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Numerical Methods

Solution of Algebraic and transcendental equation

$$f_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \text{——} a_n$$

$$x^4 + 3x^3 - 2x^2 + 5 = 0$$] Algebraic

$$x^2 + 2x\cos x - \log x = 0$$] transcendental

Solving the Algebraic and transcendental equation

(1) Bisection Method

$$f(x) = x^3 - x - 1 = 0$$

$$f(0) = -1 = -ive$$

$$f(1) = 1-1-1 = -ive$$

$$f(2) = 5 = +ve$$

$$f(a) = +ive$$

$$f(b) = -ive$$

root lies between a & b

Ist approximation

$$x_0 = \frac{a+b}{2}$$

$f(x_0) = +ive$ (let)

root lies between x_0 and b

let $f(x_0) = -ive$

root lies between x_0 and a

IInd approx.

$$x_1 = \frac{a+x_0}{2}$$

$f(x_1) = +ive$

x_1 and x_0

ations

between

IV app.

$$x_5 = \frac{1.324775 + 1.3246825}{2}$$

$$x_5 = 1.32421825$$

$f(x_5) = +ive$

app root $\approx$ 1.3247

Ques Find approximate value of the root of $x - \cos x = 0$ by bisection method, correct upto four decimal places, between 0 and 1.

(Ans. 0.7391)

~~Let $f(x) = x - \cos x$~~

~~$f(0) = 0 - \cos 0 = -1$~~

Q Find the real root of the equation $x \log_{10} x = 1.2$ by bisection method, correct upto four ~~deci~~ decimal places.

(Ans. 2.7407)

Q Use bisection method to find out the the square root of 30. Correct to four ~~de~~ decimal place

(Ans. 5.4771)

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- l defined as

Find ~~the~~ the approximate value of the root of the equation $3x - \sqrt{1+\sin x}$ by bisection method

(Ans -0.39188)

2.7

$f(x) = x\log_{10}x - 1.2$

$f(1) = 1\log_{10}1 - 1.2$
$= -1.2 = -ive$

$f(2) = 2\log_{10}2 - 1.2$
$= 0.6020 - 1.2$
$= -ive$

$f(3) = 3 \times 0.4771 - 1.2$
$= 1.431 - 1.2$
$= +ive$

Interval 2 and 3

$f(2.1) = -ive$
$f(2.2) = -ive$
$f(2.3) = -ive$
$f(2.4) = -ive$
$f(2.5) = -ive$
$f(2.6) = -ive$
$f(2.8) = +ive$

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2.7 and 2.8

Iterated:

$f(2.71) =$ ~~+ve~~ -ve

$f(2.72) =$ -ve

$f(2.73) =$ -ve

$f(2.74) =$ -ve

$f(2.75) =$ +ve

-ve

+ve

Iterval: 2.74 and 2.75

~~$f(2.743) =$ +ve~~

~~$f(2.744) =$ +ve~~

~~$f(2.745) =$ +ve~~

~~$f(2.$~~

~~(2.742)~~

$$x_0 = \frac{2.74 + 2.75}{2}$$

$$x_0 = 2.745$$

$$f(x_0) = \text{+ive}$$

Initial 2.74 and 2.745

$$x_1 = 2.7425$$

$$f(x_1) = \text{+ive}$$

Interval 2.74 and 2.7425

$x_3 = \cancel{2.74125}\ 2.740625$

$f(x_3) = +ive$

Interval 2.74 and 2.74125

$x_3 = \boxed{2.740625}$

$f(x_3) = -ive$

Interval 2.74125 and 2.740625

$$x_4 = \frac{2.740625 + 2.74125}{2}$$

$= 2.7409375$

$f(2.7409375) = -ive$

$$x_5 = \frac{2.7409375 + 2.740625}{2}$$

$x_5 = 2.74078125$

$f(x_5) = +ive$

$2.74078125 \quad 2.740625$

$x_6 = 2.740703125$

2.7407

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shreya
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Secant Method (fixed point iteration method)

Working Rule:

① Find $f(x)$.

By approximation find out the interval

② find $\phi(x)$ {Func of (x)}

③ $[\,|\phi'(x)| < 1 \;\forall\; x\,(a,b)$

④ $x_0 =$ (mid val of interval)

$x_1 = \phi(x_0)$
$x_2 = \phi(x_1)$
$x_3 = \phi(x_2)$
⋮

Qn $x^3 - 2x - 5 = 0$

$f(x) = x^3 - 2x - 5$

$f(0) = -5$ (-ive)
$f(1) =$ (-ive)
$f(2) =$ (-ive)
$f(3) =$ +ive

$(2,3)$

$x_0 = 2.5$

$$x^3 = 2x + 5 \qquad x^3 - 5 = 2x$$

$$x = (2x+5)^{1/3} \qquad x = \frac{1}{2}(x^3 - 5)$$

let $\phi(x) = (2x+5)^{1/3}$ $\qquad \phi(x) = \frac{1}{2}(x^3-5)$

$\phi'(x) = 2 \cdot \frac{1}{3}(3x+5)^{-2/3}$ $\qquad \phi'(x) = \frac{1}{2}(3x^2)$

In the interval $(2,3)$

$$|\phi'(x)| < 1 \quad \forall \, x \, (2,3)$$

$$x_1 = \phi(x_0) = (2x_0 + 5)^{1/3}$$
$$= (2 \times 2.5 + 5)^{1/3}$$
$$= (10)^{1/3}$$
$$= 2.15443469$$

$$x_2 = \phi(x_1) = (2x_1 + 5)^{1/3}$$
$$= (2 \times 2.15443469 + 5)^{1/3}$$
$$= 2.10362029$$

$$x_3 = \phi(x_2) = 2.033$$

Computer Tech. Method

Q. Find the +ve root of the eqn $xe^x = 1$ in the interval $(0,1)$ by using iteration method

$x_0 = 0.5$

$x = \frac{1}{e^x}$

$x = e^{-x}$

$\log_e(2)$

Let $\phi(x) = e^{-x}$

$\phi'(x) = -e^{-x}$

$\phi'(0) = -1$

$\phi'(1) = -\frac{1}{e}$

$e^x = \frac{1}{x}$

$\ln_e e^x = \ln_e \frac{1}{x}$

$\phi(x) = \ln_e\left(\frac{1}{x}\right)$

$\phi'(x) = \frac{1}{x}\left(\frac{-1}{x^2}\right)$

$= \frac{-1}{x^3}$

$(0, -1)$

$x_1 = \phi(x_0) = \phi(0.5)$

$= +e^{-0.5}$

$= +\frac{1}{\sqrt{e}}$

$x_1 = +0.60522$

$x_2 = \phi(x_1) = \phi(0.60522)$

$= +e^{-0.60522}$

$= +0.54595$

20th

A 0.567147

Find the root of $\cos x - xe^x = 0$ by Bisection Method

Cube root of 15 by iteration method

Find the root of $e^{-x} - x = 0$ b/w (0,1) by bisection method

$f(x) = \cos x - xe^x = 0$

$f(0) = \cos 0 = 1$ (+ive)

$f(1) = \cancel{\cos 1} = 0.99$ (+ive)

$0.99 - 2.73 =$ −ive

Root b/w 0 and 1

$f(0.1) = 0.88$ (+ive)

$f(0.5) =$ +ive

$f(0.6) =$ −ive

roots b/w 0.5 and 0.6

$f(0.51) =$ +ive

$f(0.55) =$ +ive, $f(0.56) =$ +ive

$f(0.57) =$ −ive

0.56 and 0.57

$f(0.568) = +ive$

$f(0.567) = +ive$

$f(0.568) = -ive$

0.568 and 0.567

$$x_0 = \frac{0.567 + 0.568}{2}$$

$x_0 = 0.5675$

$f(0.5675) = -ive$

between 0.567 and 0.5675

$$x_1 = \frac{0.567 + 0.5675}{2}$$

$= 0.56725$

$f(x_1) = -ive$

between 0.56725 and 0.567

$x_2 = 0.567125$

$f(x_2) = +ive$

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0.56725 and 0.567125

$x_3 = 0.5671875$

$f(x_3) =$ -ve

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Q. $\cancel{x^3}$

$x^3 = 15$

$x^3 - 15 + 20x = 20x$

$x = \frac{1}{20}(x^3 - 15 + 20x)$

$\phi'(x) = \frac{1}{20}(3x^2 + 20)$

> 1

$x^3 - 15 - 20x = -20x$

$x = \frac{-1}{20}(x^3 - 15 - 20x)$

$$\boxed{\phi(x) = \frac{1}{20}(-x^3 + 15 + 20x)}$$

$\phi'(x) = \frac{1}{20}(-3x^2 + 20)$

$\phi'(x) = 1 - \frac{3x^2}{20}$

$|\phi'(x)| < 1$

$\phi(1) = \frac{1}{20}(-1 + 15 + 20)$

+ive

$\phi(0) =$ +ive

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$$\cancel{\phi(-1) = \frac{1}{20}(1+15-20)}$$
$$\cancel{-ive}$$

$\cancel{\therefore -1 \text{ and } 0}$

$$\cancel{\phi(2) = \frac{1}{20}(-8+15+40)}$$
$$\cancel{+ive}$$

$$f(x) = x^3 - 15$$
$$f(2) = 8 - 15 = -ive$$
$$f(3) = +ive$$

$(2, 3)$

$$x_0 = 2.5$$

$$x_1 = \phi(x_0)$$
$$x_1 = \phi(2.5)$$
2.46875
$$= \cancel{+ive} \quad \cancel{4.0} \quad \cancel{49.375}$$

$$x_2 = \phi(2.46875)$$
$$x_2 = 2.46649219$$
$$x_3 = 2.4662313$$
$$x_4 = 2.466213$$

2.4662

(3) $e^{-x} - x = 0$ ~~$f(x) = e^{-x}$~~

$f(x) = e^{-x} - x$
between 0 and 1

$f(0) = 1$

$f(1) = \frac{1}{e} - 1$ $\quad f(0,5) = e^{-0,5} - 0,5$

$f(1) = -ive$ $\quad \frac{1}{\sqrt{e}} - 0,5$

$f(0,5) = +ive.$

$f(0,6) = -ive$

$f(0,7) = -ive$

~~$f(0,61) = -ive$~~

~~$f(0,62) = -ive$~~

~~$f(0,63) = -ive$~~

~~0,6 and 0,7~~

~~$f(0,64) = -ive$~~

~~$f(0,65) = -ive$~~

0,5 and 0,6

$f(0,55) = +ive$

$f(0,56) = +ive$

$f(0,57) = -ive$

0,56 and 0,57

$f(0,565) = +ive$

$f(0,566) = +ive$

$f(0.568) = +ive$

0.567 and ~~0.568~~ 0.568

$$x_0 = \frac{0.567 + 0.568}{2}$$

$x_0 = 0.5675$

$f(0.5675) = -ive$

0.567 and 0.5675

$x_1 = 0.56725$

$f(x_1) = -ive$

0.567 and 0.56725

$x_2 = 0.567125$

$f(x_2) = +ive$

0.56725 and 0.567125

$x_3 = 0.5671875$

$f(x_3) = -ive$

0.567125 and 0.5671875

0.5671 Ans

Method (3) Regular Falsi Method

Step I: ~~Dete~~ Specify $f(x)$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

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Find a positive root of $x^3 - 4x + 1 = 0$ correct upto 3 decimal places

$f(x) = x^3 - 4x + 1$

Put $x = 0$

$f(0) = 1$ (+ive)

$f(1) = 1 - 4 + 1 = -2$ (−ive)

$x_0 = 0, \quad x_1 = 1$

$f(x_0) = 1$ (+ive), $f(x_1) = -2$ (−ive)

$$x_2 = 0 - \frac{(1-0)}{(-2-1)}(1)$$

$$x_2 = \frac{1}{3} = 0.3333$$

$f(x_2) = (0.3333)^3 - 4(0.3333) + 1$

$f(x_2) = -0.2961$ (−ive)

$f(x_2) < 1, \quad f(x_0) = 1$

$f(x_2) = (0.3333)^3 - 4(0.3333) + 1$

$= -0.2961$ (−ive)

roots will lie between $\overset{(x_2, x_0)}{(0.3333, 0)}$

$$x_3 = x_0 - \frac{x_2 - x_0}{f(x_2) - f(x_0)} f(x_0)$$

$$0 - \frac{0.3333 - 0}{(-0.2461) - 1} (1)$$

$$= 0.2571$$

$$f(x_3) = -0.0114$$

roots will lie b/w $(x_3; x_0)$
$(0.2571, 0)$

$$x_4 = x_0 - \frac{x_3 - x_0}{f(x_3) - f(x_0)} f(x_0)$$

$$= 0 - \frac{0.2571 - 0}{(-0.0114 - 1)} \times 1$$

$$x_4 = 0.2542$$

$$f(x_4) = -ive$$

root will lie b/w (x_4, x_0)
$(0.2542, 0)$

$$x_5 = 0 - \frac{0.25442 - 0}{(-0.00032 - 1)} \times 1$$

$$\boxed{x_5 = 0.2541}$$

As the 4^{th} & 5^{th} approximation the root is same upto three decimal therefore $\boxed{0.254}$ will be the root of the given equation.

Find the root of the equation $2x - \log_{10} x = 7$ which lies b/w 3.5 and 4. upto 5 decimal.
(Ans - 3.78928)

$$x_0 = 3.5, \quad x_1 = 4$$

$$f(x) = 2x - \log_{10} x - 7$$

~~Put $x = 0$~~

~~$f(0) = -\log_{10} 0 - 7$~~

$$f(x_0) = 2 \times 3.5 - \log_{10} 3.5 - 7$$
$$= \cancel{7} - 0.54406 - \cancel{7} \cdot (-ive)$$

$$f(x_1) = 8 - 0.6020 - 7 = 0.3979 \ (+ive)$$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

$$= 3.5 - \frac{0.5}{0.9420} \times (-0.544\ldots)$$

$= 3.5 - 0.5307 \times (-0.54406)$

$= 3.7887$ (~~+ive~~)

$f(x_2) = -0.30197$ (-ive)

Root will lie b/w (x_2, x_1)

$(3.7887, 4)$

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_1)$$

~~$= 4 - \frac{4 - 3.7887}{}$~~

$$= 4 - \frac{3.7887 - 4}{-0.30197 - 0.3979} \times 0.3979$$

$$= 4 - \frac{(-0.2113)}{(-0.69987)} \times 0.3979$$

$4 - 0.12013$

$x_3 = 3.87986$

$f(x_3) = -0.130109$ (-ive)

Root will lie betwn (x_3, x_1)

$(3.87986, 4)$

$$x_4 = x_1 - \frac{x_3 - x_1}{f(x_3) - f(x_1)} f(x_1)$$

$$= 4 - \frac{3.87986 - 4}{-0.130109 - 0.3929} \times 0.3929$$

$$= 4 - \frac{(-0.12014)}{(-0.528009)} \times 0.3929$$

$$= 3.90946$$

~~Root btw (x_4, x_3)~~

~~$$x_5 = x_3 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_3)$$~~

$$= 3.87986 - 3$$

$f(x_4) = -0.027421$ (-ive)

$$x_5 = x_3 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_3)$$

$$= 3.87986 - \frac{3.90946 - 3.87986}{-0.02421 + 0.130109} (-0.130109)$$

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$$\frac{0.0296}{0.055899}$$

$$= 3.87986 + 0.06889$$

$$x_5 = 3.948756$$

$f(x_5) = 0.301112$ (+ive)

~~Root~~ b/w (x_4, x_5)

$$x_6 = x_4 - \frac{x_5 - x_4}{f(x_5) - f(x_4)} f(x_4)$$

$$= 3.90946 - \frac{(3.948756 - 3.90946)}{(0.301112 + 0.07421)} \times (-0.0742$$

$$= 3.90946 - \left(\frac{0.039296}{0.375322}\right) \times (-0.07421)$$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

$$= \frac{x_0 f(x_1) - \cancel{x_0 f(x_0)} - \cancel{x} f(x_0) x_1 + \cancel{x_0 f(x_0)}}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{x_0 f(x_1) - f(x_0) x_1}{f(x_1) - f(x_0)}$$

<u>Newton Raphson Method</u>

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Put $n = 0, 1, 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

if $f'(x_0) = 0$

$$\cancel{x_1 = x_0}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

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Q1 Find roots of $x \log_{10} x = 1.2$ upto 4 decimal place using Newton Rapison Method.

$$f(x) = x \log_{10} x - 1.2$$

$$f(0) = 0 - 1.2 \quad (-ve)$$

$$f(1) = 1 \log_{10} 1 - 1.2 \quad (-ive)$$

$$f(2) = 2 \times \log 2 - 1.2 \quad (-ive)$$

$$f(3) = 3 \times \log 3 - 1.2 \quad (+ive)$$

2 and 3 are interval

Interval is (2,3) root will lie b/w (2,3)

$$f(x_n) = x_n \log_{10} x_n - 1.2$$

$$f'(x_n) = \cancel{x_n} \frac{1}{\cancel{x_n}} \times 1 + \log x_n \cdot 1$$

$$f'(x_n) = 1 + \log x_n$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$= x_n - \frac{x_n \log x_n - 1.2}{1 + \log x_n}$$

$$= \frac{x_n + \cancel{x_n \log x_n} - \cancel{x_n \log x_n} + 1.2}{1 + \log x_n}$$

difference operator:
... & defined as

$$x_{n+1} = \frac{x_n + 1.2}{1 + \log x_n}$$

put $n = 0$;

$$x_1 = \frac{x_0 + 1.2}{1 + \log x_0}$$

let $x_0 = 2.5$

$$x_1 = \frac{2.5 + 1.2}{1 + \log 2.5}$$

$$= \frac{3.7}{1 + 0.3979}$$

$$= \frac{3.7}{1.3979}$$

$$= \boxed{2.6468}$$

put $n = 1$

$$x_2 = \frac{x_1 + 1.2}{1 + \log x_1}$$

$$= \frac{2.6468 + 1.2}{1 + \log 2.6468}$$

$$= \frac{3.8468}{1.4227}$$

$$= 2.7038$$

Let $x = 2$.

$$x_3 = \frac{x_2 + 1.2}{1 + \log x_2}$$

$$= \frac{2.7038 + 1.2}{1 + \log 2.7038}$$

$$= \frac{3.9038}{1.43197}$$

$$= 2.7261$$

$$\boxed{2.74065}$$

difference operator:
~ & defined as

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Newton Raphson Methods:

1) $x^2 + \ln x - 2 = 0$ (1.314)

2) $x^4 - x - 10 = 0$ (Correct upto 3 decimal places) (1.8556)

3) $\cos x - xe^x = 0$ (0.5177)

4) $x - 2\sin x = 0$ (1.8955)

5) $x^3 - 5x + 3 = 0$ (1.8556)

6) $x \tan x = 1.28$

7) $\cos x = x^2$ (0.8241)

2) $f(x) = x^4 - x - 10$

$f(0) = -10$ (-ive)

$f(1) = 1 - 1 - 10 = -10$ (-ive)

$f(2) = 16 - 2 - 10 = 4$ (+ive)

Interval: (1, 2) let $x_0 = 1.5$

$$f(x_n) = x_n^4 - x_n - 10$$

$$f'(x_n) = 4x_n^3 - 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$\frac{4x_n^4 \cancel{- x_n} - x_n^4 \cancel{+ x_n} + 10}{4x_n^3 - 1}$$

$$x_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1}$$

Let $n = 0$

$$x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1}$$

$$= \frac{3(1.5)^4 + 10}{4(1.5)^3 - 1}$$

$$= \frac{3 \times 5.0625 + 10}{4 \times 3.375 - 1}$$

$$= \frac{25.1875}{12.5}$$

$$x_1 = 2.015$$

2

~~x_2~~ Let $n = 1$

$$x_2 = \frac{3(2.015)^4 + 10}{4(2.015)^3 - 1}$$

$$= \frac{59.45628}{31.7254135}$$

$$= 1.8740$$

$$x_3 = \frac{3(1.874)^4 + 10}{4(1.874)^3 - 1}$$

$$\frac{46.9998}{25.325}$$

$$x_3 = 1.855$$

$$x_4 = \frac{3(1.855)^4 + 10}{4(1.855)^3 - 1}$$

$$\frac{45.521}{24.51}$$

$$x_4 = 1.8552$$

Root $\rightarrow$ 1.855

(2) $\cos x - xe^x = 0$

$f(x) = \cos x - xe^x$

$f(0) = 1$ (+ive)

$f(1) = \cos 1 - e$

$= -2.183$ (-ive)

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roots between 0 and 1

$$f(x_n) = \cos x_n - x_n e^{x_n}$$

$$f'(x_n) = -\sin x_n - [x_n e^{x_n} + e^{x_n}]$$

$$f'(x_n) = -\sin x_n - x_n e^{x_n} - e^{x_n}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n + \frac{\cos x_n - x_n e^{x_n}}{\sin x_n + x_n e^{x_n} + e^{x_n}}$$

Let $x_0 = 0.5$

$$x_{n+1} = \frac{x_n \sin x_n + x_n^2 e^{x_n} + \cancel{x_n e^{x_n}} + \cos x_n - \cancel{x_n e^{x_n}}}{\sin x_n + x_n e^{x_n} + e^{x_n}}$$

$$x_{n+1} = \frac{x_n \sin x_n + x_n^2 e^{x_n} + \cos x_n}{\sin x_n + x_n e^{x_n} + e^{x_n}}$$

Let $n = 0$

$$x_1 = \frac{x_0 \sin x_0 + x_0^2 e^{x_0} + \cos x_0}{\sin x_0 + x_0 e^{x_0} + e^{x_0}}$$

$$= \frac{0.5 \sin 0.5 + (0.5)^2 e^{0.5} + \cos 0.5}{\sin 0.5 + 0.5 e^{0.5} + e^{0.5}}$$

$$= \frac{0.5 \times 0.479 + 0.25 \times 1.648 + 0.877}{0.479 + \cancel{0.5 \times 1.648 + 0} \; (0.5+1) \times 1.648}$$

$$= \frac{0.2395 + 0.412 + 0.877}{0.479 + 2.472}$$

$$= \frac{1.5285}{2.951}$$

$$x_1 = 0.517$$

$$x_2 = \frac{0.517 \sin 0.517 + (0.517)^2 e^{0.517} + \cos 0.517}{\sin 0.517 + (0.5+1)\, 1.5 e^{0.517}}$$

$$= \frac{0.517 \times 0.4942 + 0.267 \times 1.676 + 0.869}{0.4942 + 1.5 \times 1.676}$$

$$\frac{0.25550 + 0.447 + 0.869}{0.4942 + 2.514}$$

$$\frac{1.5715}{3.0082}$$

$$0.5224$$

$$x_2 =$$

Margin fragments from facing page: ate, e^{x_n}], $- e^{x_n}$, $\bar{x}_n$, $-x_n e^{x_n}$, 5, 5

(4)

$$x_3 = \frac{0.5224 \sin 0.5224 + (0.5224)^2 e^{0.5224} + \cos 0.5224}{\sin 0.5224 + 1.5 \times e^{0.5224}}$$

$$x_3 = \frac{0.5224 \times 0.4989 + 0.2729 \times 1.6860 + 0.8666}{0.4989 + 1.5 \times 1.6860}$$

$$= \frac{0.2606 + 0.46010 + 0.8666}{0.4989 + 2.529}$$

$$\frac{1.5873}{3.0279}$$

$$x_3 = 0.5242$$

x

$x - 2\sin x = 0$

$f(x) = x - 2\sin x$

$f(0) = 0$

$f(1) = 1 - 2\sin 1$

$= 1 - 2 \times 0.8414$

$= -0.6$ (-ive)

$f(2) = 2 - 2\sin 2$

$= 2 - 1.81$

$= 0.19$ (+ive)

Interval $(1, 2)$ Let $x_0 = 1.5$

$f(x_n) = x_n - 2\sin x_n$

$f'(x_n) = 1 - 2\cos x_n$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n - 2\sin x_n}{1 - 2\cos x_n}$$

$$x_{n+1} = \frac{\cancel{x_n} - 2\cos x_n \cdot x_n - \cancel{x_n} + 2\sin x_n}{1 - 2\cos x_n}$$

$$x_{n+1} = \frac{2(\sin x_n - x_n \cos x_n)}{1 - 2\cos x_n}$$

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Put $n=0$

$$x_1 = \frac{2(\sin x_0 - x_0 \cos x_0)}{1-2\cos x_0}$$

$$x_1 = \frac{2(\sin 1.5 - 1.5\cos 1.5)}{1-2\cos 1.5}$$

$$x_1 = \frac{2(0.997-1.5\times 0.0707)}{1-2\times 0.707}$$

$$= \frac{2(0.997-0.10605)}{\cancel{0.414}\ 0.8585}$$

$$= \frac{1.7819}{0.8585}$$

$x_1 = 2.0755$

$x_2 = \frac{3.752}{1.960} = 1.9100$

$x_3 = 1.8956$

$x_4 = 1.895$

$\cancel{\text{Root}} \rightarrow 1.895$

(5)

$\cancel{(6)}$

(5) $x^2 - 5x + 3 = 0$

(6) $x\tan x = 1.28$

$f(x) = x\tan x - 1.28$

$f(0) = -1.28$ (-ive)

$f(1) = 0.222$ (+ive)

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$$(D^2+2D+1)\,y = x\cos x$$

$$P.I \Rightarrow \frac{1}{(D^2+2D+1)}\,x\cos x$$

$$(e^{ix} = \cos x + i\sin x)$$

$$\Rightarrow \text{Real part of } \frac{1}{(D^2+2D+1)}\,x e^{ix}$$

$$= \text{Real part of } e^{ix}\frac{1}{(D^2+2D+1)}\,x$$

$$\text{Real part of } e^{ix}\frac{1}{[(D+i)^2+2(D+i)+1]}\,x$$

$$e^{ix} = \cos x + i\sin x$$

$$P.I = \text{Real part of } e^{ix}\frac{1}{\{D^2+i^2+2Di+2D+2i+1\}}\,x$$

$$= \text{Real part of } e^{ix}\frac{1}{\{D^2-i+2Di+2D+2i+1\}}\,x$$

$$= \text{Real part of } e^{ix}\frac{1}{\{D^2+2D(1+i)+2i\}}\,x$$

$$= \text{Real part of } e^{ix}\frac{1}{2i\{\frac{D^2}{2i}+\frac{D}{i}(1+i)+1\}}\,x$$

$P.I = \text{Real part of } e^{ix} \frac{1}{2i}\left\{1+\frac{D^2}{2i}+\frac{D}{i}(1+i)\right\}^{-1} x$

$= \text{Real part of } e^{ix} \frac{1}{2i}\left\{x-\frac{1}{i}(1+i)\right\}$

$\Rightarrow \text{Real part of } \frac{e^{ix}}{2i}\left\{\frac{xi-1-i}{i}\right\}$

$= \text{Real part of } \frac{e^{ix}}{2}\{-xi+1+i\}$

$= \text{Real part of } \frac{1}{2}\{\cos x+i\sin x\}\{xi+1+i\}$

$= \text{Real part of } \left\{\frac{-1}{2}\cos x\, i+\frac{1}{2}\cos x+\frac{1}{2}i\cos x\right.$

$\left.+\frac{1}{2}x\sin x+\frac{1}{2}i\sin x-\frac{1}{2}\sin x\right\}$

$= \frac{1}{2}\cos x+\frac{1}{2}x\sin x-\frac{1}{2}\sin x$

Qn $\frac{d^2y}{dx^2}+\frac{2dy}{dx}+y=\frac{e^{-x}}{x+2}$

Solⁿ: $(D^2+2D+1)y=\frac{e^{-x}}{x+2}$

A.E: $m^2+2m+1=0$

$m=-1,\ m=-1$

$C.F = (C_1+C_2x)e^{-x}$

defined as

$$P.I = \frac{1}{(D^2+2D+1)} \frac{e^{-x}}{x+2}$$

$$= e^{-x} \frac{1}{(D^2+2D+1)} \frac{1}{(x+2)}$$

$$= e^{-x} \frac{1}{\{(D-1)^2+2(D-1)+1\}} \frac{1}{(x+2)}$$

$$= e^{-x} \frac{1}{\{D^2+1-2D+2D-2+1\}} \frac{1}{(x+2)}$$

$$= e^{-x} \frac{1}{D^2} \frac{1}{(x+2)}$$

$$= e^{-x} \frac{1}{D} \int \frac{1}{(x+2)} dx$$

$$= e^{-x} \frac{1}{D} \log(x+2)$$

$$= e^{-x} \int \log(x+2).1 \, dx$$

$$= e^{-x} \left[\log(x+2) x - \int \frac{1}{(x+2)} x \, dx \right]$$

$$= e^{-x} \log(x+2).x - \int \left\{ 1 - \frac{2}{x+2} \right\} dx$$

$$= e^{-x} \left[\log(x+2).x - x + 2\log(x+2) \right]$$

$$y = (C_1 + C_2 x) e^{-x} + e^{-x} \{ \log(x+2) x - x + 2\log(x+2) \}$$

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7 $\frac{d^3y}{dx^3} - \frac{3dy}{dx} + 2y = xe^{3x} + \sin 2x$

$D^3y - 3Dy + 2y = xe^{3x} + \sin 2x$

$(D^3 - 3D + 2)y = xe^{3x} + \sin 2x$

$m^3 - 3m + 2 = 0$

$(m-1)$ is a root.

$$(m-1)\,\overline{)\,m^3 - 3m + 2\,}\,(m^2 + m - 2$$

$$\cancel{m^3} - m^2$$
$$-\quad +$$
$$\cancel{m^2} - 3m + 2$$
$$\cancel{m^2} - m$$
$$-\quad +$$
$$-2m + 2$$
$$-2m + 2$$
$$\times$$

$(m-1)(m^2 + m - 2) = 0$

$(m-1)(m^2 + 2m - m - 2) = 0$

$(m-1)[m(m+2) - 1(m+2)] = 0$

$(m-1)(m-1)(m+2) = 0$

$m = 1, 1, -2$

$= C_1e^{-2x} + \{C_2 + C_3x\}e^{x}$

P.I. $\frac{1}{(D^3 - 3D + 2)}(xe^{3x} + \sin 2x)$

$\frac{1}{(D^3 - 3D + 2)}xe^{3x} + \frac{1}{(D^3 - 3D + 2)}\sin 2x$

backward difference operator

∇ (nebla) & defined as

$m^2 = 4$
$m = \pm 2$

Q. A body executes a dam- forced vibrations given by the eq^n $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin\omega t$

Solve the eq^n for both the cases when $\omega^2 \neq b^2 - k^2$ and $\omega^2 = b^2 - k^2$ $(k^2 < b^2)$

Q. $(D^2 - 4)y = \cosh(2x-1) + 3^x$

$$\cosh(2x-1) = \frac{e^{(2x-1)} + e^{-(2x-1)}}{2}$$

$$3^x = e^{\log_e 3^x}$$

$$(D^2-4)y = \frac{e^{(2x-1)} + e^{-(2x-1)}}{2} + e^{\log_e 3^x}$$

$$P.I = \frac{1}{(D^2-4)}\left\{\frac{e^{(2x-1)} + e^{-(2x-1)}}{2}\right\} + \frac{1}{(D^2-4)}e^{\log_e 3^x}$$

$$P.I: \frac{1}{2(D^2-4)}e^{(2x-1)} + \frac{1}{2(D^2-4)}e^{-(2x-1)} + \frac{1}{(D^2-4)}e^{\log_e 3^x}$$

Q. 4 (Ans)

$$y = C_1 e^{mx} + C_2 e^{-mx} + C_3\cos mx + C_4\sin mx - \frac{x}{4m^3}\sin mx$$

Ans (5) $y = C_1 e^{x} + e^{x}\{C_2 \cos x + C_3 \sin x\} + x e^{x} + \frac{1}{10}\{3\sin x + \cos x\}$

Ans (6) $y = e^{2x}\{C_1 \cosh\sqrt{3}x + C_2 \sinh\sqrt{3}x\} + \frac{1}{2} - \frac{1}{8}\sin x - \frac{1}{104}(3\sin 3x + 2\cos 3x) + \frac{1}{146}(8\sin 2x + 3\cos 2x)$

~~absent~~ mean μ_1

+ arbitra ~~cons~~ μ_1'

~~G~~ origin ν

backward difference:-

$y_0, y_1, y_2, \ldots y_n$ then $y_1 - y_0, y_2 - y_1, y_3 - y_2, \ldots$
$y_n - y_{n-1}$

$\nabla y_1, \nabla y_2, \nabla y_3, \ldots \nabla y_n$

$$\nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\vdots$$
$$\nabla y_n = y_n - y_{n-1}$$

where ∇ (nabla) is called backward difference operator.

backward difference operator:

denoted by ∇ (nabla) & defined as

$$\nabla f(x) = f(x) - f(x-h)$$

$$\nabla y_r = y_r - y_{r-1}$$

③ Shift operator :-

It is denoted by E and defined as

$$E f(x) = f(x+h)$$
$$E y_r = y_{r+1}$$
$$E^{-1} f(x) = f(x-h)$$
$$E^{-1} y_r = y_{r-1}$$

Relation between Δ (forward diff. operator) and E (shift operator)

$$\Delta f(x) = f(x+h) - f(x)$$
$$= E f(x) - f(x)$$
$$\Delta f(x) = (E-1) f(x)$$

$$\boxed{\Delta = E-1}$$

Relation between ∇ (backward diff. operator) and E (shift operator)

$$\nabla f(x) = f(x) - f(x-h)$$
$$\nabla f(x) = f(x) - E^{-1} f(x)$$

$$\nabla f(x) = f(x)\left[1 - E^{-1}\right]$$
$$\nabla = 1 - E^{-1}$$

backward difference table :-

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
		∇y_1				
x_1	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$	
		∇y_3		$\nabla^3 y_4$		
x_3	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$	
		∇y_4		$\nabla^3 y_5$		
x_4	y_4		$\nabla^2 y_5$			
		∇y_5				

x	10	20	30	40	50
y	1	1.3010	1.4771	1.6021	1.6990

Construct a backward difference table of $y = \log x$ given that & find the values of $\nabla^3 \log 40$ and $\nabla^4 \log 50$.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
10	1				
		0.3010			
20	1.3010		−0.1249		
		0.1761		0.0738	
30	1.4771		−0.0511		−0.0508
		0.125		0.023	
40	1.6021		−0.0281		
		0.0969			
50	1.6990				

$\nabla^3 \log 40 = 0.0738$

$\nabla^4 \log 50 = -0.0508$

The following table gives the value of y which is a polynomial of degree 5 if it is known that $y = f(3)$ is a in error. Correct the error.

x	0	1	2	3	4	5	6
y	1	2	33	254	1025	3126	7777

$E(y_r) = y_{r+1}$ $(x-a)^n$

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Let $y(3) = a$

②

$\Delta^6 y_0 = 0$

$(E-1)^6 y_0 = 0$

$(E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) y_0 = 0$

$E^6 y_0 - 6E^5 y_0 + 15E^4 y_0 - 20E^3 y_0 + 15E^2 y_0 - 6E y_0 + \ldots$

$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 =$

$7777 - 6 \times 3126 + 15 \times 1025 - 20a + 15 \times 33 - 6 \times 2 + 1 = 0$

$4880 - 20a = 0$

$a = \dfrac{4880}{20}$

$\boxed{a = 244}$

Error $= 254 - 244$
$= 10.$

Ques- Find $\Delta \tan^{-1} x$ and $\Delta^2 \cos 2x$.

① $\Delta f(x) = f(x+h) - f(x)$

$\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1} x$

$= \tan^{-1} \left\{ \dfrac{x+h-x}{1 + x(x+h)} \right\} = \tan^{-1} \left\{ \dfrac{x}{1+x^2+hx} \right\}$

(2) $\Delta^2 \cos 2x = \Delta(\Delta \cos 2x)$

$= \Delta \{\cos 2(x+h) - \cos 2x\}$

$= \Delta \cos 2(x+h) - \Delta \cos 2x$

$= \{\cos 2(x+2h) - \cos 2(x+h)\} - \{\cos 2(x+h) - \cos 2x\}$

$$= \left\{-2\sin\left(\frac{2x+4h+2x+2h}{2}\right)\sin\left(\frac{2x+4h-2x-2h}{2}\right)\right\} + \left\{2\sin\left(\frac{2x+2h+2x}{2}\right)\sin\left(\frac{2x+2h-2x}{2}\right)\right\}$$

$$= \{-2\sin(2x+3h)\sinh\} + \{2\sin(2x+h)\sinh\}$$

$$= -2\sinh\left\{2\cos\left(\frac{2x+3h+2x+h}{2}\right)\sin\left(\frac{2x+3h-2x-h}{2}\right)\right\}$$

$$= -2\sinh\{2\cos(2x+2h)\sinh\}$$

$$= -4\sin^2 h \cos 2(x+h)$$

(3) $E = (1-\nabla)^{-1}$

$\nabla = 1 - E^{-1}$

$E = (1-(1-E^{-1}))^{-1}$

$= (\cancel{1} - \cancel{1} + E^{-1})^{-1}$

$= (E^{-1})^{-1}$

$\boxed{E = E}$

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$$\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$$

ET RHS

$$\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$$

$$\frac{\Delta^2 - \nabla^2}{\nabla\Delta}$$

$$\frac{(E-1)^2 - (1-E^{-1})^2}{(E-1)(1-E^{-1})}$$

$$\frac{(E^2+1-2E) - (1+E^{-2}-2E^{-1})}{E-1-1+E^{-1}}$$

$$\frac{E^2+1-2E-1-\frac{1}{E^2}+\frac{2}{E}}{E+\frac{1}{E}-2}$$

$$\frac{\frac{E^4+\cancel{E^2}-2E^3-\cancel{E^2}-1+2E}{E^{\cancel{2}}}}{\frac{E^2+1-2E}{\cancel{E}\,1}}$$

$$\frac{E^4-2E^3+2E-1}{E^3+E-2E^2}$$

L.H.S.

$$\Delta + \nabla$$

$$(E-1) + (1 - E^{-1})$$

$$E - \cancel{1} + \cancel{1} - E^{-1}$$

$$E - E^{-1}$$

$$E - \frac{1}{E} = \frac{2E-1}{E}$$

R.H.S

$$\frac{E-1}{1-E^{-1}} - \frac{1-E^{-1}}{E-1}$$

$$= \frac{E-1}{\left(1-\frac{1}{E}\right)} - \frac{\left(1-\frac{1}{E}\right)}{(E-1)}$$

$$\frac{E(E-1)}{(E-1)} - \frac{(E-1)}{E(E-1)}$$

$$\frac{E^2(E-1) - (E-1)}{E(E-1)}$$

$$E - \frac{1}{E}$$

$$E - E^{-1}$$

$$(E-1) + (1-E^{-1})$$

$$= \Delta + \nabla$$

Q1 (i) $\left[E^{1/2}+E^{-1/2}\right](1+\Delta)^{1/2} = 2+\Delta$

(iii) $\Delta^3 y_2 = \Delta^3 y_5$

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Que-1 ~~Find the intermediate~~ interpolation:

Finding the value of y (dependent variable) for any intermediate value of independent variable x within the interval is called interpolation.

Also finding the value of y for any value of x outside the interval is called extrapolation.

Two types of interpolation:

1. for equal intervals
2. for unequal intervals

Newton's Gregory forward difference :-

$$f(x) = f(x_0 + hu) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{\lfloor 2}\Delta^2 f(x_0)$$
$$+ \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 f(x_0) + \dots \frac{u(u-1)(u-2)\dots(u-n+1)}{\lfloor n}\Delta^n f(x_0)$$

where $u = \frac{x - x_0}{h}$

Q. The population of ~~the~~ a town in decimal census was as given below. Estimate the population for the year 1895.

year x	1891	1901	1911	1921	1931
Population (in thousands)	46	66	81	93	101

$x_0 = 1891$, $x = 1895$, $h = 10$

$$u = \frac{x - x_0}{h} = \frac{1895 - 1891}{10} = 0.4$$

forward diff. table

x	y $f(x_0)$	Δy $\Delta f(x_0)$	$\Delta^2 y$ $\Delta^2 f(x_0)$	$\Delta^3 y$ $\Delta^3 f(x_0)$	$\Delta^4 y$ $\Delta^4 f(x_0)$
1891	$\boxed{46}$				
1901	66	$\boxed{20}$			
1911	81	15	$\boxed{-5}$		
1921	93	12	-3	$\boxed{2}$	
1931	101	8	-4	-1	$\boxed{-3}$

$$f(1895) = f(x_0) + u\,\Delta f(x_0) + \frac{u(u-1)}{\lfloor 2}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 f(x_0) + \frac{u(u-1)(u-2)(u-3)}{\lfloor 4}\Delta^4 f(x_0)$$

$$f(1895) = 46 + (0.4)\times 20 + \frac{(0.4)(0.4-1)}{2}(-5) + \frac{(0.4)(0.4-1)(0.4-2)}{6}(2) + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{24}(-3)$$

54.728

$= 54.85$

From the following table of half yearly premium for policies maturing at different ages. Estimate the premium for policies maturing at the age of 46.

Age	45	50	55	60	65
Premium	114.84	94.16	83.32	74.48	68.48

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$x_0 = 45$, $x = 46$, $h = 5$

$$u = \frac{x - x_0}{h} = \frac{1}{5} = 0.2$$

forward diff. table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	$f(x_0)$	$\Delta f(x_0)$	$\Delta^2 f(x_0)$	$\Delta^3 f(x_0)$	$\Delta^4 f(x_0)$
45	$\boxed{114.84}$	$\boxed{-20.68}$	$\boxed{9.84}$	$\boxed{-7.84}$	$\boxed{8.68}$
50	94.16	-10.84	2	0.84	
55	83.32	-8.84	2.84		
60	74.48	-6			
65	68.48				

$$f(46) = 114.84 + 0.2 \times (-20.68) + \frac{(0.2)(0.2-1)}{2} \times 9.84 + \frac{0.2(0.2-1)(0.2-2)}{6} \times (-7.84)$$

115.16 ~~868~~

$$+ \frac{0.2(0.2-1)(0.2-2)(0.2-3) \times 8.64}{24}$$

$= 114.877564$

4.

110.525632

Newton Gregory backward difference interpolation formula:

$$f(x) = f(x_n+hu) = f(x_n) + u\nabla f(x_n) + \frac{u(u+1)}{\lfloor 2}\nabla^2 f(x_n)$$

$$+ \frac{u(u+1)(u+2)}{\lfloor 3}\nabla^3 f(x_n) \text{\_\_\_\_\_}$$

$$\frac{u(u+1) + \text{\_\_\_\_} (u+n-1)}{\lfloor n}\nabla^n f(x_n)$$

$$u = \frac{x-x_n}{h}$$

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

Estimate the population for the year 1925.

$x_n = 1931 \quad x = 1925 \quad h = 10$

$$u = \frac{x-x_n}{h} = \frac{1925-1931}{10}$$

$$= -0.6$$

$$f(1925) = f(1931) + u\nabla f(1931) + \frac{u(u+1)}{\lfloor 2}\nabla^2 f(1931)$$

$$+ \frac{u(u+1)(u+2)}{\lfloor 3}\nabla^3 f(1931) + \frac{u(u+1)(u+2)(u+3)}{\lfloor 4}\nabla^4 f(1931)$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		$\boxed{-3}$ $\nabla^4 f(x_n)$
		12		$\boxed{-1}$ $\nabla^3 f(x_n)$	
1921	93		$\boxed{-4}$ $\nabla^2 f(x_n)$		
		$\boxed{8}$ $\nabla f(x_n)$			
1931	$\boxed{101}$ $f(x_n)$				

$$= 101 + (-0.6)\times 8 + \frac{(-0.6)(-0.6+1)}{2}(-4) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-1)}{6} + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-3)}{24}$$

$$= 96.8968$$

Divided difference:

If the values of independent variable x are not equidistant then for finding the value of dependent variable y for any intermediate value of x we will use Divided difference method.

Let $y_0, y_1, y_2 \text{-----} y_n$ be a set of values of y for $x_0, x_1, x_2 \text{----} x_n$. Then the first divided difference for $[x_0, x_1]$ is given by

$$\Delta y_0 \; [x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\Delta y_1 \; [x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$

Second divided difference for $[x_0, x_1, x_2]$ is given by

$$\Delta^2 y_0 \; [x_0, x_1, x_2] = \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0}$$

Similarly $[x_1, x_2, x_3] = \frac{[x_2, x_3] - [x_1, x_2]}{(x_3 - x_1)}$

Third divided difference for $[x_0, x_1, x_2, x_3]$ is given by

$$\Delta^3 y_0 = [x_0, x_1, x_2, x_3] = \frac{[x_1, x_2, x_3] - [x_0, x_1, x_2]}{(x_3 - x_0)}$$

Similarly

$$[x_1, x_2, x_3, x_4] = \frac{[x_2, x_3, x_4] - [x_1, x_2, x_3]}{x_4 - x_1}$$

Properties of Divided Differences:

(1) Divided differences are symmetrical in their respective arguments.

$$[x_0, x_1] = [x_1, x_0]$$

$$[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$

$$= \frac{\cancel{-}(y_0 - y_1)}{\cancel{-}(x_0 - x_1)}$$

$$\frac{y_0 - y_1}{x_0 - x_1} = [x_1, x_0]$$

(2)

$[x_0, x_1, x_2] = [x_1, x_0, x_2]$

$= [x_2, x_0, x_1]$

② The n^{th} divided difference of n^{th} degree polynomial will always be constant.

Newton's formula for divided differences:

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0$$

$+ \ldots$

Q: Using Newton Divided formulae, find the polynomial func satisfying the following data:

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

	x	$f(x)$ y		Δy	$\Delta^2 y$	$\Delta^2 y$	$\Delta^3 y$
x_0	-4	1245	y_0	$\frac{33-1245}{-1+4} = -404$	$\frac{-28+404}{0+4} = 94$	$\frac{10-94}{2+4} = -14$	$\frac{13+14}{9}$
x_1	-1	33	y_1	$\frac{5-33}{0+1} = -28$	$\frac{2+28}{2+1} = 10$	$\frac{88-10}{5+1} = 13$	
x_2	0	5	y_2	$\frac{9-5}{2} = 2$	$\frac{442-2}{5-0} = 88$		
x_3	2	9	y_3	$\frac{1335-9}{5-2} = 442$			
x_4	5	1335	y_4				

$$f(x) = 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x)(\ldots$$
$$+ (x+4)(x+1)(x)(x-2).3$$

$$= 1245 - 404 - 1616 + (x^2+5x+4)(94)$$
$$+ (x^3+5x^2+4x)(-14) + (x^3+5x^2+4x)(x-2).3$$

$$= 1245 - 404 - 1616 + 94x^2 + 470x + 376$$
$$- 14x^3 - 70x^2 - 56x + (x^4 + 5x^3 + 4x^2 -$$
$$2x^3 - 10x^2 - 8x).3$$

$$= 1245 - 404 - 1616 + 94x^2 + 470x + \cancel{376}$$
$$- 14x^3 - 70x^2 - 56x$$

43.6668

Ques Using Newton divided difference formula calculate the value of $f(6)$ from the following table :

x	1	2	7	8
$f(x)$	1	5	5	4

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	$\boxed{1}\ y_0$			
		$\boxed{4}$		
2	$5\ y_1$		$\boxed{-2/3}$	$\frac{-1/6+2/3}{8-1}$ $\boxed{=\frac{1}{14}}$
		0		
7	$5\ y_2$		$-1/6$	
		-1		
8	$4\ y_3$			

6.2381

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0$$

$$= 1 + (x-1)\times 4 + (x-1)(x-2)\left(\frac{-2}{3}\right)$$

$$+ (x-1)(x-2)(x-7)\left(\frac{1}{14}\right)$$

$$= 1 + (4x-4) - \frac{2}{3}x^2 + 2x - \frac{4}{3}$$

$$+ \frac{1}{14}x^3 - \frac{10}{14}x^2 + \frac{23}{14}x - \frac{12}{14}x$$

$$\frac{x^3}{14} - \frac{10}{14}x^2 + \frac{23}{14}x - \frac{12}{14}x$$

$$= 1 + 4x - 4 - \frac{2}{3}x^2 + 2x - \frac{4}{3} + \frac{x^3}{14} - \frac{10}{14}x^2$$

(x^2-3x+2)
$(x-7)$
$x^3 - 7x^2 - 3x^2$
$+ 21x + 2x - 14$
$x^3 - 10x^2 + 23x - 14$

Q. By mean of ~~[illegible]~~ Newton divided difference formula find the value of $f(8)$ and $f(15)$.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	20[illegible]

$5u^4 - 40u + 105u - 100u + \ldots$

$$\left(u^4 - 3u^3 + \ldots + 3u^2 \ldots\right) \quad - 3u^3 + 11u^2 - 6u) \quad \left(u^4 - 6u^3 + 11u^2 - 18u + 12u - 6\right) \quad 4u^3$$

Numerical Differentiation :-

Numerical Differentiation is a process of finding the successive derivative of a function $y = f(x)$ for the given values of x if we have to find out the derivative for x which is nearer to the initial value in the given table then we will use newton's gregory forward difference interpolation formula if we have to find out the value of the function or derivative nearer to the end of table then we will use newton gregory backward difference interpolation formula.

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \ldots \qquad (1)$$

$$u = \frac{x - x_0}{h} \qquad (2)$$

~~differentiate eq^n~~

$$y = y_0 + u\Delta y_0 + \frac{(u^2-u)}{2}\Delta^2 y_0 + \frac{(u^3 - 3u^2 + 2u)}{6}\Delta^3 y_0 \quad (n-1)$$

$$\ldots$$

diff. (1) wrt u

$$\frac{dy}{du} = \Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2 - 6u + 2)}{6}\Delta^3 y_0 + \ldots \qquad (3)$$

diff. eq^n (2) wrt x

$$\frac{du}{dx} = \frac{1}{h} \quad \text{—— (4)}$$

$$\frac{dy}{dx} = \cancel{\frac{dy}{dx}}\frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{h}\left\{ \Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^3 y_0 + \text{------} \right\} \text{—— (5)}$$

If x is not tabulated then we will use eq^n (5) for finding the derivative

If x is tabulated then put $u = 0$ in eq^n (5)

$$\left[\frac{dy}{dx}\right]_{x=a} = \frac{1}{h}\left\{ \Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 \right.$$

$$\frac{d^2y}{dx^2} = \frac{d}{du}\left(\frac{dy}{dx}\right)\frac{du}{dx}$$

$$= \frac{d}{du}\left\{\left[\frac{1}{h}\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^3 y_0 + \text{---}\right.\right\}$$

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$$= \frac{1}{h^2}\left\{ \frac{1}{2} \Delta^2 y_0 + \frac{(6u-6)}{6} \Delta^3 y_0 + \dots \right\}$$

~~Find~~ Find $\frac{dy}{dx}$ at $x = 0.1$ from the table

x	0.1	0.2	0.3	0.4
y	0.9975	0.9900	0.9776	0.9604

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0.1	$\boxed{0.9975}$			
		Δy_0 $\boxed{-0.0075}$		
			$\Delta^2 y_0$ $\boxed{-0.0049}$	
0.2	0.9900			$\Delta^3 y_0$ $\boxed{0.0001}$
		-0.0124		
			-0.0048	
0.3	0.9776			
		-0.0172		
0.4	0.9604			

$$\left[\frac{dy}{dx}\right]_{x=0.1} = \frac{1}{0.1}\left\{ -0.0075 - \frac{1}{2}(-0.0049) + \frac{1}{3}(0.0001) \right\}$$

$$= -0.050167$$

② Find $f'(1.1)$ and $f''(1.1)$ from the following table.

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.0	0.1280	0.5540	1.2960	2.4320	4.000

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	$\boxed{0.0}$ (y_0)					
		$\boxed{0.1280}$ (Δy_0)				
1.2	0.1280		$\boxed{0.298}$ ($\Delta^2 y_0$)			
		0.426		$\boxed{0.018}$ ($\Delta^3 y_0$)		
1.4	0.5540		0.316		$\boxed{0.06}$ ($\Delta^4 y$)	
		0.742		0.078		$\boxed{-0.1}$ ($\Delta^5 y_0$)
1.6	1.2960		0.394		-0.04	
		1.136		0.038		
1.8	2.4320		0.432			
		1.568				
2.0	4.000					

$$u = \frac{x - x_0}{h} = \frac{1.1 - 1.0}{0.2} = \frac{0.1}{0.2} = 0.5$$

$$\frac{dy}{dx} = \frac{1}{0.5}\left\{ 0.1280 + \cancel{\frac{(1-1)}{2} \times 0.298} + \left[\frac{3(0.5)^2 - 6(0.5) + 2}{6}\right] 0.018 + \left[\frac{4(0.5)^3 - 18(0.5)^2 + 22(0.5) - 6}{24}\right] 0.06 + \right.$$

$$\left[\frac{5(0.5)^4 - 40(0.5)^3 + 105(0.5)^2 - 100(0.5) + 24}{120}\right](-0.1)$$

$$\frac{1}{0.2}\left\{\cancel{0.1280} + \cancel{0.127525} + 0.130025\right.$$

$$\frac{0.133722}{0.2}$$

$$0.66861$$

$$\frac{d^2y}{dx^2} = \frac{d}{du}\left(\frac{dy}{dx}\right)\left(\frac{du}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d}{du}\left(\frac{1}{h}\left(\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^3 y_0\right)\right)\frac{1}{h}$$

$$\frac{1}{h^2}\left(0 + \frac{1}{\cancel{2}}\cancel{(2)}^1\Delta^2 y_0 + (u-1)\Delta^3 y_0\right)$$

0.5

Numerical Integration:

Newton Cote's quadrature formula:

1. Trapezodial Rule
2. Simson's one third Rule
3. Simpson's three Eighth's Rule

A function $y = f(x)$ is tabulated for the values of x in the given interval say (a,b) then finding the value of $\int_a y\,dx$ or $\int_a^b f(x)\,dx$ with the help of the table form or given for the function $y = f(x)$

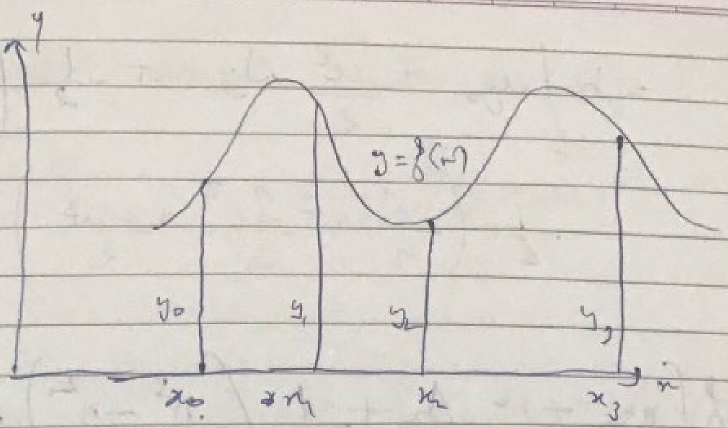
This is called numerical integration.

For finding the value of the integration we will subdivide the interval in equal parts with common width h, by using Newton forward interpolation formula.

Newton's Cote's quadrature formula:

Let the integral $I = \int_a^b y\,dx$

where y is denoted the value of the function $y_0, y_1, y_2, \ldots y_n$ for the values of x i.e. $x_0, x_1, \ldots x_n$ in the interval (a,b)



Divide the interval a, b such as

$x_0 = a$, $x_1 = x_0 + h$, $x_2 = x_0 + 2h$, $x_3 = x_0 + 3h + \ldots$

$x_n = x_0 + nh = b$

where $h = \frac{b-a}{n}$

n is total number of parts

for every $x = x_0 + hu$

$dx = h\,du$

$$\cancel{I = \int_{x_0}^{x_0+nh} h\, f(x_0 + hu)\, du}$$

n = 1
n = 2
n = 3

$$I = \int_{x_0}^{x_0+nh} f(x)\,dx$$

$$= \int_{0}^{n} f(x_0 + hu)\,h\,du$$

$$= h\left[\int_0^n \left\{ y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \ldots \right\} du\right]$$

$$= h\left[u y_0 + \frac{u^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{u^4}{4} - \frac{3u^3}{3} + \frac{2u^2}{2}\right)\Delta^3 y_0 \right]_0^n$$

$$= h\left[n y_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - \frac{3n^3}{3} + \ldots\right)\Delta^3 y_0 \right]$$

$$= nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{1}{2} n(2n-3)\Delta^2 y_0 + \frac{1}{24} n(n-2)^2 \Delta^3 y_0 + \text{———} \right]$$

For Trapezodial Rule put $n = 1$ & the curve will go through the points (x_0, y_0) & (x_1, y_1) for this the polynomial will be taken of degree 1 rest polynomials we will ignore

$$\int_{x_0}^{x_0+h} f(x)\,dx = h\left[y_0 + \frac{1}{2}\Delta y_0\right] = h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right]$$

$$= \frac{h}{2}\left[2y_0 + y_1 - y_0\right]$$

$$= \frac{h}{2}\left[y_0 + y_1\right]$$

for the ~~part~~ interval (x_0+h, x_0+2h)

$$\int_{x_0+h}^{x_0+2h} f(x)dx = \frac{h}{2}[y_1+y_2]$$

- - - - - - - - - - -

for the interval $[(x_0+(n-1)h, (x_0+nh)]$

$$\int_{x_0+(n-1)h}^{(x_0+nh)} f(x)dx = \frac{h}{2}[y_{n-1}+y_n]$$

This is called trapezodial rule.

If we will divide the interval in ~~max~~ maximum parts or sub-interval then we will get more accuracy in the value of integration as h will be smaller in this case.

~~The value of~~

Q. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezodial rule.

$(0, 6)$

$a=0, b=6$

$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

(divide in 6 parts)

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$$f(x) = \frac{1}{1+x^2}$$

x	0	1	2	3	4	5	6
$f(x)$	1	$1/2$	$1/5$	$1/10$	$1/17$	$1/26$	$1/37$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{1}{2}\left[\{y_0+y_6\} + 2\{\cancel{y_0} + y_1 + y_2 + y_3 + y_4 + y_5\}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{37} + 2\left[\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26}\right]\right]$$

$$= 1.4173$$

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Qu-1 Evaluate $\int_{0.6}^{2} y\,dx$ where y is given by

x =	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y	1.23	1.58	2.03	4.32	6.25	8.36	10.23	12.45

$\boxed{Ans - 7.922}$

Qu-2 $\int_{0}^{6} \frac{e^x}{1+x}$ by using Trapezodial Rule.

Qu-3 Find from the following table the area to bounded by the curve and x-axis from $x = 7.47$ to $x = 7.52$

x =	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

① $b = 2$, $a = 0.6$, $h = \frac{2-0.6}{7} = \frac{1.4}{7} = 0.2$

$$\int_{0.6}^{2} y\,dx = \frac{h}{2}\left[(y_0+y_7)+2(y_1+y_2+y_3+y_4+y_5+y_6\right.$$

$$= 0.1\left[(1.23+12.45)+2(1.58+2.03+4.32+6.25+8.36+10.23)\right]$$

$$= 0.1\left[13.68+2(65.54)\right]$$

$$= 0.1(79.22) = 7.922$$

(2) $I = \int_0^6 \frac{e^x}{1+x}$

$b = 6$, $a = 0$

$h = \frac{6-0}{6} = 1$

$y = \frac{e^x}{1+x}$

x	0	1	2	3	4	5	6
y	1	1.365	2.463	5.021	10.91	24.73	57.63
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$b = 57.63$, $a = 1$

$h = \frac{b-a}{56} = \frac{56.63}{6} = 1.011$

$= 9.438$

$\int_0^6 \frac{e^x}{1+x} = \frac{9.438}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$

$= 4.719 [58.63 + 2 ($

Qs-4 (i) Find the third difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.

(ii) If $f(x) = \frac{1}{x^2}$, find the first divided differences $f(a,b)$; $f(a,b,c)$; $f(a,b,c,d)$

Qs-5 Prove that $\Delta^3\left(\frac{1}{a}\right) = \frac{-1}{abcd}$

x	a	b	c	d
y	$1/a$	$1/b$	$1/c$	$1/d$

Qs-6 Given that $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$; find by the divided difference formula the value of $\log_{10} 656$.

Qs-7 Find $f'(10)$ from the table.

x	3	5	11	27	34
$f(x)$	−13	23	899	17315	35606

(4) $f(x) = x^3 - 2x$

x	2	4	9	10
$f(x)$/y	4	56	711	980

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	2	y_0 4			
			26		
x_1	4	y_1 56		15	
			131		1
x_2	9	y_2 711		23 ~~17.25~~	
			269		
x_3	10	y_3 980			

x	$f(x)$
a	$1/a^2$
b	$1/b^2$
c	$1/c^2$
d	$1/d^2$

$$f(a,b) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1/b^2 - 1/a^2}{b-a}$$

$$= \frac{\frac{a^2-b^2}{b^2a^2}}{b-a}$$

$$= \frac{(a-b)(a+b)}{b^2a^2} \times \frac{1}{(b-a)}$$

$$= \frac{-(a+b)}{b^2a^2}$$

$$f(a,b,c) = \frac{[b,c]-[a,b]}{c-a}$$

$$= \frac{\frac{f(c)-f(b)}{c-b} - \frac{f(b)-f(a)}{b-a}}{c-a}$$

$$= \frac{\frac{\frac{1}{c^2}-\frac{1}{b^2}}{c-b} - \frac{\frac{1}{b^2}-\frac{1}{a^2}}{b-a}}{c-a}$$

$$= \frac{\frac{\frac{b^2-c^2}{c^2b^2}}{c-b} - \frac{\frac{a^2-b^2}{b^2a^2}}{b-a}}{c-a}$$

$$= \frac{\left(\frac{b^2-c^2}{c^2b^2}\right)(b-a) - \left(\frac{a^2-b^2}{b^2a^2}\right)(c-b)}{(c-b)(b-a)(c-a)}$$

$$\frac{\frac{(b-c)(b+c)(b-a)}{c^2b^2} - \frac{(a-b)(a+b)(c-b)}{b^2a^2}}{(c-b)(b-a)(c-a)}$$

$$\frac{a^2(b-c)(b+c)(b-a) - c^2(a-b)(a+b)(c-b)}{(c-b)(b-a)(c-a)\,a^2b^2c^2}$$

(5) T.P. $\Delta^3\left(\frac{1}{a}\right) = \frac{-1}{abcd}$

x	a	b	c	d
y	$1/a$	$1/b$	$1/c$	$1/d$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
a	$\frac{1}{a}$			
		$-1/ab$		
b	$\frac{1}{b}$		$1/abc$	
		$-1/bc$		$-1/abcd$
c	$\frac{1}{c}$		$1/bcd$	
		$-1/cd$		
d	$\frac{1}{d}$			

$$\boxed{\Delta^3\left(\frac{1}{a}\right) = -\frac{1}{abcd}}$$

Prasad

$$\Delta^2 y = [a,b,c]$$

$$\frac{[b,c] - [a,b]}{c-a}$$

$$\frac{\dfrac{\frac{1}{c} - \frac{1}{b}}{c-b} - \dfrac{\frac{1}{b} - \frac{1}{a}}{b-a}}{c-a}$$

$$\frac{-\frac{1}{bc} + \frac{1}{ab}}{c-a}$$

$$\frac{\dfrac{-a+c}{abc}}{c-a}$$

$$\frac{-\frac{1}{cd} + \frac{1}{bc}}{d-b}$$

$$\frac{\dfrac{-b+d}{bcd}}{d-b}$$

$$\frac{\frac{1}{bcd} - \frac{1}{abc}}{d-a}$$

$$\frac{\dfrac{a-d}{abcd}}{d-a}$$

$$-(d-a)$$

$y = \log_{10} x$

x	654	658	659	661
y	2.8156	2.8182	2.8189	2.8202

x	y	Δy
6.54	2.8156	
		6.5×10^{-7}
6.58	2.8182	
659	2.8189	
661	2.8202	

Problems based on Numerical Integration

From the ~~whole~~ table of values x and y obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $f(1.2)$ and $f(2.2)$

x	1.0	1.2	1.4	1.6
y	2.7183	3.3201	4.0552	4.9530

The distance covered by an athlete for 50 ~~meters~~ race is given in the following table.

Time (sec)	0	1	2	3	4	5	6
Distance (meter)	0	2.5	8.5	15.5	24.5	36.5	50

Determine the speed of the athlete at $t=5$ sec correct upto two decimal places.

The table given below reveals the velocity 'v' of a body during the time 't' specified. Find its ~~accl~~ acceleration at $t=1.1$

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

(Ans 45.1667)

forward - backward ⇒ for equal interval

divided difference - non-equal interval

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1.0	2.7183			
		0.6018		
1.2	3.3201		0.1333	
		0.7351		0.0294
1.4	4.0552		0.1627	
		0.8978		
1.6	4.9530			

$x = 1.2$; $x_0 = 1$; $u = \frac{x - x_0}{h} = \frac{1.2 - 1}{0.2} = \frac{0.2}{0.2} = 1$

$$\left[\frac{dy}{dx}\right]_{x=1.2} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^3 y_0\right]$$

$$= \frac{1}{0.2}\left[0.6018 + \frac{1}{2} \times 0.133 - \frac{-1}{6} \times 0.0294\right]$$

$$\frac{1}{0.2}\left[\right. = 3.3175$$

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(13.13)

(2)

Time (sec)	0	1	2	3	4	5	6
Distance (meter)	0	2.5	8.5	15.5	24.5	36.3	50

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	0	2.5	3.5	-2.5	3.5	-3.5	1
1	2.5	6	1	1	0	-2.5	
2	8.5	7	2	1	-2.5		
3	15.5	9	3	-1.5			
4	24.5	12	1.5				
5	36.5	13.5					
6	50						

$x_0 = 0$, $x = 5$

$$u = \frac{x - x_0}{h} = \frac{5}{1} = 5$$

$$f(5) = 0 + 5 \times 2.5 + \frac{5(4)}{2} \times 3.5 + \frac{5(4)(3)}{6} \times (-2.5) + \frac{5(4)(3)(2)}{24} \times 3.5$$

$$+ \frac{5(4)(3)(2)(1)}{120} \times (-3.5) + \frac{5(4)(3)(2)(1)(0)}{720} \times (1)$$

$= 36.6$

"Neem has antibacterial properties which remove pimple-causing bacteria"

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Q — The table below the result of an observation Q is observed temperature in degree centigrade of a vessel of cooling water, t is the time in minutes from the beginning of observation

t	1	3	5	7	9
Q	85.3	74.5	67.0	60.5	54.3

Find approx. rate of cooling at t = 3 and 3.5.

t = 3 → −4.11667 °C/min
t = 3.5 → −3.9151 °C/min

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	85.3				
		−10.8			
3	74.5		3.3		
		−7.5		−2.3	
5	67.0		1.0		1.6
		−6.5		−0.7	
7	60.5		0.3		
		−6.2			
9	54.3				

(i) $x = 3$

$x_0 = 1$

$$u = \frac{x - x_0}{h} = \frac{3-1}{2} = 1$$

Simpson's 1/3 Rule for numerical Integration:

$$\int_{x_0}^{x_0+nh} f(x)dx = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\right] \quad \text{---} (1)$$

Put $n=2$ in eq^n (1) and taking a curve through (x_0,y_0), (x_1,y_1), (x_2,y_2)

$$\int_{x_0}^{x_0+2h} f(x)dx = 2h\left[y_0 + \frac{2}{2}\Delta y_0 + \frac{2(4-3)}{12}\Delta^2 y_0\right]$$

$$= 2h\left[y_0 + (y_1-y_0) + \frac{1}{6}(y_2-2y_1+y_0)\right]$$

$$= \frac{2h}{6}\left[6y_0 + 6y_1 - 6y_0 + y_2 - 2y_1 + y_0\right]$$

$$\int_{x_0}^{x_0+2h} f(x)dx = \frac{h}{3}\left[y_0 + 4y_1 + y_2\right] \quad \text{---} (2)$$

Similarly

$$\int_{x_0+2h}^{x_0+4h} f(x)dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4\right] \quad \text{---} (3)$$

$$\int_{x_0+(n-2)h}^{x_0+nh} f(x)dx = \frac{h}{3}\left[y_{n-2} + 4y_{n-1} + y_n\right] \quad \text{---} (4)$$

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4 \cdots y_{n-2})\right]$$

This is called Simpson's one third formula for numerical integration.

~~Simp~~ Simpson's $\frac{3}{8}$ Rule for Numerical Integration:

$$\int_{x_0}^{x_0+nh} f(x)dx = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\right] \quad \text{--- (1)}$$

Put $n=3$ in eq^n (1) and taking the curve through (x_0,y_0), (x_1,y_1), (x_2,y_2) & (x_3,y_3)

$$\int_{x_0}^{x_0+3h} f(x)dx = 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3(6-3)}{12}\Delta^2 y_0 + \frac{3(3-2)^2}{24}\Delta^3 y_0\right]$$

$$= 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0\right]$$

$$= \frac{3h}{8}\left[8y_0 + 12\Delta y_0 + 6\Delta^2 y_0 + \Delta^3 y_0\right]$$

$$\int_{x_0}^{x_0+3h} f(x)\,dx = \frac{3h}{8}\left[8y_0 + 12(y_1 - y_0) + 6(y_2 - 2y_1 + y_0) + (y_3 - 3y_2 + 3y_1 - y_0)\right]$$

$$= \frac{3h}{8}\left[8y_0 + 12y_1 - 12y_0 + 6y_2 - 12y_1 + 6y_0 + y_3 - 3y_2 + 3y_1 - y_0\right]$$

$$= \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + y_3\right]$$

$$\int_{x_0+3h}^{x_0+6h} f(x)\,dx = \frac{3h}{8}\left[y_3 + 3y_4 + 3y_5 + y_6\right]$$

$$\int_{x_0+(n-3)h}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n\right]$$

Qs-1 Using Simpson's 3/8 th rule on Integration Evaluate

$$\int_0^6 \frac{1}{(1+x)}\,dx$$

Sol: dividing the interval into 6 equal parts i.e $n=6$

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

$$f(x) = \frac{1}{1+x}$$

x	0	1	2	3	4	5	6
$f(x)$	1	$1/2$	$1/3$	$1/4$	$1/5$	$1/6$	$1/7$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By using formula for integration ($\frac{3}{8}$ Rule)

$$\int_0^6 \frac{1}{(1+x)}\,dx = \frac{3}{8}\times 1\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{8}\left[\left(\frac{1}{2}+\frac{1}{7}\right)+3\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{6}\right)+2\left(\frac{1}{4}\right)\right]$$

$= \cancel{1.95}$ 1.96607

Que. Evaluate $\int_0^1 \frac{dx}{(1+x^2)}$ using

(1) simpson's 1/3 rule taking $h = \frac{1}{4}$

(2) simpson's 3/8 rule taking $h = \frac{1}{6}$

also find approximate value of π in each case

(1) $f(x) = \frac{1}{1+x^2}$ $\qquad h = \frac{1}{4}$

$n = 4$

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x)$	1	$16/17$	$4/5$	$16/25$	$1/2$
	y_0	y_1	y_2	y_3	y_4

$\cancel{\int_0^1 f(x)\,dx} =$

$$\int_0^1 \frac{1}{(1+x^2)}dx = \frac{1}{12}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{1}{12}\left[\left(1+\frac{1}{2}\right) + 4\left(\frac{16}{17}+\frac{16}{25}\right) + 2\left(\frac{4}{5}\right)\right]$$

$$\left[\tan^{-1}x\right]_0^1 = 0.785292156$$

$$\frac{\pi}{4} = 0.785392156$$

$$\pi = 4 \times 0.785392156$$

$$\pi = 3.1414$$

Q-3

Q-4

Q-5

Q-6

Q-7

Q-8

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Q-3 $\int_0^1 \frac{dx}{1+x}$ by dividing the interval of integration in 8 equal parts. Hence find $\log_e 2$ approximately. (Ans $\log_e 2 = 0.6931548$)

Q-4 Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5+4\cos x}\,dx$ by using Simpson's $\frac{3}{8}$ rule. (Ans- 0.40219)

Q-5 Evaluate $\int_0^5 \frac{dx}{4x+5}$ by using Simpson's $\frac{1}{3}$ rule, taking 10 equal parts, find the approximate value of $\log_e 5$. (Ans 0.4026)

Q-6 Find $\int_0^{\pi/2} \sqrt{1-\frac{1}{2}\sin^2 t}\,dt$ using one of the method of numerical Integration. (Ans- 1.3496)

Q-7 Find $\int_0^{\pi/2} e^{\sin\theta}\,d\theta$ by Simpson's $\frac{3}{8}$ rule dividing the interval into six equal parts. (Ans- 3.1012)

Q-8 Compute the Integral $\int_4^{5.2} \ln x\,dx$ using the trapezoidal rule. take $h=0.2$ (Ans- 1.8278)

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Qu - 9 A rocket is launched from a ground. Its acceleration is registered during the first 80 seconds and is given as follows

$t(s)$	0	10	20	30	40	50	60	70	80
$a(m/s^2)$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Modified Euler's

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$

$$y_1^{(2)} = y_0 + hf(x_1, y_1^{(1)})$$

$$y_1^{(3)} = y_0 + hf(x_1, y_1^{(2)})$$

$$y_1^{(4)} = y_0 + hf(x_1, y_1^{(3)})$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

Rohan a

Solution of first order ordinary differential equation by Numerical methods:

1. Picard's Method
2. Taylor Series method
3. Euler's Method
4. Modified Euler's Method
5. Runge - Kutta Method

① Picard's Method:

Let an ordinary differential equation of order 1 is $\frac{dy}{dx} = f(x,y)$ with

$$y(x_0) = y_0 \quad \text{——} \quad ①$$

Integrating eqn ① from y_0 to y on left hand side & x_0 to x to right hand side

$$\int_{y_0}^{y} dy = \int_{x_0}^{x} f(x,y)\,dx$$

$$y - y_0 = \int_{x_0}^{x} f(x,y)\,dx$$

$$y = y_0 + \int_{x_0}^{x} f(x,y)\,dx \quad \text{——} \quad ②$$

for first approximation put $y = y_0$ in given diff. eqn and the put in eqn ②

$$y_2 = y_0 + \int_{x_0} f(x,y)\,dx$$

$$\boxed{y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)\, dx}$$

Qe Use Picards Method to obtain y for ~~a=0.2~~ $x = 0.2$ given $\frac{dy}{dx} = x - y$ with initial condition $y = 1$ & $x = 0$.

$$\frac{dy}{dx} = x - y, \quad x_0 = 0, \; y_0 = 1$$

Ist app.

put $y = y_0$ in given eq^n.

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$y_1 = y_0 + \int_{x_0}^{x} (x - y_0)\, dx$$

$$y_1 = 1 + \int_{0}^{x} (x - 1)\, dx$$

$$y_1 = 1 + \frac{x^2}{2} - x$$

$$y_1 = 1 - x + \frac{x^2}{2}$$

at $x = 0.2$

$$y_1 = 1 - (0.2) + \frac{(0.2)^2}{2}$$

$= 0.82$

for second approx.

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$$= y_0 + \int_{x_0}^{x} (x - y_1)\,dx$$

$$= 1 + \int_{x_0}^{x} \left[x - \left\{1 - x + \frac{x^2}{2}\right\}\right] dx$$

$$= 1 + \frac{x^2}{2} - x + \frac{x^2}{2} - \frac{x^3}{6}$$

$$= 1 - x + x^2 - \frac{x^3}{6}$$

put $x = 0.2$

$$= 1 - 0.2 + (0.2)^2 - \frac{(0.2)^3}{6}$$

$$= 0.8386$$

for third approx. put $y = y_2$

$$y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\,dx$$

$$= y_0 + \int_{x_0}^{x} f(x - y_2)\,dx$$

$$= 1 + \int_{x_0}^{x} \left[x - \left\{1 - x + x^2 - \frac{x^3}{6}\right\}\right] dx$$

$$= \left[1 + \left\{\frac{x^2}{2} - x + \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{24}\right\}\right]$$

$$= 1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{24}$$

put $x = 0.2$

$$y_3 = 0.83736$$

for fourth approx. $y = y_3$

$$y_4 = y_0 + \int_{x_0}^{x} f(x, y_3)\, dx$$

$$= y_0 + \int_{x_0}^{x} (x - y_3)\, dx$$

$$= 1 + \int_{x_0}^{x} \left[x - \left\{1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{24}\right\}\right] dx$$

$$= 1 + \frac{x^2}{2} - x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{12} - \frac{x^5}{120}$$

$$y_4 = 1 - x + x^2 + \frac{x^4}{12} - \frac{x^5}{120}$$

$$= 0.8373 \text{ Ans}$$

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Qn Using Picard's Method of successive approximation obtain a solution upto 5th approximation of the equation $\frac{dy}{dx} = x+y$ such that $y=1$ when $x=0$.

Qn If $\frac{dy}{dx} = \frac{y-x}{y+x}$ find the value of y at $x = 0.1$ using Picard's method given that $y(0)=1$

Ans -1.08062

Taylor Series Method.

Taylor series for one variable at $x = x_0$ is given by

$$y(x) = y_0 + \frac{(x-x_0)}{\lfloor 1} y_0' + \frac{(x-x_0)^2}{\lfloor 2} y''_0 + \frac{(x-x_0)^3}{\lfloor 3} y_0''' + \frac{(x-x_0)^4}{\lfloor 4} y_0^{iv} + \text{---}$$

Put $x - x_0 = h$

$$y_1 = y_0 + \frac{h}{\lfloor 1} y_0' + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + \frac{h^4}{\lfloor 4} y_0^{iv} + \text{---}$$

$$y_2 = y_1 + \frac{h}{\lfloor 1} y_1' + \frac{h^2}{\lfloor 2} y_1'' + \frac{h^3}{\lfloor 3} y_1''' + \frac{h^4}{\lfloor 4} y^{iv} + \text{---}$$

$$y_n = y_{n-1} + \frac{h}{\lfloor 1} y'_{n-1} + \frac{h^2}{\lfloor 2} y''_{n-1}$$
$$+ \frac{h^3}{\lfloor 3} y'''_{n-1} + \frac{h^4}{\lfloor 4} y^{iv}_{n-1} + \ldots$$

Q. Solve $\frac{dy}{dx} = x+y$ by Taylor series method start from $x=1, y=0$ and carry to $x=1.2$ with $h=0.1$

Sol. given $y' = x+y$, $x_0=1$, $y_0=0$

$$y'' = 1+y'$$
$$y''' = y''$$
$$y^{iv} = y'''$$

$$y_0' = x_0 + y_0 = 1+0 = 1$$

$$y_0'' = 1+y' = 1+1 = 2$$
$$y_0''' = y'' = 2$$
$$y_0^{iv} = y''' = 2$$

Now by Taylor series expansion we have

$$y_1 = y_0 + \frac{h}{\lfloor 1} y_0' + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + \frac{h^4}{\lfloor 4} y^{iv}$$

$$= 0 + \frac{0.1}{\lfloor 1} \times 1 + \frac{(0.1)^2}{\lfloor 2} \cdot 2 + \frac{(0.1)^3}{\lfloor 3} \cdot 2 + \frac{(0.1)^4}{\lfloor 4} 2$$

$y_1 = 0.1103$

$y_1' = x_1 + y_1 = 1.1 + 0.1103 = 1.2103$

$y_1''' = 1 + y_1' = 1 + 1.2103 = 2.2103$

$y_1^{iv} = y_1''' = 2.2103$

$$y_2 = y_1 + \frac{h}{\lfloor 1} y_1' + \frac{h^2}{\lfloor 2} y_1'' + \frac{h^3}{\lfloor 3} y_1''' + \frac{h^4}{\lfloor 4} y_1^{iv}$$

$$0.1103 + \frac{(0.1)}{\lfloor 1} \times 1.2103 + \frac{(0.1)^2}{2} \times 2.2103 + \frac{(0.1)^3}{\lfloor 3} \times 2.2013$$

$$= \frac{(0.1)^4}{\lfloor 4} \times 2.2103 = 0.24028$$

Q Solve $\frac{dy}{dx} = 2y + 3e^x$ with initial condition $x_0 = 0, y_0 = 1$ by taylor series method find the approx. value of y for $x = 0.1$ & $x = 0.2$ & $h = 0.1$

A
$$\boxed{\begin{aligned} y_1 &= 1.5700 \\ y_2 &= 2.303 \end{aligned}}$$

Euler's Method for finding the solution of first order ODE's

$$\frac{dy}{dx} = f(x,y)$$

$$y(x_0) = y_0$$

formula

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Q $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y=1$ for $x=0$

find the approx. value for $x = 0.1$ by Euler's Method

$h = 0.1 - 0 = 0.1$

$f(x,y) = \frac{y-x}{y+x}$, $x_0 = 0$, $y_0 = 1$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1\left(\frac{y_0 - x_0}{y_0 + x_0}\right)$$

$$y_1 = 1 + 0.1\left(\frac{1-0}{1+0}\right)$$

$$y_1 = 1.1$$

$$\boxed{y(x=0.1) = 1.1}$$

Q Solve the equation $\frac{dy}{dx} = 1-y$ with initial condition $x=0, y=0$ using Euler's Method and ~~Evaluate~~ tabulate the solution at $x=0.1, 0.2, 0.3$

Q Solve the following differential equation using Euler's method from $x=0$ to $x=$ when $h = 0.05$

$$\frac{dy}{dx} + xy = 0 \qquad y(0) = y(0)$$

A
$$\frac{dy}{dx} = 1-y$$

$$x_0 = 0, \quad y_0 = 0$$

$$y_1 = y_0 + hf(x_0, y_0)$$
$$= 0 + (0.1)(1-y_0)$$
$$0.1 \times 1$$
$$y_1(x=0.1) = 0.1$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 0.1 + 0.1(1 - y_1)$$

$$= 0.1 + 0.1(1 - 0.1)$$

$$= 0.1 + 0.1 \times 0.9$$

$$= 0.1 + 0.09$$

$$y_2 = 0.19$$

$$y_2(x = 0.2) = 0.19$$

$$\cancel{y_3(x = 0.3) =} \; 0.19$$

$$y_3(x = 0.3) = y_2 + hf(x_2, y_2)$$

$$= 0.19 + 0.1(1 - 0.19)$$

$$y_3(x = 0.3) = 0.271$$

(3) ~~Solve the following~~

$$h = 0.05, \quad x_0 = 0, \quad y_0 = 1$$

$$x_1 = 0.05, \quad x_2 = 0.1, \quad x_3 = 0.15, \quad x_4 = 0.2$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.05(1 + x_0 y_0)$$

$$y_1 = 1$$

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$y_2 = y_1 + h f(x_1, y_1)$

$= 1 + 0.05\,(f - x_1 y_1)$

$= 1 + 0.05\,(-0.05 \times 1)$

$= 0.9975$

$y_3 = y_2 + 0.05 f(x_2, y_2)$

$= 0.9975 + 0.05\left[-(.1) \times (0.9975)\right]$

$y_3(0.15) = 0.9925$

$y_4 = y_3 + 0.05 f(-x_3 y_3)$

$= 0.9925 + 0.05 \times \left[-(.15) \times 0.9925\right]$

$= 0.985$

~~Q~~ Find $y(0.1)$ using

Modified Euler's Method:

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_2, y_1^{(2)})\right]$$

Find $y(0.1)$ using modified Euler formula

$\frac{dy}{dx} = \log(x+y)$, $y_0 = 1$, $x_0 = 0$

$f(x, y) = \log(x+y)$

$x_0 = 0$

$y_0 = 1$

$x_1 = 0.1$

$h = 0.1$

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1(\log(x_0+y_0))$$

$$= 1 + \frac{0.1}{8}(\log(0+1))$$

$$y_1^{(1)} = 1$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$y_1^{(2)} = 1 + \frac{0.1}{2}\left[\log(x_0 + y_0) + \log(0.1 + 1)\right]$$

$$y_1^{(2)} = 1 + \frac{0.1}{2}\left[\log 1 + \log(1.1)\right]$$

$$= 1.0047$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + \frac{0.1}{2}\left[\log(x_0 + y_0) + (0.1 + 1.0047)\right]$$

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(3)})\right]$$

$$= 1 + \frac{h}{2}\left[\log(x_0 + y_0) + \log(x_1, y_1^{(3)})\right]$$

$$= 1.0049$$

Sol^n Last two of iterations is same 1.0049
Therefore, solution is 1.0049.

Runge Kutta fourth order Method:

Qu-1

There is no need to find the derivative like in Taylor Series Method. In this method we will introduce a new variable k such that

$$k_1 = hf(x_0, y_0)$$
$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$k_4 = hf(x_0 + h, y_0 + k_3)$$

for

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + k$$

for y_2

$$k_1 = hf(x_1, y_1)$$
$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$
$$k_4 = hf(x_1 + h, y_1 + k_3)$$
$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_2 = y_1 + k$$

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0.25 + 1.25
1.5

Q.–) Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{x+y}, \quad x_0 = 0, \ y_0 = 1$$ for all interval $(0, 1)$ choosing $h = 0.5$ by Runge Kutta Method of fourth method.

$$f(x, y) = \frac{1}{x+y}, \ x_0 = 0, \ y_0 = 1, \ y(1)$$ $h = 0.5$

$$x_1 = 0.5, \ x_2 = 1$$

for y_1

$$k_1 = h f(x_0, y_0)$$
$$= 0.5\left(\frac{1}{0+1}\right) = 0.5$$

$$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$= 0.5\left(\frac{1}{\left\{0 + \frac{0.5}{2}\right\} + \left\{1 + \frac{0.5}{2}\right\}}\right)$$
$$= 0.5\left(\frac{1}{1.5}\right) = \frac{1}{3} = 0.333$$

$$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= 0.5\left\{\frac{1}{\left(0 + \frac{0.5}{2}\right) + \left(1 + \frac{0.333}{2}\right)}\right\}$$

$$k_3 = 0.352941$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$= 0.5 \left\{ \frac{1}{(0+0.5) + (1+0.352841)} \right\}$$

$$= 0.269841$$

$$k = \frac{1}{6}\{k_1 + 2k_2 + 2k_3 + k_4\}$$

$$\frac{1}{6}\{0.5 + 2(0.333) + 2(0.352941) + \{0.269841\}\}$$

$$= 0.3570$$

$$\cancel{\boxed{y_1 = 1.3570}}$$

$$\cancel{y_1(0.5) = 1.357}$$

$$y_1 = y_0 + k$$

$$y_1 = 1.3570$$

$$\boxed{y_1(0.5) = 1.3570}$$

$$k_1 = hf(x_1, y_1)$$

$$0.5\left(\frac{1}{0.5 + 1.3570}\right) = 0.2692$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$0.5\left(\frac{1}{\left(0.5 + \frac{0.5}{2}\right) + \left(1.3570 + \frac{0.2692}{2}\right)}\right)$$

$= 0.2230$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.5\left(\frac{1}{\left\{0.5 + \frac{0.5}{2}\right\} + \left\{1.3570 + \frac{0.2230}{2}\right\}}\right)$$

$= 0.2253$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 0.5\left\{\frac{1}{(0.5 + 0.5) + \cancel{(1 + 0.2253)}}\right\}$$

$(1.3570 + 0.2253)$

$= 0.2246$

$$k = \frac{1}{6}\left\{k_1 + 2k_2 + 2k_3 + k_4\right\}$$

$$= \frac{1}{6}\left\{0.2692 + 2(0.2230) + 2(0.2253) + 0.1836\right\}$$

$k = 0.2265$

$y_2 = y_1 + k$

$= 1.3570 + 0.2265$

$y_2 = 1.5835$

$y(1) = 1.5835$

Apply the Range Kutta Method of fourth order to find an approximate value of y at x = 0.2 if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$, in step of $h = 0.1$.

$h = x - x_0$

$0.1 = x - 0$

$x_0 = 0$, $x_1 = 0.1$, $h = 0.1$

$y_0 = 1$, $x_2 = 0.2$

$$k_1 = hf(x_0, y_0)$$

$$= 0.1 \times (0 + 1^2)$$

$$k_1 = 0.1$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.1 \times f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$$

$$0.1 \times \left(0.05 + (1.05)^2\right)$$

$$= 0.1 \times (0.05 +$$

$$= 0.11525$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$\frac{h}{2}$ 1.1

$0.1\, f(0.05, 1.05762)$

$0.1\, ($

$k_3 = 0.1168$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1\, f(\cancel{0}\, 0.1, \cancel{1.0236}\ 1.1168)$

$\cancel{= 0.1\, (}$

$k_4 = \cancel{0.45808}\ \underline{0.13472}$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= \frac{1}{6}(0.1 + 2 \times 0.11525 + 2 \times 0.1168$
$\cancel{+ 0.45808)}$
0.13472

$= \cancel{0.17036}\ 0.11647$

$y_1 = y_0 + k$
$= 1.11647$

$\boxed{y_2 = 1.2734}$

for y_2

$k_1 = hf(x_1, y_1)$

$= 0.1\, f(0.1, 1.11647)$

$= 0.13465$

$$k_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 0.1 f(0.15, 1.183795)$$

$$= 0.1551$$

$$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1 (0.15, 1.19402)$$

$$= 0.15756$$

$$k_4 = h f(x_1 + h, y_1 + k_3)$$

$$= 0.1 f(0.2, 1.27403)$$

$$= 0.1823$$

$$k = \frac{1}{6}\{k_1 + 2k_2 + 2k_3 + k_4\}$$

$$k = 0.157045$$

$$y_2 = y_1 + k$$

$$= 1.273515$$

~~x_0~~ $y_n = x - x_e$

$= 1.5 - 0.1$

(3) $\frac{dy}{dx}$

Q Using Euler's Method, find the approximate value of y at $x = 1.5$ taking $h = 0.1$

$$\frac{dy}{dx} = \frac{y - x}{\sqrt{xy}} \text{ and } y(1) = 2$$

Q Using Euler's Method, find the approximate value of y at $x = 1$ taking $h = 0.2$

$$\frac{dy}{dx} = x^2 + y^2 \text{ and } y(0) = 1$$

Q Solve $\frac{dy}{dx} = x + \sqrt{xy}$ with $x_0 = 1.2$, $y = 1.6403$ by Euler's modified method for $x = 1.6$ correct upto four decimal places by taking $h = 0.2$.

Q Taylor Series Method.

$\frac{dy}{dx} = y - xy$ with $x_0 = 0$, $y_0 = 2$, $h = 0.1$. Find the value of $x = 0.2$

Q $\frac{dy}{dx} = e^x - y^2$ with $y(0) = 1$ at $x = 0.1$

$x_0 = 1$ $\quad h = 0.1$

$y_0 = 2$

$y_1 = y_0 + hf(x_0, y_0)$

$= 2 + 0.1 \times \left(\frac{2-1}{\sqrt{2}}\right)$

$= 2 + \frac{0.1}{\sqrt{2}}$

$y_1 = 2.0707$

(2)

$y_2 = y_1 + hf(x_1, y_1)$

$= 2.0707 + 0.1 \left(\frac{2.0707 - 1.1}{\sqrt{2.0707 \times 1.1}}\right)$

~~2.1302~~ 2.1350

$y_3 = y_2 + hf(x_2, y_2)$

$= 2.0707 + 0.1 \left(\frac{2.1350 - 1.2}{\sqrt{2.1350 \times 1.2}}\right)$

$= 2.1934$

$y_4 = y_3 + hf(x_3, y_3)$

$2.1934 + 0.1 \times \left(\frac{2.1934 - 1.3}{\sqrt{2.1934 \times 1.3}}\right)$

$y_4 = 2.246$

$y_5 = y_4 + hf(x_4, y_4)$

$= 2.246 + 0.1 \times \left(\frac{2.246 - 1.4}{\sqrt{2.246 \times 1.4}}\right)$

$= 2.293$

~~$y_6 = 2.293 + 0.1 \times \left(\frac{2.293 - 1.5}{\sqrt{2.293 \times 1.5}}\right)$~~

(2) $\frac{dy}{dx} = x^2 + y^2$

$y(0) = 1$

$x_0 = 0$

$y_0 = 1$

$h = 0.2$

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.2 \times (0 + 1)$

$y_1 = 1.2$

$y_2 = 1.2 + 0.2\left((0.2)^2 + (1.2)^2\right)$

$= 1.496$

$y_3 = 1.496 + 0.2\left((0.4)^2 + (1.496)^2\right)$

$= 1.9756$

$y_4 = 1.8256 + 0.2((0.6)^2 + (1.8256)^2)$

$\cancel{20.28} = \cancel{28.28}\ 2.6266$

$y_5 = 4.555$

-3 $\frac{dy}{dx} = 2 + \sqrt{xy}$

$y_1^{(1)} = y_0 + hf(x_0, y_0)$

$y_1^{(2)} = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_1, y_1^{(1)})\}$

$y_1^{(3)} = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_1, y_1^{(2)})\}$

$x_0 = 1.2, \; y_0 = 1.6403, \; x_1 = 1.4, \; x_2 = 1.6$

$y_1^{(1)} = 1.6403 + 0.2(\underbrace{2 + \sqrt{1.2 \times 1.6403}}_{3.402})$

$y_1^{(1)} = \cancel{1.9208}\ 2.3208$

$y_1^{(2)} = 1.6403 + \frac{0.2}{2}\{3.402 + 3.802\}$

$y_1^{(2)} = 2.3607$

$y_1^{(3)} = \cancel{1.603}\ 1.6403 + 0.1\{3.402 +$

$x|y, \; y|z, \; x|z$

$x|y \quad y|x$

$x \geq y$

Partial

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(ii) Antisy

(iii) Tr

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