2.1. MOMENTS

Moments are statistical tools, used in statistical investigations. The moments of distribution are the arithmetic means of the various powers of the deviations of items from given number.

2.2. MOMENTS ABOUT MEAN (Central Moments)

2.2.1. For an Individual Series

If $x_1, x_2, \ldots, x_n$ are the values of the variable under consideration, the r^{th} moment about mean $\bar{x}$ is defined as

$$\mu_r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^r}{n} \quad ; r = 0, 1, 2, \ldots$$

2.2.2. For a Frequency Distribution

If $x_1, x_2, \ldots, x_n$ are the values of a variable x with the corresponding frequencies $f_1, f_2, \ldots, f_n$ respectively then r^{th} moment μ_r about the mean $\bar{x}$ is defined as

$$\mu_r = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^r}{N} \quad ; r = 0, 1, 2, \ldots \quad \text{where} \quad N = \sum_{i=1}^{n} f_i$$

In particular, $$\mu_0 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^0 = \frac{1}{N}\sum_{i=1}^{n} f_i = \frac{N}{N} = 1$$

$\therefore$ For any distribution, $\boxed{\mu_0 = 1}$

For $r = 1$,

$$\mu_1 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i - \bar{x}\left(\frac{1}{N}\sum_{i=1}^{n} f_i\right) = \bar{x} - \bar{x} = 0$$

$\therefore$ For any distribution, $\boxed{\mu_1 = 0}$

For $r = 2$,

$$\mu_2 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2 = (\text{S.D.})^2 = \text{Variance}$$

$\therefore$ For any distribution, μ_2 coincides with the variance of the distribution.

Similarly,
$$\mu_3 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^3,\ \mu_4 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^4$$

and so on.

Note. In case of a frequency distribution with class intervals, the values of x are the mid-points of the intervals.

EXAMPLES

Example 1. *Find the first four moments for the following individual series:*

x	*3*	*6*	*8*	*10*	*18*

Sol. **Calculation of Moments**

S. No.	x	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^3$	$(x - \bar{x})^4$
1	3	– 6	36	– 216	1296
2	6	– 3	9	– 27	81
3	8	– 1	1	– 1	1
4	10	1	1	1	1
5	18	9	81	729	6561
$n = 5$	$\Sigma x = 45$	$\Sigma(x - \bar{x}) = 0$	$\Sigma(x - \bar{x})^2 = 128$	$\Sigma(x - \bar{x})^3 = 486$	$\Sigma(x - \bar{x})^4 = 7940$

Now,
$$\bar{x} = \frac{\Sigma x}{n} = \frac{45}{5} = 9$$

$$\therefore \quad \mu_1 = \frac{\Sigma(x - \bar{x})}{n} = \frac{0}{5} = 0$$

$$\mu_2 = \frac{\Sigma(x - \bar{x})^2}{n} = \frac{128}{5} = 25.6$$

$$\mu_3 = \frac{\Sigma(x - \bar{x})^3}{n} = \frac{486}{5} = 97.2$$

$$\mu_4 = \frac{\Sigma(x - \bar{x})^4}{n} = \frac{7940}{5} = 1588.$$

Example 2. *Calculate $\mu_1, \mu_2, \mu_3, \mu_4$ for the following frequency distribution:*

Marks	*5–15*	*15–25*	*25–35*	*35–45*	*45–55*	*55–65*
No. of students	*10*	*20*	*25*	*20*	*15*	*10*

Sol.

Calculation of Moments

Marks	No. of students (f)	Mid-point (x)	fx	$x - \bar{x}$	$f(x - \bar{x})$	$f(x - \bar{x})^2$	$f(x - \bar{x})^3$	$f(x - \bar{x})^4$
5–15	10	10	100	– 24	– 240	5760	– 138240	3317760
15–25	20	20	400	– 14	– 280	3920	– 54880	768320
25–35	25	30	750	– 4	– 100	400	– 1600	6400
35–45	20	40	800	6	120	720	4320	25920
45–55	15	50	750	16	240	3840	61440	983040
55–65	10	60	600	26	260	6760	175760	4569760
	N = 100		Σfx = 3400		$\Sigma f(x - \bar{x})$ = 0	$\Sigma f(x - \bar{x})^2$ = 21400	$\Sigma f(x - \bar{x})^3$ = 46800	$\Sigma f(x - \bar{x})^4$ = 9671200

Now, $\bar{x} = \dfrac{\Sigma fx}{N} = \dfrac{3400}{100} = 34$

$\therefore$

$$\mu_1 = \frac{\Sigma f(x - \bar{x})}{N} = \frac{0}{100} = 0, \qquad \mu_2 = \frac{\Sigma f(x - \bar{x})^2}{N} = \frac{21400}{100} = 214$$

$$\mu_3 = \frac{\Sigma f(x - \bar{x})^3}{N} = \frac{46800}{100} = 468, \qquad \mu_4 = \frac{\Sigma f(x - \bar{x})^4}{N} = \frac{9671200}{100} = 96712.$$

2.3. SHEPPARD'S CORRECTIONS FOR MOMENTS

While computing moments for frequency distribution with class intervals, we take variable x as the mid-point of class-intervals which means that we have assumed the frequencies concentrated at the mid-points of class-intervals.

The above assumption is true when the distribution is symmetrical and the no. of class-intervals is not greater than $\dfrac{1}{20}$th of the range, otherwise the computation of moments will have certain error called **grouping error.**

This error is corrected by the following formulae given by **W.F. Sheppard.**

$$\mu_2 \text{ (corrected)} = \mu_2 - \frac{h^2}{12}$$

$$\mu_4 \text{ (corrected)} = \mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}h^4$$

where h is the width of the class-interval while μ_1 and μ_3 require no correction.

These formulae are known as **Sheppard's corrections.**

Example 3. *Find the corrected values of the following moments using Sheppard's correction. The width of classes in the distribution is 10 :*

$\mu_2 = 214,\quad \mu_3 = 468,\quad \mu_4 = 96712.$

Sol. We have $\mu_2 = 214,\quad \mu_3 = 468,\quad \mu_4 = 96712,\quad h = 10.$

Now, $\mu_2 \text{ (corrected)} = \mu_2 - \dfrac{h^2}{12} = 214 - \dfrac{(10)^2}{12} = 214 - 8.333 = 205.667.$

$\mu_3 \text{ (corrected)} = \mu_3 = 468$

$$\mu_4 \text{ (corrected)} = \mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}h^4 = 96712 - \frac{(10)^2}{2}(214) + \frac{7}{240}(10)^4$$
$$= 96712 - 10700 - 291.667 = 86303.667.$$

2.4. MOMENTS ABOUT AN ARBITRARY NUMBER (Raw Moments)

If $x_1, x_2, x_3, \ldots, x_n$ are the values of a variable x with the corresponding frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively then r^{th} moment μ_r' about the number $x = A$ is defined as

$$\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^r \quad ; r = 0, 1, 2, \ldots \quad \text{where,} \quad N = \sum_{i=1}^{n} f_i$$

For $r = 0$, $$\mu_0' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^0 = 1$$

For $r = 1$, $$\mu_1' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i - \frac{A}{N}\sum_{i=1}^{n} f_i = \bar{x} - A$$

For $r = 2$, $$\mu_2' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^2$$

For $r = 3$, $$\mu_3' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^3 \text{ and so on.}$$

In calculation work, if we find that there is some common factor h (> 1) in values of $x - A$, we can ease our calculation work by defining $u = \frac{x - A}{h}$. In that case, we have

$$\mu_r' = \frac{1}{N}\left(\sum_{i=1}^{n} f_i u_i^r\right) h^r \; ; r = 0, 1, 2, \ldots$$

Note. For an individual series,

1. $$\mu_r' = \frac{1}{n}\sum_{i=1}^{n} (x_i - A)^r \quad ; r = 0, 1, 2, \ldots$$

2. $$\mu_r' = \frac{1}{N}\left(\sum_{i=1}^{n} u_i^r\right) h^r \; ; r = 0, 1, 2, \ldots \quad \Bigg| \text{ for } u = \frac{x - A}{h}$$

2.5. MOMENTS ABOUT THE ORIGIN

If $x_1, x_2, \ldots, x_n$ be the values of a variable x with corresponding frequencies $f_1, f_2, \ldots, f_n$ respectively then r^{th} moment about the origin ν_r is defined as

$$v_r = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^{\,r}\ ;\ r = 0, 1, 2, \ldots \quad \text{where,} \quad N = \sum_{i=1} f_i$$

For $r = 0$, $$v_0 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^{\,0} = \frac{N}{N} = 1$$

For $r = 1$, $$v_1 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \bar{x}$$

For $r = 2$, $$v_2 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^{\,2} \quad \text{and so on.}$$

2.6. RELATION BETWEEN μ_r AND μ_r'

We know that,

$$\mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i\,[(x_i - A) - (\bar{x} - A)]^r$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i\,[(x_i - A) - \mu_1']^r \qquad \mid \because \ \mu_1' = \bar{x} - A$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i\,[(x_i - A)^r - {}^rc_1 (x_i - A)^{r-1}\mu_1' + {}^rc_2 (x_i - A)^{r-2}\mu_1'^2 - \ldots\ldots + (-1)^r \mu_1'^r]$$

| using binomial theorem

$$\Rightarrow \qquad \mu_r = \mu_r' - {}^rc_1\,\mu_{r-1}'\mu_1' + {}^rc_2\,\mu_{r-2}'\mu_1'^2 - \ldots\ldots + (-1)^r \mu_1'^r$$

Putting, $r = 2, 3, 4$, we get

$$\mu_2 = \mu_2' - 2\mu_1'^2 + \mu_1'^2 = \mu_2' - \mu_1'^2 \qquad \mid \because \ \mu_0' = 1$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 3\mu_1'^3 - \mu_1'^3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Hence we have the following relations:

$$\boxed{\mu_1 = 0}$$

$$\boxed{\mu_2 = \mu_2' - \mu_1'^2}$$

$$\boxed{\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}$$

and

$$\boxed{\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4}$$

2.7. RELATION BETWEEN ν_r AND μ_r

We know that,

$$\nu_r = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^r \quad ; r = 0, 1, 2, \ldots$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A + A)^r$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i [(x_i - A)^r + {}^r c_1 (x_i - A)^{r-1} . A + \ldots + A^r]$$

$$= \mu_r' + {}^r c_1 \mu_{r-1} A + \ldots + A^r$$

If we take, $A = \bar{x}$ (for μ_r) then

$$\nu_r = \mu_r + {}^r c_1 \mu_{r-1}\bar{x} + {}^r c_2 \mu_{r-2}\bar{x}^2 + \ldots + \bar{x}^r \qquad \ldots(1)$$

Putting, $r = 1, 2, 3, 4$ in (1), we get

$$\nu_1 = \mu_1 + \mu_0 \bar{x} = \bar{x} \qquad | \because \mu_1 = 0, \mu_0 = 1$$

$$\nu_2 = \mu_2 + {}^2c_1\mu_1\bar{x} + {}^2c_2\,\mu_0\bar{x}^2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + {}^3c_1\mu_2\bar{x} + {}^3c_2\mu_1\bar{x}^2 + {}^3c_3\mu_0\bar{x}^3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + {}^4c_1\mu_3\bar{x} + {}^4c_2\mu_2\bar{x}^2 + {}^4c_3\mu_1\bar{x}^3 + {}^4c_4\mu_0\bar{x}^4$$

$$= \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Hence we have the following relations:

$$\boxed{\nu_1 = \bar{x}}$$

$$\boxed{\nu_2 = \mu_2 + \bar{x}^2}$$

$$\boxed{\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3}$$

and

$$\boxed{\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4.}$$

2.8. KARL PEARSON'S β AND γ COEFFICIENTS

Karl Pearson defined the following four coefficients based upon the first four moments of a frequency distribution about its mean :

$$\left.\begin{aligned} \beta_1 &= \frac{\mu_3^2}{\mu_2^3} \\ \beta_2 &= \frac{\mu_4}{\mu_2^2} \end{aligned}\right\} \quad (\beta\text{-coefficients})$$

$$\left.\begin{aligned} \gamma_1 &= +\sqrt{\beta_1} \\ \gamma_2 &= \beta_2 - 3 \end{aligned}\right\} \quad (\gamma\text{-coefficients})$$

The practical use of these coefficients is to measure the skewness and kurtosis of a frequency distribution. These coefficients are pure numbers independent of units of measurement.

EXAMPLES

Example 1. *The first three moments of a distribution, about the value '2' of the variable are 1, 16 and – 40. Show that the mean is 3, variance is 15 and $\mu_3 = -86$.*

Sol. We have $A = 2,\quad \mu_1' = 1,\quad \mu_2' = 16,\quad$ and $\quad \mu_3' = -40$

We know that $\mu_1' = \bar{x} - A \Rightarrow \bar{x} = \mu_1' + A = 1 + 2 = 3$

$$\text{Variance} = \mu_2 = \mu_2' - \mu_1'^2 = 16 - (1)^2 = 15$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = -40 - 3(16)(1) + 2(1)^3 = -40 - 48 + 2 = -86$$

Example 2. *The first four moments of a distribution, about the value '35' are – 1.8, 240, – 1020 and 144000. Find the values of $\mu_1, \mu_2, \mu_3, \mu_4$.*

Sol.
$$\mu_1 = 0.$$
$$\mu_2 = \mu_2' - \mu_1'^2 = 240 - (-1.8)^2 = 236.76$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = -1020 - 3(240)(-1.8) + 2(-1.8)^3 = 264.36$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 144000 - 4(-1020)(-1.8) + 6(240)(-1.8)^2 - 3(-1.8)^4 = 141290.11$$

Example 3. *Calculate the variance and third central moment from the following data:*

x_i	0	1	2	3	4	5	6	7	8
f_i	1	9	26	59	72	52	29	7	1

(U.P.T.U. 2008)

Sol.

Calculation of Moments

x	f	$u = \dfrac{x - A}{h}$ $A = 4, h = 1$	fu	fu^2	fu^3
0	1	– 4	– 4	16	– 64
1	9	– 3	– 27	81	– 243
2	26	– 2	– 52	104	– 208
3	59	– 1	– 59	59	– 59
4	72	0	0	0	0
5	52	1	52	52	52
6	29	2	58	116	232
7	7	3	21	63	189
8	1	4	4	16	64
	$N = \Sigma f = 256$		$\Sigma fu = -7$	$\Sigma fu^2 = 507$	$\Sigma fu^3 = -37$

Now, moments about the point $x = A = 4$ are

$$\mu_1' = \left(\frac{\Sigma fu}{N}\right) h = \frac{-7}{256} = -0.02734$$

$$\mu_2' = \left(\frac{\Sigma fu^2}{N}\right) h^2 = \frac{507}{256} = 1.9805$$

$$\mu_3' = \left(\frac{\Sigma fu^3}{N}\right)h^3 = \frac{-37}{256} = -0.1445$$

Moments about mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 1.9805 - (-.02734)^2 = 1.97975$$

$\therefore$ Variance $= 1.97975$

Also,

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$= (-0.1445) - 3(1.9805)(-.02734) + 2(-.02734)^3$$
$$= 0.0178997$$

$\therefore$ Third central moment $= 0.0178997$.

Example 4. *The first three moments of a distribution about the value 2 of the variable are 1, 16 and – 40 respectively. Find the values of the first three moments about the origin.*

Sol. We have $A = 2, \quad \mu_1' = 1, \quad \mu_2' = 16, \quad \mu_3' = -40$

$\therefore$

$$\nu_1 = \bar{x} = A + \mu_1' = 2 + 1 = 3$$
$$\nu_2 = \mu_2 + \bar{x}^2 = 15 + (3)^2 = 24$$
$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = -86 + 3(15)(3) + (3)^3 = 76.$$

Example 5. *The first four moments of a distribution about x = 2 are 1, 2.5, 5.5 and 16. Calculate the first four moments about the mean and about origin.*

Sol. We have $A = 2, \quad \mu_1' = 1, \quad \mu_2' = 2.5, \quad \mu_3' = 5.5, \quad \mu_4' = 16.$

Moments about mean

$$\mu_1 = 0$$
$$\mu_2 = \mu_2' - (\mu_1')^2 = 2.5 - (1)^2 = 1.5$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = 5.5 - 3(2.5)(1) + 2(1)^3 = 0$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 16 - 4(5.5)(1) + 6(2.5)(1)^2 - 3(1)^4 = 6.$$

Moments about origin

$$\nu_1 = \bar{x} = A + \mu_1', \qquad \nu_2 = \mu_2 + \bar{x}^2$$
$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3, \qquad \nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$\therefore$

$$\nu_1 = \bar{x} = 2 + 1 = 3, \qquad \nu_2 = 1.5 + (3)^2 = 10.5$$
$$\nu_3 = 0 + 3(1.5)(3) + (3)^3 = 40.5, \qquad \nu_4 = 6 + 4(0)(3) + 6(1.5)(3)^2 + (3)^4 = 168.$$

Example 6. *For a distribution, the mean is 10, variance is 16, γ_1 is 1, and β_2 is 4. Find the first four moments about the origin.*

Sol. $\bar{x} = 10, \quad \mu_2 = 16, \quad \gamma_1 = 1, \quad \beta_2 = 4$ | given

Now, $\gamma_1 = 1$ | $\because \gamma_1 = \sqrt{\beta_1}$

$\Rightarrow \quad \beta_1 = 1$

$$\Rightarrow \quad \frac{\mu_3^2}{\mu_2^3} = 1 \quad \Rightarrow \quad \mu_3^2 = \mu_2^3 = (16)^3 = (64)^2$$

$\Rightarrow \quad \mu_3 = 64$

and $\quad \beta_2 = 4$ | $\because \mu_2 = 16$

$$\Rightarrow \quad \frac{\mu_4}{\mu_2^2} = 4 \quad \Rightarrow \quad \mu_4 = 4(16)^2 = 1024$$

Moments about the origin

$$v_1 = \bar{x} = 10$$
$$v_2 = \mu_2 + \bar{x}^2 = 16 + 100 = 116$$
$$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = 64 + 480 + 1000 = 1544$$
$$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4 = 1024 + 4(64)(10) + 6(16)(100) + (10)^4$$
$$= 22184$$

Example 7. *In a certain distribution, the first four moments about the point x = 4 are – 1.5, 17, – 30 and 308. Calculate* β_1 *and* β_2.

Sol. We have, $A = 4, \mu'_1 = -1.5, \mu'_2 = 17, \mu'_3 = -30, \mu'_4 = 308$

Moments about mean

$$\mu_1 = 0$$
$$\mu_2 = \mu'_2 - \mu_1'^2 = 17 - (-1.5)^2 = 14.75$$
$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3 = -30 - 3(17)(-1.5) + 2(-1.5)^3 = 39.75$$
$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4$$
$$= 308 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4 = 342.3125$$

Calculation of β_1 and β_2

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0.492377$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 1.573398$$

Example 8. *The first four moments of a distribution about the value '4' of the variable are – 1.5, 17, – 30 and 108. Find the moments about mean, about origin ;* β_1 *and* β_2. *Also find the moments about the point x = 2.* (U.P.T.U. 2007)

Sol. We have $A = 4, \mu'_1 = -1.5, \mu'_2 = 17, \mu'_3 = -30, \mu'_4 = 108$

Moments about mean

$$\mu_1 = 0$$
$$\mu_2 = \mu'_2 - \mu_1'^2 = 14.75$$
$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3 = 39.75$$
$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4 = 142.3125$$

Also, $\bar{x} = \mu'_1 + A = -1.5 + 4 = 2.5$

Moments about origin

$$v_1 = \bar{x} = 2.5$$
$$v_2 = \mu_2 + \bar{x}^2 = 14.75 + (2.5)^2 = 21$$
$$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = 166$$
$$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4 = 1132$$

Calculation of β_1 and β_2

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0.492377$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 0.654122$$

Moments about the point x = 2

$$\mu'_1 = \bar{x} - A = 2.5 - 2 = 0.5$$
$$\mu'_2 = \mu_2 + \mu_1'^2 = 14.75 + (.5)^2 = 15$$
$$\mu'_3 = \mu_3 + 3\mu'_2\mu'_1 - 2\mu_1'^3$$
$$= 39.75 + 3(15)(.5) - 2(.5)^3 = 62$$
$$\mu'_4 = \mu_4 + 4\mu'_3\mu'_1 - 6\mu'_2\mu_1'^2 + 3\mu_1'^4 = 244$$

ASSIGNMENT

1. (*i*) Calculate first four moments about mean, for the following individual series:
   5, 5, 5, 5, 5, 5.

   (*ii*) Find the first four moments about the mean of the following series:
   1, 3, 7, 9, 10.

   (*iii*) Calculate μ_1, μ_2, μ_3, μ_4 for the series : 4, 7, 10, 13, 16, 19, 22.

2. (*i*) Find the first four moments for the following frequency distribution:

x	1	2	3	4	5	6	7	8	9
f	1	2	3	4	5	4	3	2	1

   (*ii*) Calculate the first four moments of the following distribution about the mean and hence find β_1 and β_2.

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

3. (*i*) Find the first four moments about mean for the following frequency distribution:

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	5	10	40	20	25

   (*ii*) Calculate the first four moments about mean for the following frequency distribution:

Classes	5–15	15–25	25–35	35–45	45–55
f	14	22	36	18	10

4. Calculate the first four moments about $x = 15$ and hence find the moments about the mean of the following distribution:

x	10	11	12	13	14	15	16	17	18	19	20	21
f	9	36	75	105	116	107	88	66	45	30	18	5

5. (*i*) The first three moments of a distribution about the value 4 of the variable are 1.5, 17 and – 30. Find the moments about mean.

   (*ii*) The first four moments of a distribution about $x = 4$ are 1, 4, 10 and 45. Show that the mean is 5, the variance is 3, μ_3 is 0 and μ_4 is 26.

   (*iii*) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean, variance, μ_3 and μ_4.

$$v_2 = \left[\frac{d^2}{dt^2} M_o(t)\right]_{t=0} = p\left[\frac{d}{dt}\left\{\frac{e^t}{(1-qe^t)^2}\right\}\right]_{t=0}$$

$$= p\left[\frac{(1-qe^t)^2 . e^t - e^t . 2(1-qe^t)(-qe^t)}{(1-qe^t)^4}\right]_{t=0}$$

$$= p\left[\frac{(1-q)^2 + 2q(1-q)}{(1-q)^4}\right] = \frac{1}{p} + \frac{2q}{p^2}$$

$$\text{Mean} = \bar{x} = v_1 = \frac{1}{p}$$

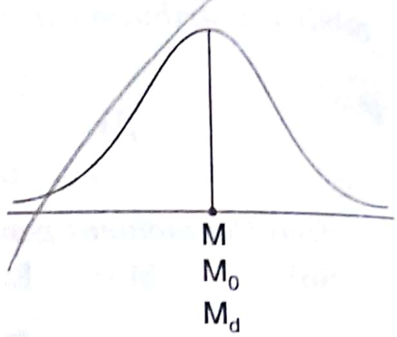
$$\text{Variance} = \mu_2 = v_2 - \bar{x}^2 = \frac{1}{p} + \frac{2q}{p^2} - \frac{1}{p^2} = \frac{q}{p^2}.$$

2.10. SKEWNESS

(U.P.T.U. 2006)

For a symmetrical distribution, the frequencies are symmetrically distributed about the mean *i.e.*, variates equidistant from the mean have equal frequencies. Also, the mean, mode and median coincide and median lies half-way between the two quartiles.

Thus, $M = M_0 = M_d$ and $Q_3 - M = M - Q_1$.



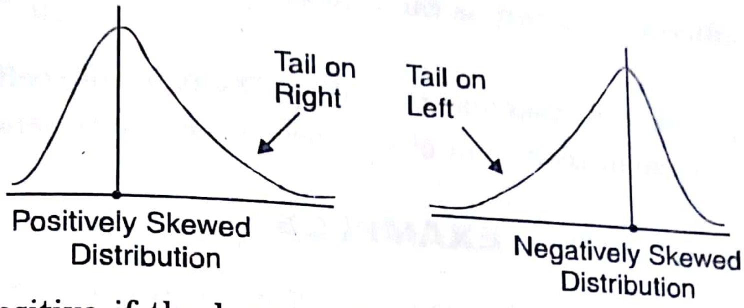
Symmetrical distribution

2.11. MEANING OF SKEWNESS

(U.P.T.U. 2006)

If the curve of the distribution is not symmetrical, it may admit of tail on either side of the distribution. **Skewness means lack of symmetry or lopsidedness in a frequency distribution.**

The object of measuring skewness is to estimate the extent to which a distribution is distorted from a perfectly symmetrical distribution. Skewness indicates whether the curve is turned more to one side than to other *i.e.*, whether the curve has a longer tail on one side. Skewness can be positive as well as negative.



Skewness is positive if the longer tail of the distribution lies towards the right and negative if it lies towards the left.

2.12. TESTS OF SKEWNESS

1. If A.M. = Mode = Median, then there is no skewness in the distribution. In other words, the curve of the frequency distribution would be symmetrical, bell-shaped.
2. If A.M. is less than (greater than), the value of mode, the tail would be on left (right) side, *i.e.*, the distribution is negatively (positively) skewed.
3. If sum of frequencies of values less than mode is equal to the sum of frequencies of values greater than mode, then there would be no skewness.
4. If quartiles are equidistant from median, then there would be no skewness.

2.13. METHODS OF MEASURING SKEWNESS (*U.P.T.U. 2007*)

Relative measures of skewness are called the **coefficient of skewness**. They are independent of the units of measurement and as such, they are pure numbers.

Following are the methods of measuring skewness:

1. Karl Pearson's Method
2. Bowley's Method
3. Kelly's Method
4. Method of Moments.

Here, we will discuss only the method of moments.

2.14. METHOD OF MOMENTS

In this method, second and third central moments of the distribution are used. This measure of skewness is called the **Moment coefficient of skewness** and is given by:

$$\boxed{\textbf{Moment coefficient of skewness} = \frac{\mu_3}{\sqrt{\mu_2{}^3}}.}$$

[*G.B.T.U. (C.O.) 2009, 2011*]

For a symmetrical distribution, its value would come out to be zero. The coefficient of skewness as calculated by this method gives the magnitude as well as direction of the skewness

Scanned by CamScanner

In Statistics, we define $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$.

$\therefore$ Moment coefficient of skewness can also be written as $= \frac{\mu_3}{\sqrt{\mu_2^3}} = \pm\sqrt{\beta_1}$.

The sign with $\sqrt{\beta_1}$ is to be taken as that of μ_3. The moment coefficient of skewness is also denoted by γ_1. The moment coefficient of skewness is generally denoted by 'SK_M'.

EXAMPLES

Example 1. *The first three central moments of a distribution are 0, 15, – 31. Find the moment coefficient of skewness.*

Sol. We have $\mu_1 = 0, \mu_2 = 15$ and $\mu_3 = -31$

$$\text{Moment coefficient of skewness} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-31}{\sqrt{(15)^3}} = -\frac{31}{58.09} = -0.53.$$

Example 2. *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moment coefficient of skewness.*

Sol. We have $A = 5, \mu_1' = 2, \mu_2' = 20, \mu_3' = 40$ and $\mu_4' = 50$

Now $\mu_2 = \mu_2' - (\mu_1')^2 = 20 - (2)^2 = 16$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 = 40 - 3(2)(20) + 2(2)^3$$
$$= 40 - 120 + 16 = -64$$

$$\text{Moment coefficient of skewness} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-64}{\sqrt{(16)^3}} = \frac{-64}{64} = -1.$$

Example 3. *Calculate the moment coefficient of skewness for the following distribution:*

Classes	*2.5–7.5*	*7.5–12.5*	*12.5–17.5*	*17.5–22.5*	*22.5–27.5*	*27.5–32.5*	*32.5–37.5*
Frequency	*8*	*15*	*20*	*32*	*23*	*17*	*5*

Sol. **Calculation of Moment Coefficient of Skewness**

Classes	*f*	*Mid-pts.* x	$d = x - A$, $A = 20$	$u = d/h$, $h = 5$	*fu*	fu^2	fu^3
2.5–7.5	8	5	– 15	– 3	– 24	72	– 216
7.5–12.5	15	10	– 10	– 2	– 30	60	– 120
12.5–17.5	20	15	– 5	– 1	– 20	20	– 20
17.5–22.5	32	20	0	0	0	0	0
22.5–27.5	23	25	5	1	23	23	23
27.5–32.5	17	30	10	2	34	68	136
32.5–37.5	5	35	15	3	15	45	135
	N = 120				$\Sigma fu = -2$	$\Sigma fu^2 = 288$	$\Sigma fu^3 = -62$

Now, $\mu_1' = \left(\frac{\Sigma fu}{N}\right)h = \left(\frac{-2}{120}\right) 5 = -0.083$

$$\mu_2' = \left(\frac{\Sigma fu^2}{N}\right)h^2 = \left(\frac{288}{120}\right) 5^2 = 60$$

$$\mu_3' = \left(\frac{\Sigma fu^3}{N}\right)h^3 = \left(\frac{-62}{162}\right) 5^3 = -64.583$$

Now, $\mu_2 = \mu_2' - \mu_1'^2 = 60 - (-0.083)^2 = 59.993$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 = -64.583 - 3(-0.083)(60) + 2(-0.083)^3$$
$$= -49.644.$$

$\therefore$ Moment coefficient of skewness $= \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-49.644}{\sqrt{(59.993)^3}} = -0.1068.$

ASSIGNMENT

1. The first-three central moments of a distribution are 0, 2.5, 0.7. Find the value of the moment coefficient of skewness.
2. In a certain distribution, the first four moments about the point 4 are – 1.5, 17, – 30 and 308. Calculate the moment coefficient of skewness.
3. The first three moments of a frequency distribution about origin '5' are – 0.55, 4.46 and – 0.43. Find the moment coefficient of skewness.
4. Calculate the moment coefficient of skewness for the following data:

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70
No. of students	8	12	20	30	15	10	5

5. Calculate the moment coefficient of skewness from the following data:

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

6. For the following frequency distribution, find the first four moments about the mean. Also find the value of β_1. Is it a symmetrical distribution?

x	2	3	4	5	6
f	1	3	7	3	1

[G.B.T.U. (C.O.) 2009]

7. Compute the coefficient of skewness from the following data:

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

(U.P.T.U. 2006)

8. (*i*) Define skewness of a distribution. [G.B.T.U. (C.O.) 2009, 2011; U.P.T.U. 2007]
   (*ii*) Define the coefficients of skewness.

Answers

1. 0.1771 2. 0.7017 3. 0.7781 4. 0.0726
5. 0 6. 0, 0.933, 0, 2.533, Yes 7. 0.0903.

2.15. KURTOSIS

[U.P.T.U. (C.O.) 2008; U.P.T.U. 200...]

Given two frequency distributions which have the same variability as measured by the standard deviation, they may be relatively more or less flat topped than the normal curve. A frequency curve may be symmetrical but it may not be equally flat topped with the normal curve. The relative flatness of the top is called **kurtosis** and is measured by β_2. Kurtosis refers to the bulginess of the curve of a frequency distribution.

Curves which are neither flat nor sharply peaked are called normal curves or **mesokurtic curves.**

Curves which are flatter than the normal curve are called **platykurtic curves.**

Curves which are more sharply peaked than the normal curve are called **leptokurtic curves.**

2.16. MEASURE OF KURTOSIS

[G.B.T.U. (C.O.) 2009, 2011; U.P.T.U. 200...]

The measure of kurtosis is denoted by β_2 and is defined as

$$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$$

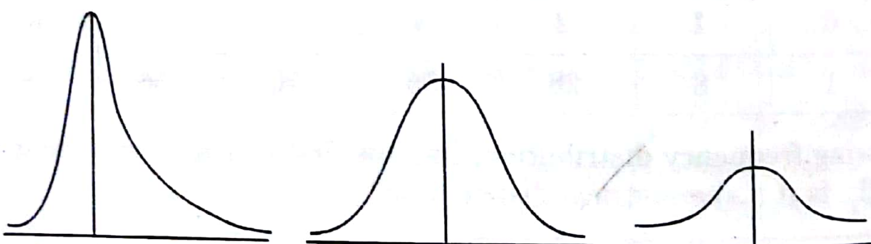
where μ_2 and μ_4 are respectively the second and fourth moments about mean of the distribution.

If $\beta_2 > 3$, the distribution is **leptokurtic.** If $\beta_2 = 3$, the distribution is ***mesokurtic.*** If $\beta_2 < 3$, the distribution is **platykurtic.** The kurtosis of a distribution is also measured by using Greek letter 'γ_2' which is defined as $\gamma_2 = \beta_2 - 3$.

$\therefore \quad \gamma_2 > 0 \Rightarrow \beta_2 - 3 > 0 \Rightarrow \beta_2 > 3 \Rightarrow$ the distribution is **leptokurtic.**

Similarly, if $\gamma_2 = 0$, then $\beta_2 = 3 \Rightarrow$ The distribution is **mesokurtic.**

$\gamma_2 < 0 \Rightarrow \beta_2 < 3 \Rightarrow$ the distribution is **platykurtic.**



Leptokurtic
$\beta_2 > 3$
$\gamma_2 > 0$

Mesokurtic
$\beta_2 = 3$
$\gamma_2 = 0$

Platykurtic
$\beta_2 < 3$
$\gamma_2 < 0$

2.17. STEPS FOR COMPUTING β_2

I. *If the value of μ_2 and μ_4 are given, then find β_2 by using the formula:* $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$.

II. *If raw moments μ_1', μ_2', μ_3' and μ_4' are given, then calculate:*

$$\mu_2 = \mu_2' - \mu_1'^2 \quad \text{and} \quad \mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Now, find $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$.

III. *If moments are not given, then first find μ_2 and μ_4 by using the given data and then use the formula:* $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$.

IV. *The given distribution is leptokurtic, mesokurtic and platykurtic according as $\beta_2 > 3$, $\beta_2 = 3$ and $\beta_2 < 3$ respectively.*

EXAMPLES

Example 1. *The first four moments about mean of a frequency distribution are 0, 100, – 7 and 35000. Discuss the kurtosis of the distribution.*

Sol. We have, $\mu_1 = 0,\ \mu_2 = 100,\ \mu_3 = -7$ and $\mu_4 = 35000$

Now, $$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{35000}{(100)^2} = 3.5 > 3$$

$\therefore$ The distribution is **leptokurtic.**

Example 2. *The first four moments of a distribution about the value '4' of the variable are – 1.5, 17,– 30 and 108. State whether the distribution is leptokurtic or platykurtic.*

(U.P.T.U. 2007)

Sol. We have, $\mu_1' = -1.5,\ \mu_2' = 17,\ \mu_3' = -30,\ \mu_4' = 108$

Moments about mean:

$$\mu_2 = \mu_2' - \mu_1'^2 = 17 - (-1.5)^2 = 14.75$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4 = 142.3125$$

Kurtosis: $$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.3125}{(14.75)^2} = 0.6541$$

Since $\beta_2 < 3$, the distribution is **platykurtic.**

Example 3. *The first four moments of a distribution about $x = 4$ are 1, 4, 10 and 45. Obtain the various characteristics of the distribution on the basis of the given information. Comment upon the nature of the distribution.*

Sol. We have $A = 4,\ \mu_1' = 1,\ \mu_2' = 4,\ \mu_3' = 10$ and $\mu_4' = 45$

Moments about mean:

$$\mu_1 = 0 \text{ (always)}$$
$$\mu_2 = \mu_2' - \mu_1'^2 = 4 - (1)^2 = 3$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 10 - 3(4)(1) + 2(1)^3 = 0$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 45 - 4(10)(1) + 6(4)(1)^2 - 3(1)^4 = 26$$

Skewness: Moment coefficient of skewness, $\gamma_1 = \dfrac{\mu_3}{\sqrt{\mu_2^3}} = \dfrac{0}{\sqrt{(3)^3}} = 0$.

$\therefore$ The distribution is **symmetrical.**

Kurtosis: $\beta_2 = \dfrac{\mu_4}{\mu_2^2} = \dfrac{26}{(3)^2} = 2.89 < 3$ $\therefore$ The distribution is **platykurtic.**

Example 4. *The standard deviation of a symmetric distribution is 5. What must be the value of the fourth moment about the mean in order that the distribution be (i) leptokurtic (ii) mesokurtic (iii) platykurtic?*

Sol. We have, $\sigma = 5 \Rightarrow \sigma^2 = 25 \Rightarrow \mu_2 = 25$

Now, $$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\mu_4}{625}$$

Thus, the distribution will be

(*i*) Leptokurtic if $\beta_2 > 3 \Rightarrow \frac{\mu_4}{625} > 3 \Rightarrow \mu_4 > 1875$

(*ii*) Mesokurtic if $\beta_2 = 3 \Rightarrow \frac{\mu_4}{625} = 3 \Rightarrow \mu_4 = 1875$

(*iii*) Platykurtic if $\beta_2 < 3 \Rightarrow \frac{\mu_4}{625} < 3 \Rightarrow \mu_4 < 1875.$

Example 5. *The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate β_1, β_2 and comment upon the skewness and kurtosis of the distribution.* (U.P.T.U. 20

Sol. We have, $\mu_1' = 0.294, \mu_2' = 7.144, \mu_3' = 42.409, \mu_4' = 454.98$

Moments about mean

$$\mu_1 = 0$$
$$\mu_2 = \mu_2' - \mu_1'^2 = 7.144 - (.294)^2 = 7.0576$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$= 42.409 - 3(7.144)(.294) + 2(.294)^3 = 36.1588$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 454.98 - 4(42.409)(.294) + 6(7.144)(.294)^2 - 3(.294)^4$$
$$= 408.7896$$

Calculation of β_1 and β_2

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 3.7193$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 8.2070$$

Skewness

Since β_1 is positive, $\gamma_1 = 1.9285$ | μ_3 is posit

$\therefore$ The distribution is **positively skewed.**

Kurtosis

Since $\beta_2 = 8.2070 > 3$

$\therefore$ The distribution is **leptokurtic.**

Example 6. *The first four moments of a distribution about the value '0' are – 0.20, 1.76, – 2.36 and 10.88. Find the moments about the mean and measure the kurtosis.* (U.P.T.U. 20

Sol. We have, $\mu'_1 = -.20, \mu'_2 = 1.76, \mu'_3 = -2.36, \mu'_4 = 10.88$

Moments about the mean:

$$\mu_1 = 0$$
$$\mu_2 = \mu'_2 - \mu_1'^2 = 1.76 - (-.20)^2 = 1.72$$
$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3$$
$$= -2.36 - 3(1.76)(-.20) + 2(-.20)^3 = -1.32$$
$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4$$
$$= 10.88 - 4(-2.36)(-.20) + 6(1.76)(-.20)^2 - 3(-.20)^4$$
$$= 9.4096$$

Kurtosis:

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3.180638$$

Since $\beta_2 > 3$ hence the distribution is **leptokurtic.**

Example 7. *The following table represents the height of a batch of 100 students. Calculate kurtosis.*

Height (in cm)	59	61	63	65	67	69	71	73	75
No. of students	0	2	6	20	40	20	8	2	2

[U.P.T.U. (C.O.) 2008]

Sol. To calculate β_2, we will have to first find the values of μ_2 and μ_4.

Height (cm) x	*No. of students f*	$u = \frac{x-67}{2}$	fu	fu^2	fu^3	fu^4
59	0	–4	0	0	0	0
61	2	–3	–6	18	–54	162
63	6	–2	–12	24	–48	96
65	20	–1	–20	20	–20	20
67	40	0	0	0	0	0
69	20	1	20	20	20	20
71	8	2	16	32	64	128
73	2	3	6	18	54	162
75	2	4	8	32	128	512
	$N = \Sigma f = 100$		$\Sigma fu = 12$	$\Sigma fu^2 = 164$	$\Sigma fu^3 = 144$	$\Sigma fu^4 = 1100$

Moments about 67

$$\mu_1' = \left(\frac{\Sigma fu}{N}\right)h = \left(\frac{12}{100}\right)(2) = 0.24$$

$$\mu_2' = \left(\frac{\Sigma fu^2}{N}\right)h^2 = \left(\frac{164}{100}\right)(4) = 6.56$$

$$\mu_3' = \left(\frac{\Sigma fu^3}{N}\right)h^3 = \frac{144}{100} \times 8 = 11.52$$

$$\mu_4' = \left(\frac{\Sigma fu^4}{N}\right) h^4 = \frac{1100}{100} \times 16 = 176$$

Moments about mean

$$\mu_2 = \mu_2' - \mu_1'^2 = 6.56 - (.24)^2 = 6.5024$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 176 - 4\,(11.52)\,(.24) + 6\,(6.56)\,(.24)^2 - 3\,(.24)^4 = 167.197$$

Kurtosis

Measure of kurtosis $\beta_2 = \dfrac{\mu_4}{\mu_2^2} = 3.9544 > 3$

Hence the distribution is **leptokurtic.**

ASSIGNMENT

1. The first four moments about mean of a frequency distribution are 0, 60, – 50 and 8020 respectively. Discuss the kurtosis of the distribution.
2. The μ_2 and μ_4 for a distribution are found to be 2 and 12 respectively. Discuss the kurtosis of the distribution.
3. The first four central moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the kurtosis of the distribution.
4. The standard deviation of symmetric distribution is 4. What must be the value of μ_4 so that the distribution may be mesokurtic?
5. If the first four moments about the value '5' of the variable are – 4, 22, – 117 and 560, find the value of β_2 and discuss the kurtosis.
6. (*i*) Calculate the value of β_2 for the following distribution:

Class	2.5–7.5	7.5–12.5	12.5–17.5	17.5–22.5	22.5–27.5	27.5–32.5	32.5–37.5
Frequency	8	15	20	32	23	17	5

(*ii*) Compute the value of β_2 for the following distribution. Is the distribution platykurtic ?

Class	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	1	20	69	108	78	22	2

7. Calculate μ_1, μ_2, μ_3, μ_4 for the frequency distribution of heights of 100 students given in the following table and hence find coefficient of skewness and kurtosis.

Height (cm.) Class interval	144.5 – 149.5	149.5 – 154.5	154.5 – 159.5	159.5 – 164.5	164.5 – 169.5	169.5 – 174.5	174.5 – 179.5
Frequency	2	4	13	31	32	15	3

(*G.B.T.U.* 2011)

8. Find the measures of skewness and kurtosis on the basis of moments for the following distribution:

x	1	3	5	7	9
f	1	[illegible]	6	4	1

[*G.B.T.U. (C.O.)* 2011]

9. Calculate β_1 and β_2 from the following data:

Profit (in lakhs of ₹)	10–20	20–30	30–40	40–50	50–60
No. of companies	18	20	30	22	10

Indicate the nature of frequency curve.

10. Prove that the frequency distribution curve of the following frequency distribution is leptokurtic.

Class	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55
Frequency	1	4	8	19	35	20	7	5	1

11. (*i*) What do you mean by kurtosis ? Explain in brief. [*U.P.T.U. (C.O.) 2008*]

(*ii*) Define kurtosis of a distribution. (*U.P.T.U. 2006*)

12. Define the coefficients of kurtosis. [*G.B.T.U. (C.O.) 2009, 2011; U.P.T.U. 2007*]

Answers

1. $\beta_2 = 2.2278$, Platykurtic **2.** $\beta_2 = 3$, Mesokurtic **3.** $\beta_2 = 3$, Mesokurtic
4. $\mu_4 = 768$ **5.** $\beta_2 = 0.8889$, Platykurtic
6. (*i*) $\beta_2 = 2.3216$, Platykurtic (*ii*) $\beta_2 = 2.7240$, Yes
7. $\mu_1 = 0$, $\mu_2 = 36.66$, $\mu_3 = -85.104$, $\mu_4 = 4373.3832$, $\gamma_1 = -.3834$, $\beta_2 = 3.2541$
8. $\gamma_1 = 0$, $\beta_2 = 2.5$ **9.** $\beta_1 = 0.0001$, $\beta_2 = 2.047$, Platykurtic.

2.18. CURVE FITTING