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UNIT

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- **Mathematical Induction :—**

- **Proposition :—**

Any statement may be true or false for each value pass to the statement.

Ex :-

i) All even number is divisible by 2.
This statement is proposition.

ii) All even number is divisible by 4
This statement is not proposition.

There are two principle to check the statement is proposite or not.

i) show P(1) is true.

ii) let P(n) is true then P(n+1) is also true

Example :-

Q $P(x) = x^3 + 3x^2 + 4$

$P(2) = 2^3 + 3 \times 2^2 + 4$

$= 8 + 12 + 4$

$= 24$

Q. $1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

for P(1)

L.H.S $P(1) = 1$

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R.H.S

$$\frac{1(1+1)}{2} = \frac{1\times \cancel{2}}{\cancel{2}} = 1$$

L.H.S = R.H.S
P(1) is true.

Let P(n) is true

$$1+2+3+\ldots+n = \frac{n(n+1)}{2} \quad \text{---} \ (1)$$

for P(n+1)

$$1+2+3+\ldots+n+(n+1) = \frac{(n+1)(n+1+1)}{2}$$

L.H.S

$$1+2+3+\ldots+n+(n+1)$$

$$\frac{n(n+1)}{2} + (n+1) \quad [\text{for } (1)]$$

$$\frac{n(n+1)}{2} + \frac{(n+1)}{1}$$

$$\frac{n(n+1)+2(n+1)}{2}$$

$$(n+1)\frac{(n+2)}{2} = \text{R.H.S}$$

Proved

Now P(n+1) is also true

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Ques:- $1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

for P(1)

L.H.S

$P(1) = 1^3 = \underline{1}$

R.H.S

$\left\{\frac{1(1+1)}{2}\right\}^2$

$= \left\{\frac{1 \times \cancel{2}}{\cancel{2}}\right\}^2 = 1^2 = \underline{1}$

L.H.S = R.H.S

P(1) is true.

Let P(n) is true

$1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$ —— ①

for P(n+1)

$1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3 = \left\{\frac{(n+1)(n+1+1)}{2}\right\}^2$

$= \left\{\frac{(n+1)(n+2)}{2}\right\}^2$

L.H.S

$1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3$

$= \left\{\frac{n(n+1)}{2}\right\}^2 + (n+1)^3$ [for ①]

$$\frac{\{n(n+1)\}^2}{4} + \frac{(n+1)^3}{1}$$

$$\cancel{\frac{\{n(n+1)\}^2 + 4(n+1)^3}{4}}$$

$$(n+1)^2 \left\{ \frac{n^2}{4} + \frac{n+1}{1} \right\}$$

$$(n+1)^2 \left\{ \frac{n^2 + 4(n+1)}{4} \right\}$$

$$(n+1)^2 \left\{ \frac{n^2 + 4n + 4}{4} \right\}$$

$$(n+1)^2 \left\{ \frac{n^2 + 2 \times 2 \times n + 2^2}{4} \right\}$$

$$(n+1)^2 \left\{ \frac{n+2}{2} \right\}^2$$

$$\left\{ \frac{(n+1)(n+2)}{2} \right\}^2 = R.H.S$$

Now P(n+1) is also true.

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Ques:- $1^2+2^2+3^2+ \dots\dots +n^2 = \frac{n(n+1)(2n+1)}{6}$

for P(1)

L.H.S

$P(1) = 1^2 = 1$

R.H.S

$\frac{1(1+1)(2\times1+1)}{6}$

$\frac{2\times3}{6} = \frac{\cancel{6}}{\cancel{6}} = 1$

L.H.S = R.H.S

Let P(n) is true

$1^2+2^2+3^2+ \dots\dots +n^2 = \frac{n(n+1)(2n+1)}{6}$ —— ①

for P(n+1)

$1^2+2^2+3^2+ \dots +n^2+(n+1)^2 = \frac{(n+1)(n+2)[2(n+1)+1]}{6}$

$= \frac{(n+1)(n+2)(2n+3)}{6}$

L.H.S

$1^2+2^2+3^2+ \dots\dots +n^2+(n+1)^2$

$\frac{n(n+1)(2n+1)}{6} + (n+1)^2$

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$$\frac{n(n+1)(2n+1)}{6} + \frac{(n+1)^2}{1}$$

$$(n+1)\left\{\frac{n(2n+1)}{6} + \frac{n+1}{1}\right\}$$

$$(n+1)\left\{\frac{n(2n+1)+6n+6}{6}\right\}$$

$$(n+1)\left\{\frac{2n^2+n+6n+6}{6}\right\}$$

$$(n+1)\left\{\frac{2n^2+7n+6}{6}\right\}$$

R.H.S

$$(n+1)\frac{(n+2)(2n+3)}{6}$$

$$(n+1)\left\{\frac{n(2n+3)+2(2n+3)}{6}\right\}$$

$$(n+1)\left[\frac{2n^2+3n+4n+6}{6}\right]$$

$$(n+1)\left[\frac{2n^2+7n+6}{6}\right] = \text{L.H.S}$$

Proved

Now P(n+1) is true.

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Ques:- $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2-1)$

for P(1)
L.H.S
$P(1) = (2\times1-1)^3 = (2-1)^3 = 1^3 = 1$

R.H.S
$n^2(2n^2-1)$
$1^2(2\times1^2-1)$
$1(2-1) = 1$
L.H.S = R.H.S
P(1) is true

let P(n) is true
$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2-1)$ —— ①

for P(n+1)
$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 + (2(n+1)-1)^3 = (n+1)^2(2(n+1)^2-1)$

$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 + (2n+1)^3 = (n+1)^2(2n^2+4n+1)$

L.H.S

$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 + (2n+1)^3$

$= n^2(2n^2-1) + (2n+1)^3$

$= (2n^4 - n^2) + (2n)^3 + (1)^3 + 3\times2n\times1(2n+1)$
$= (2n^4 - n^2) + 8n^3 + 1 + 6n(2n+1)$
$= (2n^4 - n^2) + 8n^3 + 12n^2 + 6n + 1$
$= 2n^4 - n^2 + 8n^3 + 12n^2 + 6n + 1$

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$= 2n^4 + 8n^3 + 11n^2 + 6n + 1$

R.H.S

$= (n+1)^2 (2n^2 + 4n + 1)$

$= (n^2 + 2n + 1)(2n^2 + 4n + 1)$

$= n^2(2n^2 + 4n + 1) + 2n(2n^2 + 4n + 1) + 1(2n^2 + 4n + 1)$

$= 2n^4 + 4n^3 + n^2 + 4n^3 + 8n^2 + 2n + 2n^2 + 4n + 1$

$= 2n^4 + 8n^3 + 11n^2 + 6n + 1$

= L.H.S

Proved

Now $P(n+1)$ is true.

Ques:- $5^{2n} - 1$ is divisibl by 24

$P(1)$

L.H.S

$P(1) = 5^{2\times 1} - 1$

$= 5^2 - 1 = 25 - 1 = 24$

R.H.S

$= 24 =$ L.H.S

24 is divisible by 24

Let $P(n)$ is true

$(5^{2n} - 1) \div 24 = K$

$(5^{2n} - 1) = 24K$

$5^{2n} = 24K + 1$ —— (1)

$P(n+1)$

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$= 5^{2(n+1)} - 1$

$= 5^{2n+2} - 1$

$= 5^{2n} \times 5^{2} - 1$

$= (24k+1)\,25 - 1$

$24k \times 25 + 25 - 1$

$= 24k \times 25 + 24$

$= 24(25k+1)$

It is divisible by 24

the $p(n+1)$ is also true

Ques:- $3^{4n+2} + 5^{2n+1}$ is divisible by 14

$P(1)$

L.H.S.

$3^{4\times1+2} + 5^{2\times1+1}$

$3^{6} + 5^{3}$

$729 + 125 = 854$

854 is divisible by 14

let $P(n)$ is true

$3^{4n+2} + 5^{2n+1} \div 14 = K$

$3^{4n+2} + 5^{2n+1} = 14K$

$3^{4n+2} = 14K - 5^{2n+1}$ —— ①

$P(n+1)$

$3^{4(n+1)+2} + 5^{2(n+1)+1}$

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$3^{4n+4+2} + 5^{2n+2+1}$

$3^{4n+4+2} + 5^{2n+3}$

$(14K - 5^{2n+1})\, 81 + 5^{2n+3}$

$14K \times 81 - 81 \times 5^{2n+1} + 5^{2n} \times 5^{3}$

$14K \times 81 - 81 \times 5^{2n} \times 5 + 5^{2n} \times 125$

$14K \times 81 - 5^{2n}(405 - 125)$

$14K \times 81 - 5^{2n} \times 280$

$14(81K - 5^{2n} \times 20)$

It is divisible by 14 Proved.

- Number :-

It is a character or collection of character use for measuring, counting, weighing and performing mathmatical operation.

Ex:- 4,437.

There are two types of number.

i) Integer - 43

ii) float - 43.2

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i) Integer :—

Any number which does not contain any decimal character called integer. It may be positive or negative.
Ex:— $-4, -3, 0, 1, 2, 3$

ii) float :—

Any number which contain decimal character called float. for example - 4.32, 4.75. It is not used in counting but perform all mathmatical operation.

• Power :—

It is way to represent the multiplication of same number for multiple times. for example the multiplication of two for 20 times can to be represent an. 2^{20} – in this case 2 is base value 20 is power

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$a^{-m} = \frac{1}{a^m}$$

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Simplify

Ques:- $3^4 \times 3^7 \div 3^{10} + 4^7 \div 4^8$

$= 3^{4+7} \div 3^{10} + 4^{7-8}$

$= 3^{11} \div 3^{10} + 4^{-1}$

$= 3^{11-10} + \frac{1}{4}$

$= 3 + \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4}$

Ques:- $\left\{10\left[(216)^{1/3} + (64)^{1/3}\right]^3\right\}^{3/4}$

$\left\{10\left[\left(6^{\cancel{3}}\right)^{\frac{1}{\cancel{3}}} + \left(4^{\cancel{3}}\right)^{\frac{1}{\cancel{3}}}\right]^3\right\}^{\frac{3}{4}}$

$\left\{10\left[6+4\right]^3\right\}^{\frac{3}{4}}$

$\left\{10\left(10\right)^3\right\}^{\frac{3}{4}}$

$\left\{10 \times 1000\right\}^{\frac{3}{4}}$

$\left\{10^{\cancel{4}}\right\}^{\frac{3}{\cancel{4}}}$

10^3

1000 Ans.

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Ques:- $\left[(5^2)^{\frac{1}{2}}\right]^4 \div 125$

$\left[(5^{\cancel{2}})^{\frac{1}{\cancel{2}}}\right]^4 \div 125$

$(5)^4 \div 5^3$

$5^{4-3} = 5$ Ans.

Ques:- $3^{2x-1} + 3^{2x+1} = 270$

$3^{2x} \times 3^{-1} + 3^{2x} \times 3^{+1} = 270$

$\frac{3^{2x}}{3} + 3^{2x} \times 3 = 270$

$\frac{3^{2x} + 3^{2x} \times 9}{3} = 270$

$3^{2x} + 3^{2x} \times 9 = 270 \times 3$

$3^{2x}(1+9) = 810$

$3^{2x} \times 1\cancel{0} = 81\cancel{0}$

$3^{2x} = 3^4$

$\cancel{2}x = \cancel{4}\ 2$

$x = 2$ Ans.

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logrithm :-

$\log n^a = a \log n$

$\log(n \times m) = \log n + \log m$

$\log(m \div n) = \log m - \log n$

$\log_n n = 1$

Ques:- $\log 64$

$= \log 2^6$

$= 6 \log 2$

Ques:- $\log(4 \times 5)$

$= \log 4 + \log 5$

Ques:- $\log(8 \div 2)$

$= \log 8 - \log 2$

$= \log 2^3 - \log 2$

$= 3 \log 2 - \log 2$

$= 2 \log 2$

Ques:- $\log_2 64$

$= \log_2 2^6$

$\Rightarrow 6 \log_2 2 = 6 \times 1 = 6$

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Ques:- $\log 216 - \log 36 + \log 6$

$= \log 6^3 - \log 6^2 + \log 6$

$= 3\log 6 - 2\log 6 + 1\log 6$

$= 3\log 6 - 1\log 6$

$= 2\log 6$ Ans.

Ques:- $\log 64 \div \log 8 - \log 128 + \log 32 \div \log 8$

$= \log(64 \div 8) - \log 128 + \log(32 \div 8)$

$= \log 56 - \log 128 + \log 24$

$= \log 8 \times 7 - \log 2^6 + \log 8 \times 3$

$= \log 8 + \log 7 - 6\log 2 + \log 8 + \log 3$

$= 3\log 2 + \log 7 - 6\log 2 + 3\log 2 + \log 3$

$= 6\cancel{\log} 2 + \log 7 - 6\cancel{\log} 2 + \log 3$

$= \log 7 - \log 3$

$= \log(7 - 3)$

$= \log 4$

$= 2\log 2$ Ans.

Ques:- $\log 75/16 - 2\log 5/9 + \log 32/243$.

$= \log 75 - \log 16 - 2(\log 5 - \log 9) + \log 32 - \log 243$

$= \log 25\times 3 - \log 2^4 - 2(\log 5 - \log 3^2) + \log 2^5 - \log 3^5$

$= 2\log 5 + \log 3 - 4\log 2 - 2\log 5 + 2\log 3^2 + 5\log 2 -$
$5\log 3$

$= \cancel{2\log 5} + \log 3 - 4\log 2 - \cancel{2\log 5} + 4\log 3 + 5\log 2$
$- 5\log 3$

$= \cancel{5\log 3} - 4\log 2 + 5\log 2 - \cancel{5\log 3}$

$= \log 2$ Ans.

Ques:- $4^{2x+1} = 16$, so $x = ?$

$\log_4 16 = 2x + 1$

$\log_4 4^2 = 2x + 1$

$2 \times 1 = 2x + 1$

$2 - 1 = 2x$

$x = \frac{1}{2}$.

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Que:- $3^{2x+5} = 243$

$\log_3 243 = 2x+5$

$\log_3 3^5 = 2x+5$

$5 \times 1 = 2x+5$

$\cancel{5} - \cancel{5} = 2x$

$0 = 2x$

$x = 0$ Ans.

- Integer function :—

It is a function which accept any number and return an integer. It is written as int ( ).
Example -
int (1.54) = 1
int (7) = 7

There are two types of integer.
i) ceilling ii) floor.

i) ceilling :—

This function accept any number and return integer which is greater or equal to input number.

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Example :-
ceilling $(1.54) = 2$
ceilling $(2) = 2$

ii) floor :-

This function accept any number and return integer which is smaller or equal to input number.

Example :-
floor $(1.54) = 1$
floor $(2) = 2$

- Elementary number theory :-

- Divisiblity :-

If a and b are two positive number then a/b give me a positive number.

It 6 and 2 are two positive number it gives 3.

$a/b = c \quad \therefore \quad a = bc$

Ques :- Let $a|b$, $b|a$ then prove $a|c$.

$a|b = k$

$a = bk$ — ①

$b|c = r$

$b = cr$ — ①

$\therefore a|c$

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$= bK/c$

$= \cancel{c}rK/\cancel{c}$

$= rK$ Proved.

Ques:- a/b and c/b then proved $(a+c)/b$

$a/b = K$

$a = bK$ —— (I)

$c/b = r$

$c = br$ —— (II)

$\therefore (a+c)/b$

$(bK + br)/b$

$\cancel{b}(k+r)/\cancel{b}$

$(k+r)$ Proved

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Que:- write an algorithm to enter two number and check first number is divisible by second number.

Step-1. Print " Enter two number".
2. Input a, b
3. If (a%b == 0)
4. Print " first number is divisible by second number.
else
5. Print " first number is not divisible by second number.
endif
6. end.

- Logical operator :-

This operator use when we require to take a decision on the basis of multiple condition. There are following types of logical operator :-
i) AND ii) OR iii) NOT

i) AND :-

It combine multiple expression (decision expression) and return the true if all expression return ~~there~~ true otherwise false.

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Ques:- write an algorithm to enter a number and check that number is divisible by 5 & 6.

Step. 1. Print "Enter a number".
2. Input a
3. If (a % 5 == 0 AND a % 6 == 0);
4. Print " number is divisible by 5 & 6.
else
5. Print " number is not divisible by 5 & 6.
end if.
6. end.

ii) OR :—

It combine multiple expression and return false if all expression return false otherwise return true.

Ques:- write an algorithm enter a number and check that number is divisible by 5 or 6.

Step-1. Print " Enter a number".
2. Input a
3. ~~Print~~ " If (a % 5 == 0 OR a % 6 == 0);
4. Print " number is divisible by 5 or 6";
else
5 Print " number is not divisible by 5 or 6".
end if
6. end .

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iii) NOT :—

It convert an expression which return true to false and false to true.

Ques:- write an algorithm to Enter a number and check that number is divisible by 5.

Step-1. Print "Enter a number"
2. Input a
3. If [NOT (a % 5 != 0)]
4. Print "Number is divisible by 5
else
5. Print "Number is not divisible by 5.
end it.
6. end.

Ques:- write an algorithm to enter two number and display the L.C.M of both number.

Step-1. Print "Enter Two number".
2. input a, b
3. Let x = 1
4. If (x % a == 0 AND x % b == 0)
5. Print x
6. exit()
7. end it
8. x = x + 1
9. Goto step4
10. exit

Exit:— It is a function which terminate an algorithm.

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- Permutation and factorial :—

- factorial :— factorial of a number in the product of all number between number and 1.

$\lfloor n = n(n-1)(n-2)....$

$\lfloor 3 = 3 \times 2 \times 1$

$\lfloor 0 = 1$

Ques:— $\lfloor 8 \div \lfloor 7 + \lfloor 3 \div \lfloor 2$

$\frac{\lfloor 8}{\lfloor 7} + \frac{\lfloor 3}{\lfloor 2}$

$\frac{8 \times \cancel{\lfloor 7}}{\cancel{\lfloor 7}} + \frac{3 \times \cancel{\lfloor 2}}{\cancel{\lfloor 2}}$

$8 + 3 = 11$ Ans

Ques:— $\lfloor x \div \lfloor x-1 = 5$ find x

$\frac{\lfloor x}{\lfloor x-1} = 5$

$\frac{x \times \cancel{\lfloor x-1}}{\cancel{\lfloor x-1}} = 5$

$x = 5$ Ans

Ques:— $\lfloor x \div \lfloor x-2 = 7$

$\frac{x \times (x-1)\cancel{\lfloor x-2}}{\cancel{\lfloor x-2}} = 7$

$x(x-1) = 7$

$x = 7$ | $x-1 = 7$
$x = 8$

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- $^nP_r = \dfrac{\lfloor n}{\lfloor n-r}$

n = total number of value
r = number of arrange value.

i) $^6P_3$

$$\frac{\lfloor 6}{\lfloor 6-3} = \frac{6\times5\times4\times\cancel{\lfloor 3}}{\cancel{\lfloor 3}} = 120 \text{ Ans.}$$

ii) $^nP_4 = 12 \times {}^nP_2$

$$\frac{\cancel{\lfloor n}}{\lfloor n-4} = 12 \times \frac{\cancel{\lfloor n}}{\lfloor n-2}$$

$$\frac{1}{\cancel{\lfloor n-4}} = \frac{12}{(n-2)(n-3)\cancel{\lfloor n-4}}$$

$(n-2)(n-3) = 12$
$(n-2)(n-3) = 4\times3$

$n-2 = 4$ | $n-3 = 3$
$n = 6$ | $n = 6$ Ans

iii) $^nP_5 = 20 \times {}^nP_3$

$$\frac{\cancel{\lfloor n}}{\lfloor n-5} = \frac{20\times\cancel{\lfloor n}}{\lfloor n-3}$$

$$\frac{1}{\cancel{\lfloor n-5}} = \frac{20}{(n-3)(n-4)\cancel{\lfloor n-5}}$$

$20 = (n-3)(n-4)$

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$5 \times 4 = (n-3)(n-4)$

$n-3 = 5$ | $n-4 = 4$
$n = 8$ | $n = 8$

iv) ${}^{n}P_4 : {}^{n+1}P_4 = 3:4$

$$\frac{{}^{n}P_4}{{}^{n+1}P_4} = \frac{3}{4}$$

$$4 \times \frac{\lfloor n}{\lfloor n-4} = 3 \times \frac{\lfloor n+1}{\lfloor n+1-4}$$

$$4 \times \frac{\lfloor n}{\lfloor n-4} = \frac{3 \times \lfloor n+1}{\lfloor n-3}$$

$$\frac{4 \times \cancel{\lfloor n}}{\cancel{\lfloor n-4}} = \frac{3(n+1)\cancel{\lfloor n}}{(n-3)\cancel{\lfloor n-4}}$$

$4(n-3) = 3n+3$
$4n - 12 = 3n + 3$
$4n - 3n = 3 + 12$
$n = 15$ Ans

v) ${}^{20}P_r = 6840$ find $r = ?$

$$\frac{\lfloor 20}{\lfloor 20-r} = 6840$$

$$\frac{\cancel{20 \times 19 \times 18} \times \lfloor 17}{\lfloor 20-r} = \cancel{6840}$$

$\lfloor 17 = \lfloor 20 - r$

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$17 = 20 - r$

$r = 20 - 17$

$r = 3$ Ans.

vi) How many number can be form of three digit using 1, 2, 3, 4.

$n = 4$

$r = 3$.

$$\frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor 4}{\lfloor 4-3} = \frac{4 \times 3 \times 2 \times 1}{1} = 24 \text{ Ans.}$$

vii) $^{12}P_r = 11880$

$$\frac{\lfloor 12}{\lfloor 12-r} = 11880$$

$$\frac{12 \times 11 \times 10 \times 9 \times \lfloor 8}{\lfloor 12-r} = 11880$$

$$\frac{\cancel{11880} \times \lfloor 8}{\lfloor 12-r} = \cancel{11880}$$

$\lfloor 8 = \lfloor 12-r$

$8 = 12 - r$

$r = 12 - 8$

$r = 4$ Ans.

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Ques:- program:-
write an algorithm to enter two number and display the L.C.M of both number.

```
# include <stdio.h>
# include <conio.h>
void main();
{
int   a, b, x=1, K
printf ("enter two number");
scanf ("%d %d", &a, &b);
K;
if( x%a==0 && x%b==0 )
{
printf ("%d", x);
exit();
}
goto K;
getch();
}
```

Ques:- write an algorithm to enter a number and sum the all factor of input number.

Step-1. Print("Enter a number");

2. Input a   sum=0

3. let $x=1$

4. if $(a\%x==0)$

5. sum = sum + x

6. end $x=x+1$

7. if $(x<=a)$

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8. Goto step 4.
9. Print sum
10. end

a = 6, sum = 0
let x = 1
6 % 1 == 0 T
sum = 0 + 1 = 1
x = 1 + 1 = 2
if (2 <= 6) T
if (6 % 2 == 0) T
sum = 1 + 2 = 3
x = 2 + 1 = 3
if (3 <= 6) T
if (6 % 3 == 0) T
sum = 3 + 3 = 6
x = 3 + 1 = 4
if (4 <= 6) T
if (6 % 4 == 0) F
x = 4 + 1 = 5
if (6 % 5 == 0) F
x = 5 + 1 = 6
if (6 % 6 == 0) T
sum = 6 + 6 = 12
Print 12
end

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- Binomial Coefficients :-

- Binomial theorm :-

It is a theorm which expand the power of binomial expression in series.

Any expression which contain maximum two terms called binomial expression.

Ex:- $x+y$, $4+5$, x^2+y

$$(a+b)^n = {}^nc_0 \cdot a^n \cdot b^0 + {}^nc_1 \cdot a^{n-1} \cdot b^1 + {}^nc_2 \cdot a^{n-2} \cdot b^2 + \ldots\ldots\ldots + {}^nc_n \cdot a^0 \cdot b^n$$

$${}^nc_r = \frac{\lfloor n}{\lfloor n-r \lfloor r}$$

Ques:- ${}^{12}C_{10}$

$$\frac{\lfloor 12}{\lfloor 12-10 \cdot \lfloor 10} = \frac{\lfloor 12}{\lfloor 2 \cdot \lfloor 10}$$

$$= \frac{\overset{6}{\cancel{12}} \times 11 \times \cancel{\lfloor 10}}{\cancel{2} \times 1 \times \cancel{\lfloor 10}} = 66 \text{ Ans}$$

Ques:- ${}^6C_3$

$$\frac{\lfloor 6}{\lfloor 6-3 \cdot \lfloor 3} = \frac{\lfloor 6}{\lfloor 3 \cdot \lfloor 3} = \frac{\cancel{6} \times 5 \times 4 \times \cancel{\lfloor 3}}{\cancel{\lfloor 3} \times \cancel{3} \times \cancel{2} \times 1} = 20 \text{ Ans.}$$

Ques :- $(a+b)^3$

$= {}^3C_0\, a^3 \cdot b^0 + {}^3C_1\, a^{3-1} \cdot b^1 + {}^3C_2\, a^{3-2} \cdot b^2 + {}^3C_3\, a^{3-3} \cdot b^3$

$= \dfrac{|\underline{3}}{|\underline{3-0} \cdot |\underline{0}} \cdot a^3 \cdot 1 + \dfrac{|\underline{3}}{|\underline{3-1} \cdot |\underline{1}} \cdot a^2 b^1 + \dfrac{|\underline{3}}{|\underline{3-2} \cdot |\underline{2}}\, a^1 b^2 + \dfrac{|\underline{3}}{|\underline{3-3} \cdot |\underline{3}}\, a^0 b^3$

$= \dfrac{\cancel{|\underline{3}}}{\cancel{|\underline{3}}}\, a^3 + \dfrac{3 \times \cancel{|\underline{2}}}{\cancel{|\underline{2}}}\, a^2 b + \dfrac{3 \times \cancel{|\underline{2}}}{1 \times \cancel{|\underline{2}}}\, ab^2 + \dfrac{\cancel{|\underline{3}}}{\cancel{|\underline{3}}}\, b^3$

$= a^3 + 3a^2 b + 3ab^2 + b^3$ Ans.

- Binomial Coefficient :-

r^{th} term of binomial expension $(a+b)^n$

$t_r = {}^nC_{r-1} \cdot (a)^{n-r+1} \cdot b^{r-1}$

Example :-

$(a+b)^2$ find 2nd term.

$t_2 = {}^2C_{2-1} \cdot a^{\cancel{2}-\cancel{2}+1} \cdot b^{2-1}$

$= \dfrac{|\underline{2}}{|\underline{2-1} \cdot |\underline{1}} \cdot a^1 \cdot b^1$

$= \dfrac{|\underline{2}}{|\underline{1} \cdot |\underline{1}} \cdot a \cdot b = 2ab.$

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Ques:- $(a+b)^{10}$ find 4^{th} term.

$$t_4 = {}^{10}C_{4-1} \cdot a^{10-4+1} \cdot b^{4-1}$$

$$= \frac{\lfloor 10}{\lfloor 10-3 \cdot \lfloor 3} \cdot a^{10-3} \cdot b^{3}$$

$$= \frac{\lfloor 10}{\lfloor 7 \cdot \lfloor 3} \cdot a^{7} b^{3}$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4} \times \cancel{\lfloor 7}}{\cancel{\lfloor 7} \times \cancel{3} \times \cancel{2} \times 1} \; a^{7} b^{3}$$

$$= 120\, a^{7} b^{3} \quad \text{Ans.}$$

Ques:- $\left(2x + \frac{1}{x}\right)^{9}$ find the coefficient of x.

let the r^{th} term be the coefficient of x

$$t_r = {}^{9}C_{r-1} \cdot (2x)^{9-r+1} \cdot \left(\frac{1}{x}\right)^{r-1}$$

$$= {}^{9}C_{r-1} \cdot 2^{10-r} \cdot x^{10-\cancel{r}} \cdot x^{-r+1}$$

$$= {}^{9}C_{r-1} \left(2^{10-r}\right) x^{10-r+(-r+1)}$$

$$= {}^{9}C_{r-1} \cdot 2^{10-r} \, x^{10-r-r+1}$$

$$= {}^{9}C_{r-1} \cdot 2^{10-r} \, x^{11-2r}$$

$$\therefore x^{11-2r} = x^{1}$$

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$11 - 2r = 1$

$11 - 1 = 2r$

$\cancel{10} = 2r$

$r = \cancel{6}\,5$

coefficient of x

$= {}^{9}C_{5-1} \cdot 2^{10-5} \cdot x^{11-2\times5}$

$= \frac{\lfloor 9}{\lfloor 9-4 \lfloor 4} \cdot 2^{5} \cdot x^{11-10}$

$= \frac{9 \times \cancel{8} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{5} \cdot \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times 2^{5} \cdot x^{1}$

$= 126 \times 32 \times x^{1}$

$= 4032x$

Ques:- $\left(3x^{2} + \frac{1}{5x}\right)^{11}$ find the coefficient of x^{7}

let the r^{th} term be the cofficient of x^{7}

$t_r = {}^{11}C_{r-1} \cdot (3x^{2})^{11-r+1} \cdot \left(\frac{1}{5x}\right)^{r-1}$

$= {}^{11}C_{r-1} \cdot 3^{12-r} \cdot x^{24-2r} \cdot (5x^{-1})^{r-1}$

$= {}^{11}C_{r-1} \cdot 3^{12-r} \cdot x^{24-2r} \cdot 5^{-r+1} \cdot x^{-r+1}$

$= {}^{11}C_{r-1} \cdot 3^{12-r} \cdot 5^{-r+1} \cdot x^{24-2r+(-r+1)}$

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$= {}^{11}C_{r-1} \cdot 3^{12-r} \cdot 5^{-r+1} \cdot x^{25-3r}$

$x^{25-3r} = x^7$

$25-3r = 7$

$25-7 = 3r$

$\cancel{18}^{6} = \cancel{3}r$

$r = 6$

coefficient of x

$= {}^{11}C_{6-1} \cdot 3^{12-6} \cdot 5^{-6+1} \cdot x^{25-3\times 6}$

$= {}^{11}C_5 \cdot 3^6 \cdot 5^{-5} \cdot x^{25-18}$

$= \frac{\lfloor 11}{\lfloor 11-5 \cdot \lfloor 5} \cdot 729 \times \frac{1}{3125} \times x^7$

$= \frac{11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8} \times 7 \times \cancel{6}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times 729 \times \frac{1}{3125} \times x^7$

$= 421 \times 729 \times \frac{1}{3125} \times x^7$

$= 107.77\, x^7$

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Ques:- $(x+y)^{11}$ find the coefficient of x^7

$t_r = {}^{11}C_{r-1} \cdot x^{11-r+1} \cdot y^{r-1}$

$= {}^{11}C_{r-1} \cdot x^{12-r} \cdot y^{r-1}$

$= x^{12-r} = x^7$

$12-r = 7$

$12-7 = r$

$r = 5$

coefficient of x

$= {}^{11}C_{5-1} \cdot x^{12-5} \cdot y^{5-1}$

$= \frac{\lfloor 11}{\lfloor 11-4 \cdot \lfloor 4} \cdot x^7 \cdot y^4$

$= \frac{11 \times 10 \times \cancel{9}^{3} \times \cancel{8} \times \cancel{\lfloor 7}}{\cancel{\lfloor 7} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} x^7 y^4$

$= 330 x^7 y^4$

Ques:- $\left(x+\frac{1}{x}\right)^{11}$ find the coefficient of x.

$$t_r = {}^{11}C_{r-1} \cdot x^{11-r+1} \cdot \left(\frac{1}{x}\right)^{r-1}$$

$$= {}^{11}C_{r-1} \cdot x^{12-r} \cdot (x^{-1})^{r-1}$$

$$= {}^{11}C_{r-1} \cdot x^{12-r} \cdot x^{-r+1}$$

$$= {}^{11}C_{r-1} \cdot x^{12-r+(-r+1)}$$

$$= {}^{11}C_{r-1} \cdot x^{13-2r}$$

$$\cancel{x}^{13-2r} = \cancel{x}^{1}$$

$$13 - 2r = 1$$

$$13 - 1 = 2r$$

$$6\ \cancel{12} = \cancel{2}r$$

$$r = 6$$

$$= \cancel{{}^{11}C_{6-1} \cdot x^{11-6+1}} \cdot \cancel{\frac{x}{x}}\ x$$

$${}^{11}C_{6-1} \cdot x^{13-2\times 6}$$

$$= \frac{\lfloor 11}{\lfloor 11-5 \cdot \lfloor 5} \cdot x^{13-12}$$

$$= \frac{11 \times \cancel{10} \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times 7 \times \cancel{\lfloor 6}}{\cancel{\lfloor 6} \times \cancel{5} \times 4 \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 462x$$

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- Harmonic number :-

Reciprocal of any natural number Called harmonic number.
example - $1, \frac{1}{2}, \frac{1}{3} \dots$

Ques :- write an algorithm to display series of N harmonic number.

Step-1. Print "Number of element in series"
2. input n
3. let $x = 1$
4. Print " $\frac{1}{x}$ "
5. $x = x + 1$
6. if $(x <= n)$
7. Goto step-4
8. end

Ques :- write an algorithm to sum N harmonic number.

Step-1. Print "Number of element in series".
2. input N
3. let $x = 1$, sum = 0.
4. $sum = sum + \frac{1}{x}$
5. $x = x + 1$
6. if $(x <= n)$
7. Goto step-4
8. Print "sum"
9. end.

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- Fibonancci Number :—

If each element of a series is the sum of its previous two element called fibonancci series.
Example:— 0, 1, 1, 2, 3, 5, 8, 13, ...