Sarthak Taneja

Mathematical logic

Proposition:

A Proposition (or statement) is a sentence which can be judged to be true or false but not both.

For eg:

1. Delhi is the capital of India. ✓
2. The sun rises in the east. ✓
3. Close the door. ✗
4. What is your name? ✗
5. Oh My God! ✗

Only 1 & 2 are statements while 3, 4 & 5 are sentences.

Note: A statement is generally denoted by small letter p, q, r etc.

Truth ~~Tabl~~ value:

The truth or falsity of a given statement p is denoted by T or F, depending on the fact that it is true or false.

For eg:

p : 2 is an even number. (T)

$n(n+1)$
2×7

q : $n^2 - n$ is not divisible by 2. (F)

Negation :

Given a statement p, the negation of p is a new statement having a different truth value as that of p.
It is actually "not p"
It is denoted by $\sim$; $\neg$

For eg:
p : 2 is an even number.
then
$\sim p$: 2 is not an even number.
Remark: $\sim(\sim p) = p$ (law of double negation)

Connectives :

~~Connectives~~ Two simple statements can ~~be~~ be joined together with the help of connectives "And, Or, if ---- then", if and only if".

① Conjunction (And)
If two simple statements p and q are joined together with the help of connectives "And", then the resulting statement is denoted by

$p \wedge q$ (read as 'p and q')

For eg : p : 2 is an even number.
q : 7 is prime number.

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then

$p \wedge q$: 2 is an even number and 7 is prime number.

Remark : $p \wedge q$ is true if and only if both p and q are true

Truth Table :

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (Or) :

If two simple statements p and q are joined together with the help of connectives ~~(OR)~~ 'Or' then the resulting statement is denoted by

$p \vee q$ (read as p or q)

For eg:

p : 2 is an even number.

q : 7 is ~~a~~ prime number.

then

$p \vee q$: 2 is an even number or 7 is prime number, Remark: ~~pVq~~ $p \vee q$ is true if either of them is true.

Truth Table :

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional (if ----- then) (implication) :

If two simple statements p & q are joined together with the help of connective "if --- then", then the resulting statement is denoted by

$p \Rightarrow q$ (read as 'p implies q')

or

$p \rightarrow q$ (another notation)

Remark : $p \Rightarrow q \equiv \sim p \vee q$

Consider an example :

A Father declared "I shall buy a watch for my son if he stands Ist in class"

Or equivalently

"If my son stands first in class then I shall buy a watch for him".

Here p: My son stands 1^{st} in class (anticidant)
q: I shall buy a watch for him. (consequent)

Then the given conditional ~~be~~ becomes $p \Rightarrow q$.

Truth Table:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	~~F~~ T	T
F	F	T

Biconditional (double implication):

When two simple statements p & q are joined ~~toget~~ together with the help of connective "if and only if", then the resulting compound statement is denoted by $p \Leftrightarrow q$ or $p \leftrightarrow q$

Consider following scenario

A father declared, "I shall buy a watch for my son, if & only if he stands first in class."

Here,
p : The son stands first in class,
q : Father buys a watch for him

Then it becomes $p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ques- Determine the truth values of the given statements.

① If Tigers have wings, then RDX is dangerous. $(F \Rightarrow T \approx T)$

② Ice is hot if and only if 2 is an odd no. $(F \Leftrightarrow F \approx T)$

③ $2+3=5$ and $n(n+1)$ is an odd no. for $n \in N$. $(T \wedge F \approx F)$

④ $x+2=5$ given $x=3$ or Area of similar Δ is same. $(T \vee F \approx T)$

⑤ $5^n - 1$ is divisible by 4 and every square is a rectangle. $(T \wedge T \approx T)$

⑥ n^{th} derivative of x^n is $(n-1)$ and the base of cylinder is a circle. $(F \wedge T \approx F)$

Tautology, Fallacy (Contradiction) and Contigency:

Tautology: A Compound Statement is said to be a tautology if it is always true irrespective of the truth or falsity of its prime statements.

For eg:

$p \vee \sim p$ is a tautology.

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Fallacy: (Contradiction)

A compound statement is said to be a fallacy if it is always false irrespective of the truth or falsity of its prime statements.

For eg: $p \wedge \sim p$ is a fallacy.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Contigency:

A compound statement is said to be a contigency if its is neither a tautology nor a fallacy.

For eg: $p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Remark: A ~~tautology~~ tautology is denoted by t and a fallacy is denoted by f or c

Que Identify the following statements as tautology, fallacy or contigency.

1. $(p \Rightarrow q) \Rightarrow (\sim p \vee q)$
2. $(p \Leftrightarrow q) \Rightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$
3. $(p \Rightarrow (q \vee r)) \Rightarrow (p \Rightarrow q) \vee (p \Rightarrow r)$
4. $(p \Rightarrow (q \wedge r)) \Rightarrow (p \Rightarrow q) \wedge (p \Rightarrow r)$
5. $[(p \wedge q) \Rightarrow r] \Rightarrow r$

(4) $(p \to (q \wedge r)) \to ((p \to q) \wedge (p \to r))$

1 p	2 q	3 r	4 $q\wedge r$	5 $p\to q$	6 $p \to r$	7 $(p\to q)\wedge(p\to r)$	8 $p \to (q \wedge r)$	9 $8 \to 7$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	T	F	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

$p \to (q \wedge r) \to (p \to q) \wedge (p \to r)$ Tautology

(1) $(p \to q) \to (\sim p \vee q)$

p	q	$\sim p$	$\sim p \vee q$	$p \to q$	$(p \to q) \to (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Tautology.

(2) $(p \leftrightarrow q) \to (p \to q) \wedge (q \to p)$

① p	② q	③ $p \Leftrightarrow q$	④ $p \Rightarrow q$	⑤ $q \Rightarrow p$	⑥ $(p \Rightarrow q) \wedge (q \Rightarrow p)$	③ ⇒ ⑥
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

Contigency. Tautology.

② $(p \Rightarrow (q \vee r)) \Rightarrow (p \Rightarrow q) \vee (p \Rightarrow r)$

① p	② q	③ r	④ $q \vee r$	⑤ $p \Rightarrow (q \vee r)$	⑥ $p \Rightarrow q$	⑦ $p \Rightarrow r$	⑧ $(p \Rightarrow q) \vee (p \Rightarrow r)$	⑤ ⇒ ⑧ $p \Rightarrow (q \vee r) \Rightarrow (p \Rightarrow q) \vee$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Tautology

(5) $[(p \wedge q) \Rightarrow r] \Rightarrow r$

p	q	r	$p \wedge q$	$(p \wedge q) \Rightarrow r$	$[(p \wedge q) \Rightarrow r] \Rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	F

Contigency

Equivalent statements

Two compound statements are said to be equivalent if their truth values are same for all the possible combinations of their prime statements.

For eg: $p \Rightarrow q$ is equivalent to $\sim p \vee q$

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Identical

Q1 Prove the following:

(i) $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (p \vee \sim q)$

(ii) $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$

(iii) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$

(iv) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(v) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$\left.\right\}$ Distributive laws.

Algebra of Propositions:

There are certain laws which are followed by these connectives called as algebra of propositions.

① Associative Law:

(i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$

(ii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

② Distributive Law:

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(iii) <u>Commutative Laws</u>

$$p \vee q \equiv \cancel{qp\wedge}\; q \vee p$$

$$p \wedge q \equiv q \wedge p$$

(iv) De Morgan's Law:

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

(v) Idem-potent laws:

$$p \vee p \equiv p$$

$$p \wedge p \equiv p$$

(vi) Identity laws:

$$p \vee T \equiv T \qquad (T \to \text{True})$$

$$p \wedge T \equiv p$$

$$p \vee F \equiv p \qquad (F \to \text{False})$$

$$p \wedge F = F$$

(vii) Complementary laws:

$$p \vee \sim p \equiv T$$

$$p \wedge \sim p \equiv F$$

(viii) ~~Double~~ Law of Double Negation:

$$\sim(\sim p) \equiv p$$

Converse, Inverse and Contrapositive statements :-

Let $p \Rightarrow q$ be a given conditional. Then the converse, inverse & contrapositive of it are defined as ~~follow~~ follows.

Converse:

The Converse of $p \Rightarrow q$ is $q \Rightarrow p$.

Inverse:

The Inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$.

Contrapositive Statement:

The contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$.

For eg:
Consider a given condition is "If a no. is divisible by 6, then it is divisible by 3".

<u>Converse:</u> "If a no. is divisible by 3, then it is divisible by 6".

∴ Converse is not always true.

<u>Inverse:</u> "If a no. is not divisible by 6, then it is not divisible by 3".

∴ Inverse is not always true.

~~Contrapositive~~
<u>Contrapositive:</u> "If a no. is not divisible by 3, then it is not divisible by 6.

<u>Remark</u>

① The contrapositive $(\sim q \Rightarrow \sim p)$ of a given conditional $(p \Rightarrow q)$ has same meaning as the original condition $(p \Rightarrow q)$.

				Original	Converse	Inverse	Contrapositive
p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$q \Rightarrow p$	$\sim p \Rightarrow \sim q$	$\sim q \Rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Ques- Consider the following two statements written on the welcome board of two different restaurant.

I: Cheap food is not good.

II: Good food is not cheap.

Are the two statement have same meaning?

Statement I can be rewritten as:

If the food is cheap then it is not good

Contrapositive If the food is good then it is not cheap

$\equiv$ Good food is not cheap.

Both the statements are same.

Validity of an arguement:

An arguement is a set of statement in which all the statements except the final one are called premises or hypothesis, and final statement is called conclusion.

For eg: All dogs are cats.
Some cats are elephants.
All elephants fly.
} premises
Therefore some dogs fly. } conclusion.

An arguement is said to be valid if and only if the conjuction of the premises implies the conclusion.

To check the validity of a given arguement the following method is used.

$p \Leftrightarrow q \equiv (\sim p \vee q) \wedge (p \vee \sim q)$

① p	② q	③ $\sim p$	④ $\sim q$	⑤ $p \Leftrightarrow q$	⑥ $\sim p \vee q$	⑦ $p \vee \sim q$	⑥ $\wedge$ ⑦
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	F	F	T	F	F
F	F	T	T	T	T	T	T

~~$p \wedge (q \wedge r)$~~ $\equiv$ $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

① p	② q	③ r	④ $p \Rightarrow q$	⑤ $p \Rightarrow r$	⑥ $q \vee r$	① $\Rightarrow$ ⑥	④ $\vee$ ⑤
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

~~$p \Rightarrow (q \vee r)$~~ $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$

① p	② q	③ r	④ $q \wedge r$	⑤ $p \Rightarrow q$	⑥ $p \Rightarrow r$	① $\Rightarrow$ ④	⑤ $\wedge$ ⑥
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

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(iv) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

① p	② q	③ r	④ $q \wedge r$	⑤ $p \vee q$	⑥ $p \vee r$	① ∨ ④	⑤ ∧ ⑥
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

(v) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

① p	② q	③ r	④ $q \vee r$	⑤ $p \wedge q$	⑥ $p \wedge r$	① ∧ ④	⑤ ∨ ⑥
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

classmate

Date ____
Page ____

(5) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

H.P.

$p \to q$

Test the validity of the following argument

1) If Yesterday was wednesday then today is thrusday. Today is thursday. Therefore yesterday was wednesday.

Sol. Let

p : Yesterday was a wednesday

q : Today is thrusday

symbolically we can write

$$\left.\begin{array}{l} p \Rightarrow q \\ q \end{array}\right\} \text{Premisses}$$

$$\therefore p \quad \} \text{ Conclusion}$$

p	q	Premise $p \Rightarrow q$	q	p
T	T	T	T	T ✓
T	F	F	F	T
F	T	T	T	F ✗
F	F	T	F	F

2 critical rows

Since all conclusion in 2 critical rows are not true, hence the arg. is not valid.

2.

If

Sol

Same

classmate

Date

Page

2\. Test the validity of the following arg.

If I study then I pass in examination. ~~If~~ I passed in examination. Therefore I studied.

Sol Let

p: I study

q: I pass examination

Symbolically we can write

$$\begin{array}{c} p \Rightarrow q \\ q \\ \hline p \end{array}$$

Since

3. Test the validity

If a man is batchelor, he is unhappy.
If a man is unhappy. He dies young.
Therefore bachelor die young.

Let
p: Man is bachelous
q: Man is unhappy
r: Man dies young

Symbol

$p \Rightarrow q$
$q \Rightarrow r$
$\therefore p \Rightarrow r$

			Premises		Conclu
p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$
T	T	T	T	T	T ✓
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T ✓
F	T	F	T	F	T
F	F	T	T	T	T ✓
F	F	F	T	T	T ✓

Since all conclusion in critical rows are true, hence the argument is valid.

4. Test the validity.

Either I work late in office or I go to cinema.
If I miss my train then I go to cinema.
Either it is housefull or I work late in office.
Therefore I go to cinema.

Sol Let
p: I work late in office
q: I goto cinema
r: I miss my train
s: cinema is housefull
Symbol

$$\left.\begin{array}{l} p \vee q \\ r \Rightarrow q \\ s \vee p \end{array}\right\} \text{prim}$$

$\therefore q$

				P			Con
p	q	r	s	p∨q	r⇒q	s∨p	q
T	T	T	T	T	T	T	T ✓
T	T	T	F	T	T	T	T ✓
T	T	F	T	T	T	T	T ✓
T	T	F	F	T	T	T	T ✓
T	F	T	T	T	F	T	F
T	F	T	F	T	F	T	F
T	F	F	T	T	T	T	F
T	F	F	F	T	T	T	F ✗
F	T	T	T	T	T	T	T ✓
F	T	T	F	T	T	F	T
F	T	F	T	T	T	T	T ✓
F	T	F	F	T	T	F	T
F	F	T	T	F	F	T	F
F	F	T	F	F	F	F	F
F	F	F	T	F	T	T	F
F	F	F	F	F	T	F	F

Since all the critical row are not true in conclu, so it is invalid statement

5. Test the validity

White is black. Black is not blue. Therefore blue is not white.

Let:

p: ~~White is black~~ colour is white

q: ~~Blac~~ Colour is black

r: colour is blue

Symbo

$p \Rightarrow q$

$q \Rightarrow \neg r$

$\therefore r \Rightarrow \neg p$

$\neg p$	p	q	r	$\neg r$	$p \Rightarrow q$	$q \Rightarrow \neg r$	Concl $r \Rightarrow \neg p$
F	T	T	T	F	T	F	F
F	T	T	F	T	T	T	T
F	T	F	T	F	F	T	F
F	T	F	F	T	F	T	T
T	F	T	T	F	T	F	T
T	F	T	F	T	T	T	T
T	F	F	T	F	T	T	T
T	F	F	F	T	T	T	T

In Concl

Since all the critical rows are true, so the statement is valid

Quantifiers

There are two types of quantifier

- Universal quantifiers

The symbol $\forall$ used for the phrases

"for all", "for every", "for each", "for any" is called universal quantifier

For Examples-

~~All wor~~

All birds fly

This can be rewritten as

"for every x, if x is a bird then x must fly"

Let

$p(x)$: x is a bird

$q(x)$: x must fly

Symbolically

$\forall x, p(x) \Rightarrow q(x)$

Rem

R

Remarks:- A universal quantifier is always followed by implication ($\Rightarrow$)
(if $\forall \Rightarrow \Rightarrow$)

- Existential quantifier

The symbol $\exists$ ~~is~~ used for the phrases "For some", "There exist", "for few", is called existential quantifier.

For eg.

Some students are poor.
This can be rewritten as:-
"For some x, x is a ~~th~~ student and x is poor"

Let
$p(x)$: x is a student
$q(x)$: x is poor

Symbol
$$\exists x,\ p(x) \wedge q(x)$$

Remarks
Existential quantifier is always followed by conjunction (and, $\wedge$)

Negation of a quantified Statement

The negation of a statement is a new statement having the meaning opp. to the original statement. To negate a given quantifier statements are the following point

→ $\sim(\forall x) \equiv \exists x$ & $\sim(\exists x) \equiv \forall x$

→ Demorgan's laws

$\sim(p \vee q) = \sim p \wedge \sim q$

$\sim(p \wedge q) = \sim p \vee \sim q$

→ $p \Rightarrow q \equiv \sim p \vee q$

For example

Consider the statement

"All birds fly"

Symbolically we can write.

$\forall x, p(x) \Rightarrow q(x)$

Where $p(x)$: x is a bird

$q(x)$: x must fly

Negation $\sim[\forall x, p(x) \Rightarrow q(x)]$

$\equiv \sim(\forall x), \sim[p(x) \Rightarrow q(x)]$

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$$\exists x, \sim[\sim p(x) \vee q(x)]$$

$$\exists x, p(x) \wedge \sim q(x)$$

i.e for some x, x is a bird and x doesn't fly

or "Some birds do not fly."

Consider another statement
"Some students are poor."
In Symbols,

$$\exists x, p(x) \wedge q(x)$$

where $p(x)$: x is a student.
$q(x)$: x is poor.

its negation $\sim[\exists x, p(x) \wedge q(x)]$

$$\equiv \sim(\exists x), \sim[p(x) \wedge q(x)]$$

$$\equiv \forall x, \sim p(x) \vee \sim q(x)$$

$$\equiv \forall x, p(x) \Rightarrow \sim q(x)$$

i.e for all x, if x is a student then x is not poor

OR

"All students are not poor"

Rewrite the following statement in quantified format & write their negation

- for every real x, if $x > 0$ then $x^2 > 0$

Ans $\exists x$, $p(x) \wedge \sim q(x)$

Alg- For some x, $x > 0$ & $x^2 \leq 0$

- Their exist some real x, such that $x < 1$ but $x^2 > 1$

- For all x, if $x < 1$ then $x^2 \leq 1$

"Some student of b-tech know physics but not mathematics"
It can be written as
"for some x, x is a student of b-tech and x know physics but not mathematics"
$p(x)$: x is a student of b-tech
$q(x)$: x know phy
$r(x)$: x doesn't know mathematics

Symbolically
$$\exists x, p(x) \wedge q(x) \wedge r(x)$$
$$\sim [\exists x (p(x) \wedge q(x) \wedge r(x))]$$
$$\equiv \forall x, \sim p(x) \vee \sim \{q(x) \wedge r(x)\}$$
$$\equiv \forall x, p(x) \Rightarrow \sim [q(x) \wedge r(x)]$$
$$\equiv \forall x, p(x) \Rightarrow (\sim q(x) \vee \sim r(x))$$

"for every x, if x is a student of b-tech then either x doesn't know Physics or known Maths"
OR "Every student of b-tech either know Maths or doesn't know Phy."

Normal forms

for a given compound statement, an equivalent statement can be obtained containing only the basic connectives like '∧', '∨' and '¬'.
Since the normal form of the given compound statement is not unique, we are interested to find principle normal form, which is unique.
~~It~~ Their are two types of principle normal form.

→ Principle disjunctive normal form (PDNF)
for two given statement p & q, consider the following combination
$p \wedge q$, $\neg p \wedge q$, $p \wedge \neg q$, $\neg p \wedge \neg q$

Each of these terms are called <u>minterms</u>

The principle disjunctive normal form of a given compound statement is constructed by the disjunction of its all minterms.
PDNF is also called as "<u>sum of product</u> form"

To write the PDNF of a given compound statement following method is used:

Step-1: Construct a truth table with the given compound statement 'S' as the last coloumn of it.

Step-2: Identify the rows in which the truth value is 'T' in the coloumn of 'S'.

Step-3: Write the minterm for each of these rows.

Step-4: The disjunction of these minterms is required PDNF.

For - example

Consider the statement

$p \Rightarrow q$

p	q	$p \Rightarrow q$	Minterms
T	T	T →	$p \wedge q$
T	F	F	
F	T	T →	$\sim p \wedge q$
F	F	T →	$\sim p \wedge \sim q$

PDNF: $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

Page

Principle conjunctive Normal form (PCNF)

For two given statements p & q, consider the following combination
$p \vee q$, $\sim p \vee q$, $p \vee \sim q$, $\sim p \vee \sim q$

Each of these terms are called maxterms.

The PCNF of a compound statement is given by conjunction of its all max-terms.
PCNF is also called Product of Sum form.

To write PCNF of a given compound statement following method is used.

Step-1 Construct a truth table with the given compound statement 'S' as the last coloumn of it.
Step-2. Identify the rows in which the truth value is 'F' in the coloumn of 'S'
Step-3 Write the max-terms of each of these rows

Step-4

Rem

Step-4 The conjunction of these maxterms is the required PCNF.

for eg:- Consider the statement

$\sim(p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	Maxterms
T	T	T	F $\rightarrow$	$\sim p \vee \sim q$
T	F	F	T	
F	T	T	F $\rightarrow$	$p \vee \sim q$
F	F	T	F $\rightarrow$	$p \vee q$

~~PCNF = $(p \vee q) \wedge (\sim p \vee q) \wedge ($~~

PCNF = $(\sim p \vee \sim q) \wedge (p \vee \sim q) \wedge (p \vee q)$

Remarks

- PCNF & PDNF of a given statement is always unique.
- However the normal form of a given statement is not unique.

for eg:- If given compound statement is $p \Rightarrow q$ then the possible DNF of it

are as follows
$p \Rightarrow q$

i) $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ (PDNF) an NF

DNF

ii) $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim p)$

iii) $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge \sim q)$

iv) $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim p) \vee (q \wedge \sim q)$

<u>Ques.</u>

Obtain the PDNF & PCNF of the follow

i) $[(p \Rightarrow q) \wedge r] \Rightarrow \sim r$

ii) $(p \wedge q) \vee (\sim p \vee \sim q) \wedge (\sim p \wedge q)$

iii) $[(p \Leftrightarrow q) \Rightarrow r] \wedge \sim r$

PDNF - $(p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r)$

PCNF - $(\sim p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee r) \wedge$
$(\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r)$
$\wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r)$

1. $[(p \Rightarrow q) \wedge r] \Rightarrow \sim r$

$\sim r$	p	q	r	$p \Rightarrow q$	$\wedge$	$\Rightarrow$	Minterm	Maxterm
F	T	T	T	T	T	F		$\sim p \vee \sim q \vee \sim r$
T	T	T	F	T	F	T-	$p \wedge q \wedge \sim r$	
F	T	F	T	F	F	T-	$p \wedge \sim q \wedge r$	
T	T	F	F	F	F	T-	$p \wedge \sim q \wedge \sim r$	
F	F	T	T	T	T	F		$p \vee \sim q \vee \sim r$
T	F	T	F	T	F	T-	$\sim p \wedge q \wedge \sim r$	
F	F	F	T	T	T	F		$p \vee q \vee \sim r$
T	F	F	F	T	F	T-	$\sim p \wedge \sim q \wedge \sim r$	

PDNF - $(p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r)$
$\vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$

PCNF - $(\sim p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r)$

3. $[(p \Leftrightarrow q) \Rightarrow r] \wedge \sim r$

p	q	r	$\sim r$	$p \Leftrightarrow q$	$\Rightarrow r$	$\wedge \sim r$	Minterm	Maxterm
T	T	T	F	T	T	F		$\sim p \vee \sim q \vee \sim r$
T	T	F	T	T	F	F		$\sim p \vee \sim q \vee r$
T	F	T	F	F	T	F		$\sim p \vee q \vee \sim r$
T	F	F	T	F	T	T →	$p \wedge \sim q \wedge \sim r$	
F	T	T	F	F	T	F		$p \vee \sim q \vee \sim r$
F	T	F	T	F	T	T →	$\sim p \wedge q \wedge \sim r$	
F	F	T	F	T	T	F		$p \vee q \vee \sim q$
F	F	F	T	T	F	F		$p \vee q \vee r$

Date
Page

18/10/16

Q. $(p \wedge q) \vee (\sim p \vee \sim q) \wedge (\sim p \wedge q)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$\sim p \wedge q$	$\vee$	$\wedge$	Mint	Ma
T	T	F	F	T	F	F	T	~~F~~		$\sim p\vee$
T	F	F	T	F	T	F	T	F		$\sim p\vee$
F	T	T	F	F	T	T	T	F	$\sim p \wedge q$	
F	F	T	T	F	T	F	T	F		$p\vee$

PDNF - $(\sim p \wedge q)$

PCNF - $(\cancel{\sim p \vee \sim q}) \wedge (\sim p \vee q) \wedge (p \vee q)$

Cond. 1.

Cond. 2,

Not

Pe