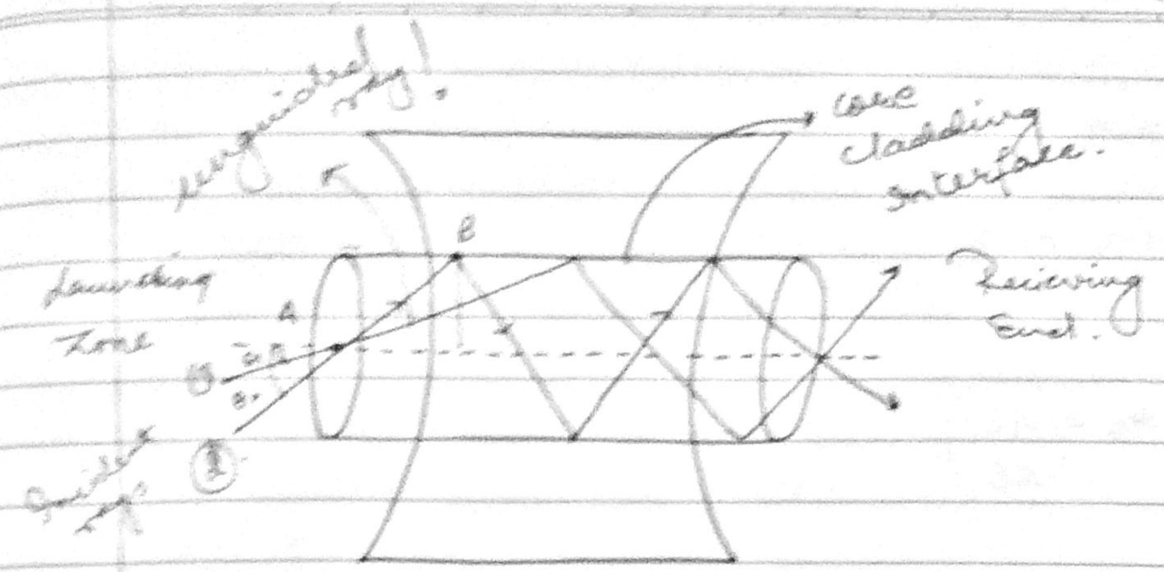
That particular angle of incidence where ~~θ_i~~ Refraction angle is $90°$.

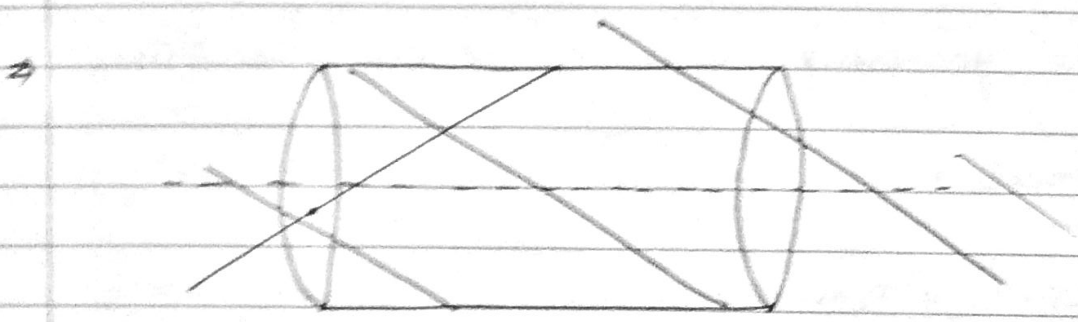
Snell's Law

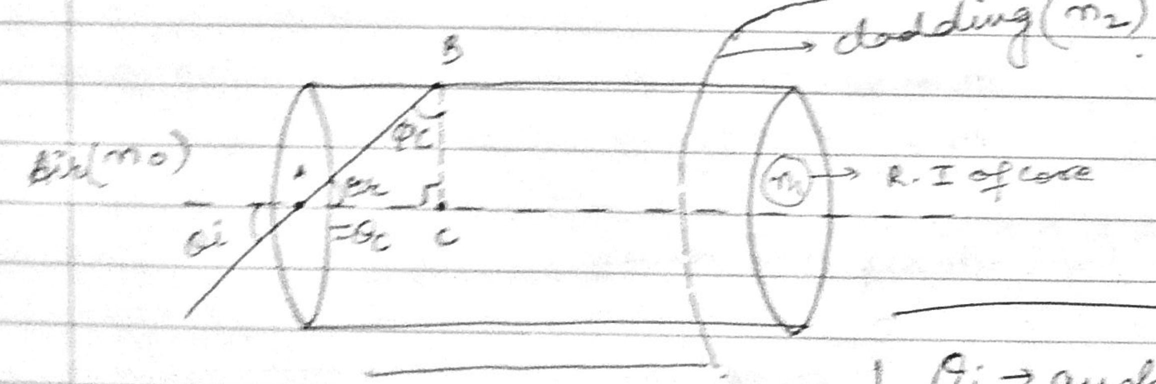
$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$\mu_1 \sin\theta_c = \mu_2$

$\Rightarrow \sin\theta_c = \left(\frac{\mu_2}{\mu_1}\right) \Rightarrow \boxed{\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)}$







θ_i → angle of incidence
θ_r → angle of refracting

$$\theta_r + \phi_c + 90^\circ = 180^\circ$$

$$\boxed{\theta_r = 90^\circ - \phi_c}$$

$$\boxed{\theta_c = 90^\circ - \theta_r}$$

ϕ_c = Critical angle

θ_c = Critical propagation angle (angle which is responsible for the critical angle)

30 January θ_i = angle of incidence

In ΔABC

$$\sin\phi_c = \left(\frac{AC}{AB}\right) = \left(\frac{n_2}{n_1}\right)$$

$$\cos\theta_c = \left(\frac{AC}{AB}\right)$$

→ NUMERICAL APERTURE (NA) (maximum angle of incidence)

* light gathering capability of an optical fibre.

Snell's law

$$n_0 \sin\theta_i = n_1 \sin\theta_r$$

$$\sin\theta_i = \left(\frac{n_1}{n_0}\right)\sin\theta_r$$

from ΔABC

$$\theta_r + \phi_c + 90^\circ = 180^\circ$$

$$\theta_r = 90^\circ - \phi_c.$$

Substituting the value.

$$\sin\theta_i = \left(\frac{n_1}{n_0}\right)\sin(90^\circ - \phi_c]$$

$$= \frac{n_1}{n_0}\cos\phi_c.$$

$\sin\phi_c$ will be defined when we will find θ_i which will be max for ϕ_c. thus $\theta_i = (\theta_i)_{max}$.

Angle of incidence.

$$\boxed{\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

$$\sin\phi_c = \frac{n_2}{n_1}$$

So now from this value for $\cos\phi_c = \sqrt{1 - \frac{n_2^2}{n_1^2}}$

$$= \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\therefore \sin(\theta_i)_{max} = \left(\frac{\cancel{n_1}}{n_0}\right) \frac{\sqrt{n_1^2 - n_2^2}}{\cancel{n_1}}$$

$$\sin(\theta_i)_{max} = \sqrt{n_1^2 - n_2^2} \quad (\because n_0 = 1 \text{ (for air)})$$

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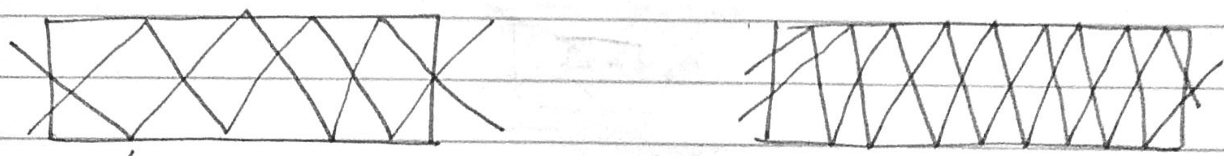
$$NA \longrightarrow 0.13 - 0.50.$$

→ Now consider two optical fibres



Low NA   High NA

In an optical fibre with High Numerical Aperture (NA) no. of modes are more as compared to the one with Low NA.

RELATIVE REFRACTIVE INDEX (Δ)
(fractional)

$$\boxed{\Delta = \frac{n_1 - n_2}{n_1}}$$

($\Delta \rightarrow 0 - 0.01$)

$\rightarrow$ $NA = \sqrt{n_1^2 - n_2^2}$.

$$(NA)^2 = n_1^2 - n_2^2$$
$$= (n_1 + n_2)(n_1 - n_2)$$

$$= 2n_1 \frac{(n_1 + n_2)}{2} \left(\frac{n_1 - n_2}{n_1}\right)$$

$$= 2n_1 \left(\frac{2n_1}{2}\right) \left(\frac{n_1 - n_2}{n_1}\right)$$

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Let $n_1 = 1.558$
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$$= 2n_1^2 [\Delta]$$

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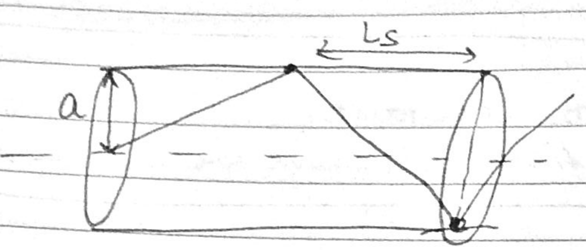

→ $$L_s = 2a\left[\left(\frac{n_1}{n_0 \sin\theta_1}\right)^2 - 1\right]^{1/2}$$

→ N_r (Total no. of Reflection) $= \dfrac{\overbrace{L}^{\text{Length of the optical fibre}}}{L_s}$

$$= \frac{L}{2a}\left[\frac{1}{\left[\left(\frac{n_1}{\sin\theta_i}\right)^2 - 1\right]^{1/2}}\right]$$

$$\boxed{N_r = \frac{L}{2a}\,\frac{1}{\left[\left(\frac{n_1}{\sin\theta_i}\right)^2 - 1\right]^{1/2}}}$$

V-Number

(normalised Frequency)

$$V = \frac{\pi d}{\lambda}[NA]$$

$$= \frac{\pi d}{\lambda}\left[\sqrt{n_1^2 - n_2^2}\right]$$

$$= \frac{\pi d}{\lambda}\left[n_1\sqrt{2\Delta}\right]$$

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1/Feb/2018

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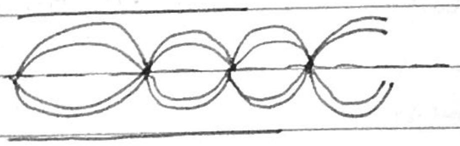
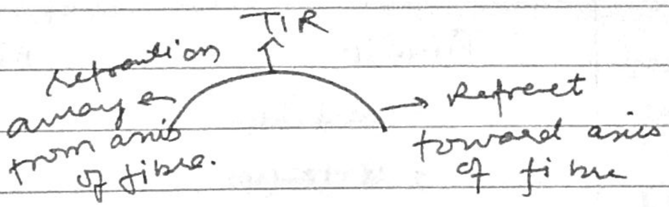
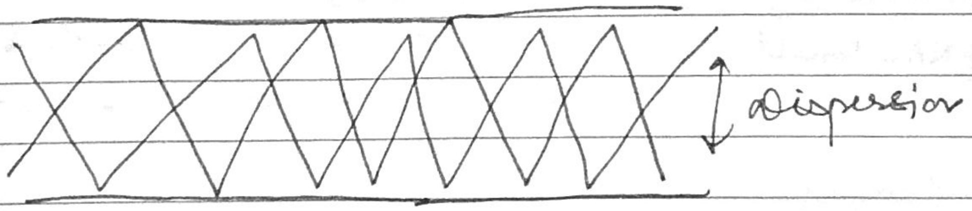




Skew helical / self periodic focusing

step:



Graded

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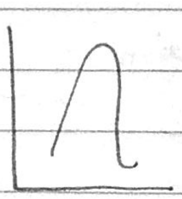
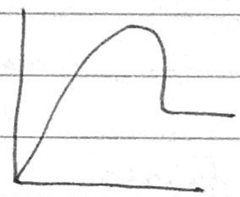
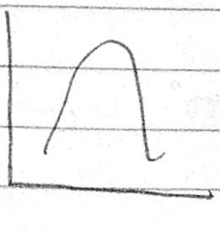
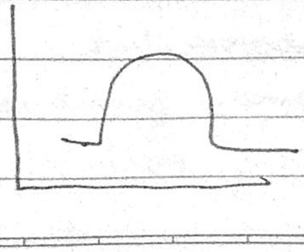
SIF

I/P  O/P 

GIF

I/P  O/P 

	SMSIF	MMSIF	MM GRIN
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	$d_{core} \rightarrow 18-12\ \mu m$	50 - 100 μm	125 - 400 μm
	$d_{clad} \rightarrow 125\ \mu m$	150 - 250 μm	$n(r) = n_1\sqrt{1-\left\{2\Delta\left(\frac{r}{a}\right)\right\}}$ $(r>a)$ where $\alpha \geq 2$
	$n(r) = n_1 (r<a)$ $= n_2 (r>a)$	Finite no. of modes signal degradation which leads to high attenuation	$r = n_2\ (r<a)$
	only zero order mode		
	Δ & NA = small	Δ & NA high	$NA = n_1\sqrt{2\Delta\left[1-\left(\frac{r}{a}\right)^2\right]}$

Rays marking larger angles with axis traverse longer path but they travel in a region of lower refractive index & hence have higher speed of propagation.

Glass fibres

		or	or
Core	SiO_2	SiO_2	$GeO_2 SiO_2$ (Ge is added to n_1)
Cladding	$GeO_2 SiO_2$	$B_2O_3 SiO_2$	SiO_2

n_1 (silica) = 1.458 at λ = 850 nm.

used in long distance Communication

Plastic fibres

Core - Polystyrene (1.6)	PMMA	(1.49)
Clad - PMMA (1.49)	copolymer	(1.40)

Polymethylon

Cost effective

Temp. sensitive upto 80°C, after 80°, copolymer may melt.

PCS.

Core - High purity quartz material (1.05)

Clad - Teflon.

Short distance Communication app.

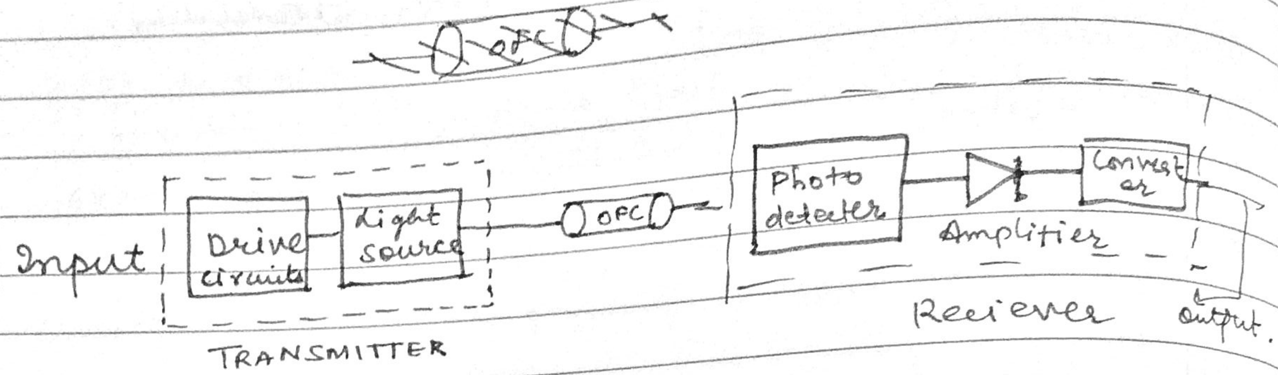
used in SIF only.

Less Expensive & have v. high attenuation.

5/2.

"OPTICAL FIBRE COMMUNICATION"

1. Transmitter
2. channel — optical fibre cable (OFC)
3. Reciever



Non Electrical Signal → Transducer ↓ Electrical Signal

These electrical signals In Transmitter are coupled with light source

- LEDs
- Laser diodes

↓

IR Region

850 nm

1300 nm

1500 nm

ANALOG MODULATION

↓

DIGITAL MODULATION

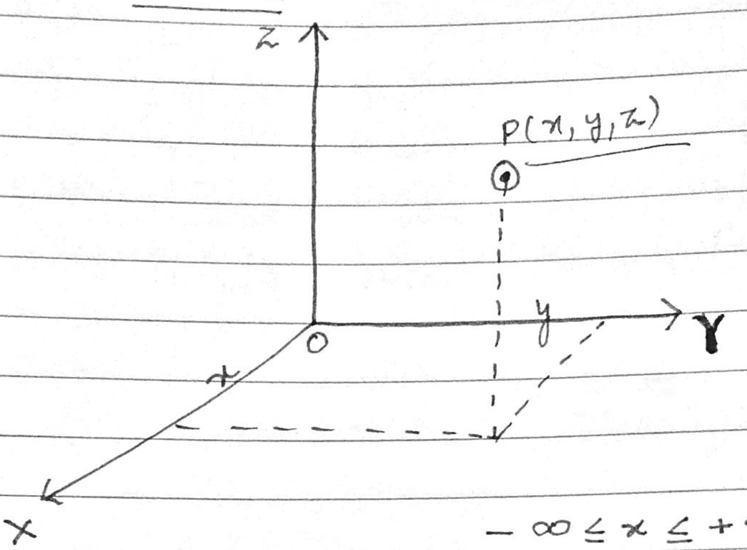
↓

Signal Transmission to longer distance with Same power, photo detectors detects the signal then it's amplified and then ~~converted~~ converts it back to it's original form of signal.

"CO-ORDINATE SYSTEMS"

1. Cartesian (x, y, z)
2. Cylindrical (ρ, ϕ, z)
3. Spherical (r, θ, ϕ)

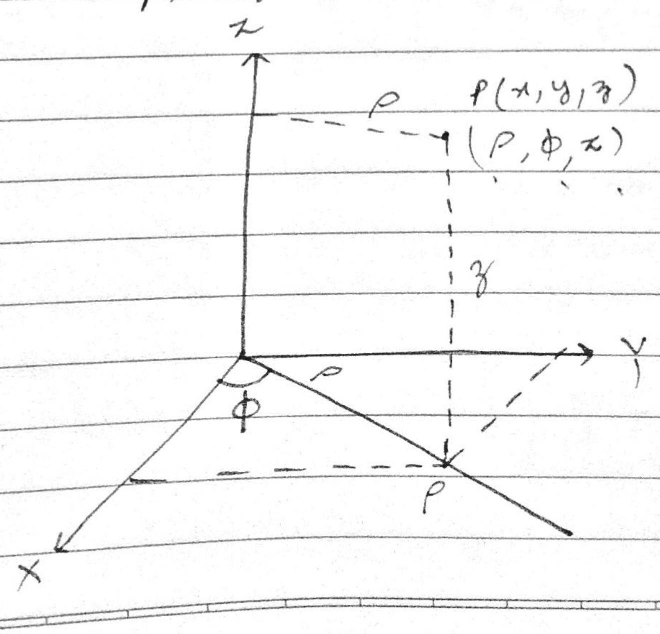
Cartesian



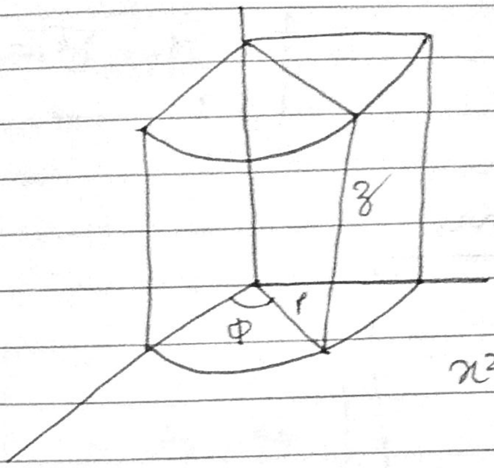
$$-\infty \leq x \leq +\infty$$
$$-\infty \leq y \leq +\infty$$
$$-\infty \leq z \leq +\infty$$

Cylindrical

(ρ, ϕ, z)

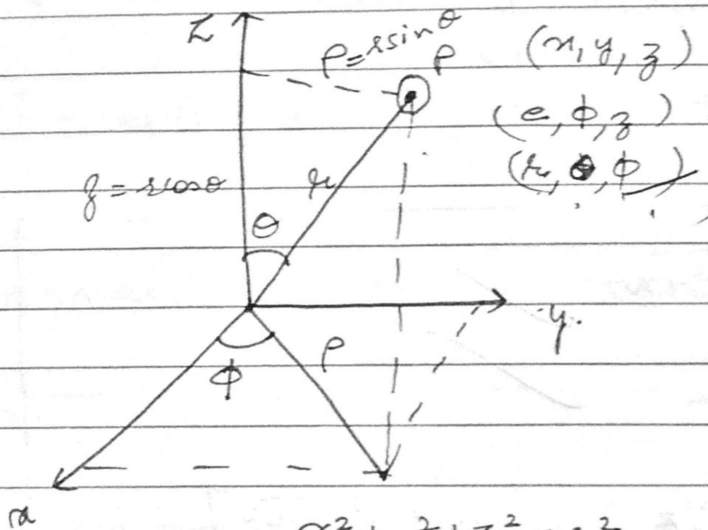


$$0 < \rho \leq \infty$$
$$0 < \phi \leq 2\pi$$
$$-\infty \leq z \leq \infty$$



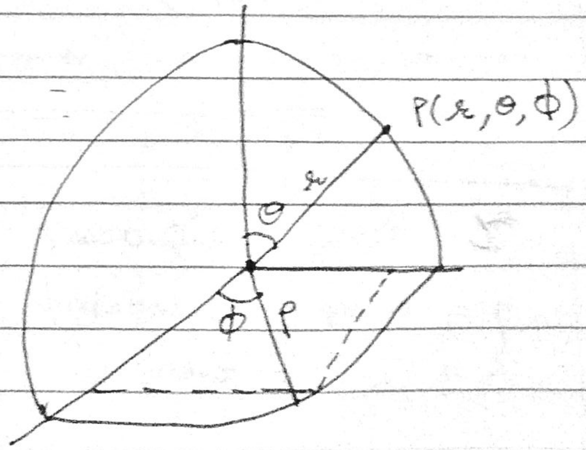
$x^2+y^2=\rho^2.$

→ "**SPHERICAL**"
(r, θ, ϕ)



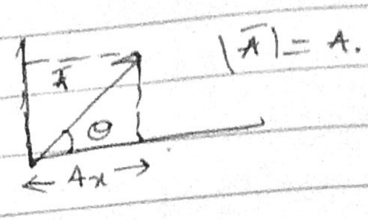
$x^2+y^2+z^2=r^2$
$\rho^2+z^2=r^2.$

$0 < r \leq \infty$
$0 < \theta \leq \pi$
$0 < \phi \leq 2\pi$



→ $A_x = A\cos\theta$

→ $A_y = A\sin\theta$



Conversions

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z.$

$\boxed{\text{Cart} \leftarrow \text{cy}}$

$\rho = \sqrt{x^2+y^2}$

$\phi = \tan^{-1}(y/x)$

$z = z.$

cy ← cart

$r = \sqrt{x^2+y^2+z^2}$

$= \sqrt{\rho^2+z^2}$

$\theta = \tan^{-1}(\rho/z) = \tan^{-1}\left[\frac{\sqrt{x^2+y^2}}{z}\right]$

$\phi = \tan^{-1}(y/x)$

$x = \rho\cos\phi = r\sin\theta\cos\phi$

$y = \rho\sin\phi = r\sin\theta\sin\phi$

$z = z = r\cos\theta.$

8/2

$$\bar{A} = A_x\bar{i} + A_y\bar{j} + A_z\bar{k}$$

$$\bar{B} = B_x\bar{i} + B_y\bar{j} + B_z\bar{k}$$

$$\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z.$$

$$\bar{c} = \bar{A}\times\bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\bar{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z \longrightarrow$$

$$\bar{A} = A_\rho\bar{a}_\rho + A_\phi\bar{a}_\phi + A_z\bar{a}_z$$

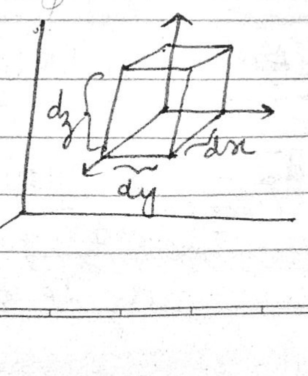
$$\bar{A} = A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\phi\bar{a}_\phi$$

→ Elemental Displacement (vector)
length

$$\bar{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

$$= d\rho\,\bar{a}_\rho + (\rho\,d\phi)\bar{a}_\phi + dz\,\bar{a}_z$$

$$= dr\,\bar{a}_r + (r\,d\theta)\bar{a}_\theta + (\rho\,d\phi)\bar{a}_\phi$$



Hi

Hilo

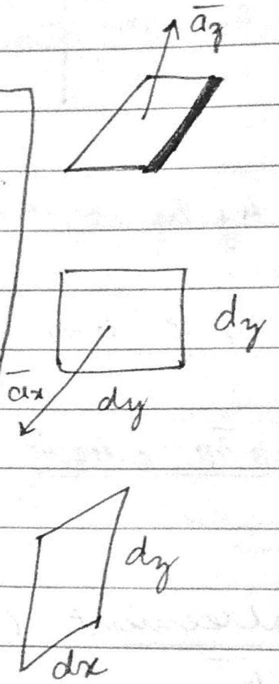
How u doin!

→ Differential elemental area.

$$\bar{ds} = ds\,\hat{a}_n.$$

1. Cartesian co-ordinate System.

$$\boxed{\begin{aligned}\bar{ds} &= (dx\,dy)\,\bar{a}_z \\ &= (dy\,dz)\,\bar{a}_x \\ &= (dz\,dx)\,\bar{a}_y\end{aligned}}$$

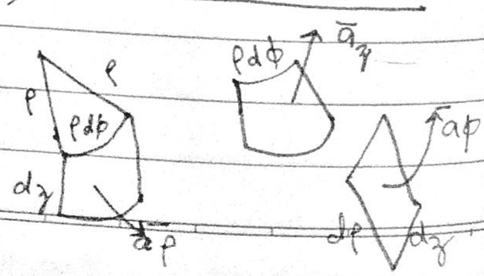


→ volume → Scalar.

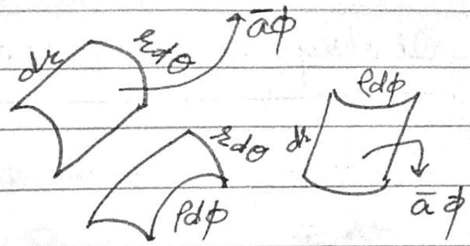
Cartesian = $dv = dx\,dy\,dz$.

→ 2. Area in Cylindrical co-ordinate system.

$$\begin{aligned}\bar{ds} &= d\rho(\rho d\phi)\,\bar{a}_z \\ &= (\rho d\phi)\,dz\,\bar{a}_\rho \\ &= dz\,d\rho\,\bar{a}_\phi\end{aligned}$$



3. Spherical.



$$d\bar{S} = dr\,(r d\theta)\,\bar{a}_\phi$$
$$= (r d\theta)(\rho d\phi)\,\bar{a}_r$$
$$= (\rho d\phi)\,dr\,\bar{a}_\theta$$

Volumes.

1. Cartesian $\rightarrow$ $dV = dx\,dy\,dz$.

2. Cylindrical $\rightarrow$ $(dr)(r d\theta)(r\sin\theta)d\phi$

3. Spherical $\rightarrow$ $r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\rightarrow$ DEL OPERAPOR. (∇)

It is a vector operator. It is a Mathematical operator. It is used to convert a parameter from vector to scalar or vice versa.

$$\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z. \quad \text{(CART)}$$

$$\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}a_\phi + \frac{\partial}{\partial z}a_z. \quad \text{(cyl)}$$

$$\nabla = \frac{\partial}{\partial r}a_r + \frac{1}{r}\frac{\partial}{\partial \theta}a_\theta + \frac{1}{\underset{r\sin\theta}{\rho\ \text{or}}}\frac{\partial}{\partial \phi}\bar{a}_\phi \quad \text{(sph)}$$

② Divergence of a vector field.

~~$div\bar{A} = \nabla.\bar{A} = $ scalar~~

~~$\nabla\bar{A} =$~~ ~~Scal~~ ~~cyl~~

~~cyl~~

$div\bar{A} = \nabla.\bar{A} \longrightarrow$ scalar

$\nabla.\bar{A} =$ cart

cyl.

sph.

③ Curl of a vector field.

curl $\bar{A} = \nabla\times\bar{A} \rightarrow$ vector $\nabla\times\bar{A} =$ Cart cyl sph.	$curl A = \nabla\times A = \left[\lim_{\Delta S\to 0} \frac{\oint_L A.dl}{\Delta S}\right]_{max}$

① Gradient of a Scalar field

grad $\phi = \nabla\phi \rightarrow$ Vector

$\nabla\phi =$ (cart) $\rightarrow \frac{\partial v}{\partial x}\bar{a}_x + \frac{\partial v}{\partial y}\bar{a}_y + \frac{\partial v}{\partial z}\bar{a}_z$

$\nabla\phi$ (cyl)

(sph.)

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Page No.:

Line, Surface & volume Integral

→ $\oint_L A \cdot dl$ → Line Integral

→ $\oint_S A \cdot ds$ → Surface Integral

→ $\int_V v \cdot dV$ → volume Integral.

→ A Curl of A is an axial vector whose magnitude is the maximum circulation of A per unit area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maxium.

☆ $\boxed{\text{Divergence or Gauss' Theorem.}}$

The divergence theorem states that the total outward flux of a vector field A through the closed surface, S is the same as the volume integral of the divergence of A.

$$\oint_S ds = \int_V \nabla \cdot A \, dv$$

Stokes Theorem.

Stokes's Theorem state that circulation of a vector field A around a closed ~~...~~ path L is equal to the ~~...~~ surface integral of the curl A over the open surface S bounded by L, provided A and $\nabla \times A$ are continuous set.

$$\oint_L A \cdot dl = \int_S (\nabla \times A) \cdot ds$$

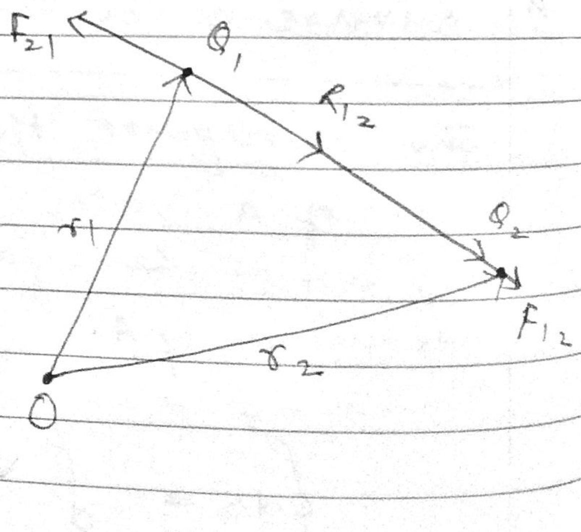
"ELECTROSTATICS"

Coulomb's Law

It states that the force F between two point charges Q_1 and Q_2 is

$$F = \frac{kQ_1Q_2}{R^2}$$

In vector form



$$F_{12} = \frac{Q_1Q_2}{4\pi\epsilon_0 R^3} R_{12}$$

or

$$F_{12} = \frac{Q_1Q_2(r_2-r_1)}{4\pi\epsilon_0 |r_2-r_1|^3}$$

$$\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^3} \vec{r}$$

$$\overline{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R^2} \overline{a}_{R_{12}} \quad ; \overline{F_{21}} = -\overline{F_{12}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R^2} \frac{\overline{R}}{|\overline{R}|}$$

→ If we have more than two point charges.

$$E = \frac{1}{4\pi\epsilon_0} \sum_{K=1}^{N} \frac{Q_K (r - r_K)}{|r - r_K|^3}$$

→ Electric field Intensity

Electric field intensity is the force per unit charge when placed in the electric field

$$E = \frac{F}{q}$$

In vector Form

$$E = \frac{Q(r - r')}{4\pi\epsilon_0 |r - r'|^3}.$$

→ $\vec{E} = -\text{grad } V.$

$$E = \lim_{Q \to 0} \left(\frac{\overline{F}}{Q} \right)$$

$$E = \left(\frac{V}{d} \right)$$

ρ_l = linear charge density (C/m)

ρ_s = Surface charge density (C/m²)

ρ_v = volume charge density (C/m³)

→ $\underline{E} = \int \frac{\rho_L \, dl}{4\pi\epsilon_0 R^2} \, a_R$ (line charge)

→ $= \int \frac{\rho_s \, dS}{4\pi\epsilon_0 R^2} \, a_R$ (Surface charge)

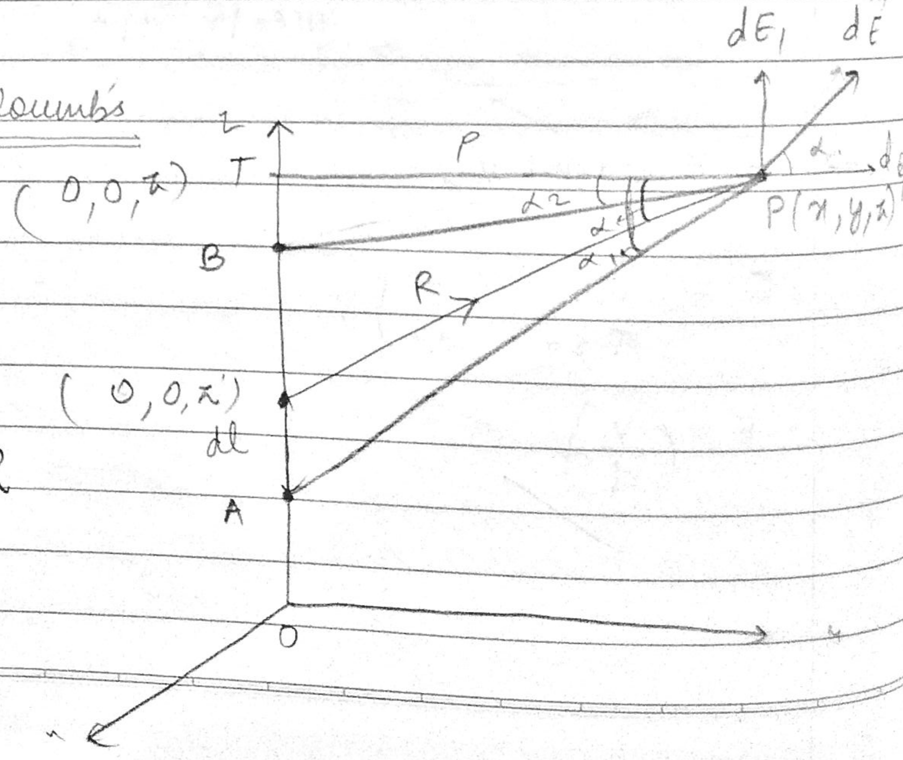
→ $= \int \frac{\rho_v \, dV}{4\pi\epsilon_0 R^2} \, a_R$ (Volume charge)

☆ Application of coloumb's

~~LINE CHARGE~~

$d\vec{E} = \frac{\rho_l \, dl}{4\pi\epsilon_0 R^2} \, \bar{a}_r$

$= \frac{\rho_l \, dl}{4\pi\epsilon_0} \, \frac{R}{|R|^3}$



$dQ = \rho_l \, dl$

$Q = \int_L dQ = \int_L \rho_l \, dl$.

$dl = dz = dz'$

$E = \bar{E}_\rho - \bar{E}_z$

$$\vec{E} = \int d\vec{E} = \int \frac{\rho_l \, dl \, \bar{R}}{4\pi\epsilon_0 |R|^3} = \frac{\rho_l}{4\pi\epsilon_0} \int \frac{dz' \, \bar{R}}{|\bar{R}|^3}$$

$\bar{R} = (x, y, z) - (0, 0, z')$

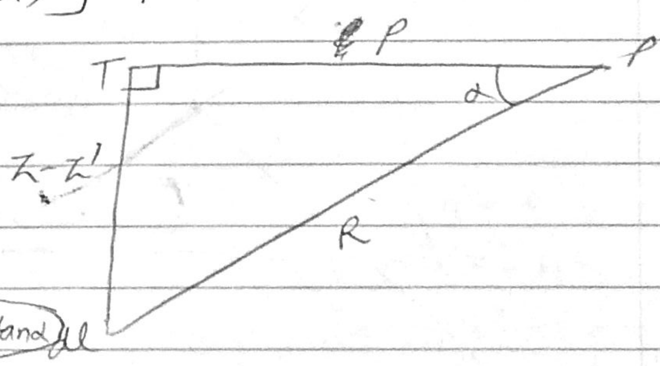
$\bar{R} = x\bar{a}_x + y\bar{a}_y + (z - z')\bar{a}_z$

$\bar{R} = \rho\bar{a}_\rho + (z - z')\bar{a}_z$

$|\bar{R}| = \sqrt{\rho^2 + (z - z')^2}$

$$\vec{E} = \frac{\rho_l}{4\pi\epsilon_0} \int \frac{\rho\bar{a}_\rho + (z - z')\bar{a}_z}{[\rho^2 + (z - \bar{z}')^2]^{3/2}} dz'$$

In Δle $PTdl$



$\sec\alpha = \frac{R}{\rho}$, $R = \rho\sec\alpha$

$\tan\alpha = \frac{(z - z')}{\rho}$, $(z - z') = \rho\tan\alpha$

$$\vec{E} = \frac{\rho_l}{4\pi\epsilon_0} \int \frac{\rho\bar{a}_\rho + (\rho\tan\alpha)\bar{a}_z}{(\rho^2 + \rho^2\tan^2\alpha)^{3/2}} dz'$$

Now $z - z' = \rho\tan\alpha$

$-dz' = \rho\sec^2\alpha \, d\alpha$

$$\vec{E} = \frac{-\rho_l}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\rho(\bar{a}_\rho + \tan\alpha \, \bar{a}_z) \, \rho\sec^2\alpha \, d\alpha}{\rho^3(1 + \tan^2\alpha)^{3/2}} = \frac{\rho_l}{\cancel{4\pi}}$$

$$\vec{E} = \frac{-\rho_l}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\rho[\cos\alpha \, \bar{a}_\rho + \sin\alpha \, \bar{a}_z] \, \cancel{\rho} \, \cancel{\sec^2\alpha} \, d\alpha}{\cancel{\cos\alpha} \; \rho^{\cancel{3}} \cancel{\sec^3}\alpha} = \frac{-\rho_l}{4\pi\epsilon_0\rho} \int_{\alpha_1}^{\alpha_2} [\cos\alpha \, \bar{a}_\rho + \sin\alpha \, \bar{a}_z] \, d\alpha$$

We applied limits here

$$= \frac{-\rho_l}{4\pi\epsilon_0 \rho}\left[(\sin\alpha_2 - \sin\alpha_1)\bar{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\bar{a}_z\right]$$

For infinite length

$A = (0, 0, -\infty)$, $\alpha_1 = \pi/2$

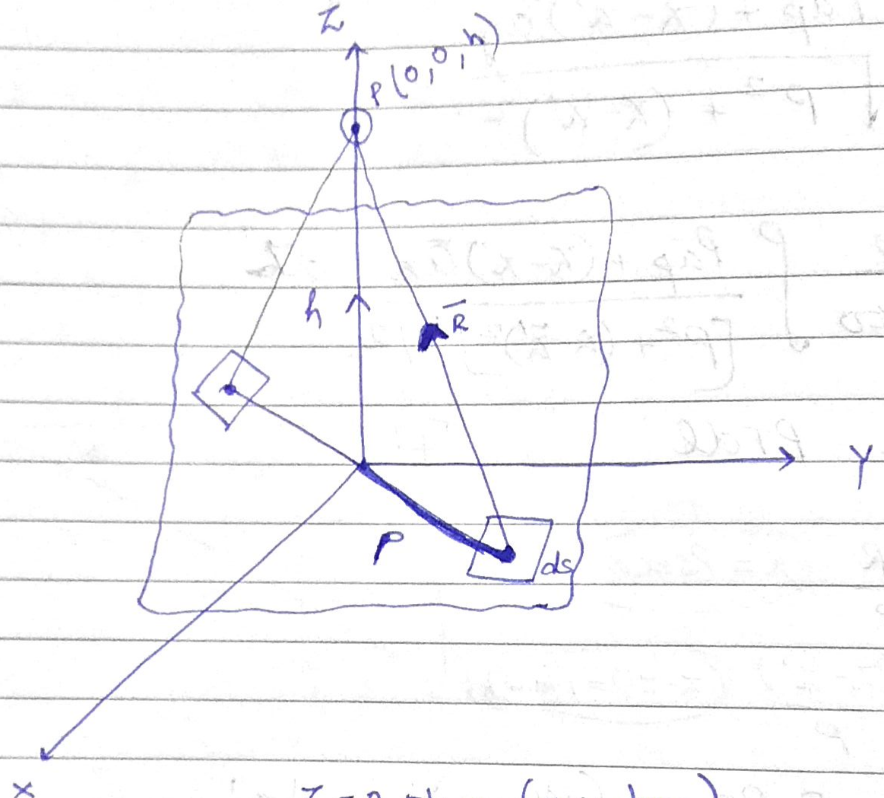
$B = (0, 0, \infty)$, $\alpha_2 = -\pi/2$

$$\bar{E} \rightarrow \frac{\rho_l}{4\pi\epsilon_0 \rho}(2\bar{a}_\rho]$$

$$\boxed{\vec{E} = \frac{2\rho_l}{4\pi\epsilon_0 \rho}[2\bar{a}_\rho]}$$

Surface charge (ρ_s)

$$dE = \frac{\rho_s ds}{4\pi\epsilon_0 R^2}\bar{a}_R$$



Z = 0 plane (x-y plane)

$\rho\bar{a}_\rho + \bar{R} = h\bar{a}_z$

$\therefore \bar{R} = h\bar{a}_z - \rho\bar{a}_\rho$

$|\bar{R}| = R = \sqrt{\rho^2 + h^2}$

$\boxed{d\bar{s} = ds\,\hat{a}_n}$

For -ve Direction

$\rho(-\bar{a}_\rho) + \bar{R} = h\bar{a}_z$

$\therefore \bar{R} = h\bar{a}_z + \rho\bar{a}_\rho$

~~due to symmetry~~

Now in this problem we want dz as our unit vector so from $\bar{dl} = \underline{d\rho}\,\bar{a}_\rho + \underline{(\rho\, d\phi)}\,\bar{a}_\phi + dz\,\bar{a}_z$

Thus

$$\bar{ds} = ds\,\hat{a}_n$$

$$= (d\rho)(\rho\, d\phi)\,\bar{a}_z.$$

$$\therefore ds = d\rho(\rho\, d\phi)$$

$$\therefore \bar{E} = \int d\bar{E} = \frac{\rho_s}{4\pi\epsilon_0}\int_0^{\infty}\int_0^{2\pi} \frac{d\rho(\rho\, d\phi)\,[h\bar{a}_z - \rho\bar{a}_\rho]}{(h^2+\rho^2)^{3/2}}$$

due to symmetry $\rho\bar{a}_\rho$ is cancelled.

$$\bar{E} = \frac{\rho_s h(2\pi)}{4\pi\epsilon_0}\int_0^{\infty} \frac{\rho\, d\rho\ \bar{a}_z}{(h^2+\rho^2)^{3/2}}.$$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0}\int_0^{\infty} \frac{\rho\, d\rho}{(\rho^2+h^2)^{3/2}}\,\bar{a}_z$$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0}\int_{x=h}^{\infty} \frac{\cancel{x}\, dx}{x^{\cancel{3}2}}\,\bar{a}_z$$

$\rho^2+h^2 = x^2$

$\Rightarrow \rho^2 = x^2 - h^2$

$\rho = 0,\ x^2 - h^2 = 0$

$x = h$

$\rho = \infty,\ x^2 - h^2 = \infty$

$x = \infty$

$$\bar{E} = \frac{\rho_s h}{2\epsilon_0}\left[\frac{-1}{x}\right]_h^{\infty}\bar{a}_z$$

$$\bar{E} = \frac{-\rho_s h}{2\epsilon_0}\left[-\left(\frac{1}{h}\right)\right]\bar{a}_z$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\,\bar{a}_z}$$

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Electric flux density

The Electric field Intensity depends on the Medium in which the charges are placed.

Suppose a vector field D Independent of the Medium is defined by $\boxed{D = \varepsilon_0 E}$

The Electric flux (ψ) in Terms of D can be defined as

$$\boxed{\psi = \int D \cdot ds.}$$

The vector field D is called the <u>E.F.D.</u> & is measured in coloumbs per square metre.

Gauss law

It states that the total electric ~~field~~ flux ψ through any closed Surface is equal to the total charge enclosed to that Surface.

$$\psi = Q_{enclosed}.$$

$$\psi = \oint d\psi = \oint_s D \cdot ds$$

Total charge enclosed $Q = \int \rho_v dV$

$$Q = \oint_s D \cdot ds = \int_v \rho_v dV \quad -(1)$$

using Divergence Theorem

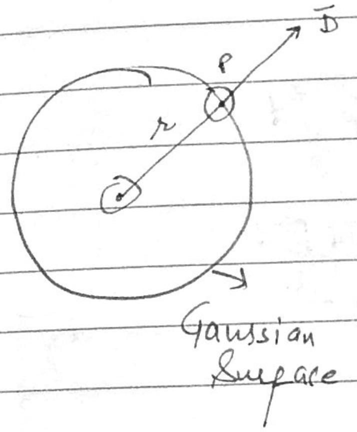
$$\oint D \cdot ds = \int \nabla \cdot D dv \quad -(2)$$

comparing the two volume integrals in (1) and (2)

$$\rho_v = \nabla \cdot D$$

This is the first maxwell equation.
It states that the volume charge density is the same as the divergence of the electric flux density.

Point charge.



$$\bar{D} = D_r \bar{a}_r$$

$$\boxed{\bar{D} = \epsilon_0 \bar{E}}$$

$$Q = \Psi = \int_S \bar{D} \cdot \bar{ds}$$

$$= \int_S D_r \bar{a}_r \cdot ds\, \bar{a}_r$$

$$= \int_S D_r \, ds$$

$$Q = D_r (4\pi r^2)$$

$$\therefore D_r = \frac{Q}{4\pi r^2}$$

or $\boxed{\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r}$

$\bar{dl} = dr\,\bar{a}_r + (r\,d\theta)\,\bar{a}_\theta + (\rho\,d\phi)\,\bar{a}_\phi$

$\bar{ds} = (r\,d\theta)(\rho\,d\phi)\,\bar{a}_r$

$= (r\,d\theta)(r\sin\theta)\,d\phi\,\bar{a}_r$

$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$

$= 2\pi r^2 \int_{\theta=0}^{\pi} \sin\theta \, d\theta.$

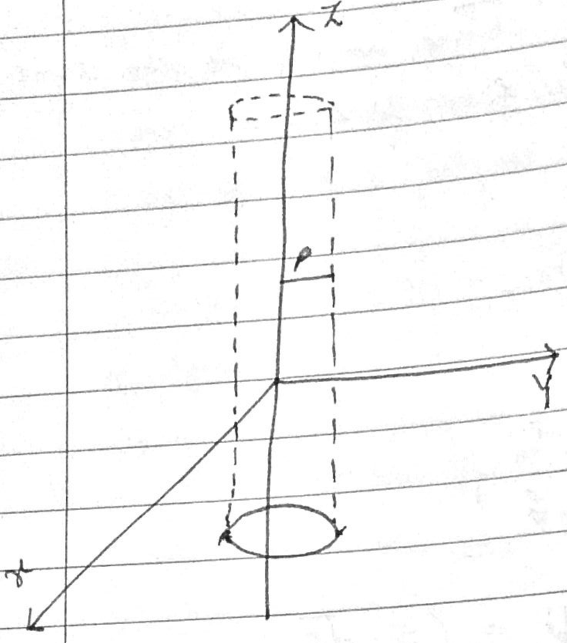
$= -2\pi r^2 \left[\cos\theta\right]_0^{\pi}$

$= -2\pi r^2 [-1 - 1].$

$= 4\pi r^2.$

(This is how we got this)

Line charge:



$$\bar{D} = D_\rho \bar{a}_\rho$$

$$Q = \int_S \bar{D} \cdot \bar{ds}$$

$$= \int_S D_\rho \bar{a}_\rho \cdot ds\, \bar{a}_\rho$$

$$= D_\rho \int_S ds$$

$$\cancel{ds = (2\pi \rho) l}$$

$$= D \rho \int_{\phi=0}^{2\pi} \int_{z=0}^{l} (\rho\, d\phi)\, dz$$

$$\bar{dl} = dl\, \bar{a}_\rho + (\rho\, d\phi)\bar{a}_\phi + dz\, \bar{a}_z$$

$$\bar{ds} = (\rho\, d\phi)\, dz\, \bar{a}_\rho$$

$$ds = (\rho\, d\phi)\, dz$$

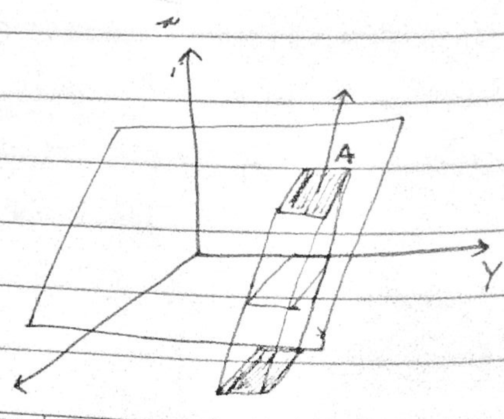
$$Q = D_\rho (2\pi \rho) \int_0^l dz$$

$$= D_\rho (2\pi\rho) l$$

$$\therefore D_\rho = \frac{Q}{(2\pi\rho) l}$$

$$\therefore \bar{D} = \frac{Q}{(2\pi\rho) l} \bar{a}_\rho$$

→ Surface charge



$$dQ = \rho_s\, ds$$

$$Q = \int_S \rho_s\, ds = (\rho_s A)$$

$$Q = \int_S \bar{D} \cdot \bar{ds}$$

$$\bar{E} = -\nabla V$$
$$= -\text{grad } V.$$

$$\bar{D} = D_z \bar{a_z}$$

$$Q = \int_S D_z \bar{a_z} \cdot ds\, \bar{a_z}$$

$$= \int D_z\, ds$$

$$= D_z \left[\int_{bottom} ds + \int_{top} ds \right]$$

$$= D_z (2A)$$

$$\therefore D_z = \frac{Q}{2A} = \frac{\rho_s A}{2A} = \left(\frac{\rho_s}{2}\right)$$

$$\boxed{\therefore \bar{D} = \left(\frac{\rho_s}{2}\right) \bar{a_z}}$$

→ <u>Electric potential</u>

Test charge

~~Some~~ amount of work done to bring a unit charge from infinity to point p. into the Electric field.

$$V = \left(\frac{W}{Q}\right)$$

$$E\, dr = -dV$$

$$V = -\int_{\infty}^{r} \bar{E} \cdot \bar{dr}$$

$$W = -Q \int_{A}^{B} \vec{E} \cdot dl$$

$$\frac{W}{Q} = -\int_{A}^{B} \vec{E} \cdot dl = V_{AB}$$

Potential is a Scalar

$$V_{AB} = V_B - V_A$$

$$= -\int \frac{Q}{4\pi\epsilon_0 r^2} \bar{a}_r \cdot dr\, \bar{a}_r .$$

$$= -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2}\, dr .$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$\downarrow \qquad \downarrow$$

$$r \qquad A \to \infty$$

$$\Rightarrow \boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$$

$$dV = -\bar{E}\cdot \bar{dl}$$

$$= -\left[E_x \bar{a}_x + E_y \bar{a}_y + E_z \bar{a}_z\right] \cdot \left[dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z\right]$$

$$\left(\frac{\partial V}{\partial x}\right) dx + \qquad = -E_x dx - E_y dy - E_z dz$$

Electric dipole

dipole moment = $\vec{P} = Q\vec{d}$.

Imp

~~Polarization in Dielectric~~

→ For the measurement of intensity of polarization, we define polarization $\vec{P}$ as dipole moment per unit volume.

$$P = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} Q_k d_k}{\Delta v.}$$

$$dV = \frac{\bar{P} \cdot \nabla'(1/R)\, dv'}{4\pi\epsilon_0}$$

$$\cancel{\nabla' \cdot f\bar{A} = f\nabla' \cdot \bar{A} + \bar{A} \cdot \nabla' f}$$

$$f\bar{A} = f\nabla'\bar{A} + \bar{A} \cdot \nabla f$$

$$f = \frac{1}{R}, \quad \bar{A} = \bar{P}$$

$$\bar{A} \cdot \nabla' f = \nabla' \cdot f\bar{A} - f\nabla' A$$

$$dV = \frac{\bar{P} \cdot \nabla'\left(\frac{1}{R}\right) dv}{4\pi\epsilon_0} = \frac{\left[\nabla' \cdot \frac{1}{R}\bar{P} - \frac{1}{R}\nabla'\bar{P}\right] dv'}{4\pi\epsilon_0.}$$

$$V = \int_V dV = \frac{1}{4\pi\epsilon_0}\int_{v'} \nabla \cdot \frac{1}{R}\bar{P}\, dv' - \frac{1}{4\pi\epsilon_0}\int_V \frac{1}{R}\nabla' \cdot \bar{P}\, dv'$$

$$\boxed{\int \bar{A} \cdot \bar{ds} = \int_V \nabla \cdot A\, dv}$$ Gauss divergence

$$= \frac{1}{4\pi\epsilon_0 R} \oint_S \vec{P} \cdot \overline{ds}$$

$$= \frac{1}{4\pi\epsilon_0 R} \oint_S \vec{P} \cdot ds\, \bar{a}_n$$

$\bar{P}$ ⇒

→ $1 + \chi_e = \epsilon_r$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\boxed{\bar{P} = \epsilon_0 [\chi_e - 1] \bar{E}}$$

&

P ⇒

$$\frac{\epsilon}{\epsilon_0} = \epsilon_R$$

$$\boxed{\epsilon = \epsilon_r(\epsilon_0)}$$

We know that

$D = \epsilon_0 E + P$ and $P = \chi_e \epsilon_0 E$

Thus $D = \epsilon_0 (1 + \chi_e) E = \epsilon_0 \epsilon_r E_r$

Current Continuity equation.

$$\int_V \nabla \cdot J \, dv = -\int_V \frac{\partial \rho_v}{\partial t} \, dV$$

or

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

Relaxation time is the time in which a charge placed in the interior of a material to drop to $e^{-1} = 36.8\%$ of it's initial value.

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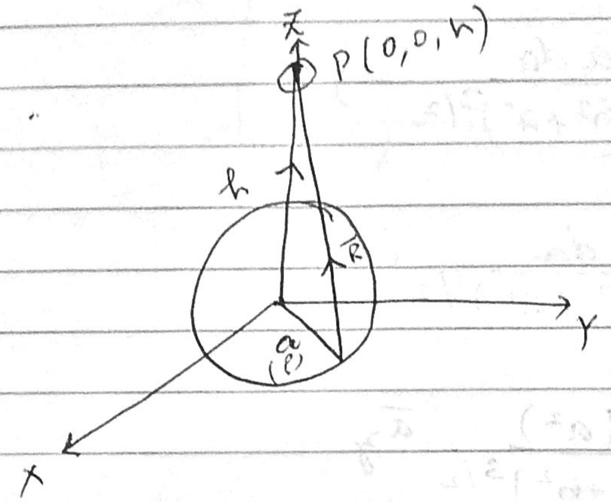
Electric field circular disk.

(Coulomb's law) (ρ_s) C/m²

$z = 0$ (xy plane)



$$\bar{E} = \int_s \frac{\rho_s ds}{4\pi\epsilon_0 R^2} \, a_{\bar{R}}$$

$$= \int_s \frac{\rho_s \, ds}{4\pi\epsilon_0 R^3} \, \bar{R}$$

$$ds = (da)(a d\phi)$$

$$a\bar{a}_p + \bar{R} = h\bar{a}_z .$$

$$\therefore \bar{R} = h\bar{a}_z - a\bar{a}_p$$

$$|\bar{R}| = R = \sqrt{h^2+a^2}$$

$$\therefore \bar{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_0^a \int_{\phi=0}^{2\pi} \frac{(da)(ad\phi)}{[h^2+a^2]^{3/2}} \left[h\bar{a}_z - a\bar{a}_p\right]$$

$$\bar{E} = \frac{\rho_s (2\pi)}{4\pi\epsilon_0} \int_0^a \frac{a\,da}{(h^2+a^2)^{3/2}} \left[h\bar{a}_z - a\bar{a}_p\right]$$

Due to symmetry components get cancelled out

$$= \frac{\rho_s}{2\epsilon_0}\int_0^a \frac{a\,da}{[h^2+a^2]^{3/2}}\, h\,\bar{a}_z .$$

$$= \frac{\rho_s h}{2\epsilon_0}\int_0^a \frac{a\,da}{(h^2+a^2)^{3/2}}\,\bar{a}_z .$$

$$= \frac{\rho_s h}{4\epsilon_0}\int_0^a \frac{2a\,da}{(h^2+a^2)^{3/2}}\,\bar{a}_z$$

$$= \frac{\rho_s h}{4\epsilon_0}\int_0^a \frac{d(a^2)}{[a^2+h^2]^{3/2}}\,\bar{a}_z$$

$$= \frac{\rho_s h}{4\epsilon_0}\left[\frac{(a^2+h^2)^{-3/2+1}}{-\frac{3}{2}+1}\right]_0^a \bar{a}_z$$

$$= \frac{\rho_s h}{2\cancel{4}\epsilon_0}\times\cancel{2}\left[(a^2+h^2)^{-1/2}\right]_0^a \bar{a}_z .$$

$$= \frac{\rho_s h}{2\epsilon_0}\left[\frac{1}{\sqrt{a^2+h^2}}\right]_0^a \bar{a}_z ,$$

$$= \frac{-\rho_s h}{2\epsilon_0}\left[\frac{1}{\sqrt{a^2+h^2}}-\frac{1}{h}\right]\bar{a}_z .$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\left[1-\frac{h}{\sqrt{a^2+h^2}}\right]\bar{a}_z}$$

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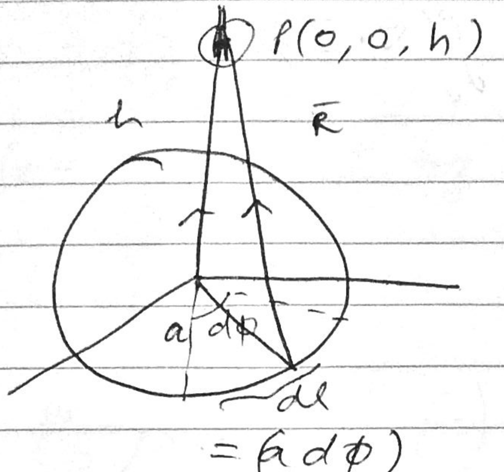
For circular Ring



$dl = a\,d\phi$

$\bar{R} = h\bar{a}_z - a\bar{a}_\rho$

$|\bar{R}| = R = \sqrt{h^2 + a^2}$

$$\bar{E} = \int \frac{\rho_l\,dl}{4\pi\epsilon_0 R^2}\,\bar{a}_R$$

$$\therefore\ \bar{E} = \frac{\rho_l}{4\pi\epsilon_0}\int \frac{a\,d\phi}{(a^2+h^2)^{3/2}}\left[h\bar{a}_z - a\bar{a}_\rho\right]$$

$$= \frac{\rho_l}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi} \frac{a\,d\phi}{(a^2+h^2)^{3/2}}\,h\,\bar{a}_z$$

$$= \frac{\rho_l\,ah\,(2\pi)}{4\pi\epsilon_0\,(a^2+h^2)^{3/2}}\,\bar{a}_z.$$

$$\boxed{\bar{E} = \frac{\rho_l\,ah}{2\epsilon_0(a^2+h^2)^{3/2}}\,\bar{a}_z.}$$

Q What is the max. height where it is possible for $\vec{E}$ to be +ve & max!

→ find $\left(\frac{dE}{dh}\right)$ and equate to 0.

continuity equation,

$$I = \oint_s \bar{J} \cdot \bar{ds} = \int_v (\nabla \cdot \bar{J})\, dv$$

$$I = -\left(\frac{dQ_{in}}{dt}\right)$$

$$dQ = \rho_v\, dv$$

$$Q = \int_v \rho_v\, dv$$

$$\int_v \nabla \cdot \bar{J}\, dv = -\int_v \frac{d}{dt}(\rho_v\, dv$$

$$\int_v \nabla \cdot \bar{J} dv = -\int_v \left(\frac{d\rho_v}{dt}\right) dv.$$

$$\nabla \cdot \bar{J} = -\left(\frac{\partial \rho_v}{dt}\right)$$

$$\nabla \cdot \bar{J} + \left(\frac{\partial \rho_v}{dt}\right) = 0.$$

$\boxed{\nabla \cdot \bar{J} = 0}$ (Steady currents)

$$\nabla \cdot \bar{J} = -\left(\frac{\partial \rho_v}{dt}\right)$$

$\bar{J} \propto \bar{E} \Rightarrow \boxed{\bar{J} = \sigma \bar{E}}$

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot \epsilon \bar{E} = \rho_v$$

$$\nabla \cdot \bar{E} = \left(\frac{\rho_v}{\epsilon}\right)$$

$$\nabla \cdot \sigma \bar{E} = -\left(\frac{\partial \rho_v}{dt}\right)$$

$$\sigma\left(\frac{\rho_v}{\epsilon}\right) = -\left(\frac{\partial \rho_v}{\partial t}\right)$$

$$\frac{\partial \rho_v}{\rho_v} = -\left(\frac{\sigma}{\epsilon}\right)dt.$$

$$\log \rho_v = -\left(\frac{\sigma}{\epsilon}\right)t + \log \rho_{v0}.$$

$$\log\left(\frac{\rho_v}{\rho_{v0}}\right) = \left(-\frac{\sigma}{\epsilon}\right)t$$

$$\therefore \left(\frac{\rho_v}{\rho_{v0}}\right) = Exp\left(\left(\frac{-\sigma}{\epsilon}\right)t\right)$$

$$= Exp\left(\frac{-t}{t_r}\right) \qquad t_r = \frac{\epsilon}{\sigma}$$

$$\therefore \boxed{\rho_v = \rho_{v0}\, Exp\left(-\frac{t}{t_r}\right)}$$

cyl → cart

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 1 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Cart → cyl

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

~~SP → Cart~~ cart → SP,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

'Basic formulae'

$$dB = \frac{\mu_0 I\, dl \times a_R}{4\pi R^2}$$

→ $F = \frac{Q_1 Q_2\, a_r}{4\pi \epsilon_0 r^2}$

$\oint H \cdot dl = I_{enc.}$

→ $\oint D \cdot ds = Q_{enc.}$

$F = Qu \times B$

→ $F = QE$

$Qu = I\,dl$.

→ dQ

→ $E = \frac{V}{l}$ (V/m).

→ Biot savart's law:

It states that magnetic field Intensity dH produced at a point P by the differential Current element $I\,dl$ is proportional to the product of current element $I\,dl$ and the sine of angle α between the element and line Joining point P

$$dH = \frac{\mu_0 I\, dl \times a_R}{4\pi R^2}$$

$$I\,dl = K\,ds = J\,dv$$

→ $H = \int_L \frac{I\,dl \times a_R}{4\pi R^2} = \int \frac{I\,dl \times R}{4\pi R^3}$ line Current

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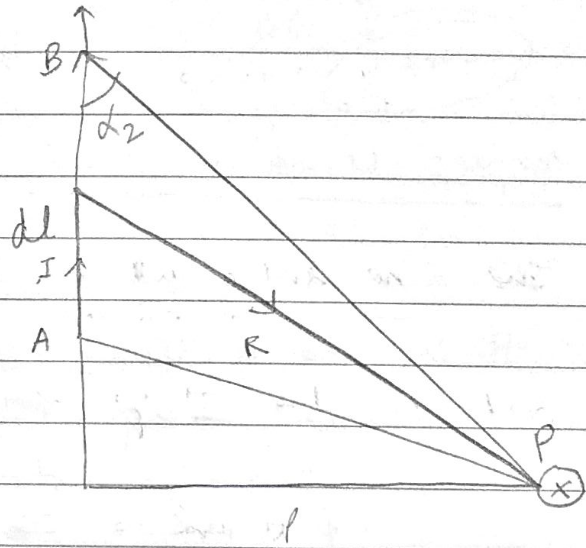
$$H = \int_S \frac{K\,dS \times a_R}{4\pi R^2} = \int_S \frac{K\,dS \times a_R}{4\pi R^2}$$ Surface current.

$$H = \int_V \frac{J\,dv \times a_R}{4\pi R^2} = \int_V \frac{J\,ds \times R}{4\pi R^2}$$

→ what is H at point P.?



$$dH = \frac{I\,dl \times a_R}{4\pi R^2}$$

$$= \frac{I\,dl \times R}{4\pi R^3}$$

$dl = dz \cdot a_z$

$R = \rho a_y - z a_z$

$$= \int \frac{I\rho\,dz}{4\pi\left(\rho^2 + z^2\right)^{3/2}}\, a_\phi$$

$z = \rho \cot\alpha$, $dz = -\rho \operatorname{cosec}^2\alpha\, d\alpha$

So above equation becomes $H = \frac{-I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2\alpha\, d\alpha}{\rho^3 \operatorname{cosec}^3\alpha}\, a_\phi$

$$= \frac{-I}{4\pi\rho}\, a_\phi \int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha$$

$$H = \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right) a_\phi$$

Another special case is when the conductor is infinite in length. For this case, point A is at $(0,0,\infty)$ while B is at $(0,0,\infty)$; $\alpha_1 = 180°$, $\alpha_2 = 0°$ so eq. reduces to

$$H = \frac{1}{2\pi\rho} a_\phi$$

→ Ampere circuital law.

The line integral of the tangential component of H around a close path is the same as the net current I_{enc} enclosed by the path.

$$\oint H \cdot dl = I_{enc}.$$

using stoke's law

$$I_{enc} = \oint_L H \cdot dl = \int_S (\nabla \times H) \cdot ds$$

But

$$I_{enc} = \int_S J \cdot ds.$$

By comparing we get

$$\nabla \times H = J$$

Third maxwell equation

Ampere's law

$\bar{D} = \epsilon \bar{E}$

$\downarrow \qquad \downarrow$

$\bar{B} = \mu \bar{H}$

$\bar{H} = \frac{I}{2\pi\rho} \bar{a}_\rho$

$I = \int_S \bar{J} \cdot ds = \int \bar{H} \cdot d\bar{l}$

$\oint \vec{H} \cdot d\vec{l} = \int_S \nabla \times \bar{H} \cdot d\vec{S}$

$\boxed{\nabla \times \bar{H} = \bar{J}}$

Biot-Savart law

$d\bar{H} = I d\bar{l} \times \bar{R} = Am$ (Am)

$= \bar{K} dS \times \bar{R}$

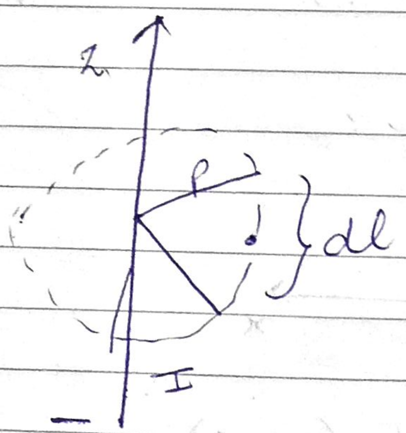
$\frac{A}{m} \; m^2 = A-m$

$= \bar{J} dv \times R$

$= \frac{A}{m^2} \times m^3 \times R$

$= A-m$

Long conductor



$\bar{dl} = \rho d\phi \bar{a}_\phi$

$I = \oint \bar{H} \cdot \bar{dl} = \oint \frac{H_\phi \bar{a}_\phi}{2\pi} \cdot (\rho d\phi \, a_\phi)$

$= H_\phi \int_{\phi=0}^{2\pi} \rho d\phi = H_\phi [2\pi\rho]$

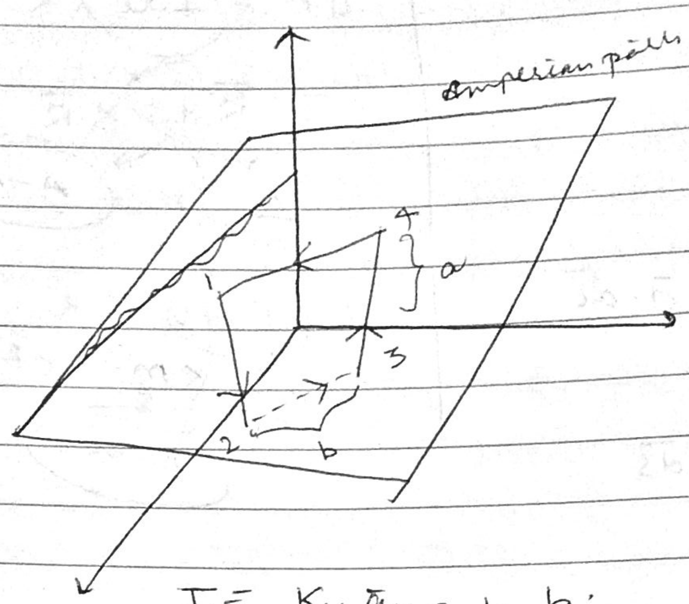
$\therefore H_\phi = (2\pi\rho)$

$\therefore H_\phi = \frac{I}{2\pi\rho}$

$\Rightarrow H = H_\phi a_\phi$

$\bar{H} = \frac{I}{2\pi\rho} \bar{a}_\phi$

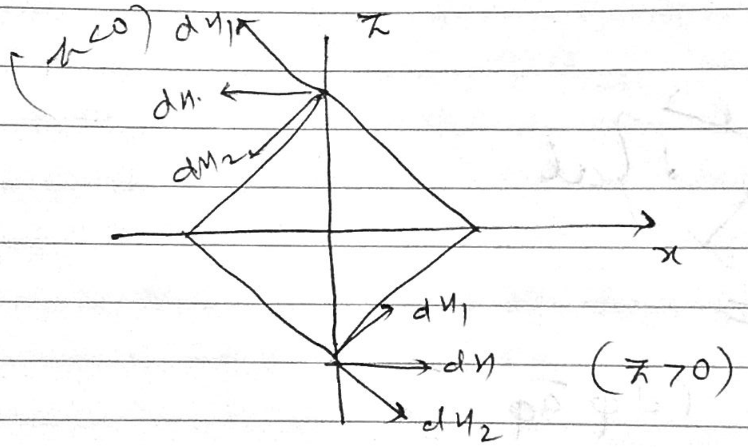
Surface plane $(z=0(x,y)) \rightarrow xy$-plane



$$I = K_y \bar{a}_y = k_y b.$$

$$\bar{K}_y = K_y \bar{a}_y$$

$$K_y = \bar{K}_y$$



$$\boxed{\begin{aligned}\bar{H} &= H_0 \bar{a}_x \quad (z>0) \\ &= H_0(-\bar{a}_x) \quad (z<0)\end{aligned}}$$

$$I = \oint \bar{H} \cdot \bar{dl} = \oint_1^2 \cancel{0 \bar{a}_z / a a_z} + \int_2^3 (-H_0)(-b) + \int_3^4 0(a) + \int_4^1 H_0 b$$

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$I = H_0 b + H_0 b$

$K_y b = 2 H_0 b$

$\therefore \quad H_0 = \frac{1}{2} K_y$

$\bar{H} = \frac{1}{2} K_y \bar{a}_x$

$= \frac{1}{2} K_y (-\bar{a}_x)$

$\bar{H} = \frac{1}{2} \bar{K}_y \times \bar{a}_x$

as here we took xy plane!!

$$\boxed{\bar{H} = \frac{1}{2} \bar{K} \times \bar{a}_n}$$

depending on the plane we are considering.

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$\epsilon_r = \frac{\epsilon}{\epsilon_0}$

Magnetic flux density

Electric flux density

Permittivity

$\bar{D} = \epsilon_0 \bar{E}$

Permeability

↳ $\bar{B} = \mu_0 \bar{H}$

$\frac{\text{weber}}{m^2}$ $\quad 4\pi \times 10^{-7}$ Henry/metre

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

Integral form $\quad$ Diff. form

Gauss's law $\int_S \bar{D} \cdot \bar{ds} = Q = \int_V \rho_v dv.$ $\quad \nabla \cdot \bar{D} = \rho_v$

$$\psi = \int_S \bar{B} \cdot \bar{ds}$$

~~For~~ If there are no Isolated magnetic charges.

$$\boxed{\int_S \bar{B} \cdot \bar{dS} = 0.}$$

Integral form $\quad$ Diff. Form

→ $\int_S \bar{B} \cdot \bar{ds} = 0$ $\quad \nabla \cdot \bar{B} = 0$

Maxwell's Equation

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Maxwell's Equations

	Integral Form	Diff. Form
Gauss's law in Electrostats	$\int_S \bar{D}\cdot \bar{ds} = Q = \int_V \rho_v \, dv$	$\nabla \cdot \bar{D} = \rho_v$
Non-Existence of magnetic poles (Gauss law in magnetostatics)	$\int_S \bar{B}\cdot \bar{ds} = 0$	$\nabla \cdot \bar{B} = 0.$
Conservative nature of Electric field.	$\oint \bar{E}\cdot \bar{dl} = 0$	$\nabla \times \bar{E} = 0$
Ampere's law.	$\oint \bar{H}\cdot \bar{dl} = I = \int_S \bar{J}\cdot \bar{ds}$	$\nabla \times \bar{H} = \bar{J}$

→ $\bar{E}(x, y, z)$ – Electrostatics – Static charges

$\bar{H}(x, y, z)$ – magnetostatics – moving charges

* Time Invariant fields
* Independent
* STATIC Fields

with reference to time fields will change

→ Time VARIANT fields — $\bar{E}(x, y, z, t)$, $\bar{H}(x, y, z, t)$

Interdependent

Dynamic fields

Accelerated charges

Faraday's law

No. of Turns in a coil

$$V_{emf} \propto -\left(\frac{d\lambda}{dt}\right) \quad \text{or} \quad V_{emf} = -N\left(\frac{d\psi}{dt}\right)$$

→ magnetic flux through each turn.

The net value for a voltage or Induced Emf is proportional to $-\left(\frac{d\lambda}{dt}\right)$.

→ Change in magnetic flux Passing through a coil produces an Emf → Faraday's law.

$$\rightarrow \psi = \int_S \bar{B} \cdot \bar{ds}$$

$$\rightarrow E = \left(\frac{dB}{dt}\right)$$ → Time Rate of Change of Magnetic field is Electric field.

or

$$B = \left(\frac{dE}{dt}\right)$$

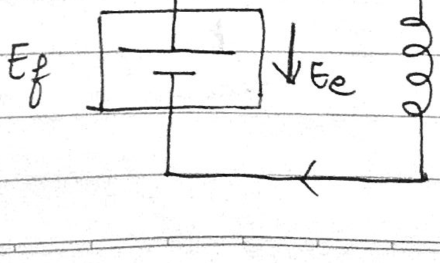
Now $V_{emf} = (-)N\left(\frac{d\psi}{dt}\right)$

due to Lenz's Law

or

Neumann's Law!

→



E_f → Due to chemical Reactions.

E_e → Electrostatic field.

negative Terminal $\rightarrow$ ~~[illegible]~~ Excess of electrons
positive Terminal $\longrightarrow$ deficit of Electrons

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$$E = E_f + E_e \qquad E_f = -E_e$$

$$V_{emf} = -\oint \bar{E}\cdot dl$$

E_f & E_e are equal but opp. in a battery.

$$= -\left[\int \bar{E_f}\cdot d\bar{l} + \int \bar{E_e}\cdot dl\right]$$

$$\int \bar{E_e}\cdot d\bar{l} = 0$$ [Conservative nature maxwell's eqn].

or

$$\nabla \times E_e = 0.$$

$$\Rightarrow V_{emf} = \int \bar{E_F}\cdot d\bar{l}$$

$$V_{emf} = -\int \bar{E_e}\cdot d\bar{l} \qquad (\because E_f = -E_e)$$

$$= -\int \bar{E}\cdot d\bar{l} \qquad (E_e = E)$$

$$V_{emf} = -N\left(\frac{d\Psi}{dt}\right)$$

$$= -N\int_s \frac{d}{dt}(\bar{B}\cdot\bar{ds}) = -N\int_S \left(\frac{\partial \bar{B}}{\partial t}\right)\cdot\bar{ds}$$

~~& also~~ also $V_{emf} = -\oint \bar{E}\cdot dl$

applying stokes Theorm,

$$V_{emf} = -\int_s (\nabla\times\bar{E})\cdot\bar{ds}$$

Equating the Equations

$$\boxed{\nabla \times \bar{E} = \left(\frac{\partial \bar{B}}{dt}\right)}$$

The variation of flux with time may be caused in 3 ways

1. Transfor and motional Emf.

$$V_{emf} = -\frac{d\psi}{dt}$$

$$\text{or } V_{emf} = \oint_L E \cdot dl = -\frac{d}{dt}\int_S B \cdot dS$$

This equation says that in time varying situation, both electric and magnetic fields are present and Interrelated.

By applying stokes.

$$\nabla \times E = -\frac{dB}{dt}$$

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Moving Loop in Static B field (motional emf)

$$F_m = Q\bar{u} * \bar{B}$$

$$E_m = \frac{F_m}{Q} = u \times B$$

Consider a Conducting loop moving with uniform velocity u, emf is Induced in the loop

$$V_{emf} = \oint_L E_m \cdot dl = \oint_L (u \times B) \cdot dl \quad (1)$$

$$\int_S \nabla \times \bar{E}_m \cdot d\bar{s} = \int \nabla \times (\bar{u} \times \bar{B}) \cdot d\bar{s}$$

$$\boxed{E_m = \bar{u} \times \bar{B}}$$

Now $\because$ $f_m = Q\bar{u} \times \bar{B}$ $\left(I = \frac{Q}{t} \times l \; u. \right.$

$$= I\bar{l} \times \bar{B}$$

moving loop in time varing field.

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J = Current density

Displacement Current

For static EM fields

$$\nabla \times H = J \quad -(1)$$

But divergence of curl of a vector is zero. So

Taking Dot product both sides $\nabla . (\nabla \times H) = 0 = \nabla . J \quad -(2)$

But continuity of current requires

$$\nabla . J = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad -(3)$$

From (2) & 3

Equation 2 & (3) are incompatible for ~~time~~ time-varying conditions, so we need to modify equation (i) to agree with (3)

Add a term to (1) so that it becomes.

$$\nabla \times H = J + J_d \quad -(4)$$

Where J_d is to defined & determined.

↓

Displacement current density

Now

$$\nabla . (\nabla \times H) = 0 = \nabla . J + \nabla . J_d \quad -(5)$$

In order for eqn (5) to agree with (3)

$$\nabla . J_d = -\nabla . J \Rightarrow \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla . D) \quad (\because \nabla . D = \rho_v)$$

$$= \nabla . \frac{\partial D}{dt}$$

or $$J_d = \frac{\partial D}{dt} \quad -(6).$$

By Putting we get $\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$

This is maxwell's equation for a time varying field.

The term $J_d = \frac{\partial D}{\partial t}$ is known as Displacement Current density and J is the conduction Current density $J = \sigma E$

Maxwell's equations in final form.

	Integral form	differential form
Gauss's law	$\oint_S D \cdot ds = \int_V \rho_v \, dv$	~~Gauss's law~~ $\nabla \cdot D = \rho_v$ $\cancel{\nabla \times D = \rho_v}$
Non-Existence of Isolated magnetic charge	$\oint_S B \cdot ds = 0$	$\cancel{\nabla \times B = 0}$ $\nabla \cdot B = 0$
Faraday's law	$\oint_L E \cdot dl = -\frac{\partial}{\partial t} \int_S B \cdot ds$	$\nabla \times E = -\frac{\partial B}{\partial t}$
Ampere's Circuit law	$\oint_L H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot \partial s$	$\nabla \times H = J + \frac{\partial D}{\partial t}$

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UNIT-3.

SPECIAL THEORY OF RELATIVITY

→ "Relative" → Length, Size, Mass, time, velocity
Each & every parameter is Relative parameter.

→ Light - Em wave

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/sec} \simeq 2.997975 \pm 0.00003 \times 10^8 \text{ m/s}$$

Consider, satellite, $u = 180000$ mph
$u/c = 0.000027$.

Sound, $u = 331$ m/sec
$u/c = 0.0000010$.

These concepts are Explained by Newtonian Mechanics or classical Mechanics. These are for Macroscopic world. But if we want to study the microscopic world classical Mechanics doesn't help.

$\boxed{\text{Velocity of electron} = V = 0.99\,c}$ (microscopic).

→ Special Theory of Relativity deals with the motion of bodies whose velocity's are moving with velocity comparable to velocity of light.

→ General Theory " deals with the bodies. Accelerating or not

$\cdot\ v = \sqrt{\frac{2eV}{m}}$, $eV = \frac{1}{2}mv^2$

$\Rightarrow$ $V = 10\ MeV$

$v = 0.99c$.

suppose $V = 40\ meV$

$\Rightarrow v = 1.98c$ ✗ not possible v must be $\underline{v = 0.99c}$

Bohr's Theory $E_n = \frac{-mz^2e^4}{8\epsilon_0^2 n^2 h^2}\left[1 + \frac{\alpha^2 z^2}{n}\left[\frac{1}{n\phi} - \frac{3}{4n}\right]\right]$

$\alpha^2 = \frac{e^2}{2\epsilon_0 hc}$

Principal Quantum no. $= n = n_r + n_\phi$

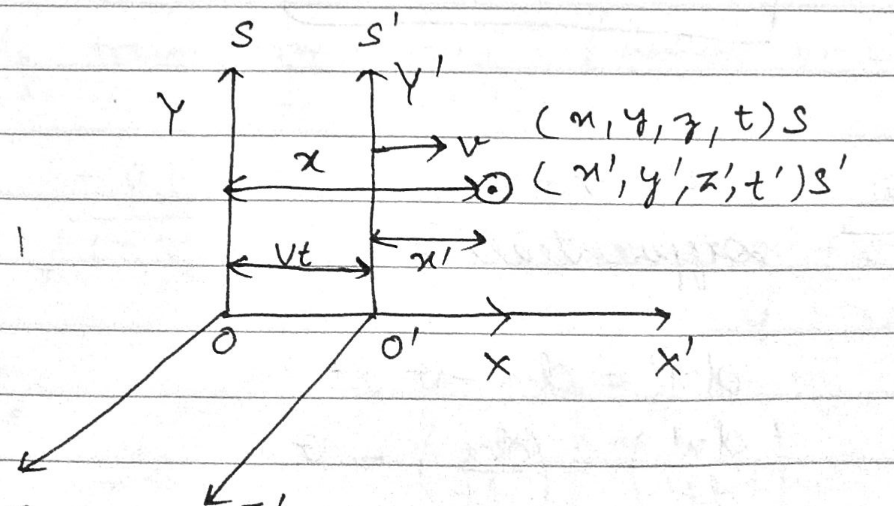
radial, azimuthal

→ FRAME OF REFRENCE → To define motion of bodies we need frame of refrence

also called (unaccelrated frames of refrences)

1. Intertial → satisfies Newtons law
2. non intertial → don't satisify newton's law.
   also called as (accelrated frames of refrences).

→ Intertial



In this S' is moving along +ve x direction with velocity V

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Galilean Transformation equations

$$\boxed{\begin{aligned} x - x' &= OO' \\ &\text{or} \\ x - x' &= vt \\ x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t. \end{aligned}}$$

$S' \rightarrow v \rightarrow S \rightarrow (+x)$

S' is moving - - - - - -

(Since frame is moving only along x).

GALILEAN TRANSFORMATIONS.

Now here $S \rightarrow (-v) \rightarrow S' \rightarrow (-x)$

here S is moving with negative velocity v along S' a[l]so in $-ve$ x direction.

So

$$\boxed{\begin{aligned} x &= x' + vt' \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}}$$

Inverse Galilean Transformation

Now $x' = x - vt$

Differentiating

$$dx' = dx - v\,dt$$

$$\left(\frac{dx'}{dt'}\right) = \left(\frac{dx}{dt}\right) - v.$$

$\frac{dx'}{dt'}$ = velocity of the object as measured from the S' Frame.

~~so $u_x' = \frac{dx'}{dt}$~~ ~~so $u_x' = u_x - v$~~

$$\boxed{u_x' = u_x - v}$$ → Galilean velocity Transformation.

Diff. once again:

$$\frac{du_x'}{dt'} = \frac{du_x}{dt}$$

$$\boxed{a_x' = a_x}$$ → ~~Galilean~~ Accelration remains Same.

That is why inertial Frame of Refrence is unaccelrated.

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If $V << C$ then → Galileau Transformation.

$V \sim C$ → Lorentz transformation.

comparable

Lorentz transformations (when $v \sim c$)

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Inverse Lorentz transform

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{x'v}{c^2}}{\sqrt{1 - v^2/c^2}}$$

If velocity is comparable to c use Lorentz
for Ex:- 2.4×10^8 m/s.

If $V << c$

$\frac{V}{c} << 1$

$$\Rightarrow K = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} = 1.$$

→ Galilean & Lorentz Transformation will give same answer

Q. An Event occurs at $x=100$ m, $y=20$ m, $z=10$ m & $t = 1\times10^{-4}$ sec in frame S. find coordinates of this event in frame ~~S prime~~ S' which is moving with velocity 2.4×10^8 m/s. w.r.t frame S along common axis XX', using (i) Galilean Transformation & (ii) Lorentz Transformations

Sol: (i) Galilean Transformation

$x' = x - vt$
$= 100 - 2.4\times10^8 \times 1\times10^{-4}$
$= 100 - 2.4\times10^{4}$
$= -2.3\times10^4$ m

$y' = 20$ m
$z' = 10$ m
$t = 1\times10^{-4}$ sec

(ii) Lorentz Transformation.

$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-\frac{2.4\times10^8}{3\times10^8}}$$

$$= \sqrt{0.2}$$

$$= \cancel{0.48}$$

$$= 0.447$$

$$x' = \frac{x-vt}{0.44} = \frac{-2.3\times10^4}{0.44}$$

$$= \cancel{-5.22\times10^4\ m}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-\frac{(2.4\times10^8)^2}{(3\times10^8)^2}}$$

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} = -39830\ m.$$

$$t' = \frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

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$$\boxed{\overset{\text{moving}}{m} = \frac{\overset{\text{Rest}}{m_0}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Q2. At what speed mass of an object will be double of it's value at rest.

Sol:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

A.T.Q

$$2\cancel{m_0} = \frac{\cancel{m_0}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$2\sqrt{1-\frac{v^2}{c^2}} = 1$$

S.B.S.

$$4\left(1-\frac{v^2}{c^2}\right) = 1.$$

$$4 - \frac{4v^2}{c^2} = 1.$$

$$4c^2 - 4v^2 = c^2.$$

$$-4v^2 = c^2 - 4c^2$$

$$-4v^2 = -3c^2.$$

$$4v^2 = 3c^2$$

$$v^2 = \frac{3}{4}c^2.$$

$$v = \sqrt{\frac{3}{4}c^2}$$

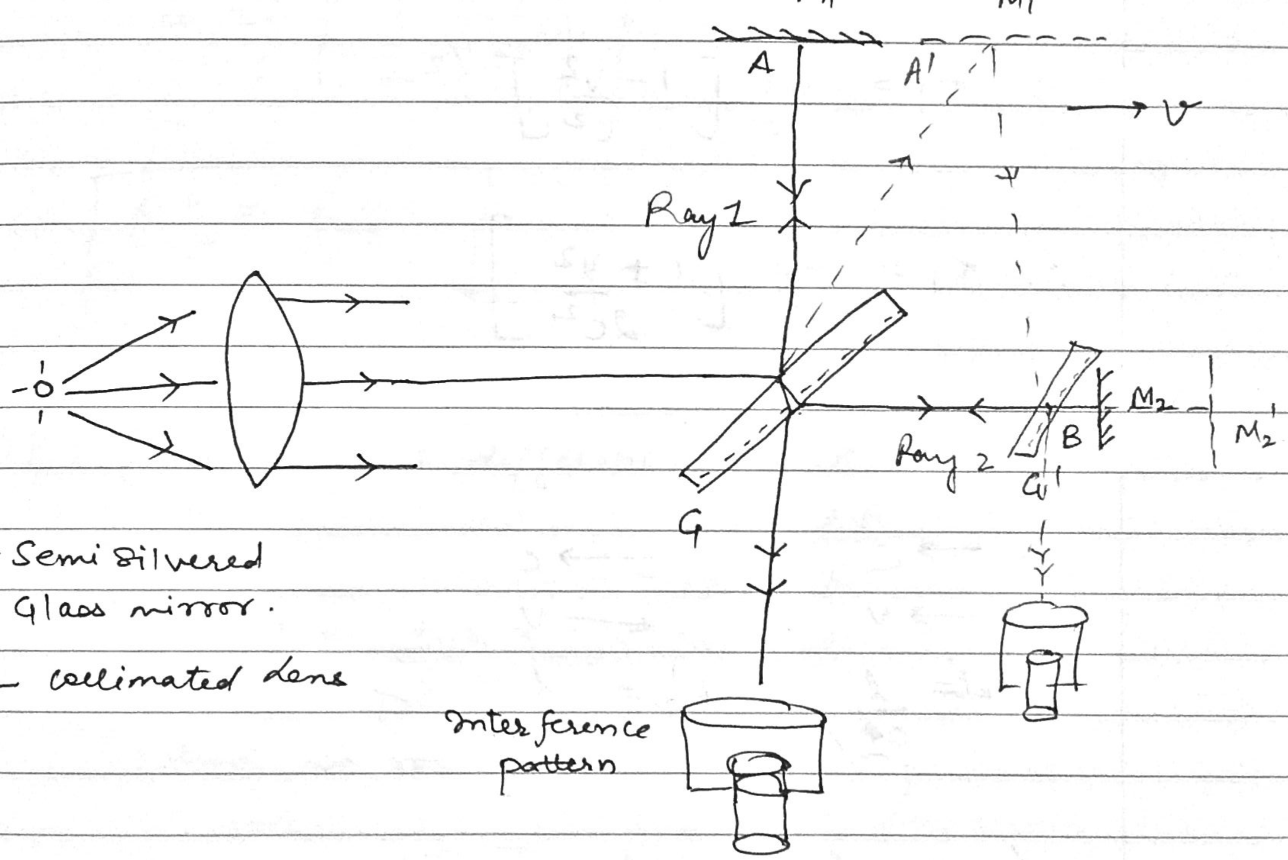
$$\boxed{v = 2.6 \times 10^8}$$

MICHELSON - MORLEY EXPT. - Postulates

* Absolute Frame of Refrence

Ether hypothesis

$(v)_{earth}$ w.r.t $(Ether)_{rest}$.



G - Semi silvered Glass mirror.

L - collimated lens

$GM_1 = GM_2 = l.$

Δ^{le} $GA'D$

$(GA')^2 = (GD)^2 + (M_1'D)^2$

$= (AA')^2 + (GM_1)^2$

$(ct)^2 = (vt)^2 + l^2.$

$[c^2 - v^2]t^2 = l^2$

$$\Rightarrow \quad t^2 = \frac{l^2}{c^2\left[1 - v^2/c^2\right]}$$

$$t = \frac{l}{c\left[1 - \frac{v^2}{c^2}\right]^{1/2}}.$$

$$t_1 = 2t.$$

$$t_1 = \frac{2l}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1/2}$$

$$\therefore \; t_1 = \frac{2l}{c}\left[1 + \frac{v^2}{2c^2}\right]$$

$\rightarrow c$ $\qquad\qquad \longrightarrow c$

$\rightarrow v$ $\qquad\qquad \longleftarrow v$

$$\cancel{t_2}\, t' = \frac{l}{c - v} \qquad t_2' = \frac{l}{c+v}$$

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$= \frac{lc + \cancel{lv} + lc - \cancel{lv}}{(c^2 - v^2)}$$

$$t_2 = \frac{2lc}{c^2\left[1 - \frac{v^2}{c^2}\right]} = \frac{2l}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1}$$

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$$\boxed{t_2 = \frac{2l}{c}\left[1+\frac{v^2}{c^2}\right]}$$

$$\therefore \Delta t = t_2 - t_1$$

$$= \frac{2l}{c}\left[\left(1+\frac{V^2}{c^2}\right) - \left(1+\frac{v^2}{2c^2}\right)\right]$$

$$= \frac{2l}{c}\left[\frac{1}{2}\left(\frac{V^2}{c^2}\right)\right]$$

$$\boxed{\Delta t = \frac{lv^2}{c^3}}$$

OPD (optical path difference) $= c \times \Delta t$

$$= \frac{lv^2}{c^2}$$

$$OPD(\lambda) = \left(\frac{lv^2}{\lambda c^2}\right)$$

When we rotate $\rightarrow \left(-\frac{lv^2}{\lambda c^2}\right)$

$$\text{Shift} = \frac{lv^2}{\lambda c^2} - \left(-\frac{lv^2}{\lambda c^2}\right)$$

$$\delta = \frac{2lv^2}{\lambda c^2}$$

Expected value
For Shift $= \delta = 0.4$
$= 0.01$

$l = 1.0 \times 10^3$ cm
$v = 3.0 \times 10^5$ cm/sec
$c = 3 \times 10^{10}$ cm/sec
$\lambda = 5 \times 10^{-5}$ cm.

Q) Explain postulates of special theory of Relativity?

Imp.

1. Ether Drag theory.

2. ~~Loren~~ Lorentz Fitzgerald length contraction

3. Light velocity hypothesis.

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postulates of STOR (special Theory of Relativity).

$$\delta = \frac{2lv^2}{\lambda c^2}$$

$= 0.4$

$= 0.01$

NEGATIVE RESULT

1. Ether - drag Theory
2. Lorentz - Fitzgerald length contraction.
3. Light velocity Hypothesis.

1. Absolute Motion
   — Meaningless

2. Light velocity

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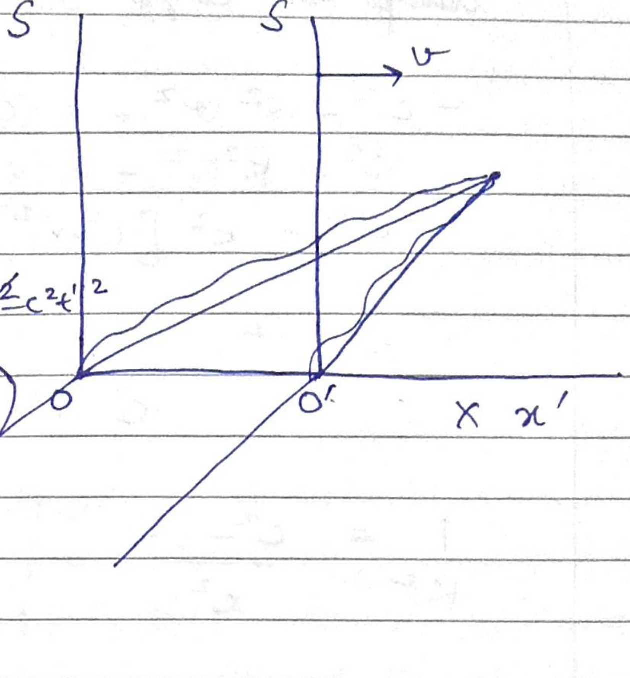
→ Lorentz transformation equations.



$d = v \times t$

$S \rightarrow x^2 + y^2 + z^2 - c^2t^2 = 0$

$S' = x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$

$x^2 + \cancel{y^2} + \cancel{z^2} - c^2t^2 = x'^2 + \cancel{y'^2} + \cancel{z'^2} - c^2t'^2$

$\Rightarrow x^2 - c^2t^2 = x'^2 - c^2t'^2 \quad — (1)$

$x' = K(x - vt) \quad — (2)$

$x = K[x' + vt']$

$= K[K(x - vt) + vt']$

$\Rightarrow \frac{x}{K} = K(x - vt) + vt'$

$vt' = \frac{x}{K} - \{K(x - vt)\}$

$\therefore t' = \frac{x}{Kv} - \frac{Kx}{v} + Kt$

$t' = K\left[t - \frac{x}{v}\left(1 - \frac{1}{K^2}\right)\right] \quad — (3)$

Substituting (2) & (3) in (1),

$\therefore x^2 - c^2t^2 = x'^2 - c^2t'^2$

$x^2 - c^2t^2 = K^2[x - vt]^2 - c^2K^2\left[t - \frac{x}{v}\left(1 - \frac{1}{K^2}\right)\right]^2$

$= K^2[x^2 + v^2t^2 - 2xvt]$

$- c^2K^2\left[t^2 + \frac{x^2}{v^2}\left(1 - \frac{1}{K^2}\right)^2 - 2t\frac{x}{v}\left(1 - \frac{1}{K^2}\right)\right]$

Comp the Coeff of t^2

$$-c^2 = k^2 v^2 - k^2 c^2$$

$$c^2 = k^2 c^2 - k^2 v^2$$

$$c^2 = k^2 [c^2 - v^2]$$

$$\Rightarrow k^2 = \frac{c^2}{c^2 - v^2}$$

$$\frac{1}{k^2} = \frac{c^2 - v^2}{c^2}$$

$$= 1 - \frac{v^2}{c^2}$$

or $\boxed{k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$

$$\boxed{\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - (vx/c^2)}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}}$$

$$\boxed{\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + (vx'/c^2)}{\sqrt{1 - v^2/c^2}} \end{aligned}}$$

Lorentz Transformations

$v = 0.99c$

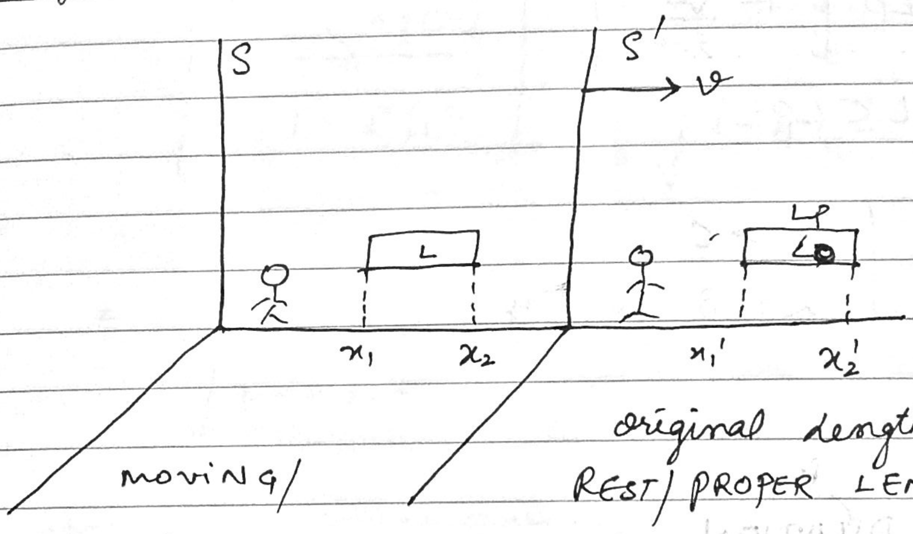
$v < c$

$(v/c) < 1$

$(v^2/c^2) \simeq 0$.

$x' = x - vt$

1. <u>Length Contraction</u>



MOVING/ Improper

$L = x_2 - x_1$

original length REST/PROPER LENGTH

$L_p = x_2' - x_1'$

$$L_p = x_2' - x_1'$$

$$= \left[\frac{x_2 - \cancel{vt}}{\sqrt{1 - \frac{v^2}{c^2}}}\right] - \left[\frac{x_1 - \cancel{vt}}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

$$= \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

The proper length (The length observed by Rest Observer in the moving Frame)

$$= \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \boxed{L = L_p \sqrt{1 - \frac{v^2}{c^2}}}$$

Length observed by the Rest Observer

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1. $V < C$

$V/C < 1$

$V^2/C^2 \simeq 0$

$L = L_p$

2. V comparable to c.

$$L = L_p\left[1 - \frac{V^2}{2C^2}\right]$$

$L < L_p$

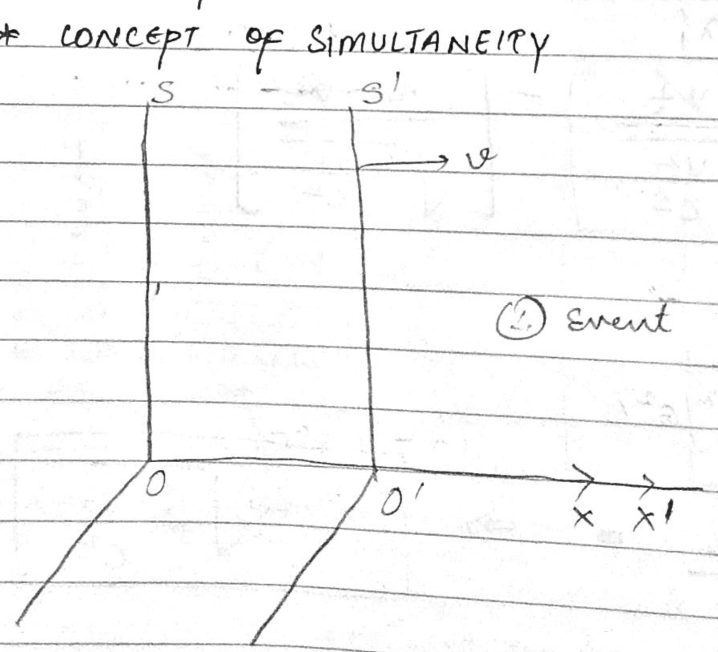
3. $V = C$ | $v > C$

$L = 0$ | $L =$ Imaginary

* TIME DILATION → Lengthening of time

* TWIN PARADOX

* CONCEPT OF SIMULTANEITY



S' - Event - Time interval, $\underset{(T_0)}{\Delta t'} = t_2' - t_1'$

S - " " " $\underset{(T)}{\Delta t} = t_2 - t_1$

$$\Delta t = t_2 - t_1$$

$$= \left[\frac{t_2' + (vx'/c^2)}{\sqrt{1 - v^2/c^2}}\right] - \left[\frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-v^2/c^2}}\right]$$

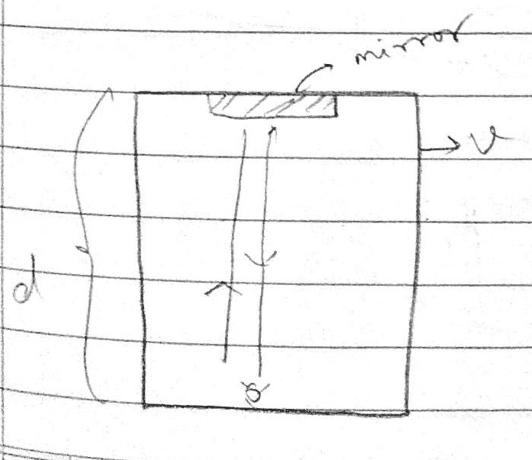
$$= \frac{t_2' - t_1'}{\sqrt{1-v^2/c^2}}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-v^2/c^2}} = \Delta t'\left[1 + \frac{v^2}{2c^2}\right]$$

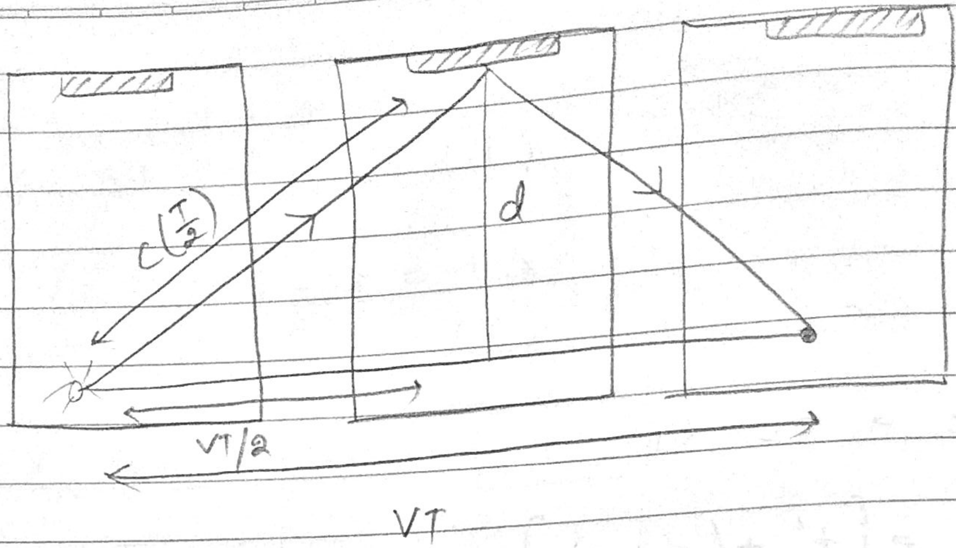
$$\boxed{\Delta t > \Delta t'}$$

$$\boxed{T = \frac{T_0}{\sqrt{1-v^2/c^2}}}$$

→



$$T_0 = \left(\frac{2d}{c}\right)$$



$$c^2\left(\frac{T^2}{4}\right) = v^2\left(\frac{T^2}{4}\right) + d^2$$

$$(c^2 - v^2)\frac{T^3}{4} = d^2$$

$$c^2\left(\frac{1-v^2}{c^2}\right)T^2 = \left(\frac{2d}{c}\right)^2$$

$$T^2 = \frac{T_0^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

1. $v < c \Rightarrow v/c < 1 \Rightarrow \left(\frac{v^2}{c^2}\right) \simeq 0 \Rightarrow T = T_0$

2. v comp. c, $T > T_0$

3. $v = c$ or $v > c$

$T = \infty$ T = Imaginary

3/4

Addition of velocities.

$$S' \rightarrow u'_x = \left(\frac{dx'}{dt'}\right) = \frac{x'_2 - x'_1}{dt'}$$

$$S \rightarrow u_x = \left(\frac{dx}{dt}\right)$$

$$x'_1 = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x'_2 = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dx' = x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{t - \left(\frac{vx}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} = dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dx' = \frac{dx - v\,dt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$u'_x = \frac{dx'}{dt'} = \frac{dx - v\,dt}{\cancel{\sqrt{1 - \frac{v^2}{c^2}}}} \times \frac{\cancel{\sqrt{1 - v^2/c^2}}}{dt - \frac{v\,dx}{c^2}}$$

$$= \frac{dx - v\,dt}{dt - \frac{v\,dx}{c^2}} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

$$u'_x = \frac{u_x - v}{1 - \frac{V u_x}{c^2}} \quad — (1)$$

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$$y' = y$$

$$dy' = dy$$

$$\frac{dy'}{dt'} = \frac{dy}{dt'}$$

$$= \left(\frac{dy'}{dt'}\right) = \left[\frac{dy}{\frac{dt - (v\,dx/c^2)}{\sqrt{1-\frac{v^2}{c^2}}}}\right]$$

$$u_y' = \frac{(dy/dt)\sqrt{1-v^2/c^2}}{1-\left(\frac{v}{c^2}\right)\left(\frac{dx}{dt}\right)} = \frac{u_y\sqrt{1-v^2/c^2}}{1-(u_x v/c^2)}$$

$$z = z'$$

$$dz' = dz.$$

$$\frac{dz'}{dt'} = \frac{dz\sqrt{1-v^2/c^2}}{dt - \left(\frac{v\,dx}{c^2}\right)} = \frac{u_z\sqrt{1-v^2/c^2}}{1-\frac{v}{c^2}u_x}.$$

Equation (1) can be represented as

$$\boxed{u = \frac{u'+v}{1+(u'v/c^2)}}$$

Relativistic Law of addition of velocities.

1. $v << c$

$v/c << 1$

$v/c^2 \simeq 0$

$u = u' + v.$

2. $v \simeq c.$

$u = c.$

3. $u' = c$ & $v = c$

$$u = \frac{c+c}{1+c^2/c^2} = \frac{2c}{2} = c. \quad u = c$$

Variation of mass with velocity

A B

$S' \rightarrow u' \quad -u'$

$S \rightarrow u_1 \quad u_2$

(A) $u_1 = \dfrac{u'+v}{1+(u'v/c^2)}$ $\qquad u_2 = \dfrac{-u'+v}{1-(u'v/c^2)}$

$$m_1 v_1 + m_2 v_2 = (m_1+m_2)v$$

(Before) (After)

$$m_1\left[\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)-v\right] = m_2\left[v-\left[\frac{-u'+v}{1-\frac{u'v}{c^2}}\right]\right]$$

$$m_1\left[\frac{u'+\cancel{v-v-u'v^2/c^2}}{1+\frac{u'v}{c^2}}\right] = m_2\left[\frac{v-\cancel{(u'v^2/c^2)+u'-v}}{(1-u'v/c^2)}\right]$$

$$\frac{m_1}{m_2} = \frac{1+(u'v/c^2)}{1-u'v/c^2}$$

Now $1-\dfrac{u_1^2}{c^2} = 1-\left[\dfrac{(u'+v/c)^2}{[1+(u_1 v/c^2)]^2}\right]$

$$= \left[1+(u'v/c^2)\right]^2 - \left[\frac{u'+v}{c}\right]^2 \Big/ \left[1+(u'v/c^2)\right]^2$$

$$= \frac{\left[1+(u'^2v^2/c^4)+\cancel{(2u'v/c^2)}\right]-\left[\frac{u'^2}{c^2}+\frac{v^2}{c^2}+\cancel{\frac{2u'v}{c^2}}\right]}{\left[(1+u'v/c^2)\right]^2}$$

$$= \frac{\left[1-\frac{u'^2}{c^2}+\frac{u'^2v^2}{c^4}-\frac{v^2}{c^2}\right]}{[1+u'v/c^2]^2} \Rightarrow \frac{\left[1-\frac{u'^2}{c^2}\right]-\frac{v^2}{c^2}\left[1-\frac{u'^2}{c^2}\right]}{\left[1+\frac{u'v}{c^2}\right]^2}$$

$$= \frac{\left[1-\frac{u'^2}{c^2}\right]\left[1-\frac{v^2}{c^2}\right]}{\left[1+\frac{u'v}{c^2}\right]^2}$$

$$\left[1+\frac{u'v}{c^2}\right] = \left\{\frac{\left[1-\frac{u'^2}{c^2}\right]\left[1-\frac{v^2}{c^2}\right]}{\left[1-\frac{u_1^2}{c^2}\right]}\right\}^{1/2}$$

Similarly

$$\left[1-\frac{u'v}{c^2}\right] = \left\{\frac{\left[1-\frac{u'^2}{c^2}\right]\left[1-\frac{v^2}{c^2}\right]}{\left[1-\frac{u_2^2}{c^2}\right]}\right\}^{1/2}$$

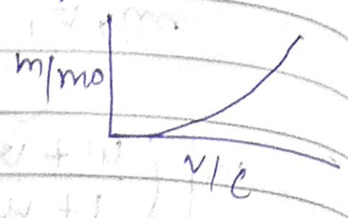
$$\therefore \frac{m_1}{m_2} = \left[\frac{\left(1-\frac{u_2^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}\right]^{1/2}$$

Assuming $u_2 = 0$, $u_1 = v$
$m_2 = m_0$ (rest mass), $m_1 = m$ (moving mass)

$$\boxed{\frac{m_1}{m_0} = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}}}$$

1. $v < c$
   $m = m_0$
2. v comp c
   $m = \dfrac{m_0}{1-\frac{v^2}{2c^2}}$
3. $v = c$
   $m = \infty$

$v > c$
$m =$ Imaginary.



Mass-Energy Equivalence.

$$F = \frac{dP}{dt} = \frac{d(mv)}{dt} = m\left(\frac{dv}{dt}\right) + v\left(\frac{dm}{dt}\right)$$

$$dw = F.dx$$
$$= m\left(\frac{dv}{dt}\right)dx + v\left(\frac{dm}{dt}\right)dx \qquad \left(\because \frac{dx}{dt} = v\right)$$

change in kinetic energy

$$\boxed{dK = m\,v\,dv + v^2\,dm} \quad — (1)$$

Take the equation

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow m^2 = \frac{m_0^2}{\left(1-\frac{v^2}{c^2}\right)}$$

$$m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2$$

$$m^2\left(\frac{c^2-v^2}{c^2}\right) = m_0^2$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

differentiate

$$\left\{[2m\,dm]c^2 + m^2[2c\,dc]\right\} - \left\{(2m\,dm)v^2 - m^2(2v\,dv)\right\}$$

$$\boxed{m\,v\,dv + v^2\,dm = c^2\,dm} \quad — (2)$$

Equating (1) and (2)

$dK = c^2 dm$

m_0 = Rest mass

m = moving mass.

$$K = \int_{m_0}^{m} c^2 dm$$

$$K = (m - m_0) c^2$$

Rest mass Energy $= m_0 c^2$

$\Rightarrow \boxed{E = mc^2}$

$\rightarrow \quad E^2 = p^2c^2 + m_0^2 c^4$

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UNIT-4

QUANTUM MECHANICS.

wave Mechanics

* clasical Mechanics vs wave mechanics.
* particle $\Longleftrightarrow$ Wave

particle	Wave
Mass	Disturbance
position	Frequency (ν)
velocity	wavelength (λ)
Momentum	Energy ($E = h\nu$)
Energy	

$$C = \nu\lambda$$

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

→ Newton corpuscular Theory of light

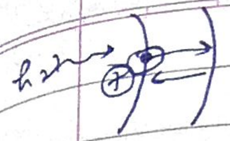
* Hygen's wave Theory

* Interference
* diffraction
* polarisation.

1. Stability of Hydrogen atom

$$m v r = \frac{nh}{2\pi}$$

$$E_n = \frac{-m Z^2 e^4}{8 \epsilon_0^2 n^2 h^2} = \frac{-13.6 \text{ eV}}{n^2}$$



$$E_1 + h\nu = E_2$$

$$\boxed{E_2 - E_1 = h\nu}$$

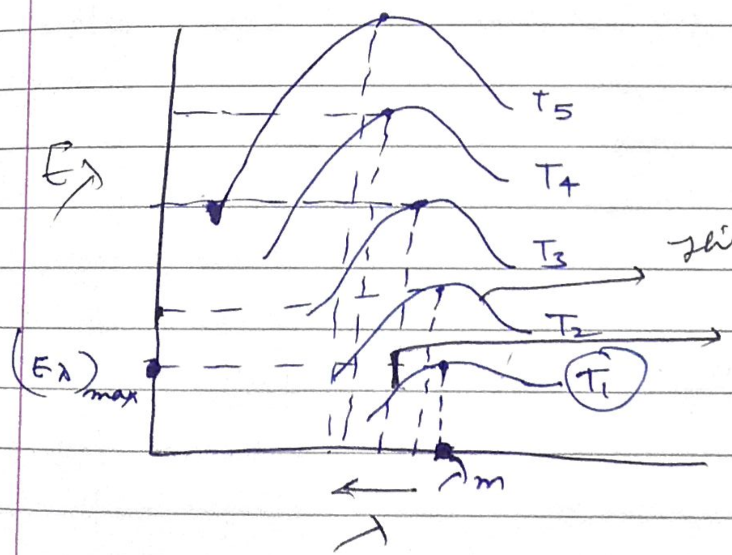
$$\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

* Black Body Radiation

$$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{d\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}\right]$$

- ultra voilet catastrophe.

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{d\lambda}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}\right]$$



This upper region is Explained by → WIEN's Law

This Lower Region is Explained by RAYLEIGH-JEANS Law. $= E_\lambda d\lambda = \frac{8\pi kT\, d\lambda}{\lambda^4}$

$\lambda m \propto \frac{1}{T}$ (WIEN's displacement Law).

Increase in Temprealure decreas the wavelength.

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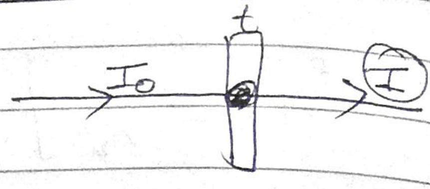
$\lambda = \frac{h}{mv}$ de Broglie

here h Relates Partical with wave mechanics

→ Interaction of Radiation with matter

$$\boxed{I = I_0 \, Exp(-\mu t)}$$



- Low Energy ⟶ PEE (photo Electric effect)

Medium Energy ⟶ Compton Effect

High Energy ⟶ pair production

&

pair Annihilation

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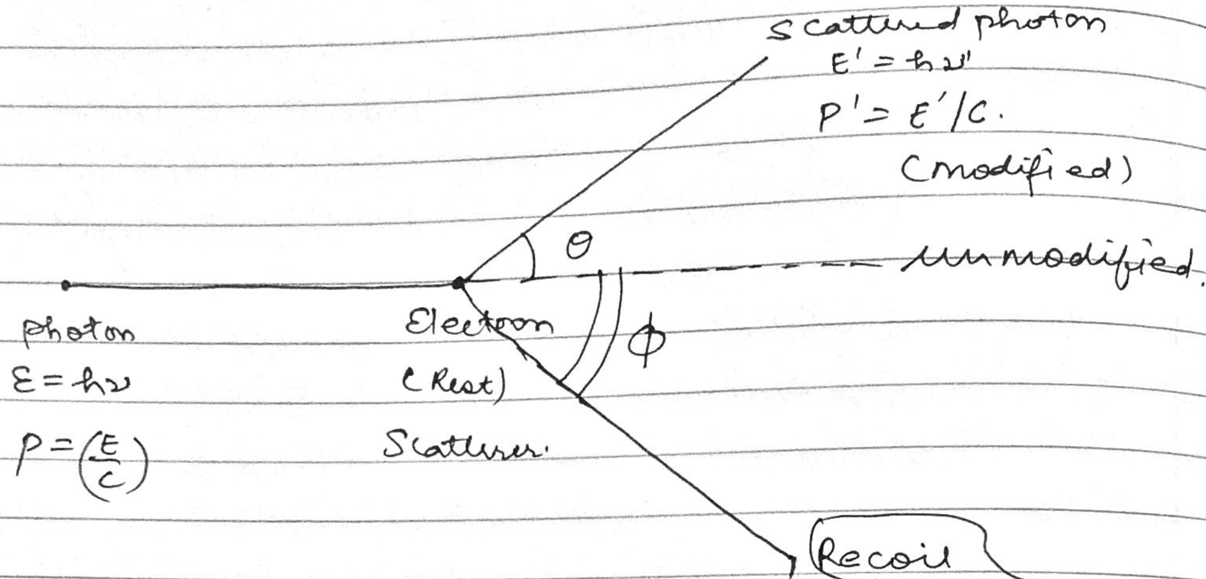
'COMPTON EFFECT'



	Before collision		After collision	
	Photon	Electron	Photon	Electron
Energy	$E = h\nu$	$m_0 c^2$	$E' = h\nu'$	mc^2
Momentum	$p = \frac{E}{c}$	0	$p' = E'/c$	mv

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

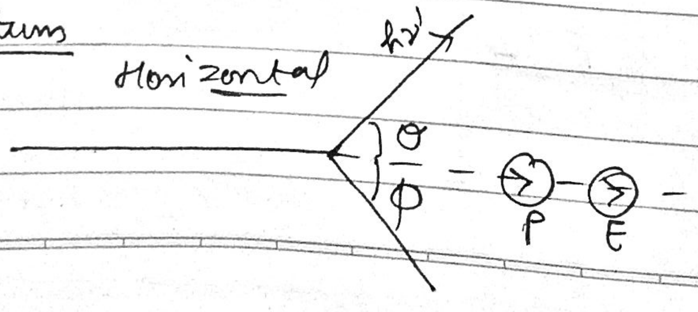
Energy

$$\underbrace{\underset{(\text{Photon})}{h\nu} + \underset{(\text{Electron})}{m_0 c^2}}_{\text{Before}} = \underbrace{\underset{(\text{photon})}{h\nu'} + \underset{(\text{Electron})}{mc^2}}_{\text{after}}$$

$$\therefore \boxed{mc^2 = h(\nu - \nu') + m_0 c^2} \quad \text{— (A)}$$

Momentum

Horizontal

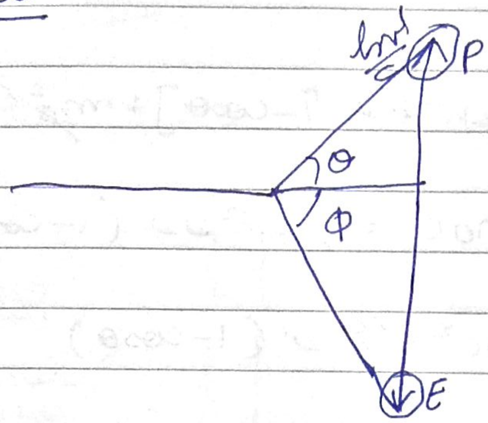


we will take the components here

$h\nu + 0 = \left(\frac{h\nu'}{c}\right)\cos\theta + mv\cos\phi$

$\therefore mvc\cos\phi = h\nu - h\nu'\cos\theta$ — (1)

<u>vertical</u>



there's no momentum in vertical direction before collision.

$0 + 0 = \left(\frac{h\nu'}{c}\right)\sin\theta - mv\sin\phi$

$\therefore mvc\sin\phi = h\nu'\sin\theta$ — (2)

$(1)^2 + (2)^2$

$m^2v^2c^2 = [h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta] + [h^2\nu'^2\sin^2\theta]$

$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta$ — (B)

$(A)^2 = m^2c^4 = [h^2[\nu^2 + \nu'^2 - 2\nu\nu') + m_0^2c^4 + 2h(\nu-\nu')m_0c^2]$ — (5)

Equation $A^2 - B$

$A^2 - B = m^2c^4 - m^2v^2c^2 = -2h^2\nu\nu' + m_0^2c^4 + 2h(\nu-\nu')m_0c^2 + 2h^2\nu\nu'\cos\theta$

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$$\Rightarrow m^2c^2(c^2 - v^2] = -2h^2\nu\nu'[1-\cos\theta] + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$$

$$\frac{m_0^2c^2}{\left(1-\frac{v^2}{c^2}\right)}(c^2-v^2) = -2h^2\nu\nu'[1-\cos\theta] + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$$

$$\frac{m_0^2 \cancel{c^4}}{\cancel{c^2-v^2}}[\cancel{c^2-v^2}] = -2h^2\nu\nu'[1-\cos\theta] + \cancel{m_0^2c^4} + 2h(\nu-\nu')m_0c^2$$

$$\cancel{2}h(\nu-\nu')m_0c^2 = \cancel{2}h^2\nu\nu'(1-\cos\theta]$$

$$(\nu-\nu')m_0c^2 = h\nu\nu'(1-\cos\theta)$$

$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\left(\frac{1}{\nu'}-\frac{1}{\nu}\right) = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\left(\frac{c}{\nu'}-\frac{c}{\nu}\right) = \frac{h}{m_0c}[1-\cos\theta]$$

$$\lambda'-\lambda = \frac{h}{m_0c}[1-\cos\theta]$$

$$\boxed{\Delta\lambda = \lambda_c[1-\cos\theta]} \longrightarrow$$ Equation for the Compton shift

compton's wavelength

where $\lambda_c = \frac{h}{m_0c} = 0.024 A^\circ$

$$\Delta\lambda = \lambda_c[2\sin^2\theta/2]$$

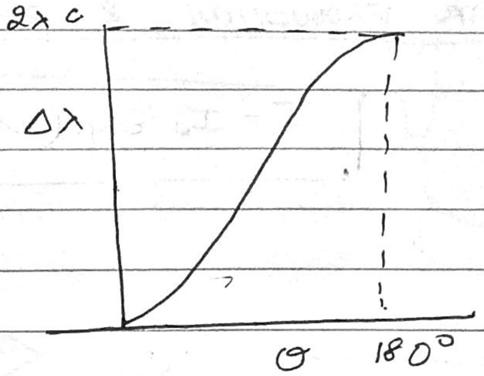
$$\boxed{\Delta\lambda = 2\lambda_c\sin^2(\theta/2)}$$

If collision Head on

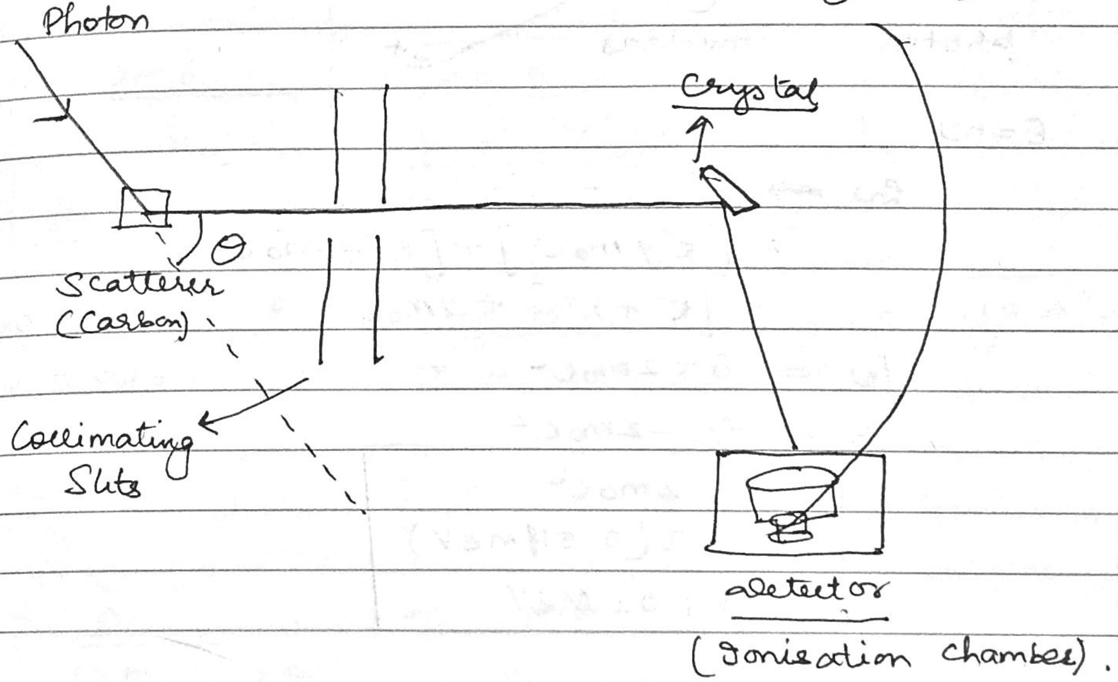
$\Delta\lambda = 2\lambda_C$

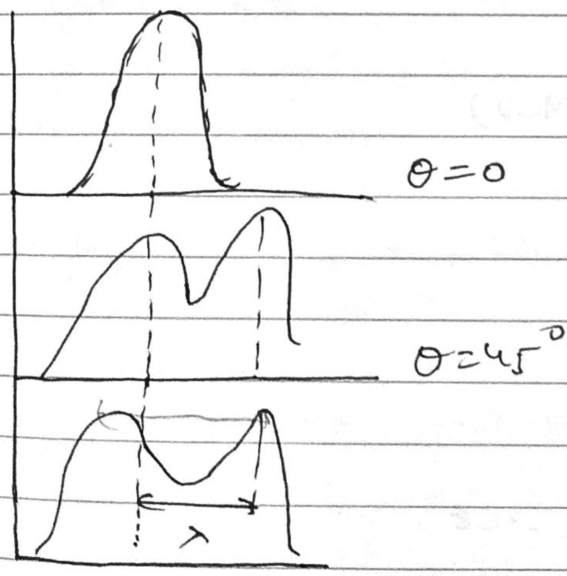
$= 0.048 A^\circ$

$\simeq 0.05 A^\circ$



$\boxed{2d\sin\theta = n\lambda}$

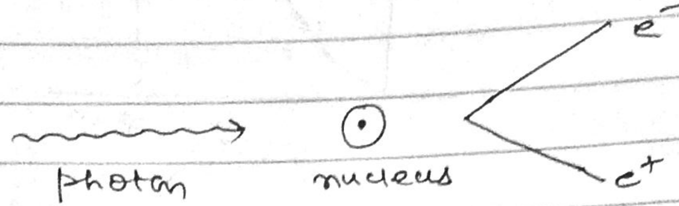




Now For High Energy:

→ **PAIR PRODUCTION & PAIR ANNIHLATION**

$$\boxed{I = I_0 \exp(-\mu t)}$$



$E = h\nu$

$h\nu \rightarrow [E^-] + [E^+]$

$= [K^- + m_0c^2] + [K^+ + m_0c^2]$

$= [K^- + K^+] + 2m_0c^2$

$h\nu = E + 2m_0c^2$

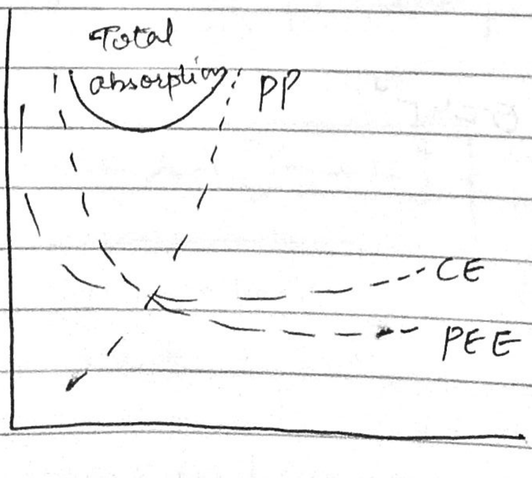
$E = h\nu - 2m_0c^2$

$$\boxed{\begin{aligned} h\nu &> 2m_0c^2 \\ &> 2(0.51\,\text{meV}) \\ &> 1.02\,\text{MeV} \end{aligned}}$$

$e^- + e^+ \longrightarrow 2\gamma$

$(0.51\ \text{MeV})$

$\lambda = 0.012\ \text{Å}$



De - Broglie's wave hypothesis

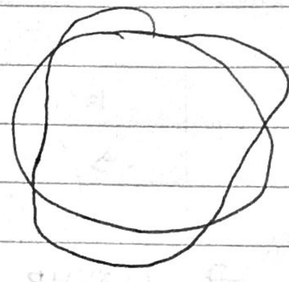
$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$= \frac{h}{\gamma m v}$$

$$2\pi r = n\lambda$$

$$= n\left(\frac{h}{mv}\right)$$

$$mvr = \frac{nh}{2\pi}$$



$E = mc^2$

$E = h\nu$

$p^2 = 2mE$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

$$eV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$$

also $\lambda = \frac{h}{mv} = \frac{h\sqrt{m}}{\cancel{m}\sqrt{2eV}} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}}\ A^\circ = \frac{12.26}{\sqrt{V}}\ A^\circ$

$= E = \frac{3}{2}KT$

$= \lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}\right)KT}} \Rightarrow \lambda = \frac{h}{\sqrt{3mKT}}$

→ MATTER WAVES.

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}} = \frac{12.26}{\sqrt{V}}\ A^\circ$$

$$= \frac{h}{\sqrt{3mKT}}$$

$y = A\sin(\omega t - kx)$

$\omega = 2\pi\nu$

$k = \frac{2\pi}{\lambda}$

$v_p = \left(\frac{\omega}{k}\right)$

$$\frac{2\pi\nu}{(2\pi/\lambda)} = \nu\lambda$$

$$= \left(\frac{mc^2}{\cancel{h}}\right)\left(\frac{\cancel{h}}{\cancel{m}v}\right) = \left(\frac{c^2}{v}\right).$$

$E = mc^2,\ E = h\nu$

$$\nu = \left(\frac{E}{h}\right) = \left(\frac{mc^2}{h}\right)$$

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$$d(\omega t - Kx) = 0$$

$$\omega(dt) - K(dx) = 0$$

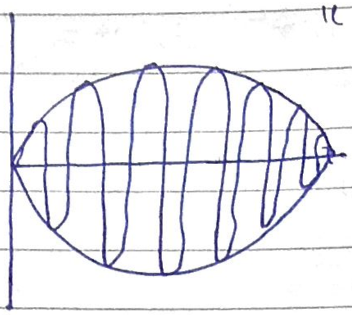
$$K(dx) = \omega(dt)$$

$$\Rightarrow \left(\frac{dx}{dt}\right) = \left(\frac{\omega}{K}\right)$$

→ Group velocity (v_g)(G)

"BEATS"

wave packet



$v_g = v$

The velocity with which the wave packet is advancing in a forward direction is ~~wave packet~~. Group velocity

→ $v_p = \left(\frac{\omega}{K}\right)$, $v_p = \frac{h}{\sqrt{2m\lambda}}$

⇒ Relation b/w phase velocity (v_p) & Group velocity (v_g)

$$v_p = \frac{\omega}{K}, \quad v_g = \left(\frac{d\omega}{dK}\right) \quad ; \quad v_g = d(uK)$$

$$v_g = \frac{d}{dK}(uK)$$

$$= u + K\left(\frac{du}{dK}\right)$$

$$= u + \left(\frac{K}{dK}\right)du$$

$$K = \frac{2\pi}{\lambda}$$

$$dK = -\frac{2\pi}{\lambda^2} d\lambda.$$

$$= \frac{d}{dK}\left[\frac{2\pi \nu}{2\pi/\lambda}\right]$$

$$v_g = \frac{d}{dK}[\nu\lambda] = \nu\left(\frac{d\lambda}{dK}\right) + \lambda\left(\frac{d\nu}{dK}\right)$$

$$= u - \frac{\lambda}{d\lambda} du$$

$$\therefore \frac{K}{dK} = \frac{-\lambda}{d\lambda}$$

$u = \nu\lambda$

$$\Rightarrow \boxed{G = u - \lambda\left(\frac{du}{d\lambda}\right)}$$

$G \rightarrow v_g$, $\quad u \rightarrow v_p$

$$y_1 = a\sin[\omega_1 t - k_1 x]$$
$$y_2 = a\sin[\omega_2 t - k_2 x]$$
$$y = y_1 + y_2$$
$$= a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$
$$= 2a\left[\left\{\sin\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right\}\left\{\cos\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right\}\right]$$

$\omega_1 = \omega_2 = \omega \qquad \omega_1 - \omega_2 = \Delta\omega$

$k_1 = k_2 = k \qquad k_1 - k_2 = \Delta k$

$$y = 2a\left[\sin(\omega t - kx)\cos\left\{\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x\right\}\right]$$

$$\boxed{y = 2a\cos\left\{\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x\right\}.\sin(\omega t - kx)}$$

$\rightarrow \quad G = \frac{\Delta\omega}{\Delta k} = \frac{\partial\omega}{\partial k}$

$$= \frac{\partial(2\pi\nu)}{\partial(2\pi/\lambda)} = \frac{\partial\nu}{\partial\left(\frac{1}{\lambda}\right)}$$

$$G = -\lambda^2\left(\frac{d\nu}{d\lambda}\right)$$

$\rightarrow \quad G = \lambda^2\left[\frac{u}{\lambda^2} - \frac{1}{\lambda}\left(\frac{du}{d\lambda}\right)\right]$

$$= -\lambda^2\left[\frac{d}{d\lambda}\left(\frac{u}{\lambda}\right)\right]$$

$$G = -\lambda^2\left[\frac{d\nu}{d\lambda}\right] \longrightarrow$$

$$G = -\lambda^2\left(\frac{d\nu}{d\lambda}\right)$$

$$\frac{1}{G} = -\frac{1}{\lambda^2}\left(\frac{d\lambda}{d\nu}\right)$$

Tushy

$\rightarrow \quad E = V + \frac{1}{2} m v^2$

$E - V = \frac{1}{2} m v^2$

$\Rightarrow v = \sqrt{\frac{2(E-V)}{m}}$

& $\lambda = \frac{h}{mv} \Rightarrow \frac{1}{\lambda} = \frac{mv}{h}$ & $v = \frac{h}{\lambda m}$

Substitute the value for V

$\Rightarrow \frac{1}{\lambda} = \frac{\sqrt{m}}{h} \sqrt{\frac{2(E-V)}{m}}$

$= \frac{1}{h}\sqrt{2m(E-V)} = \frac{1}{h}\sqrt{2m(h\nu - V)}$

$\frac{1}{G} = \frac{1}{\partial \nu}\left[d\left\{\frac{1}{h}\sqrt{2m(h\nu - \nu}\right\}\right]$

$= \frac{1}{\cancel{h}} \times \frac{1}{\cancel{2}} \left\{ \{2m(h\nu - \nu)\right\}^{-1/2} (\cancel{2}m\cancel{h})$

$\frac{1}{G} = \frac{1}{[2 \cancel{m} (E-V)]^{1/2}} \sqrt{m}$ (here we made the numeritor's m $\sqrt{m}$ and cancelled the denominator m).

$\frac{1}{G} = \sqrt{\frac{m}{2(E-V)}} = \frac{1}{v}$.

$\boxed{G = v}$

Important derivation $\hbar = \frac{h}{2\pi}$

$$\boxed{(KE)_{max} = \frac{2E^2/E_0}{1 + 2E/E_0}}$$

Imp. derivation done in tut - 11.

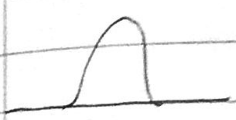
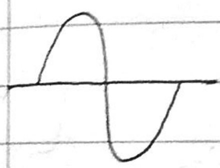
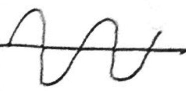
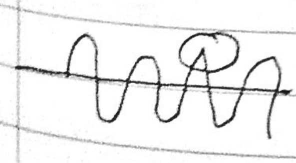
* Some Important derivations in lect-notes

→ HEISENBERG's uncertainity principle.

$$y = 2a\cos\left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x\right]\sin(\omega t - kx).$$

$\Delta p . \Delta x > \hbar \rightarrow \frac{h}{2\pi}$

$\Delta E . \Delta t \geq \frac{\hbar}{2}$

	P (Position)	M (momentum)
	very well defined	very poorly defined.
	well defined	poorly defined.
	less well defined	well defined.
	Poorly defined	very well defined.
	very poorly defined	very poorly defined.

$\cos\theta = 0.$

$\theta = 0$

$\Rightarrow \cos 0 = 1.$

$\theta = (2n+1)\pi/2$

$n=1 \quad \left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x_1\right] = (2n+1)\frac{\pi}{2}.$

$n = n+1 \quad \left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x_2\right] = \left[2(n+1)+1\right]\pi/2.$

$\Rightarrow \frac{\Delta k}{2}(x_1 - x_2) = (2n+3)\frac{\pi}{2} - (2n+1)\frac{\pi}{2}$

$= \pi.$

$\left(\frac{\Delta k}{2}\right)(\Delta x) = \pi$

$k = \frac{2\pi}{\lambda}$

$= \frac{2\pi}{h} p.$

$\therefore \Delta k = \left(\frac{2\pi}{h}\right)\Delta p$

$\left(\frac{2\pi}{h}\right)\Delta p . \Delta x = 2\pi$

$\therefore \Delta p . \Delta x = h$

$\Delta p . \Delta x \geq h$

$\geq \frac{h}{2}.$

→ WAVE FUNCTION : (ψ)

$\psi (x, y, z)$

$\psi (x, y, z, t)$

$|\psi|^2 = 1.$

→ Properties. characteristics physical significance } of a wave function.

→ $\iiint |\psi|^2 dv = 1$

→ Characteristics of a wave Function

→ Finite

Single valued

differentiable

continuous.

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Schroedinger's Time Independent equation

~~The Reed~~

$m \longrightarrow v \quad \lambda = \frac{h}{mv}$

$$\boxed{\left(\frac{\partial^2 y}{\partial t^2}\right) = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)}$$

$$\left(\frac{\partial^2 \psi}{\partial t^2}\right) = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$$

$$\left(\frac{\partial^2 \psi}{dt^2}\right) = v^2\left[\frac{d^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right] = v^2\left[\nabla^2\psi\right]$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \rightarrow \text{Laplacian operator}$$

$$\nabla = \bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}.$$

$$\psi = \psi_0 \sin(\omega t)$$

$$= \psi_0 \sin\left[(2\pi\nu)t\right]$$

$$\left(\frac{\partial\psi}{dt}\right) = \psi_0 \cos\left[(2\pi\nu)t\right](2\pi\nu)$$

$$\left(\frac{\partial^2\psi}{\partial t^2}\right) = -\psi_0 \sin\left[(2\pi\nu)t\right]\left[4\pi^2\nu^2\right)$$

$$= -4\pi^2\nu^2\psi.$$

$$\Rightarrow -4\pi^2\nu^2\psi = v^2\left[\nabla^2\psi\right]$$

$$-4\pi^2\left(\frac{\cancel{v^2}}{\lambda^2}\right)\psi = \cancel{v^2}\left[\nabla^2\psi\right]$$

$$\nabla^2\psi + \left(\frac{4\pi^2}{\lambda^2}\right)\psi = 0.$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0.$$

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$$E = V + \frac{1}{2} m v^2$$

$$\Rightarrow 2(E - V) = m v^2$$

$$2m[E - V] = m^2 v^2$$

$$\nabla^2 \psi + \frac{4\pi^2}{h^2}[2m(E-V)]\psi = 0.$$

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}[E - V]\psi = 0.$$ Time independent wave eq^n,

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2}[E - V]\psi = 0.}$$

$$\hbar^2 = \left(\frac{h}{2\pi}\right)^2$$

$$\hbar = \frac{h}{2\pi}$$

for a free particle, $V=0$.

$$\nabla^2 \psi + \frac{2m}{\hbar^2} E = 0.$$

Now $$\nabla^2 \psi = -\frac{2m}{\hbar^2}[E - V]\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi = E\psi$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

$$\boxed{\hat{H}\psi = E\psi}$$

$$\boxed{\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V}$$ HAMILTONIAN OPERATOR.

→ TIME DEPENDENT EQUATION.

$$\nabla^2 \psi + \frac{2m}{\hbar^2}[E - V]\psi = 0.$$ that's iota

$$\psi = \psi_0(x, y, z) \cdot e^{-i\omega t}$$

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$$\left(\frac{\partial\psi}{\partial t}\right) = \psi_0 (x, y, z)\, e^{-i\omega t} (-i\omega).$$

$$= \psi\left[-i(2\pi\nu)\right]$$

$E = h\nu$

$$\nu = \left(\frac{E}{h}\right) \Rightarrow \left(\frac{\partial\psi}{\partial t}\right) = (-1)\, i (2\pi) \left(\frac{E}{h}\right)\psi$$

$$= -i\frac{E}{\hbar}\psi$$

$$\Rightarrow -\frac{\hbar}{i}\left(\frac{\partial\psi}{\partial t}\right) = E\psi$$

$$(i^2)\frac{\hbar}{\cancel{i}}\left(\frac{\partial\psi}{\partial t}\right) = E\psi$$

$$i\hbar\left(\frac{\partial\psi}{\partial t}\right) = E\psi$$

in time independent wave eqn.

Substituting $\rightarrow \nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\left(\frac{\partial\psi}{\partial t}\right) - V\psi\right] = 0.$

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\left(\frac{\partial\psi}{\partial t}\right) - V\psi\right]$$

$$\Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t} - V\psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\boxed{\hat{H}\psi = \hat{E}\psi}$$

$\Rightarrow \hat{O}\,[f(x)] = g(x)$

operator — operand — New Function.

Eigen Function.

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ψ

Page No.:

T.I = $\hat{H}\psi = E\psi$

T.D. $\rightarrow$ $\hat{A}\psi = \hat{E}\psi$

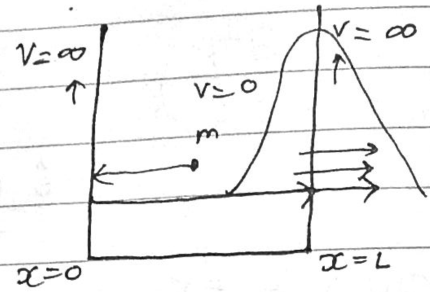
$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V \quad , \quad \hat{E} = i\hbar\frac{\partial}{\partial t}$$

$$\hat{O}[f(x)] = \lambda[g(x)]$$

FREE PARTICLE IN A BOX.

$V=0 \quad 0 \le x \le L$

$V=\infty \quad x<0 \;\&\; x>L.$



Boundary conditions

$\psi_n(x) = 0 \quad x=0$

$\psi_n(x) = 0 \quad x=L$

$$\frac{d^2\psi_n}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0.$$

$$\frac{\partial^2\psi_n}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0.$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

$$= \frac{2\pi\sqrt{2mE}}{h} = \frac{\sqrt{2mE}}{\hbar}$$

$\therefore E = p^2/2m$

$p = \sqrt{2mE}$

$$\psi(x) = A\sin kx + B\cos kx$$

$x=0 \;,\; \psi(x) = 0.$

$0 = A\sin k(0) + B\cos k(0)$

$= 0 + B$

$\boxed{B=0}$

$\psi(x) = A\sin kx$

$x=L,\; \psi(x) = 0$

$0 = A\sin k(L)$

$\boxed{A=0}$ $\times$ $A \neq 0.$

$\therefore \sin kL = 0$

$$k = \left(\frac{n\pi}{L}\right)$$

$$k^2 = \frac{n^2\pi^2}{L^2}, \cancel{n=L,\ \psi(x)=0}$$

$\cancel{0 = A}$ $\quad \dfrac{2mE}{\hbar^2} = \dfrac{n^2\pi^2}{L^2}$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad \therefore \hbar = \frac{h}{2\pi}.$$

$$\boxed{E_n = \frac{n^2 h^2}{8mL^2}}$$

$n = 1, 2, 3 \ldots$

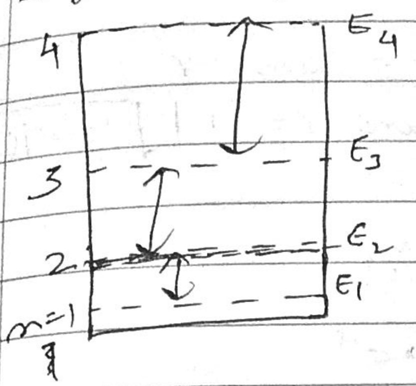
$n=1, \quad E_1 = \dfrac{h^2}{8mL^2}$

$=2 \quad E_2 = \dfrac{4h^2}{8mL^2} = 4E_1$

$=3, \quad E_3 = 9E_1$

$=4, \quad E_4 = 6E_1$

Eigen Energy Levels



value for n can never be zero!

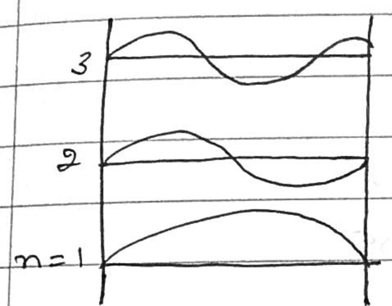
NORMALISATION

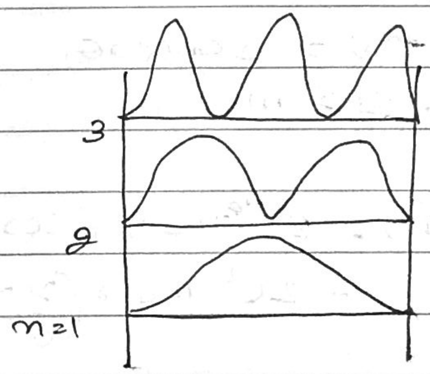
Now $\int_0^L \psi\psi^* = 1 \Rightarrow \int_0^L (A\sin kx)(A\sin kx)\,dx = 1$

$= \int_0^L A^2 \sin^2 kx\,dx = 1 \Rightarrow A^2 \int_0^L \left[\frac{1-\cos(2kx)}{2}\right] dx = 1$

After putting limits $\Rightarrow \frac{A^2}{2}[L] = 1 \Rightarrow A^2 = \frac{2}{L}$

$\therefore \boxed{A = \sqrt{\frac{2}{L}}}$ normalisation constant.





- Applications of Heisenberg uncertainity principle.

$e^- = 4\,MeV$
$p = 52\,MeV$

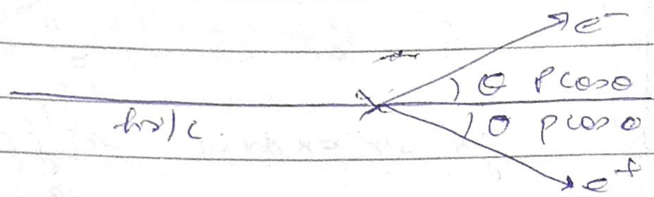
$P \cdot x = \hbar = \frac{h}{2\pi}$

$P = \frac{h}{2\pi x}$

→ To exist in a nucleus an e^- must have a K.E of 20 MeV or greater but experiment shows the energy is only 3 - 4 eV. Thus we can conclude that the electrons are not present in nucleus.

→ pair production cannot occur in free space

From conservation of energy $h\nu = 2m_0c^2\gamma$
here m_0 is the rest mass and $\gamma = 1/\sqrt{1-v^2/c^2}$.



In the direction of motion of photon, the momentum is conserved if

$$\frac{h\nu}{c} = 2p\cos\theta$$

$$h\nu = 2cp\cos\theta.$$

and $p = m_0 v\gamma$.

$$h\nu = 2c\, m_0 v\gamma \cos\theta$$
$$= 2c^2 m_0 \gamma \left(\frac{v}{c}\right) \cos\theta.$$

But $v/c < 1$ and $\cos\theta \leq 1$

$$h\nu < 2m_0c^2\gamma$$

But conservation of energy Required
$$h\nu = 2m_0c^2\gamma.$$

Hence it's Impossible for pair production to conserve both the E & P unless some other object is Involved in the process to carry away part of the initial photons momentum. Therefore pair production cannot occur in empty space.

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Imp

Q → Why the pair production cannot happen in free space?

Q → product of Group velocity & phase velocity

$$\boxed{v_p \cdot v_g = c^2}$$

2nd Important derivation

$$\boxed{\tan\phi = \frac{\cot(\theta/2)}{1+(h\nu/m_oc^2)}}$$

→ Taking momentums

- $m v c \cos\phi = h\nu - h\nu'\cos\theta$

$m v c \sin\phi = h\nu'\sin\theta$

$$\tan\phi = \frac{h\nu'\sin\theta}{h\nu - h\nu'\cos\theta} = \frac{\nu'\sin\theta}{\nu-\nu'\cos\theta} = \frac{\nu'\sin\theta}{\nu'\left(\frac{\nu}{\nu'}-\cos\theta\right)}$$

$$= \frac{\sin\theta}{\left(1+\left(\frac{h\nu}{m_oc^2}\right)[1-\cos\theta]\right\}-\cos\theta} = \frac{\sin\theta}{\left(1+\frac{h\nu}{m_oc^2}\right)-\frac{h\nu}{m_oc^2}(\cos\theta)-\cos\theta}$$

$$= \frac{\sin\theta}{[1+(h\nu/m_oc^2)]-\cos\theta\left(1+\frac{h\nu}{m_oc^2}\right)} = \frac{\sin\theta}{\left(1+\frac{h\nu}{m_oc^2}\right)(1-\cos\theta)}$$

$$= \frac{\sin\theta}{\left(1+\frac{h\nu}{m_oc^2}\right)\left\{1-\{1-2\sin^2\theta/2\}\right]} = \frac{2\sin\theta/2\cos\theta/2}{\left(1+\frac{h\nu}{m_oc^2}\right)(2\sin^2\theta/2)}$$

$$= \boxed{\tan\phi = \frac{\cot\theta/2}{1+(h\nu/m_oc^2)}}$$

3rd derivation → and also this can be written as

$$\phi = \tan^{-1}\left[\frac{\sin\theta}{\frac{\lambda'}{\lambda}-\cos\theta}\right]$$

$$\left[\begin{aligned} \lambda'-\lambda &= \frac{h}{m_oc}[1-\cos\theta] \\ \lambda' &= \lambda+\frac{h}{m_oc}[1-\cos\theta] \\ \frac{\lambda'}{\lambda} &= 1+\frac{h}{m_oc\lambda}(1-\cos\theta) \end{aligned}\right.$$

how / see this

$$\tan\phi = \frac{\nu'\sin\theta}{\nu-\nu'\cos\theta} = \frac{(c/\lambda')\sin\theta}{\frac{c}{\lambda}-\frac{c}{\lambda'}\cos\theta} = \frac{\left(\frac{1}{\lambda'}\right)\sin\theta}{\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\theta\right)}$$

$$= \frac{\frac{1}{\lambda'}\sin\theta}{\frac{1}{\lambda'}\left[\frac{\lambda'}{\lambda}-\cos\theta\right]}$$

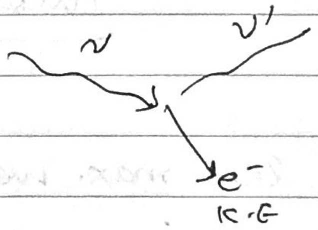
$$\boxed{\tan\phi = \frac{\sin\theta}{\frac{\lambda'}{\lambda}-\cos\theta}}$$

$$\frac{1}{\nu'} = \frac{1 + \left(\frac{h\nu}{m_0c^2}\right)(1-\cos\theta)}{\nu}$$

$$\Rightarrow \nu' = \frac{\nu}{1 + \frac{h\nu}{m_0c^2}(1-\cos\theta]}$$

$$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{m_0c^2}\left[1 - [1 - 2\sin^2(\theta/2)\}\right]$$

$$= \left[1 + \frac{2h\nu}{m_0c^2}\{\sin^2(\theta/2)\}\right]$$



$$K.E = \underline{h\nu - h\nu'}$$

$$K.E = h\nu\left(1 - \frac{\nu'}{\nu}\right) \quad —(1)$$

$$\frac{\nu}{\nu'} = 1 + \frac{2h\nu}{m_0c^2}\sin^2\frac{\theta}{2} \quad —(2)$$

Put (2) in (1)

$$K.E = h\nu\left[1 - \frac{1}{1 + \frac{2h\nu}{m_0c^2}\sin^2\frac{\theta}{2}}\right] = h\nu\left[\frac{\cancel{1} + \frac{2h\nu}{m_0c^2}\sin^2\frac{\theta}{2} - \cancel{1}}{1 + \frac{2h\nu}{m_0c^2}\sin^2\frac{\theta}{2}}\right]$$

$K.E \longrightarrow (KE)_{max}$ when $\theta = 180^\circ$, $\sin\frac{\theta}{2} \rightarrow 1$

$$(KE)_{max} = h\nu\left[\frac{\frac{2h\nu \cdot 1}{m_0c^2}}{1 + \frac{2h\nu}{m_0c^2}}\right]$$

$\Rightarrow E = h\nu$ = Energy of incident Photon

$E_0 = m_0c^2$ = rest mass energy of e^-

$$\boxed{[KE]_{max} = \frac{2E^2/E_0}{1 + 2E/E_0}}$$

6.63 ×10

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&

Sol:- here $E = 500\ keV$

$= 0.5\ MeV$

$E = mc^2$

$0.5 = m \times 3\times10^8$

$m = \frac{0.5\times10^{-8}}{3}$

$= 0.17\times10^{-8}$

Tutorial - 11

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Q Find the max. wavelength than can liberate electron from potassium. work function of pottasium is 2.24 eV

Sol: $\lambda_0 = \frac{12400}{W_0}\ A^0$

$= \frac{12400}{2.24}$

$= 5535.7\ A^0$

Q A photon of Energy E is scattered by an electron initially at rest (rest mass Energy E_0) Show that the max. kinetic energy of the recoil electrons can be calculated as $\boxed{(KE)_{max} = \frac{2E^2/E_0}{1+2E/E_0}}$

→ ~~Derivation in~~ fair class notes.

$\lambda' - \lambda = \lambda_c[1-\cos\theta]$

$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}[1-\cos\theta] \Rightarrow \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}[1-\cos\theta]$

$\therefore \frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0c^2}[1-\cos\theta]$

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Q. The density of Gold 19.3×10^3 kg/m³ in a frame S which is at Rest, calculate it's density as an Observer in Frame S' would determine if Frame S' is moving with a velocity of 0.9c along x-axis

Sol: $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ & $d = \left(\dfrac{m}{v}\right)$

$d_0 = \dfrac{m_0}{x_0 y_0 z_0}$ and $d = \dfrac{m}{xyz}$

Now $d = \dfrac{m}{xyz}$

$d = \dfrac{m_0}{\left[x_0\sqrt{1-v^2/c^2}\right] y_0 z_0 \sqrt{1-v^2/c^2}} = \dfrac{m_0}{x_0 y_0 z_0 (1-v^2/c^2)}$

$d = \dfrac{d_0}{1-v^2/c^2} \Rightarrow \boxed{\dfrac{d}{d_0} = \dfrac{1}{1-v^2/c^2}}$

$d = \dfrac{19.3 \times 10^3}{1 - \dfrac{0.9 \times 0.9 \times \cancel{c^2}}{\cancel{c^2}}}$

$= \dfrac{19.3 \times 10^3}{1 - 0.81} = \dfrac{19.3 \times 10^3}{0.19} = 101.5 \times 10^3$

$= 10.1 \times 10^4$

Q. Calculate the Relativistic momentum in units of MeV/c having a kinetic energy of 500 keV.

Sol: $\boxed{p = \dfrac{1}{c}\sqrt{K.E\,(K.E + m_0c^2)}}$

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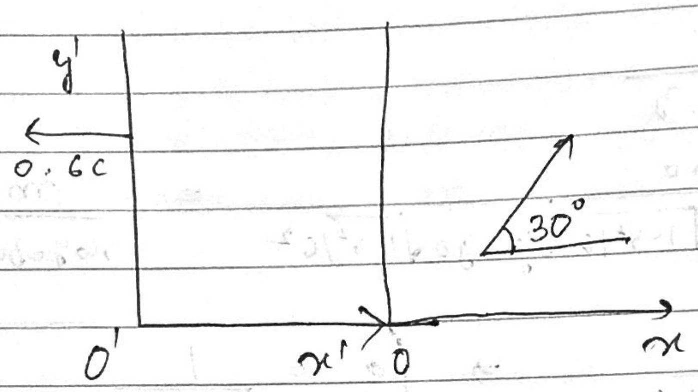
Tutorial - 10

Q. A particle moves with $v = 0.8c$ at angle of 30° to x-axis as determined by O. What is velocity of particle as determined by O' moving with $-0.6c$ along $x - x'$ axis.

Solution →

$$u = \frac{u' + v}{1 + u'v/c^2} \quad , \quad u' = \frac{u - v}{1 - uv/c^2}$$



$$u_x = 0.8c \cos 30^\circ = 0.693c$$
$$u_y = 0.8c \sin 30^\circ = 0.4c.$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0.693c - (-0.6c)}{1 + \frac{0.6c}{c^2} \times 0.693c}$$

$$= \frac{1.293c}{1.4158} = 0.913c.$$

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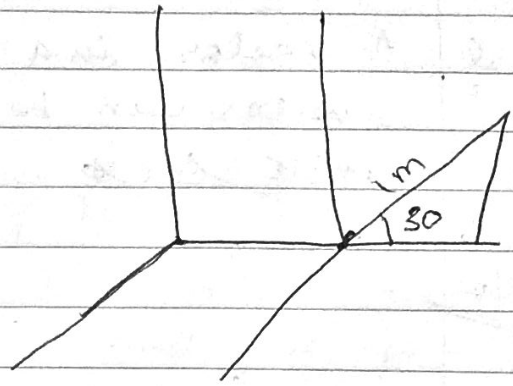
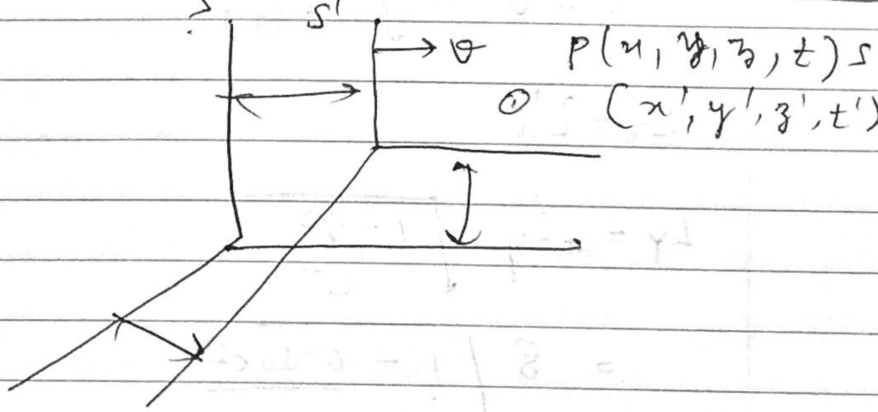
Sol^n: $L' = L'_x \hat{i} + L'_y \hat{j}$

$L'_x = 1 \cdot \cos 30^\circ$

$L'_y = 1 \cdot \sin 30^\circ = \frac{1}{2} m.$





$x' = x - vt$

$u'_x = u_x - v$

$a'_x = a_x$

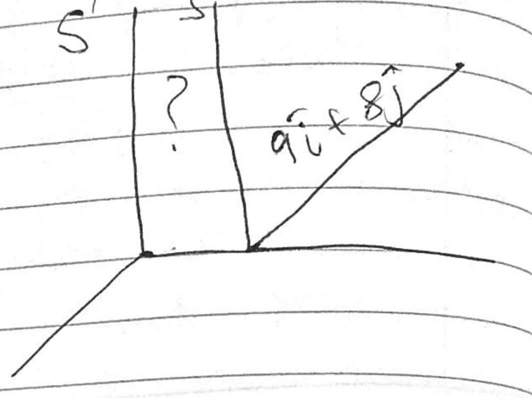
$x' = x - v_x t$ $dx' = dx - v_x dt$ $\frac{dx'}{dt'} = \frac{dx}{dt} - v_x$ $u'_x = (u_x - v_x)$	$y' = y - v_y t$ $u'_y = u_y - v_y$.	$z' = z - v_z t$ $u'_z = (u_z - v_y)$.

u'_x

Q2 A vector in a Frame ~~S~~ S' $9\bar{i}+8\bar{j}$ How this vector can be represented in a Frame ~~S~~ S

$v = 0.6c\,\bar{j}$

~~while in C~~



Solve

$L = L_x\bar{i} + L_y\bar{j} + L_z\bar{k}$ — in S

$L' = L'_x\bar{i} + L'_y\bar{j} + L'_z\bar{k}$ — in S'

$v = 0.6c\,\bar{j} \rightarrow$ along Y

$L_x = L'_x$

$L_Y = L'_y\sqrt{1-\frac{v^2}{c^2}}$

$= 8\sqrt{1-\frac{0.36\cancel{c^2}}{\cancel{c^2}}}$

$= 8 \times 0.8$

$= 6.4$.

The vector in S would be $\rightarrow 9\hat{i} + 6.4\hat{j}$

Q An observer O' holds ~~a~~ 1m stick at an angle 30° w.r.t the positive ~~x-axis~~ x-axis. O' is moving in the +ve xx' direction with a velocity of $0.8C$ w.r.t the observer O. What are the length and the angle of the stick as measured by observer O.

28/march/18

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Tutorial - 8.

Q2 As determined by O' lighting bolt strikes at $x' = 60\,m$, $y' = z' = 0$, $t' = 8\times10^{-8}\,s$. O' has $v = 0.6c$ along x axis. What are space-time co-ordinates of strikes as determined by O.

$$x = \frac{x' + vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$x' = 60\,m$

$y' = z' = 0$

$t' = 8\times10^{-8}\,sec.$

$$x = \frac{60 + 0.6\times3\times10^{8}\times8\times10^{-8}}{0.8}$$

$$= \cancel{\frac{60 + 1.8\times10^{8}}{0.8}} = \frac{60 + 0.6\times3\times8}{0.8}$$

$$= \frac{60 + 14.4}{0.8}$$

$$= \underline{\underline{90}}$$

also find other parameters.

Page No.:

Q2 A Conducting circular loop of radius 20 Cms lies in the $Z=0$ plane in a Magnetic field
$\bar{B} = 10 \cos 377t \ \bar{a}_z$ mWb/m²
Calculate the induced voltage in the loop.

$$V_{emf} = -\left(\frac{d\psi}{dt}\right)$$

$$= -\frac{d}{dt}\int_s \bar{B}\cdot\bar{ds}$$

$$= -\frac{dB}{dt}\int_s ds.$$

$$= -\frac{dB}{dt}(s)$$

$$= -\frac{dB}{dt}(\pi r^2)$$

Now $\bar{B} = 10\cos 377t \ \bar{a}_z$
$r = 20$ cm $= 0.2$ m.

$$= -\frac{d}{dt} 10\cos 377t \times \pi r^2$$

$$= \cancel{10 \sin 377t \times 0.628}.$$

$$= 0.4738 \sin(377t) \text{ volts}.$$

I

(ii)

(iii)

$(5, 2, -3)$ $\quad 40 \times 4 = 160 \quad 80 \times 8 \quad 160+50+8 \quad 25\times 2$

$= (8x^2y + y^2x)\,\bar{a}_x - (8xy^2 + x^2y + y^3)\,\bar{a}_y + (y^2z + zx^2 + 3y^2)\,\bar{a}_z$

$= \cancel{(8\times 25\times 2}$

$= (400+20)\,\bar{a}_x - (202)\,\bar{a}_y - (-75)\,\bar{a}_z$

$\bar{J} = 420\,\bar{a}_x - 202\,\bar{a}_y + 125\,\bar{a}_z$

(ii) yz plane

$dS = dy\,dz$

$\bar{dS} = dy\,dz\,\bar{a}_x$

Can I please teach You physics?
I don't want you to fail

$I = \iint \bar{J} \cdot \bar{dS}$

$= \iint (8x^2y + xy^2)\,\bar{a}_x \cdot dy\,dz\,\bar{a}_x$

$= \int_0^2 \int_0^2 \left\{ (8x^2y + xy^2)\,dy\,dz \right\}_{x=-1}$

Solve it

$x^2yz + y^3z$

(iii) $\nabla \cdot \bar{B} = \mu [\nabla \cdot \bar{H}]$

$2xyz$

$\frac{\partial}{\partial x} yz(x^2+y^2) - \frac{\partial}{\partial y}[y^2xz] + \frac{\partial}{\partial z}(4x^2y^2)$

$= 2xyz - 2xyz + 0$

$= 0.$

Fail

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$\frac{\partial}{\partial y} xy = x$.

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Tutorial - 7

Q2 In a certain conducting Region $\bar{H} = yz(x^2+y^2)\bar{a}_x - y^2xz\,\bar{a}_y + 4x^2y^2\bar{a}_z$ A/m

(i) determine $\bar{J}$ at $(5, 2, -3)$

(ii) find current passing through $x=-1,\ 0<y,\ z<2$.

(iii) show that $\nabla\cdot\bar{B} = 0$

Ans. $\nabla\times\bar{H} = \bar{J}$

$\bar{J} = \cancel{\frac{\partial}{\partial x} yz(x^2+y^2)} - \cancel{\frac{\partial}{\partial y} y^2xz}$

$$\nabla\times\bar{H} \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz(x^2+y^2) & -y^2xz & 4x^2y^2 \end{vmatrix}$$

$x^2 y z + y^3 z$

$$= \left[\frac{\partial}{\partial y}4x^2y^2 - \frac{\partial}{\partial z}(-y^2xz)\right]\bar{a}_x + \left[\frac{\partial}{\partial x}4x^2y^2 - \frac{\partial}{\partial z}yz(x^2+y^2)\right] + \left[\frac{\partial}{\partial x}(-y^2xz) - \frac{\partial}{\partial y}(yz(x^2+y^2))\right]\bar{a}_z$$

$$= \cancel{[8yx^2 + 1]\bar{a}_x + [8x - 2]\bar{a}_y + [x - 1 + 2y]\bar{a}_z}$$

$$= (8x^2y + y^2x)\,\bar{a}_x - (8xy^2 + x^2y + y^3)\,\bar{a}_y - (y^2z + zx^2 + 3y^2)\,\bar{a}_z$$

W

Page No.:

Q 2 $\bar{T} = \frac{1}{r^2} \cos\theta \, \bar{a}_r + r\sin\theta\cos\phi \, \bar{a}_\theta + \cos\theta \, \bar{a}_\phi$

$\nabla \cdot T = ?$

$$\nabla \cdot T = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(A_\phi)$$

Q 2 $\bar{A} = y\bar{a}_x + (x+z)\bar{a}_y$

$$= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} y\sin\theta\cos\phi + (x+z)\sin\theta\sin\phi \\ y\cos\theta\cos\phi + (x+z)\cos\theta\sin\phi \\ -y\sin\phi + (x+z)\cos\phi \end{bmatrix}$$

=

Q 2 $B = r^2 a_r + \sin\theta a_\phi$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} =$$

③

$P = \sqrt{x^2+y^2}$
$\phi = \tan^{-1} y/x$
$z = z$

$\overline{ds} = ds\,\bar{a}_n$ → surface area
↓ surface area vector

Q Convert the vector

$$\bar{C} = \boxed{z\sin\phi}\,\bar{a}_\rho - \boxed{\rho\cos\phi}\,\bar{a}_\phi + \boxed{2\rho z}\,\bar{a}_z$$
$$A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z$$

Sol:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ +\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \begin{matrix} = z\sin\phi \\ = -\rho\cos\phi \\ = 2\rho z \end{matrix}$$

~~Kno~~ Basing on
$x = \rho\cos\phi$
$y = \rho\sin\phi$
$z = z$

$$\boxed{\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z}$$

Solving the matrix

$$\begin{bmatrix} z\cos\phi\sin\phi + \rho\sin\phi\cos\phi + 2\rho z \\ z\sin^2\phi - \rho\cos^2\phi \\ 2\rho z \end{bmatrix} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$A_x =$

~~$\bar{C} = z\sin\phi\,\bar{a}_x - \rho\cos\phi\,\bar{a}_y + 2\rho z\,\bar{a}_y$~~

$$\bar{C} = (z\sin\phi\cos\phi + \rho\cos\phi\sin\phi + 2\rho z)\,\bar{a}_x + (z\sin^2\phi - \rho\cos^2\phi)\bar{a}_y + (2\rho z)\,\bar{a}_z$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

z
x
z

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Tutorial → 5

E.M.T

Q.Q.

$\rho = 3$ | $\rho = 5$
$\phi = 100$ | $\phi = 130$
$z = 3$ | $z = 4.5$

1. enclosed volume
2. Total area.

$$\bar{ds} = (d\rho)(\rho d\phi)\ \bar{a}_z$$
$$(\rho d\phi)(dz)\ \bar{a}_\rho$$
$$(dz)(d\rho)\ \bar{a}_\phi$$

$$V = \int dV = \int_3^{4.5}\int_{100}^{130}\int_3^{5} (d\rho)(\rho d\phi)(dz)$$

$$S = \int ds = \int_{100}^{130}\int_3^{5} 2(d\rho)\rho d\phi + \int_3^{4.5}\int_{100}^{130} 2(\rho d\phi)\, dz.$$

$$+ \int_3^{4.5}\int_3^{5} 2(d\rho)\, dz.$$

Q. Calculate the core diameter necessary for single mode operation at 850 nm in a step index fibre with $n_1 = 1.480$ & $n_2 = 1.47$. ~~calculate the core~~ also calculate the numerical aperture, the max. angle of the fibre.

Sol:- For single mode $\boxed{V = 2.405}$

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}.$$

~~V = 3.14 ×~~.

$$d = \frac{V\lambda}{\pi\sqrt{n_1^2 - n_2^2}}.$$

$$d = \frac{2.405 \times 850 \times 10^{-9}}{\pi \times \sqrt{(1.480)^2 - (1.47)^2}}$$

$$= \frac{2.405 \times 850 \times 10^{-9}}{\pi \times 0.171}$$

$$= \frac{2044.2}{0.537} \times 10^{-9}$$

$$= 3.8 \times 10^{-7}.$$

$$= 3.8 \, \mu m.$$

Q. Determine the Temperature at which the Rate of Spontaneous & the stimulated emissions are equal for a wavelength of 5000 Å.

Tutorial - 4

Q. A step index fibre has $n_1 = 1.466$ & $n_2 = 1.460$. If the $\lambda = 0.85\ \mu m$ is propagate through the fibre of core $d = 50\ \mu m$. Find normalised freq & no. of modes.

Solving :-

$n_1 = 1.466$

$n_2 = 1.460$

$\lambda = 0.85\ \mu m$

$d = 50\ \mu m$.

$$V = \frac{\pi d}{\lambda} [NA]$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(1.466)^2 - (1.460)^2}$$

$$= \sqrt{2.094}$$

$$V = \frac{\pi \times 50 \times 0.13 \times \cancel{10^{-6}}}{0.85 \times \cancel{10^{-6}}}$$

$$\boxed{V = 24.48}$$

$$\text{no. of modes} = \frac{V^2}{2} = \frac{(24.48)^2}{2} = 299.6 \approx 300.$$

Now here.

$$\alpha = \frac{10}{L} \log\left(\frac{P_i}{P_o}\right)$$

$$\alpha = \frac{10}{1Km} \log\left(\frac{P_i}{P_o}\right)$$

$$3 = 10 \log\left(\frac{P_i}{P_o}\right)$$

$$\log\left(\frac{P_i}{P_o}\right) = 0.3$$

$$\boxed{\frac{P_i}{P_o} = 2}$$

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$\Delta = \cancel{\frac{n_1 - n_2}{n_1}}$ here $\Delta = 0.02$

$NA = 0.25$

$NA = n_1\sqrt{2\Delta}$

$0.25 = n_1 \times \sqrt{2 \times 0.02}$

$$\boxed{n_1 = \frac{0.25}{0.2} = 1.25}$$

$\frac{n_1 - n_2}{n_1} = 0.02$

$1.25 - n_2 = 0.02 \times 1.25$

$$\boxed{\begin{aligned} n_2 &= 1.25 - 0.025 \\ &= 1.225 \end{aligned}}$$

Q) The signal Retrieved in an optical fibre Reduces at it's output to one ~~length~~ 10th of it's Input power, what is the overall attenuation. what shall be the power at the output in a fibre having a loss of 3db/km.

Ans

$P_o = \frac{P_i}{10}$ $\alpha = 3db/km.$

$\alpha = \frac{10}{L} \log\left(\frac{P_i}{P_o}\right) db/km$

$3 = \frac{10}{L} \log \frac{10 P_i}{P_i}$

$3 = \frac{10}{L}$

$$\boxed{L = 3.33}$$

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from Tut sheet

Q. 2. ~~QA~~ Calc. the numerical aperture of an O.F where core & cladding are made up of material of R.I = 1.6 & 1.5.

$n_1 = 1.6$

$n_2 = 1.5$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{2.56 - 2.25}$$

$$= \sqrt{0.31} = 0.556$$

Q.3. ~~NA @ 0.25~~

The Numerical aperture of Fibre is 0.25 and refractive ~~angle~~ index is 0.02. determine the Refractive Indices of the core & cladding of a fiber

$$NA = n_1^2 - n_2^2 \qquad \Delta = 0.02$$

$$(0.25)^2 = (n_1 - n_2)(n_1 + n_2)$$

$$0.0625 = (n_1 - n_2)(0.02)$$

$$\frac{0.0625}{0.02} = n_1 - n_2$$

$$n_1 - n_2 = 1.5625$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$0.02 = \frac{1.5625}{n_1}$$

$$n_1 = \frac{1.5625}{0.02}$$

(iii) Acceptance Angle :

$= \sin^{-1} NA$

$= \cancel{\sin^{-1} 0.1964}$

$= \cancel{10.16}$. 24.83

(iv) $\phi c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$= \sin^{-1}\left(\frac{1.44}{1.50}\right)$

$= \cancel{10.16}$

$\boxed{= 73.73}$

Q.1 A step index fibre has a numerical aperture of 0.26 ; core refractive index of 1.5 and a core of d of 100 micrometer. Calc. r.I. of cladding

Ans. $NA = 0.26$
$n_1 = 1.5$
$d = 100$.

$NA = \sqrt{n_1^2 - n_2^2}$

$0.26 = \sqrt{(1.5)^2 - n_2^2}$

$0.0676 = (1.5)^2 - n_2^2$

$0.0676 - 2.25 = -n_2^2$

$n_2^2 = 2.1824$

$n_2 = \sqrt{2.18}$

$n_2 = 1.4776$

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Tutorial - 3

Q. 5 → The Refractive index of the core is 1.50 & the fractional change in the Refractive index is 4%.
Estimate → (i) R.I of cladding
(ii) Numerical Aperture
(iii) Acceptance angle in A
(iv) The critical angle at core-cladding interface

Sol:

$NA = \sqrt{n_1^2 - n_2^2}$

$\sin i_0 = \sqrt{n_1^2 - n_2^2}$

$\Delta = \left(\frac{n_1 - n_2}{n_1}\right)$ $\quad\quad \Delta = 4\%$

$= 0.04$

$NA = n_1 \sqrt{2\Delta}$ $\quad\quad n_1 = 1.50$

$\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

→ $NA = 1.50 \times \sqrt{1.414 \times 0.04}$ $= \boxed{0.1764}$ $= \underline{0.42}$

$\boxed{= \cancel{0.084}}$

→ $\Delta = \left(\frac{n_1 - n_2}{n_1}\right)$

$0.04 \times 1.50 = 1.50 - n_2$

$0.06 = 1.50 - n_2$

$\boxed{n_2 = 1.44}$

$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{5890\times10^{-10}} = 5.09\times10^{14}$

Now $N_E = \nu \times T_c = 5.09\times10^{14} \times 9.81\times10^{-11}$

$= \cancel{49932.9}$

$= 50016.9$

$= 50017$

no. of oscillations $= \underline{5\times10^{4}}$

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$$S_1 = \frac{1}{\exp\left(\frac{h\nu}{K_B T}\right) - 1}$$

$$= \frac{1}{\exp\left(\frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 573 \times 5893 \times 10^{-10}}\right) - 1}$$

$$= \frac{1}{e(\cancel{0.025} \times 10} = \underline{2.9 \times 10^{-19}}$$

$$= \frac{1}{e^{(42.6)\cancel{\times 10}} - 1} = \frac{1}{3.43} = 2.93 \times 10^{-19}$$

Q.4. The coherence length of sodium light is 2.945×10^{-2} m and it's wavelength is 5890 Å.
Calculate (i) number of oscillations corresponding to coherence length and (ii) the coherence time.

Sol: $l_c = 2.943 \times 10^{-2}$ m

5,890 Å

1. $N_k = \frac{l_c}{\lambda} = \frac{2.945 \times 10^{-2}}{5,890 \times 10^{-10}} = 50,000 = 5 \times 10^{4}$

2. $T_c =$

$l_c = c T_c$

$$T_c = \frac{l_c}{c} = \frac{2.943 \times 10^{-2}}{3 \times 10^{8}} = 9.81 \times 10^{-11}$$

1. $N_k = \nu \times T_c = \cancel{9.81 \times 10^{-11}}$

Tutorial 1 is in Regular notes.

Tutorial - 2

Q2 Calculate the population ratio of two states in He-Ne laser that produces light of wavelength 6000A° at 300 K.

sol:- $N_r = N_0 \, Exp\left(\frac{-E_r}{KT}\right)$

Boltzman constant = $K = 1.38 \times 10^{-23} J$

$\lambda = 6000A° = 6000 \times 10^{-10}$

$T = 300 K.$

$N_r = $ $E_r = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{6000 \times 10^{-10}}$

$= \frac{6.63 \times 10^{-26+10}}{6 \times 10^{3}}$

$= 6.63 \times 10^{-16-3} / 6$

$= \frac{6.63 \times 10^{-19}}{6}$.

$\frac{N_r}{N_0} = e^{\left(\frac{6.63 \times 10^{-19}}{6 \times 1.38 \times 10^{-23} \times 300}\right)}$

$= e^{-80}$

Q2 Calculate the Rate of the stimulated to spontaneous emission at a temp. of 300°C for sodium D line.

Sol:- ~~x for~~ $\nu_1(\lambda) = 5890$ $\nu_2(\lambda) = 5896$

Sodium D line $= \lambda = 5893 A°$