**Partial Correlation**

data=Boston

# install.packages(“ppcor”)

library(ppcor)

pcor(data) # partial correlation

$estimate

crim zn indus chas nox

crim 1.000000000 0.107320500 -0.034494422 -0.028606382 -0.087796674

zn 0.107320500 1.000000000 -0.107388499 -0.006528860 -0.003695524

indus -0.034494422 -0.107388499 1.000000000 0.099891608 0.261414980

chas -0.028606382 -0.006528860 0.099891608 1.000000000 0.061086716

nox -0.087796674 -0.003695524 0.261414980 0.061086716 1.000000000

rm 0.031627156 0.085165894 -0.090749145 -0.024369491 -0.006593319

age 0.003651004 -0.119638374 0.001066199 0.049950310 0.259979877

dis -0.155989438 0.425310340 -0.195463957 0.029235923 -0.326690576

rad 0.288382408 -0.133103597 -0.275586571 0.075973031 0.178003096

tax -0.033036642 0.240321536 0.435391971 -0.098358984 0.042201060

ptratio -0.065406563 -0.245256280 0.128114974 -0.044693701 -0.370516902

black -0.092116128 -0.012128605 -0.035185106 0.028702975 -0.040702759

lstat 0.074929873 0.111313784 0.075410555 0.005518575 -0.025767447

medv -0.146567512 0.150711871 0.015070149 0.139218384 -0.205231212

rm age dis rad tax

crim 0.031627156 0.003651004 -0.155989438 0.28838241 -0.033036642

zn 0.085165894 -0.119638374 0.425310340 -0.13310360 0.240321536

indus -0.090749145 0.001066199 -0.195463957 -0.27558657 0.435391971

chas -0.024369491 0.049950310 0.029235923 0.07597303 -0.098358984

nox -0.006593319 0.259979877 -0.326690576 0.17800310 0.042201060

rm 1.000000000 0.190350771 0.003084494 0.06574683 -0.012358746

age 0.190350771 1.000000000 -0.275593156 -0.07469425 0.029972173

dis 0.003084494 -0.275593156 1.000000000 0.04969634 -0.028014512

rad 0.065746826 -0.074694248 0.049696341 1.00000000 0.792941644

tax -0.012358746 0.029972173 -0.028014512 0.79294164 1.000000000

ptratio -0.021564801 0.075027449 -0.014321808 0.23924715 0.025293091

black -0.154411133 0.062599159 0.028272708 -0.10144210 -0.003454294

lstat -0.287682457 0.310128503 -0.100703526 0.11770789 -0.075578023

medv 0.380134947 0.002362477 -0.316393756 0.20360917 -0.146283506

ptratio black lstat medv

crim -0.06540656 -0.092116128 0.074929873 -0.146567512

zn -0.24525628 -0.012128605 0.111313784 0.150711871

indus 0.12811497 -0.035185106 0.075410555 0.015070149

chas -0.04469370 0.028702975 0.005518575 0.139218384

nox -0.37051690 -0.040702759 -0.025767447 -0.205231212

rm -0.02156480 -0.154411133 -0.287682457 0.380134947

age 0.07502745 0.062599159 0.310128503 0.002362477

dis -0.01432181 0.028272708 -0.100703526 -0.316393756

rad 0.23924715 -0.101442095 0.117707889 0.203609171

tax 0.02529309 -0.003454294 -0.075578023 -0.146283506

ptratio 1.00000000 0.079658593 -0.092769874 -0.311938474

black 0.07965859 1.000000000 -0.073546403 0.154420357

lstat -0.09276987 -0.073546403 1.000000000 -0.422750535

medv -0.31193847 0.154420357 -0.422750535 1.000000000

$p.value

crim zn indus chas nox

crim 0.000000e+00 1.702489e-02 4.442940e-01 0.52586696 5.115200e-02

zn 1.702489e-02 0.000000e+00 1.695462e-02 0.88491214 9.347029e-01

indus 4.442940e-01 1.695462e-02 0.000000e+00 0.02640975 3.670592e-09

chas 5.258670e-01 8.849121e-01 2.640975e-02 0.00000000 1.752434e-01

nox 5.115200e-02 9.347029e-01 3.670592e-09 0.17524342 0.000000e+00

rm 4.830888e-01 5.855108e-02 4.379450e-02 0.58895767 8.837839e-01

age 9.354878e-01 7.769487e-03 9.811418e-01 0.26782708 4.495428e-09

dis 5.022039e-04 4.031535e-23 1.210573e-05 0.51679526 9.477116e-14

rad 6.460451e-11 3.035895e-03 4.641862e-10 0.09165225 6.946604e-05

tax 4.637927e-01 6.397236e-08 2.868771e-24 0.02882276 3.492699e-01

ptratio 1.466113e-01 3.354691e-08 4.343888e-03 0.32151663 1.604030e-17

black 4.070233e-02 7.880076e-01 4.352210e-01 0.52446980 3.666605e-01

lstat 9.620842e-02 1.330489e-02 9.408700e-02 0.90262447 5.677596e-01

medv 1.086810e-03 7.781097e-04 7.382881e-01 0.00192503 4.245644e-06

rm age dis rad tax

crim 4.830888e-01 9.354878e-01 5.022039e-04 6.460451e-11 4.637927e-01

zn 5.855108e-02 7.769487e-03 4.031535e-23 3.035895e-03 6.397236e-08

indus 4.379450e-02 9.811418e-01 1.210573e-05 4.641862e-10 2.868771e-24

chas 5.889577e-01 2.678271e-01 5.167953e-01 9.165225e-02 2.882276e-02

nox 8.837839e-01 4.495428e-09 9.477116e-14 6.946604e-05 3.492699e-01

rm 0.000000e+00 2.053051e-05 9.454809e-01 1.445174e-01 7.840835e-01

age 2.053051e-05 0.000000e+00 4.637273e-10 9.726217e-02 5.062894e-01

dis 9.454809e-01 4.637273e-10 0.000000e+00 2.702707e-01 5.344692e-01

rad 1.445174e-01 9.726217e-02 2.702707e-01 0.000000e+00 6.195429e-108

tax 7.840835e-01 5.062894e-01 5.344692e-01 6.195429e-108 0.000000e+00

ptratio 6.325484e-01 9.577472e-02 7.508430e-01 7.348932e-08 5.749116e-01

black 5.733005e-04 1.647799e-01 5.307079e-01 2.414773e-02 9.389567e-01

lstat 7.215181e-11 1.788498e-12 2.520345e-02 8.827331e-03 9.335675e-02

medv 1.979441e-18 9.582293e-01 6.013491e-13 5.070529e-06 1.111637e-03

ptratio black lstat medv

crim 1.466113e-01 0.0407023319 9.620842e-02 1.086810e-03

zn 3.354691e-08 0.7880076394 1.330489e-02 7.781097e-04

indus 4.343888e-03 0.4352210457 9.408700e-02 7.382881e-01

chas 3.215166e-01 0.5244697971 9.026245e-01 1.925030e-03

nox 1.604030e-17 0.3666605165 5.677596e-01 4.245644e-06

rm 6.325484e-01 0.0005733005 7.215181e-11 1.979441e-18

age 9.577472e-02 0.1647798854 1.788498e-12 9.582293e-01

dis 7.508430e-01 0.5307078979 2.520345e-02 6.013491e-13

rad 7.348932e-08 0.0241477289 8.827331e-03 5.070529e-06

tax 5.749116e-01 0.9389566709 9.335675e-02 1.111637e-03

ptratio 0.000000e+00 0.0769227191 3.929001e-02 1.308835e-12

black 7.692272e-02 0.0000000000 1.025277e-01 5.728592e-04

lstat 3.929001e-02 0.1025276535 0.000000e+00 7.776912e-23

medv 1.308835e-12 0.0005728592 7.776912e-23 0.000000e+00

$statistic

crim zn indus chas nox rm

crim 0.00000000 2.39431224 -0.76557890 -0.6347800 -1.95497373 0.70187538

zn 2.39431224 0.00000000 -2.39584699 -0.1448202 -0.08197124 1.89595934

indus -0.76557890 -2.39584699 0.00000000 2.2268410 6.00736171 -2.02125355

chas -0.63478004 -0.14482020 2.22684099 0.0000000 1.35750410 -0.54070204

nox -1.95497373 -0.08197124 6.00736171 1.3575041 0.00000000 -0.14625007

rm 0.70187538 1.89595934 -2.02125355 -0.5407020 -0.14625007 0.00000000

age 0.08098372 -2.67290556 0.02364944 1.1093363 5.97198512 4.30081992

dis -3.50289299 10.42358429 -4.42087507 0.6487615 -7.66702695 0.06841771

rad 6.68044796 -2.97888634 -6.35905215 1.6900478 4.01237762 1.46149735

tax -0.73318838 5.49152788 10.72763988 -2.1923385 0.93689944 -0.27415118

ptratio -1.45390099 -5.61143018 2.86533994 -0.9923459 -8.84822874 -0.47844169

black -2.05195893 -0.26904526 -0.78092694 0.6369252 -0.90357967 -3.46658043

lstat 1.66671043 2.48449956 1.67746348 0.1224098 -0.57173946 -6.66276846

medv -3.28651687 3.38157628 0.33431004 3.1183809 -4.65125741 9.11614020

age dis rad tax ptratio black

crim 0.08098372 -3.50289299 6.680448 -0.73318838 -1.4539010 -2.05195893

zn -2.67290556 10.42358429 -2.978886 5.49152788 -5.6114302 -0.26904526

indus 0.02364944 -4.42087507 -6.359052 10.72763988 2.8653399 -0.78092694

chas 1.10933626 0.64876146 1.690048 -2.19233847 -0.9923459 0.63692521

nox 5.97198512 -7.66702695 4.012378 0.93689944 -8.8482287 -0.90357967

rm 4.30081992 0.06841771 1.461497 -0.27415118 -0.4784417 -3.46658043

age 0.00000000 -6.35921659 -1.661440 0.66511378 1.6688932 1.39124510

dis -6.35921659 0.00000000 1.103682 -0.62163592 -0.3177056 0.62736980

rad -1.66143982 1.10368190 0.000000 28.86651266 5.4654833 -2.26176194

tax 0.66511378 -0.62163592 28.866513 0.00000000 0.5612074 -0.07662041

ptratio 1.66889316 -0.31770565 5.465483 0.56120744 0.0000000 1.77254586

black 1.39124510 0.62736980 -2.261762 -0.07662041 1.7725459 0.00000000

lstat 7.23574386 -2.24512540 2.629165 -1.68121008 -2.0666476 -1.63576814

medv 0.05240243 -7.39800360 4.612900 -3.28000914 -7.2825106 3.46679256

lstat medv

crim 1.6667104 -3.28651687

zn 2.4844996 3.38157628

indus 1.6774635 0.33431004

chas 0.1224098 3.11838086

nox -0.5717395 -4.65125741

rm -6.6627685 9.11614020

age 7.2357439 0.05240243

dis -2.2451254 -7.39800360

rad 2.6291645 4.61289977

tax -1.6812101 -3.28000914

ptratio -2.0666476 -7.28251056

black -1.6357681 3.46679256

lstat 0.0000000 -10.34714580

medv -10.3471458 0.00000000

$n

[1] 506

$gp

[1] 12

$method

[1] "pearson"

**Multiple Linear Regression**

model=lm(medv~crim+zn+indus+chas+nox+rm+age+dis+rad+tax+ptratio+black+lstat, data) # multiple linear regression

model

Call:

lm(formula = medv ~ crim + zn + indus + chas + nox + rm + age +

dis + rad + tax + ptratio + black + lstat, data = data)

Coefficients:

(Intercept) crim zn indus chas nox

3.646e+01 -1.080e-01 4.642e-02 2.056e-02 2.687e+00 -1.777e+01

rm age dis rad tax ptratio

3.810e+00 6.922e-04 -1.476e+00 3.060e-01 -1.233e-02 -9.527e-01

black lstat

9.312e-03 -5.248e-01

summary(model)

Call:

lm(formula = medv ~ crim + zn + indus + chas + nox + rm + age +

dis + rad + tax + ptratio + black + lstat, data = data)

Residuals:

Min 1Q Median 3Q Max

-15.595 -2.730 -0.518 1.777 26.199

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 3.646e+01 5.103e+00 7.144 3.28e-12 \*\*\*

crim -1.080e-01 3.286e-02 -3.287 0.001087 \*\*

zn 4.642e-02 1.373e-02 3.382 0.000778 \*\*\*

indus 2.056e-02 6.150e-02 0.334 0.738288

chas 2.687e+00 8.616e-01 3.118 0.001925 \*\*

nox -1.777e+01 3.820e+00 -4.651 4.25e-06 \*\*\*

rm 3.810e+00 4.179e-01 9.116 < 2e-16 \*\*\*

age 6.922e-04 1.321e-02 0.052 0.958229

dis -1.476e+00 1.995e-01 -7.398 6.01e-13 \*\*\*

rad 3.060e-01 6.635e-02 4.613 5.07e-06 \*\*\*

tax -1.233e-02 3.760e-03 -3.280 0.001112 \*\*

ptratio -9.527e-01 1.308e-01 -7.283 1.31e-12 \*\*\*

black 9.312e-03 2.686e-03 3.467 0.000573 \*\*\*

lstat -5.248e-01 5.072e-02 -10.347 < 2e-16 \*\*\*

---

Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.745 on 492 degrees of freedom

Multiple R-squared: 0.7406, Adjusted R-squared: 0.7338

F-statistic: 108.1 on 13 and 492 DF, p-value: < 2.2e-16

**Quantile Regression**

library(quantreg) model=rq(medv~crim+zn+indus+chas+nox+rm+age+dis+rad+tax+ptratio+black+lstat, data, tau=0.75) # quantile regression

model

Call:

rq(formula = medv ~ crim + zn + indus + chas + nox + rm + age +

dis + rad + tax + ptratio + black + lstat, tau = 0.75, data = data)

Coefficients:

(Intercept) crim zn indus chas

21.170102381 -0.080576504 0.067067434 -0.035744856 3.027856394

nox rm age dis rad

-13.351317244 5.485821488 0.005293005 -1.300071480 0.280815565

tax ptratio black lstat

-0.008719632 -1.031571808 0.014904265 -0.379773814

Degrees of freedom: 506 total; 492 residual

summary(model)

Call: rq(formula = medv ~ crim + zn + indus + chas + nox + rm + age +

dis + rad + tax + ptratio + black + lstat, tau = 0.75, data = data)

tau: [1] 0.75

Coefficients:

coefficients lower bd upper bd

(Intercept) 21.17010 3.01523 31.75960

crim -0.08058 -0.17530 0.16707

zn 0.06707 0.03283 0.08393

indus -0.03574 -0.12150 0.07128

chas 3.02786 0.38695 7.11675

nox -13.35132 -18.23297 -5.97567

rm 5.48582 3.83761 7.13095

age 0.00529 -0.02061 0.01804

dis -1.30007 -1.54959 -0.99592

rad 0.28082 0.12242 0.37647

tax -0.00872 -0.01471 -0.00396

ptratio -1.03157 -1.19289 -0.76890

black 0.01490 0.00213 0.01868

lstat -0.37977 -0.46092 -0.22963