

Let  $(x_1, x_2, \dots, x_n)$  be sample of size 'n' taken  
 mean  $= \theta_1$   
 variance  $= \theta_2$   

$$L(\theta_1, \theta_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum (x_i - \theta_1)^2$$

for  $\theta_1$  diff  $\log L(\theta_1, \theta_2)$  w.r.t  $\theta_1$  & set it to zero

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^n x_i$$

MLE of  $\theta_1$  is sample mean

for  $\theta_2$  diff. w.r.t  $\theta_2$  & put zero.

$$\theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \theta_1)^2$$

## 2) Binomial distribution

$n$  = no. of trials

$\theta = (0, 1)$  prob. of success

$$L_\theta = \prod_{i=1}^n f(x_i, n, \theta)$$

PMF

$$f(x, n, \theta) = {}^nC_x \cdot \theta^x \cdot (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^n ({}^nC_{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{n-x_i})$$

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^n u_i - \frac{1}{1-\theta} \sum_{i=1}^n (n - u_i) = 0$$

$$\Rightarrow \frac{1}{\theta} \sum_{i=1}^n u_i - \frac{1}{1-\theta} \sum_{i=1}^n (m - u_i)$$

Multiply by  $\theta(1-\theta)$

$$\Rightarrow (1-\theta) \sum_{i=1}^n u_i = \theta \sum_{i=1}^n (m - u_i)$$

$$\theta = \frac{\sum_{i=1}^n u_i}{m}$$

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SCO 8