End-Semester Project

Bivariate Analysis between GDP per capita, Sanitation and Life Expectancy across Nations in 2010

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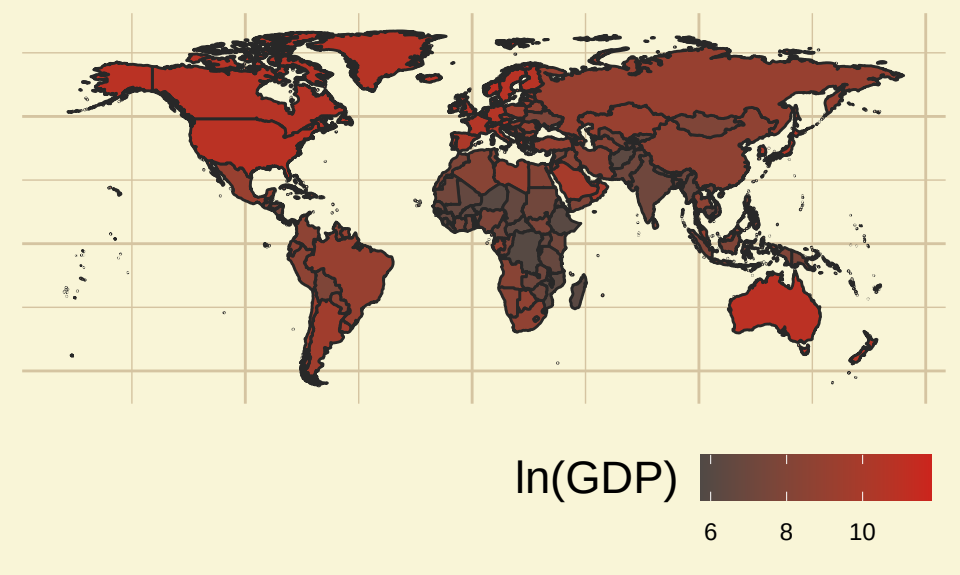
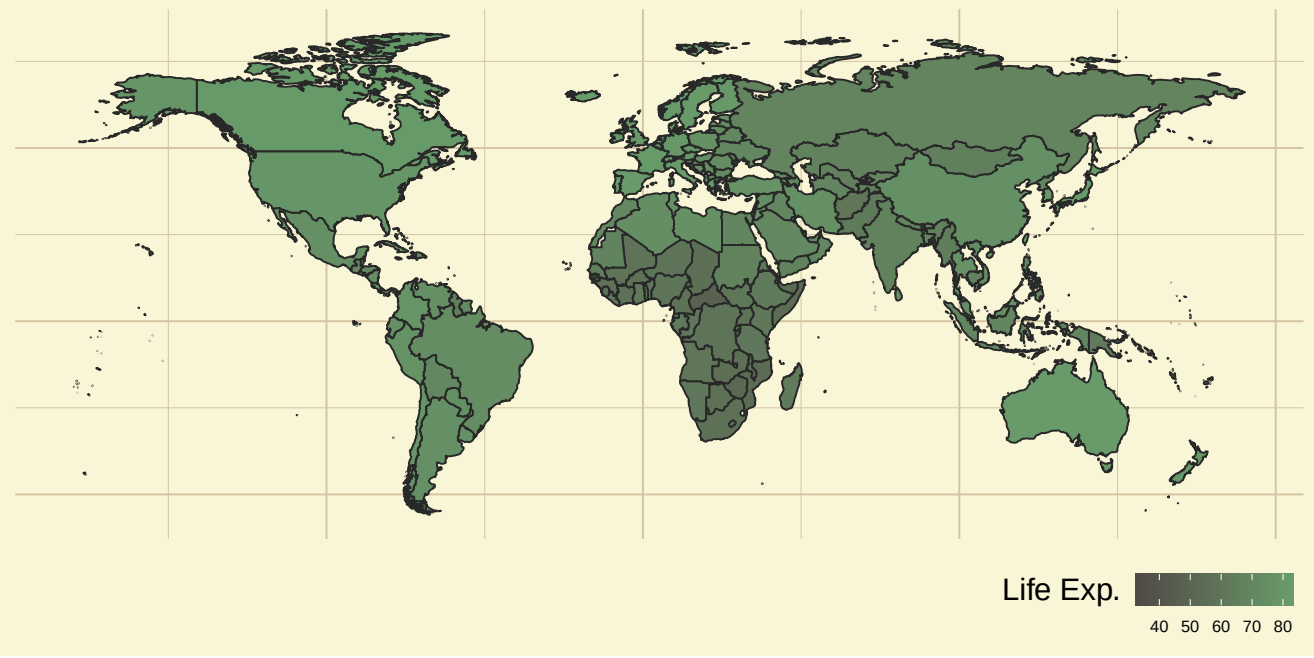
1 Introduction

1.1 Overview

Our *Aim* is to determine which features more directly affect the *Life Expectation* of the citizens.




Sanitation
25
50
75
100


Life Exp.
40
50
60
70
80

1.2 Variables

This presentation used various methods of *Bivariate Analysis* to infer relationship between various features of Nations.

- **log of GDP per capita**: Logarithm (base e) of Gross Domestic Product (in $) per citizen. Adjusted for Inflation. [*lngdp*]

  The Gross Domestic Product per capita of a country basically tells about the wealth of the citizens of the country.

- **Sanitation Access %**: Percentage of people using at least basic Sanitation facilities, not shared with other households. [*snt*]

  Sanitation of oneself is one of the basic tasks of a human being. We know poor sanitation is linked to transmission of diarrheal diseases such as cholera and dysentery, as well as typhoid, intestinal worm infections and polio which directly affect the health of an individual.

- **Life Expectancy**: The average number of years a newly born child would live, provided current mortality patterns hold. [*lfx*]

  Life expectancy is calculated based on the assumption that probability of death at a certain age stays constant into the future. Hence we can use Life Expectation as a measurable proxy for health in the current year.

1.3 Data

```
script.dir <- getSrcDirectory(function(x) {x})
setwd(script.dir)

numerise = function(x){
  x[grepl("k$", x)] <- as.numeric(sub("k$", "", x[grepl("k$", x)]))*10^3
  x <- as.numeric(x)
  return(x)
}

d1_raw = read.csv(file.path(".","Data","gdp.csv"), fileEncoding = 'UTF-8-BOM')
d2_raw = read.csv(file.path(".","Data","sanitation.csv"), fileEncoding = 'UTF-8-BOM')
d3_raw = read.csv(file.path(".","Data","life_expectancy.csv"), fileEncoding = 'UTF-8-BOM')

yearname = "X2010"

d1 = d1_raw[!is.na(numerise(d1_raw[, yearname])),][,c("country", yearname)]
colnames(d1)[2] = "lngdp"
d2 = d2_raw[!is.na(numerise(d2_raw[, yearname])),][,c("country", yearname)]
colnames(d2)[2] = "snt"
d3 = d3_raw[!is.na(numerise(d3_raw[, yearname])),][,c("country", yearname)]
colnames(d3)[2] = "lfx"
```

```
dtemp = merge(x = d1, y = d2, by = "country")
d = merge(x = dtemp, y = d3, by = "country")

d$lngdp = log(numerise(d$lngdp))

write.csv(d, "./Data/assembled.csv")

kable(head(d, 6L))
```

country	lngdp	snt	lfx
Afghanistan	6.265301	34.9	60.5
Albania	8.183118	95.2	78.1
Algeria	8.273847	87.0	74.5
Andorra	10.454495	100.0	81.8
Angola	8.291547	41.1	60.2
Antigua and Barbuda	9.546813	86.3	75.9

2 Elementary Univariate Analysis

2.1 Measures of Central Tendency

Mean or Arithmetic Mean $\bar{x}$, *Median* $\text{median}(x)$ and *Mode* $\text{mode}(x)$ are some measures of *central tendency* in the sample.

Formulae

$$x = \{x_1, x_2, \dots, x_{n-1}, x_n\} \quad \text{mean}(x) = \bar{x} = \frac{1}{n}\sum_{i=1}^{n}(x_i)$$

$$\text{median}(x) = \frac{x_{\lfloor \frac{n+1}{2} \rfloor} + x_{\lfloor \frac{n+2}{2} \rfloor}}{2} \quad \text{mode}(x) = x_i \text{ s.t. } \Pr(x_i) = \sup(\Pr(x))$$

Note: f_i is the frequency of the ith observation. $x_{(i)}$ is the ith largest observation.

```
getmode <- function(v) {
 uniqv <- unique(v)
 freq = max(tabulate(match(v, uniqv)))
 res = uniqv[which.max(tabulate(match(v, uniqv)))]
 if (freq == 1) res = NULL
 return(res)
}
```

```
d_central = data.frame(
  row.names = "Variable",
  Variable = c(
    "*ln(GDP)*",
    "*Sanitation*",
    "*Life Exp.*"
  ),
  Mean = c(
    mean(d$lngdp),
    mean(d$snt),
    mean(d$lfx)
  ),
  Median = c(
    median(d$lngdp),
    median(d$snt),
    median(d$lfx)
  ),
  Mode = c(
    getmode(d$lngdp),
    getmode(d$snt),
    getmode(d$lfx)
  )
)

kable(
  d_central,
  col.names = c(
    "$\\quad \\quad \\bar{x}$",
    "$\\operatorname{median}(x)$",
    "$\\operatorname{mode}(x)$"
  ),
  digits=5
)
```

	$\bar{x}$	median(x)	mode(x)
ln(GDP)	8.54124	8.48673	9.23014
Sanitation	72.43857	85.60000	100.00000
Life Exp.	70.54603	72.40000	73.20000

- Notice that mode of Sanitation is 100. Thus a large number of countries have universal access to basic sanitation infrastructure.

2.2 Measures of Dispersion

Range $\text{range}(x)$, *Semi-Interquartile Range* $\text{SIR}(x)$, *Mean Deviation about x'* $\text{MD}_{(x')}(x)$, *Variance* s_x^2, *Standard Deviation* s_x are some measures of *dispersion* in the sample.

Formulae

$$\text{range}(x) = |x_{(n)} - x_{(1)}| \quad Q_1 = \text{median}(x_{(1)}, \dots, x_{(\lfloor \frac{n+1}{2} \rfloor)})$$
$$Q_3 = \text{median}(x_{(\lfloor \frac{n+2}{2} \rfloor)}, \dots, x_{(n)}) \quad \text{MD}_{(x')}(x) = \text{mean}(|x_i - x'|)$$
$$\text{SIR}(x) = \frac{|Q_1 - Q_3|}{2} \quad s_x = \sqrt{\text{mean}([x_i - \bar{x}]^2)} \quad s_x^2 = (s_x)^2$$

```
getmd = function(x, center = mean(x)){
  md = mean(
    abs(
      x - rep(center, length(x))
      )
    )
  return(md)
}
d_disp = data.frame(
  row.names = "Variable",
  Variable = c(
    "*ln(GDP)*",
    "*Sanitation*",
    "*Life Exp.*"
  ),
  Range = c(
    max(d$lngdp) - min(d$lngdp),
    max(d$snt) - min(d$snt),
    max(d$lfx) - min(d$lfx)
  ),
  SIR = c(
    IQR(d$lngdp)/2,
    IQR(d$snt)/2,
    IQR(d$lfx)/2
  ),
  MD = c(
    getmd(d$lngdp),
    getmd(d$snt),
    getmd(d$lfx)
  ),
  variance = c(
    (sd(d$lngdp))^2,
    (sd(d$snt))^2,
```

```
    (sd(d$lfx))^2
  ),
  SD = c(
    sd(d$lngdp),
    sd(d$snt),
    sd(d$lfx)
  )
)

kable(
  d_disp,
  col.names = c(
    "$\\operatorname{range}(x)$",
    "$\\operatorname{SIR}(x)$",
    "$\\operatorname{MD}_{(\\bar{x})}(x)$",
    "$\\quad \\quad \\quad \\quad s_x^2$",
    "$\\quad \\quad \\quad \\quad s_x$"
  ),
  digits=5
)
```

	range(x)	SIR(x)	$\text{MD}_{(\bar{x})}(x)$	s_x^2	s_x
ln(GDP)	6.04435	1.06914	1.17229	2.01791	1.42053
Sanitation	94.03000	24.65000	25.50487	872.29346	29.53461
Life Exp.	50.80000	6.00000	6.98712	75.33494	8.67957

- We can compare s_x to $\bar{x}$ and observe that there is a high variation in Sanitation amongst countries.
- GDP per capita varies drastically across the $e6.044354$ range.

2.3 Measures of Shape

Coefficients of *Skewness* g_1 and *Kurtosis* g_2 describe the symmetry and extremity of tails of the sample distribution.

Formulae

$$m_k = \text{mean}([x - \bar{x}]^k) \quad g_1 = \frac{m_3}{{m_2}^{\frac{3}{2}}} \quad g_2 = \frac{m_4}{{m_2}^2}$$

```
d_shape = data.frame(
  row.names = "Variable",
  Variable = c(
```

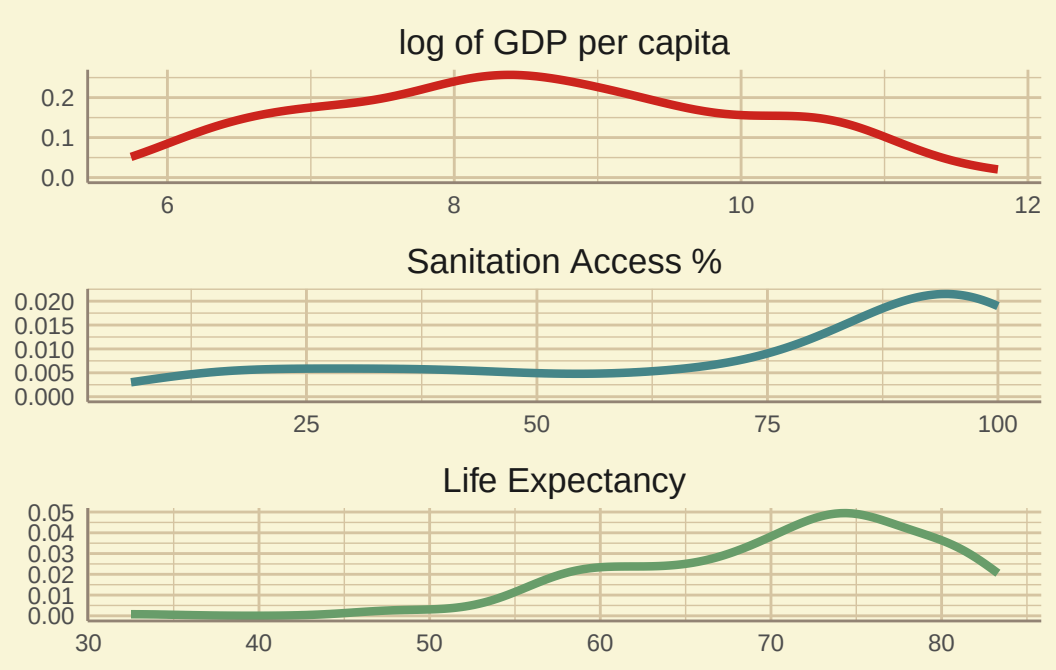
```
    "*ln(GDP)*",
    "*Sanitation*",
    "*Life Exp.*"
  ),
  Skewness = c(
    skewness(d$lngdp),
    skewness(d$snt),
    skewness(d$lfx)
  ),
  Kurtosis = c(
    kurtosis(d$lngdp),
    kurtosis(d$snt),
    kurtosis(d$lfx)
  )
)

kable(
  d_shape,
  col.names = c(
    "Skewness $g_1$",
    "Kurtosis $g_2$"
  ),
  digits=5
)
```

	Skewness g_1	Kurtosis g_2
ln(GDP)	0.09619	2.14435
Sanitation	-0.85989	2.27111
Life Exp.	-0.87903	4.02465

- ln(GDP) is nearly symmetrical, while Sanitation and Life Exp. are highly left-skewed. This indicates majority of countries have good sanitation system and citizen health.
- ln(GDP) and Sanitation are platykurtic, while Life Exp. is leptokurtic.

2.4 Density Plot

```
labelfunction = function(val1){
  return(list(c(
    "log of GDP per capita",
    "Sanitation Access %",
    "Life Expectancy"
  )))
}
ggplot(stack(d[2:4]), mapping = aes(x = values))+
geom_density(aes(color=ind), linewidth=rel(1.5))+
labs(
  x=NULL,
  y=NULL
  )+
mytheme+
mycolor+
facet_wrap(~ind, scales="free", labeller = labelfunction, ncol=1)+
  theme(legend.position="none",
        strip.text.x = element_text(size = rel(1.5))
  )
```



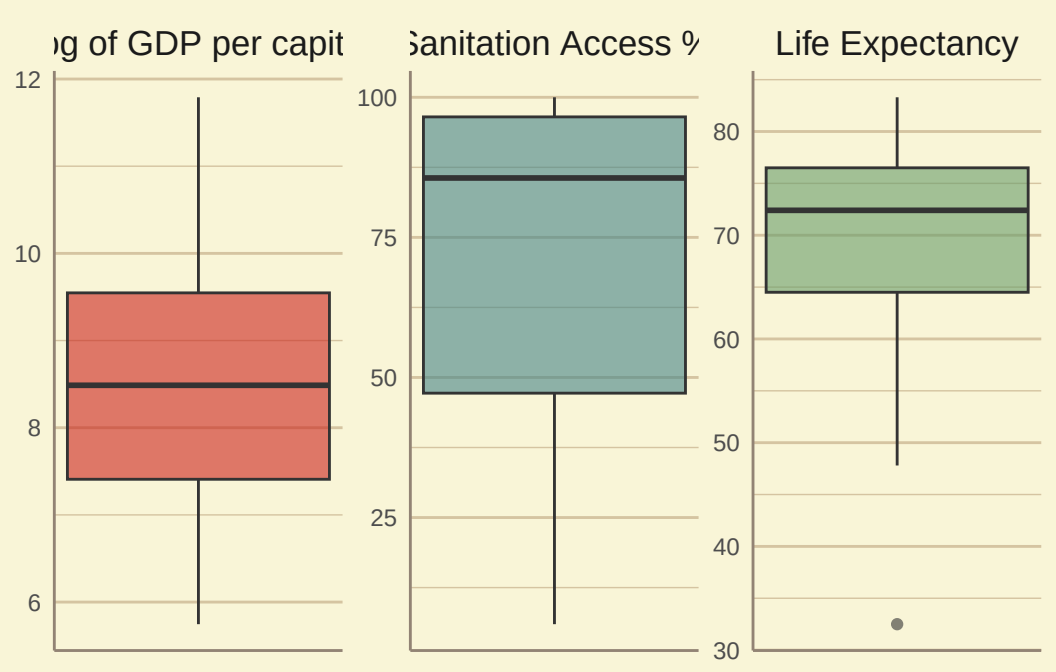
- The density of the log of GDP per capita is symmetric as inferred previously. The other two density plots

appear left skewed as supported by the negative skewness.

2.5 Box Plot

Box plots help us detect potential outliers. They also help us in estimating location and skewness of the distribution.

```
labelfunction = function(val1){
  return(list(c(
    "log of GDP per capita",
    "Sanitation Access %",
    "Life Expectancy"
  )))
}
ggplot(stack(d[2:4]), mapping = aes(y = values))+
geom_boxplot(aes(fill=ind), alpha=0.6)+
labs(
  x=NULL,
  y=NULL
  )+
mytheme+
mycolor+
facet_wrap(~ind, scales="free", labeller = labelfunction)+
  theme(axis.text.x=element_blank(),
        legend.position="none",
        strip.text.x = element_text(size = rel(1.5)),
        panel.grid.minor.x = element_blank(),
        panel.grid.major.x = element_blank()
  )
```



- We observe one potential outlier in the Life Expectancy dataset. It is Haiti at 32.5 years. It is due to a cholera outbreak and an earthquake increasing the mortality rate in the nation in 2010.

3 Scatter Plot

A *Scatter plot* helps us estimate the type of relationship between variables.

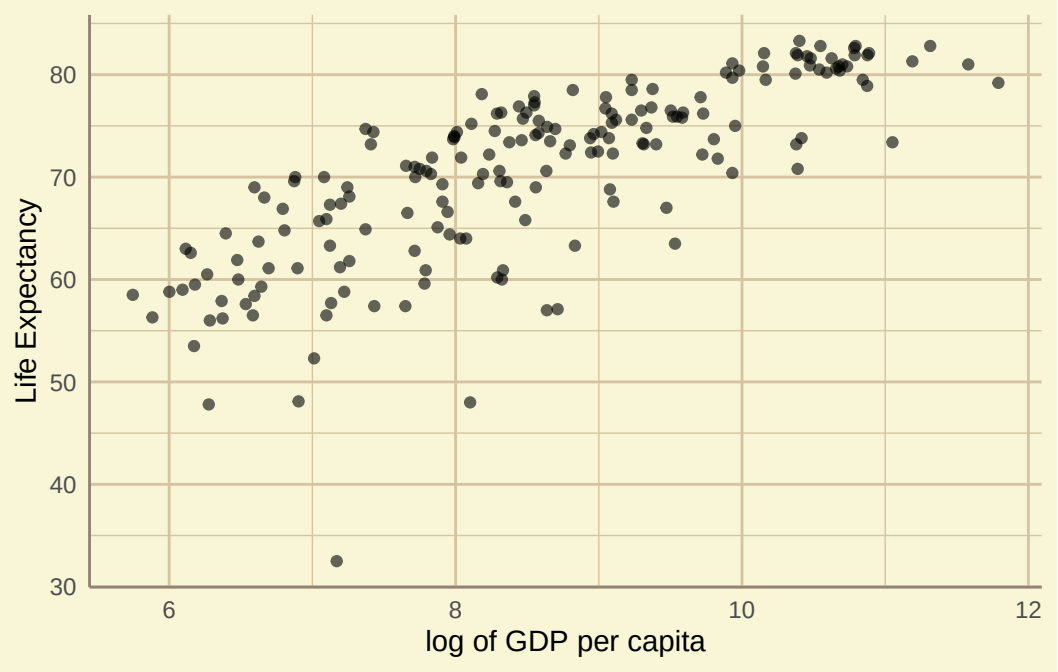
```
sctrplot = function(
    d, x_map, y_map,
    x_lab=waiver(), y_lab=waiver(),
    title=waiver()
){
  plot1 = ggplot(d, mapping = aes(x = x_map, y = y_map))+
    geom_point(
      alpha=0.6
    )+
    mytheme+
    labs(
      x=x_lab,
      y=y_lab,
      title=title
    )
```

```
  return(plot1)
}
```

3.1 Sanitation vs. log of GDP

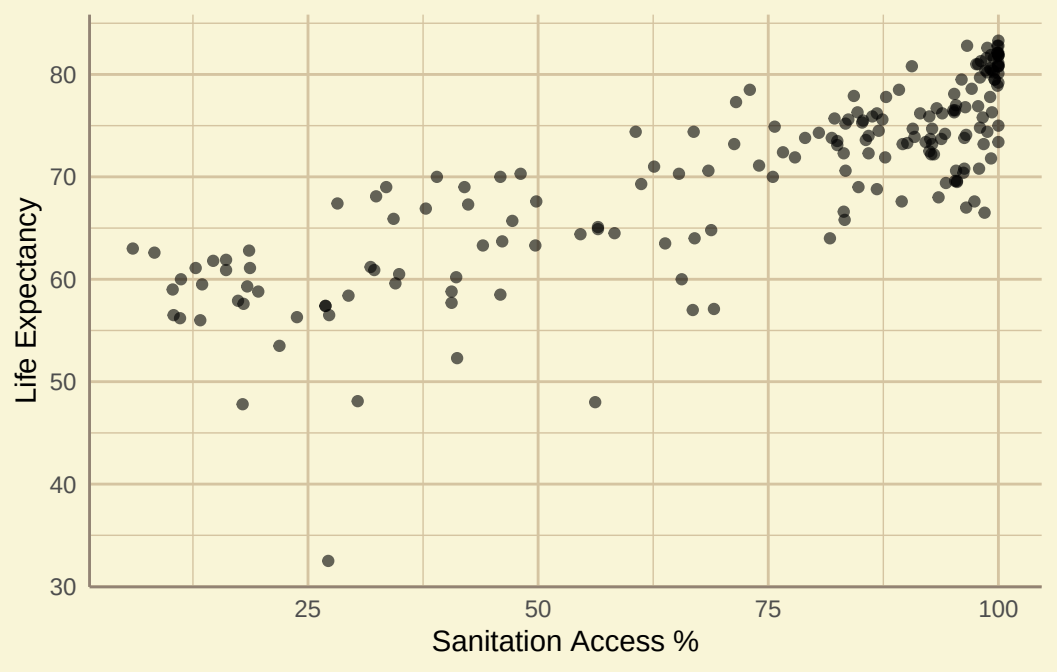
- We observe from the scatter plot that the correlation between ln(GDP) and sanitation access appears to be mostly Linear for countries with low ln(GDP).
- Also, the sanitation access is very close to 100% for countries with high ln(GDP).

3.2 Life Expectancy vs. log of GDP



- We can see that except for some countries with very low life expectancies, the correlation between the variables is appearing to be linear.

3.3 Life Expectation vs. Sanitation



- We can see that the correlation appears to be linear between life expectation and sanitation. We also observe that there is clustering around the top right side of the plot, which is supported by the box plots of both the distributions.

3.3.1 Inferences

We observe a fairly strong positive Linear Correlation between the three features in the Scatter Plot.

We subsequently compute the Correlation Coefficients to quantify the Linear Correlation.

4 Bivariate Statistics

4.1 Correlation Coefficients

Correlation is any relationship, causal or spurious, between two random variables x, y.

Pearson's r, *Spearman's* r_s, and *Kendall's* τ are some correlation coefficients. These estimate the linear correlation between two variables.

```
d_cor = data.frame(
  row.names = "Variable",
```

```
  Variable = c(
    "*Sanitation vs. ln(GDP)*",
    "*Life Exp. vs. ln(GDP)*",
    "*Life Exp. vs. Sanitation*"
  ),
  Pearson = c(
    cor(d$snt, d$lngdp, method="pearson"),
    cor(d$lfx, d$lngdp, method="pearson"),
    cor(d$lfx, d$snt, method="pearson")
  ),
  Spearman = c(
    cor(d$snt, d$lngdp, method="spearman"),
    cor(d$lfx, d$lngdp, method="spearman"),
    cor(d$lfx, d$snt, method="spearman")
  ),
  Kendall = c(
    cor(d$snt, d$lngdp, method="kendall"),
    cor(d$lfx, d$lngdp, method="kendall"),
    cor(d$lfx, d$snt, method="kendall")
  )
)

avg_cor = round(mean(d_cor[, 1]), digits=2)

kable(
  d_cor,
  digit = 5,
  col.names = c(
    "*Pearson's* $r$",
    "*Spearman's* $r_s$",
    "*Kendall's* $\\tau$"
  )
)
```

	Pearson's r	*Spearman's* r_s	*Kendall's* τ
Sanitation vs. ln(GDP)	0.80659	0.85920	0.67458
Life Exp. vs. ln(GDP)	0.77229	0.81639	0.62168
Life Exp. vs. Sanitation	0.81351	0.83513	0.63744

4.2 Covariance and Correlation Matrices

Covariance $\text{cov}(x, y)$ is a measure of the joint variability of two random variables x, y.

Formulae

$$\text{cov}(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n} \quad r_{x,y} = \frac{\text{cov}(x,y)}{s_x s_x}$$

```
cov_mat = cov(d[, 2:4])

kable(cov_mat, digits=5)
```

	lngdp	snt	lfx
lngdp	2.01791	33.84045	9.52202
snt	33.84045	872.29346	208.54155
lfx	9.52202	208.54155	75.33494

$$A_{i,j} = \text{cov}(x_i, x_j)$$

```
cor_mat = cor(d[, 2:4])

kable(cor_mat, digits=5)
```

	lngdp	snt	lfx
lngdp	1.00000	0.80659	0.77229
snt	0.80659	1.00000	0.81351
lfx	0.77229	0.81351	1.00000

$$A_{i,j} = r_{x_i,x_j}$$

4.2.1 Inferences

We observe fairly good Linear Correlation of around 0.8 between all three variables. Thus a slight increase in ln(GDP), Sanitation or Life expectancy tends to accompany rise in the other two features increasing too.

Let us try to create a Linear Model to best estimate the relationship between the three variables.

5 Regression

5.1 Simple Linear Regression

Simple Univariate Linear Regression is a method for estimating the relationship $y_i = f(x_i)$ of a *response* variable y with a *predictor* variable x, as a line that closely fits the y vs. x *scatter plot*.

$$y_i = \hat{a} + \hat{b}x_i + e_i$$

Where $\hat{a}$ is the *intercept*, $\hat{b}$ is the *slope*, and e_i is the ith residual *error*. We aim to minimize e_i for better fit.

5.2 Ordinary Least Squares

Ordinary Least squares method reduces e_i by minimizing *error sum of squares* $\sum {e_i}^2$.

Coefficient of Determination R^2 is the proportion of the variation in y predictable by the model.

Formulae

$$\hat{b} = r\frac{s_y}{s_x} \quad \hat{a} = \bar{y} - \hat{b}\bar{x} \quad R^2 = 1 - \frac{\sum {e_i}^2}{\sum (y - \bar{y})^2}$$

```
olssmry = function(
    d, x_map, y_map,
    x_lab=waiver(), y_lab=waiver(),
    title=waiver()
){
  model = lm(formula=y_map~x_map)
  smry = summary(model, signif.stars=TRUE)

  smryvec = c(
    as.numeric(model$coefficients["(Intercept)"]),
    as.numeric(model$coefficients["x_map"]),
    smry$r.squared
  )

  return(smryvec)
}

olstab = t(data.frame(
  SvG = olssmry(d, d$lngdp, d$snt),
  LvG = olssmry(d, d$lngdp, d$lfx),
  LvS = olssmry(d, d$snt, d$lfx)
))

row.names(olstab) = c(
  "*Sanitation vs. ln(GDP)*",
  "*Life Exp. vs. ln(GDP)*",
```

```
  "*Life Exp. vs. Sanitation*"
)

avg_r2 = round(mean(olstab[, 3]), digits = 1)

kable(
  olstab,
  digit = 5,
  col.names=c(
  "$\\hat{a}$",
  "$\\hat{b}$",
  "$R^2$"
  )
)
```

	$\hat{a}$	$\hat{b}$	R^2
Sanitation vs. ln(GDP)	-70.79844	16.77006	0.65059
Life Exp. vs. ln(GDP)	30.24203	4.71876	0.59643
Life Exp. vs. Sanitation	53.22795	0.23907	0.66180

- The R^2 values of all three models are all near 0.6 . Thus the models explain the variation in the response y_i fairly well.

5.3 Least Absolute Deviation

Least absolute Deviation method reduces e_i by minimizing the *sum of absolute deviations* $\sum |e_i|$.

```
ladsmry = function(
    d, x_map, y_map,
    x_lab=waiver(), y_lab=waiver(),
    title=waiver()
){
  model = rq(formula=y_map~x_map)
  smry = summary(model)

  smryvec = c(
    as.numeric(model$coefficients[1]),
    as.numeric(model$coefficients[2])
  )

  return(smryvec)
}
```

```
olstab = t(data.frame(
  SvG = ladsmry(d, d$lngdp, d$snt),
  LvG = ladsmry(d, d$lngdp, d$lfx),
  LvS = ladsmry(d, d$snt, d$lfx)
))

row.names(olstab) = c(
  "*Sanitation vs. ln(GDP)*",
  "*Life Exp. vs. ln(GDP)*",
  "*Life Exp. vs. Sanitation*"
)

kable(
  olstab,
  digit = 5,
  col.names=c(
  "$\\hat{a}$",
  "$\\hat{b}$"
  )
)
```

	$\hat{a}$	$\hat{b}$
Sanitation vs. ln(GDP)	-71.23153	16.80472
Life Exp. vs. ln(GDP)	31.99047	4.61340
Life Exp. vs. Sanitation	53.73041	0.23963

6 Line Fitting

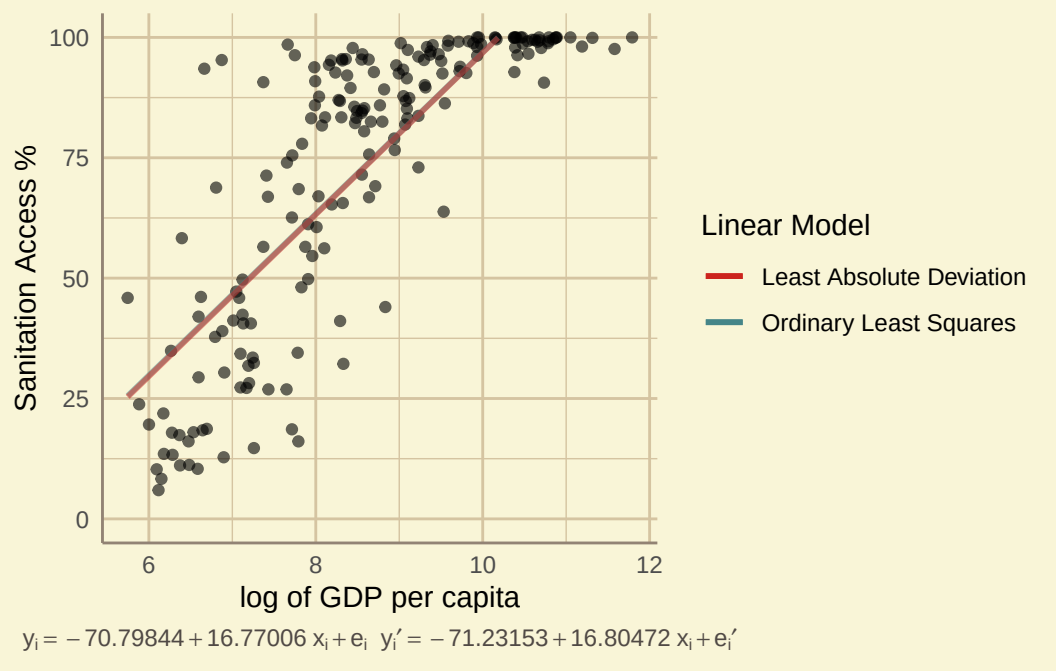
Plotting the estimated *Linear Models* on the Scatter Plots.

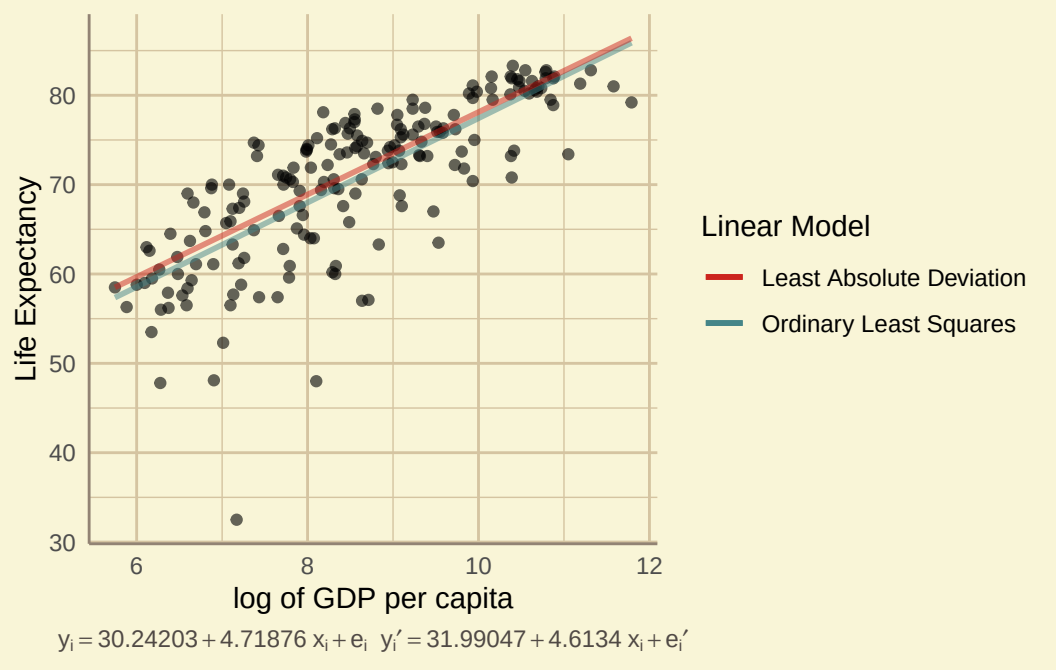
```
linearplot = function(
    d, x_map, y_map,
    x_lab=waiver(), y_lab=waiver(),
    title=waiver()
){
  olsvec = round(olssmry(d, x_map, y_map), digit=5)
  ladvec = round(ladsmry(d, x_map, y_map), digit=5)
  capstr = TeX(paste("$y_i =", olsvec[1], "+", olsvec[2], "~x_i + e_i$", "\t\t",
                     "$y_i' =", ladvec[1], "+", ladvec[2], "~x_i + e_i'$"))

  plot1 = ggplot(d, mapping = aes(x = x_map, y = y_map))+
    geom_point(
```

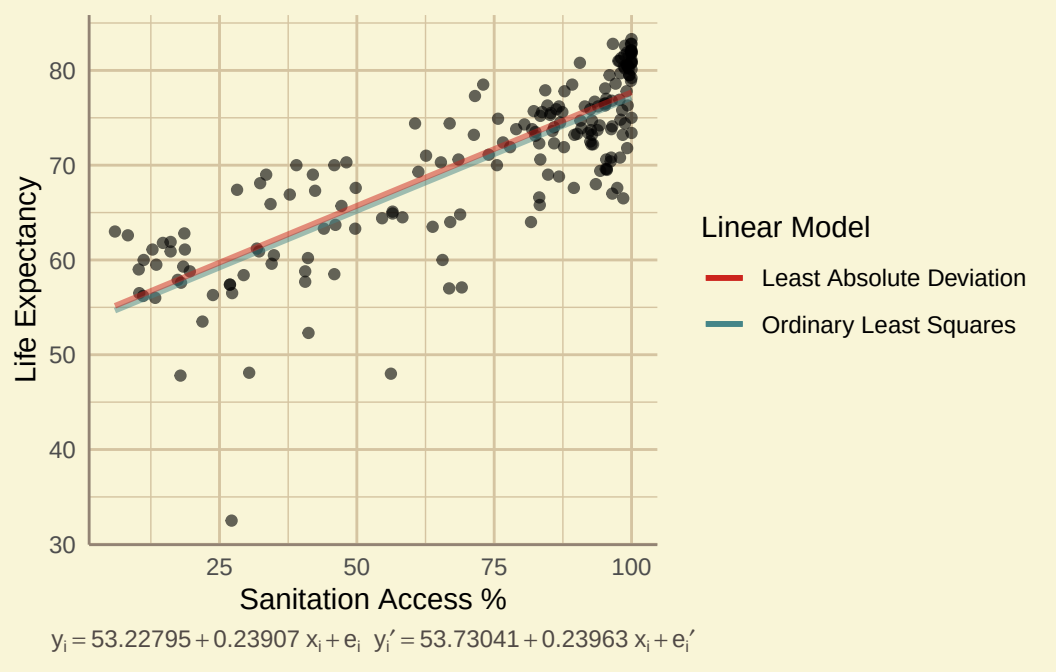
```
      alpha=0.6
    )+
    mytheme+
    labs(
      x=x_lab,
      y=y_lab,
      title=title,
      caption=capstr,
      parse=TRUE
    )+
    geom_smooth(
      method="lm",
      formula=y~x,
      se=FALSE,
      aes(color = "Ordinary Least Squares")
    )+
    geom_smooth(
      method="rq",
      formula=y~x,
      se=FALSE,
      aes(color = "Least Absolute Deviation")
    )+
    labs(
      color="Linear Model"
    )+
    scale_color_manual(
      values=c(
        "#cc241d80",
        "#45858880",
        "#689d6a80",
        "#d65d0e80"
        )
      )+
    theme(
      plot.caption = element_text(hjust=0.5, color="#504945")
    )

  return(plot1)
}
```

6.1 Sanitation vs. log of GDP

6.2 Life Expectancy vs. log of GDP


Sanitation Access %
100
75
50
25
0
6
8
10
12
log of GDP per capita
Linear Model
Least Absolute Deviation
Ordinary Least Squares
$y_i = -70.79844 + 16.77006\, x_i + e_i$ $y_i' = -71.23153 + 16.80472\, x_i + e_i'$


Life Expectancy
80
70
60
50
40
30
6
8
10
12
log of GDP per capita
Linear Model
Least Absolute Deviation
Ordinary Least Squares
$y_i = 30.24203 + 4.71876\, x_i + e_i$ $y_i' = 31.99047 + 4.6134\, x_i + e_i'$



6.3 Life Expectancy vs. Sanitation

6.3.1 Inferences

Both OLS and LAD models fit the scatter plots very well. The OLS model is affected in the Life Expectancy plots due to the existence of Haiti as an outlier.

Our aim is to deduce which features more directly affect the Life Expectancy. Thus we should remove the effect of the other features when computing correlation.

7 Partial Correlation

Partial Correlation is the relationship between two variables x, y of interest, after removing effect of some other related variable z.

Formulae

$$x_i = \hat{a_x} + \hat{b_x} z_i + e_{x,i} \quad y_i = \hat{a_y} + \hat{b_y} z_i + e_{y,i}$$
$$\Rightarrow r_{x,y;z} = r_{e_x, e_y}$$

```r
partcor = pcor(d[, 2:4])$estimate

pcortab = data.frame(
  row.names = "Variable",
  Variable = c(
    "*Sanitation vs. ln(GDP)*",
    "*Life Exp.$\\quad$ vs. ln(GDP)*",
    "*Life Exp. vs. Sanitation*"
  ),
  PCor = c(
    partcor[2, 1],
    partcor[3, 1],
    partcor[3, 2]
  )
)

kable(pcortab,
      col.names = c(
        "Partial Correlation"
      ))
```

	Partial Correlation
Sanitation vs. ln(GDP)	0.4826925
Life Exp. vs. ln(GDP)	0.3377892
Life Exp. vs. Sanitation	0.5075384

- We observe that the partial correlation between Life Exp. and Sanitation is higher than that of Life. Exp and ln(GDP).

Thus Life Expection is more directly improved by better Sanitaion access, than increase in wealth.

- We also note that there is fairly high partial correlation between Sanitation and ln(GDP).

This indicates that wealthier countries tend to improve their sanitation infrastructure.

8 Considerations

8.1 Time

```r
library(knitr)
library(ggplot2)
library(quantreg)
library(ppcor)
library(psych)
```

```r
library(moments)
library(latex2exp)
library(maps)
library(dplyr)
```

```
Attaching package: 'dplyr'
```

```
The following objects are masked from 'package:data.table':

    between, first, last
```

```
The following object is masked from 'package:MASS':

    select
```

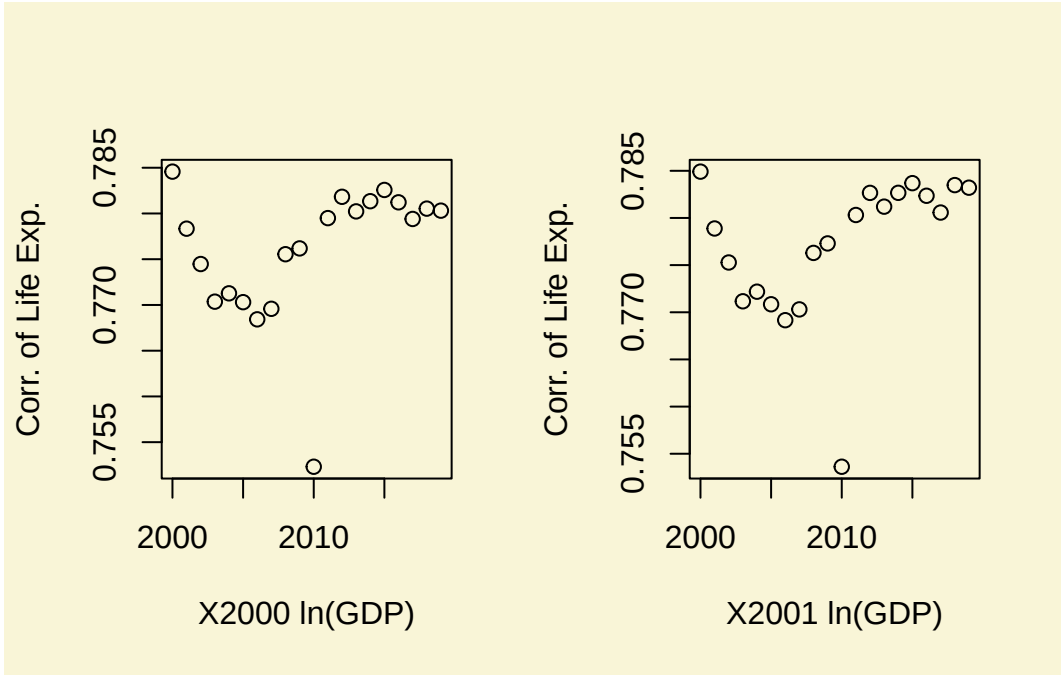
```
The following objects are masked from 'package:stats':

    filter, lag
```

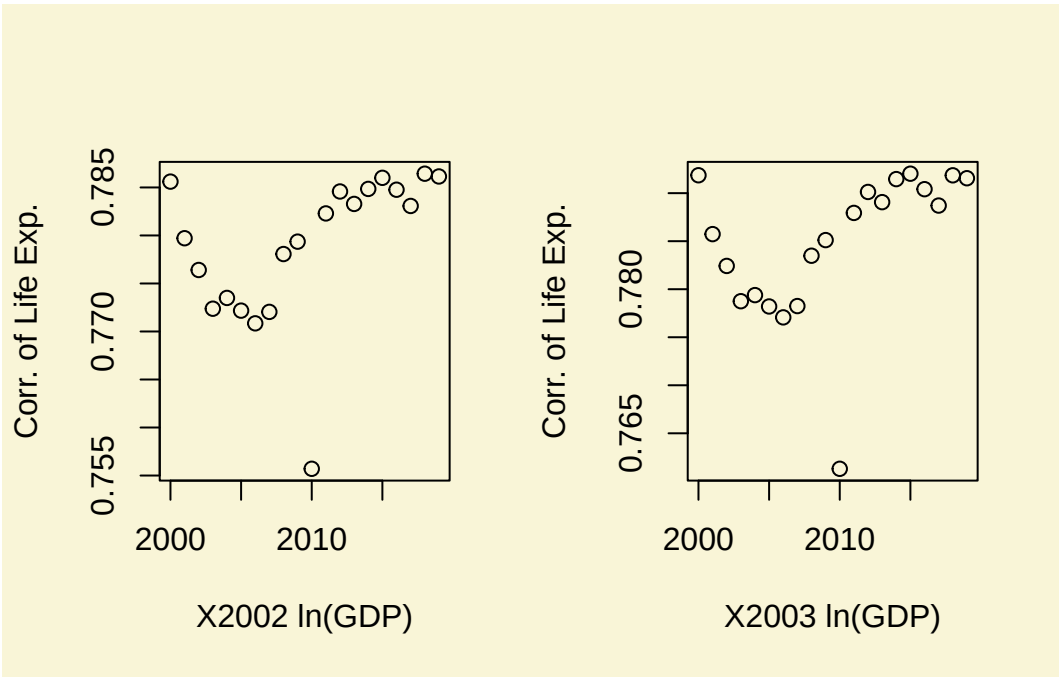
```
The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

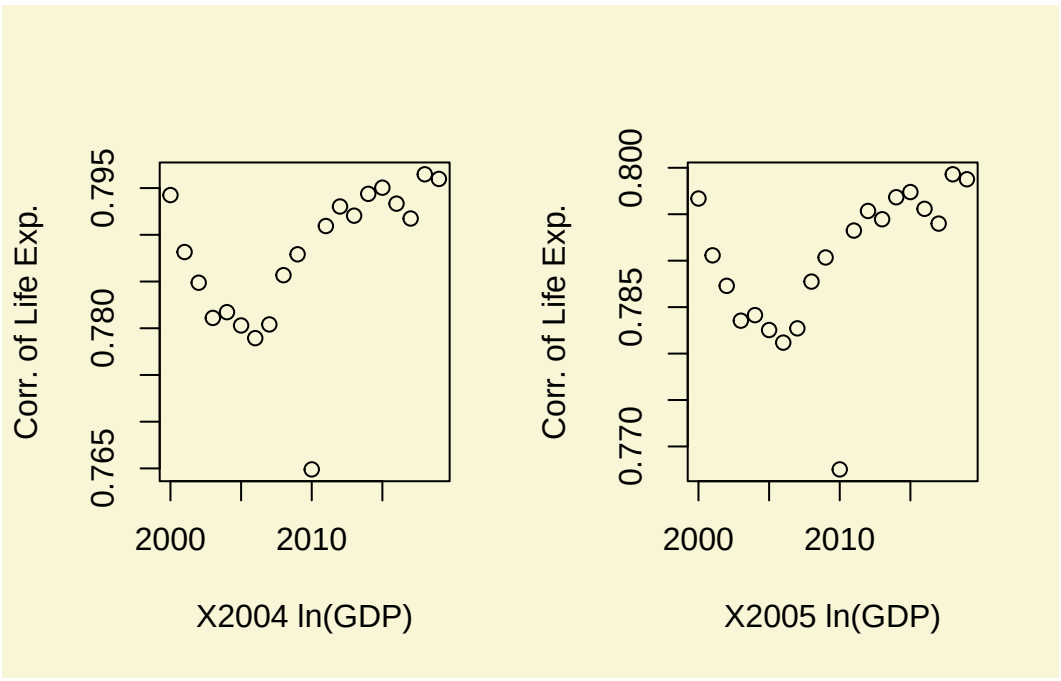
```r
numerise = function(x){
  x[grepl("k$", x)] <- as.numeric(sub("k$", "", x[grepl("k$", x)]))*10^3
  x <- as.numeric(x)
  return(x)
}

d1_raw = read.csv(file.path(".","Data","gdp.csv"), fileEncoding = 'UTF-8-BOM')
d2_raw = read.csv(file.path(".","Data","sanitation.csv"), fileEncoding = 'UTF-8-BOM')
d3_raw = read.csv(file.path(".","Data","life_expectancy.csv"), fileEncoding = 'UTF-8-BOM')

years = 2000:2019
yearnames = paste0("X", years)

makedata = function(yearname){
  d1 = d1_raw[!is.na(numerise(d1_raw[, yearname])),][,c("country", yearname)]
  colnames(d1)[2] = "lngdp"
  d2 = d2_raw[!is.na(numerise(d2_raw[, yearname])),][,c("country", yearname)]
  colnames(d2)[2] = "snt"
  d3 = d3_raw[!is.na(numerise(d3_raw[, yearname])),][,c("country", yearname)]
  colnames(d3)[2] = "lfx"

  dtemp = merge(x = d1, y = d2, by = "country")
  d = merge(x = dtemp, y = d3, by = "country")
```

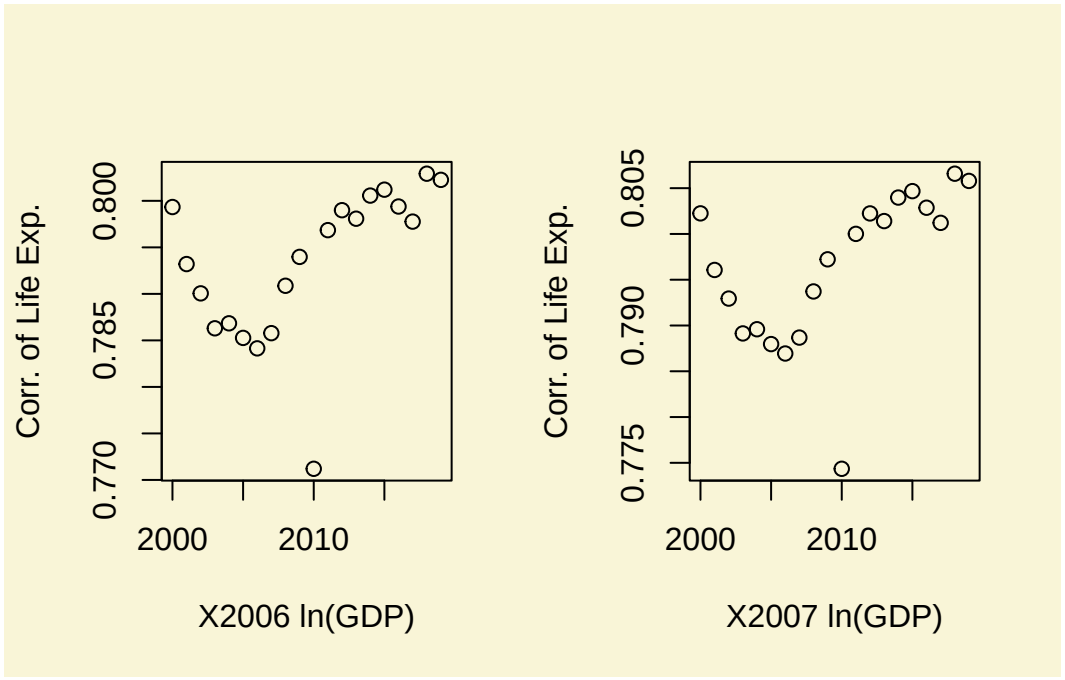
```
  d$lngdp = log(numerise(d$lngdp))

  return(d)
}
```

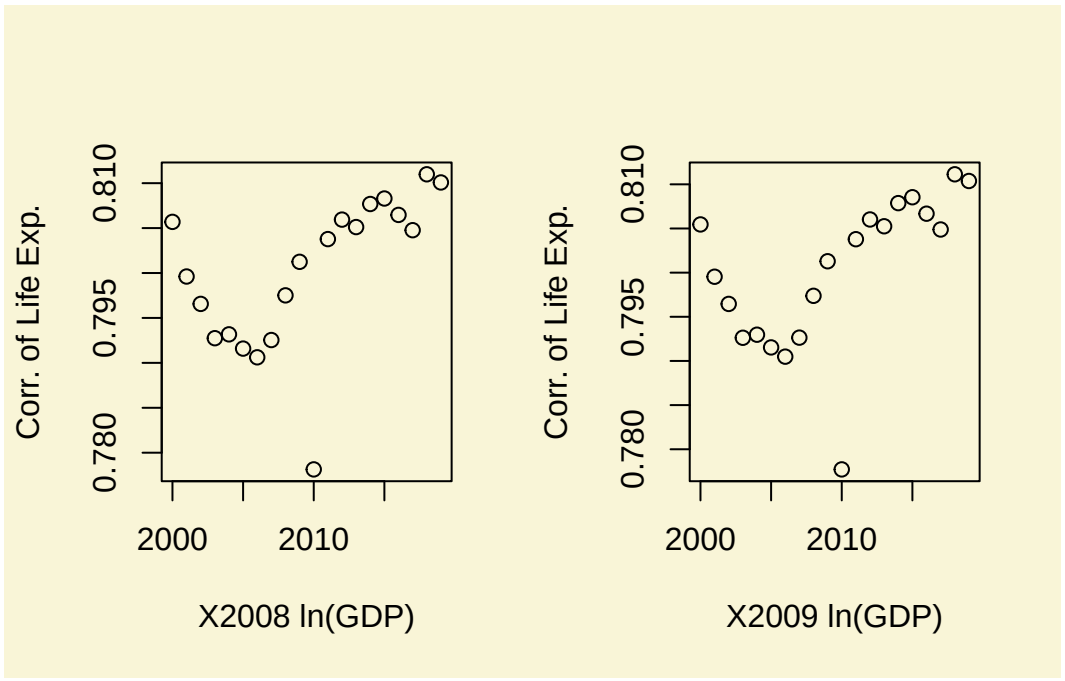
How the years' life expectancies an year's GDP correlate with.

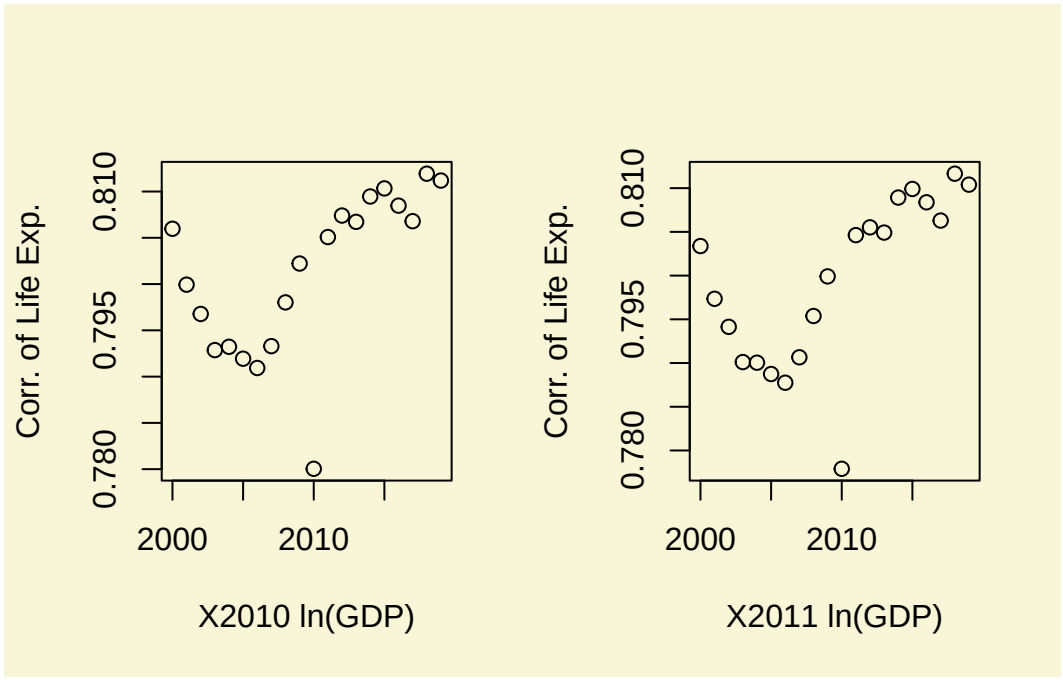


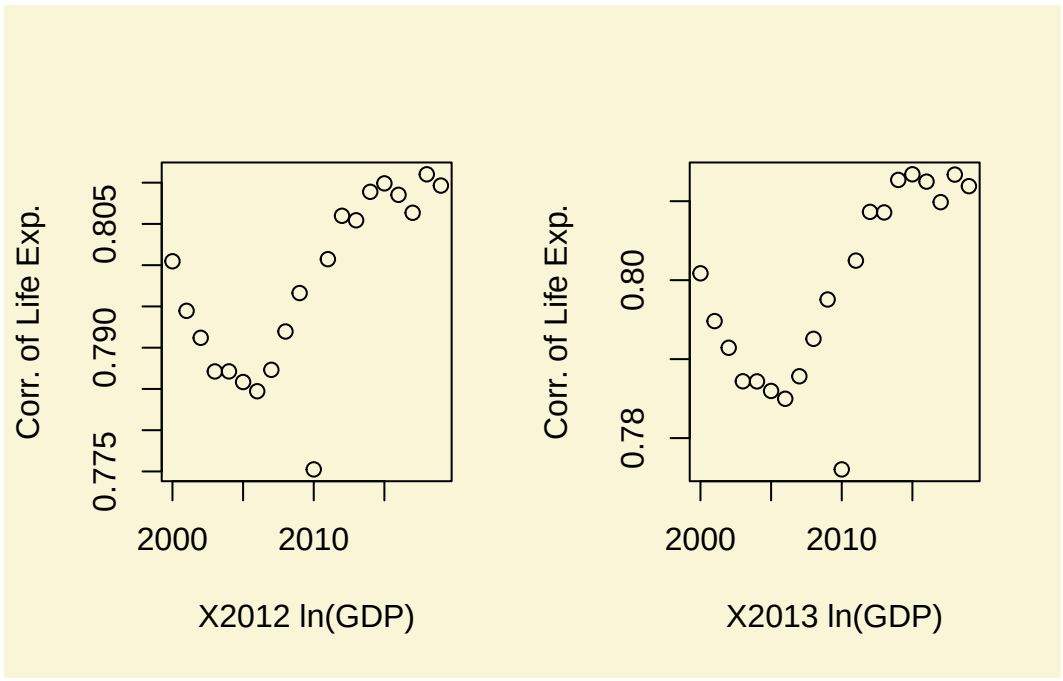

Corr. of Life Exp.
0.755
0.770
0.785
2000
2010
X2002 ln(GDP)
Corr. of Life Exp.
0.765
0.780
2000
2010
X2003 ln(GDP)

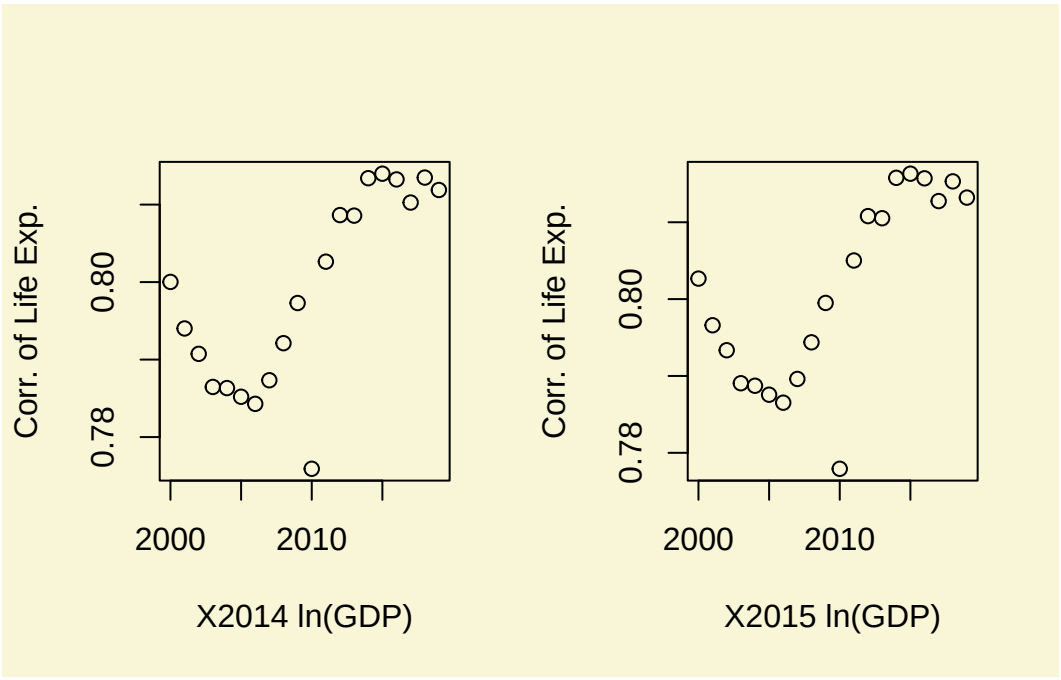

Corr. of Life Exp.
0.765
0.780
0.795
2000
2010
X2004 ln(GDP)
Corr. of Life Exp.
0.770
0.785
0.800
2000
2010
X2005 ln(GDP)

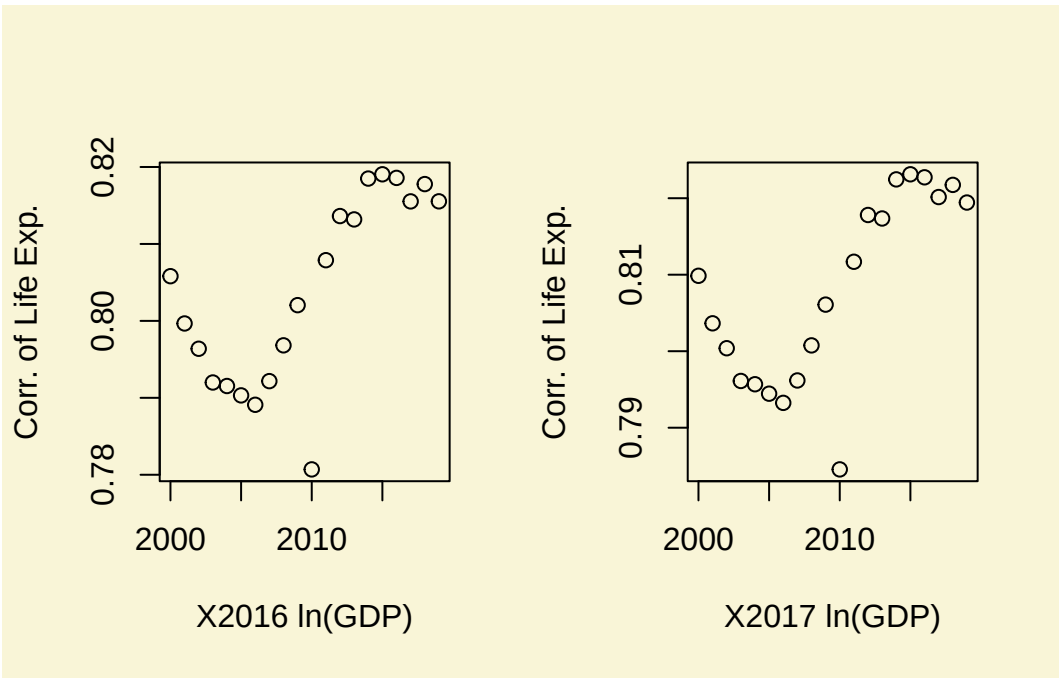

Corr. of Life Exp.
0.770
0.785
0.800
2000
2010
X2006 ln(GDP)
Corr. of Life Exp.
0.775
0.790
0.805
2000
2010
X2007 ln(GDP)

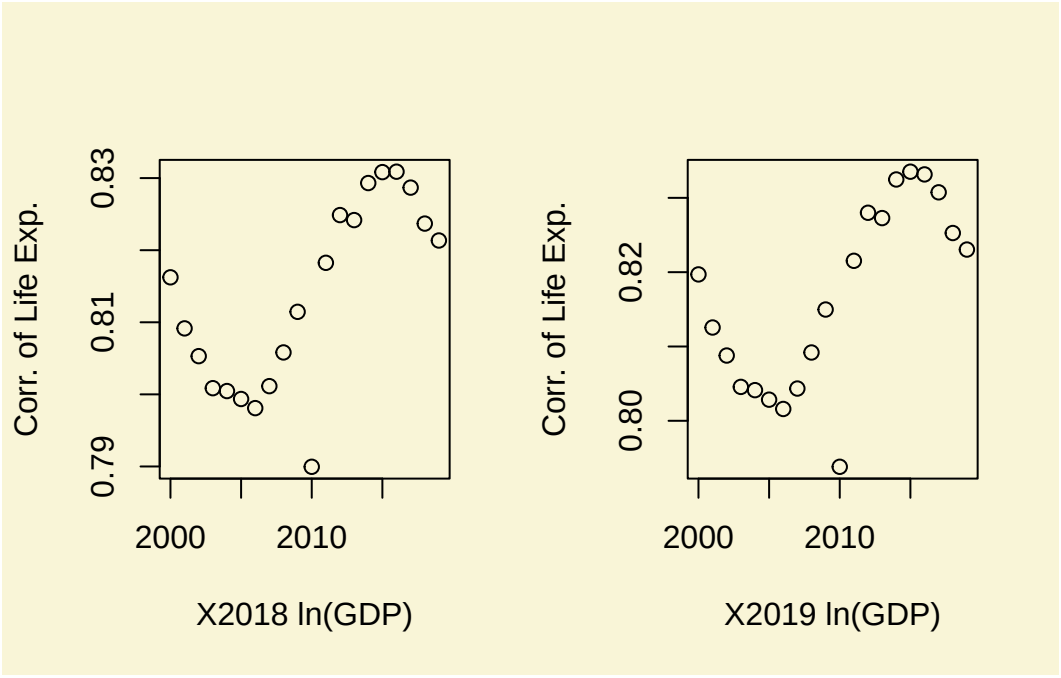

Corr. of Life Exp.
0.780
0.795
0.810
2000
2010
X2008 ln(GDP)
Corr. of Life Exp.
0.780
0.795
0.810
2000
2010
X2009 ln(GDP)


Corr. of Life Exp.
0.780
0.795
0.810
2000
2010
X2010 ln(GDP)
Corr. of Life Exp.
0.780
0.795
0.810
2000
2010
X2011 ln(GDP)

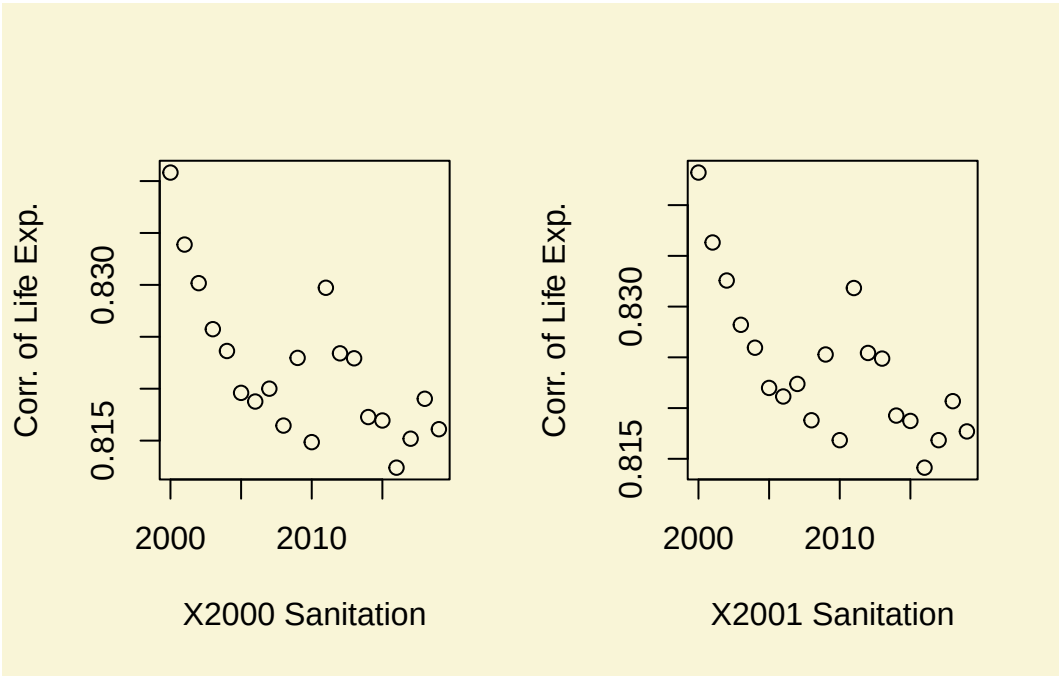

Corr. of Life Exp.
0.775
0.790
0.805
2000
2010
X2012 ln(GDP)
Corr. of Life Exp.
0.78
0.80
2000
2010
X2013 ln(GDP)

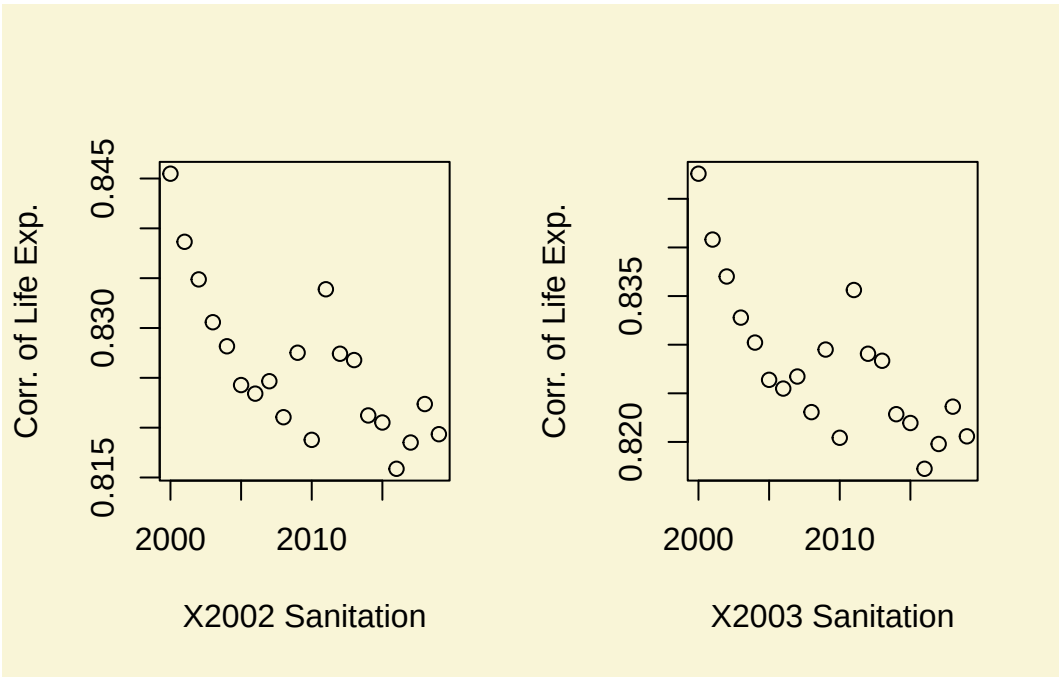

Corr. of Life Exp.
0.78
0.80
2000
2010
X2014 ln(GDP)
Corr. of Life Exp.
0.78
0.80
2000
2010
X2015 ln(GDP)

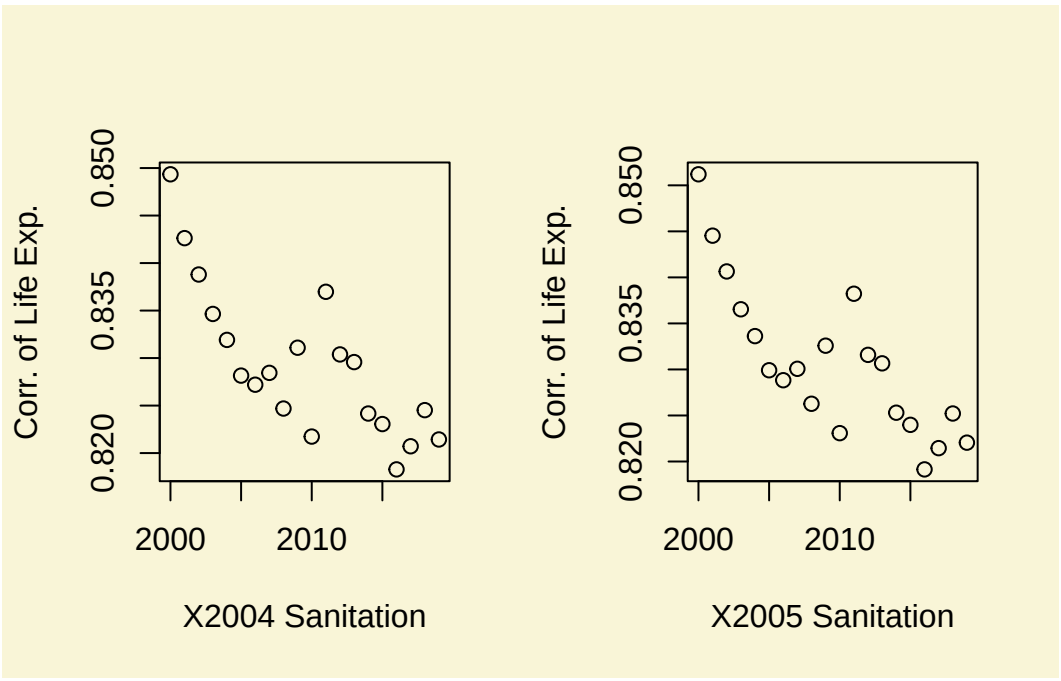

Corr. of Life Exp.
0.78
0.80
0.82
2000
2010
X2016 ln(GDP)
Corr. of Life Exp.
0.79
0.81
2000
2010
X2017 ln(GDP)



How the years' life expectancies an year's Sanitation correlate with.



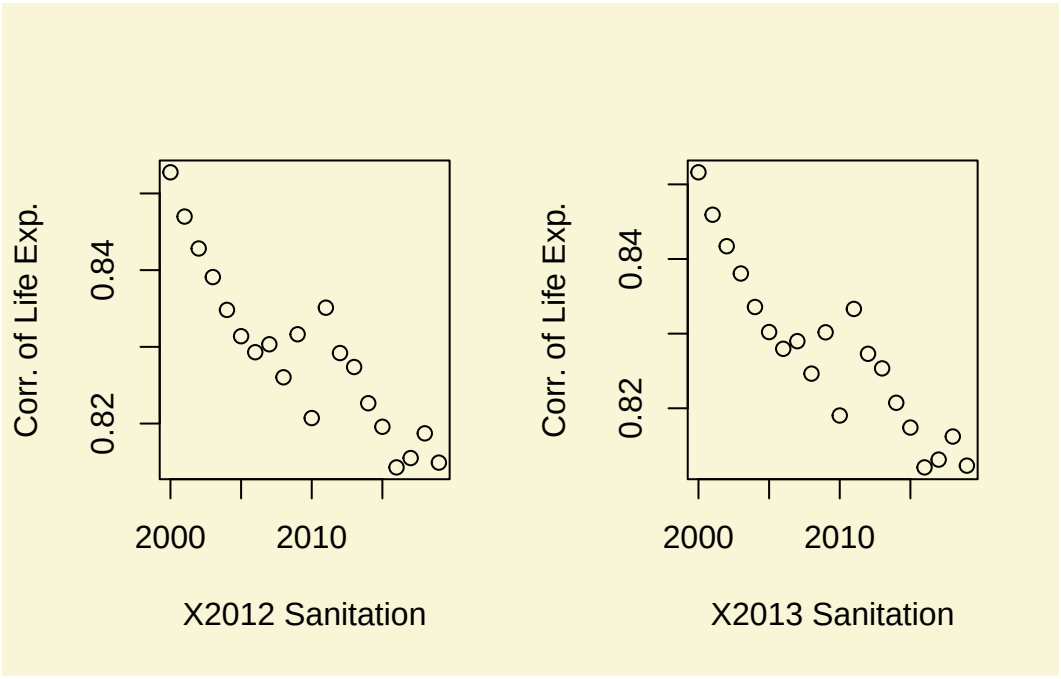

Corr. of Life Exp.
0.815
0.830
0.845
2000
2010
X2002 Sanitation
Corr. of Life Exp.
0.820
0.835
2000
2010
X2003 Sanitation


Corr. of Life Exp.
0.820
0.835
0.850
2000
2010
X2004 Sanitation
Corr. of Life Exp.
0.820
0.835
0.850
2000
2010
X2005 Sanitation


Corr. of Life Exp.
0.820
0.835
0.850
2000
2010
X2006 Sanitation
Corr. of Life Exp.
0.820
0.835
0.850
2000
2010
X2007 Sanitation

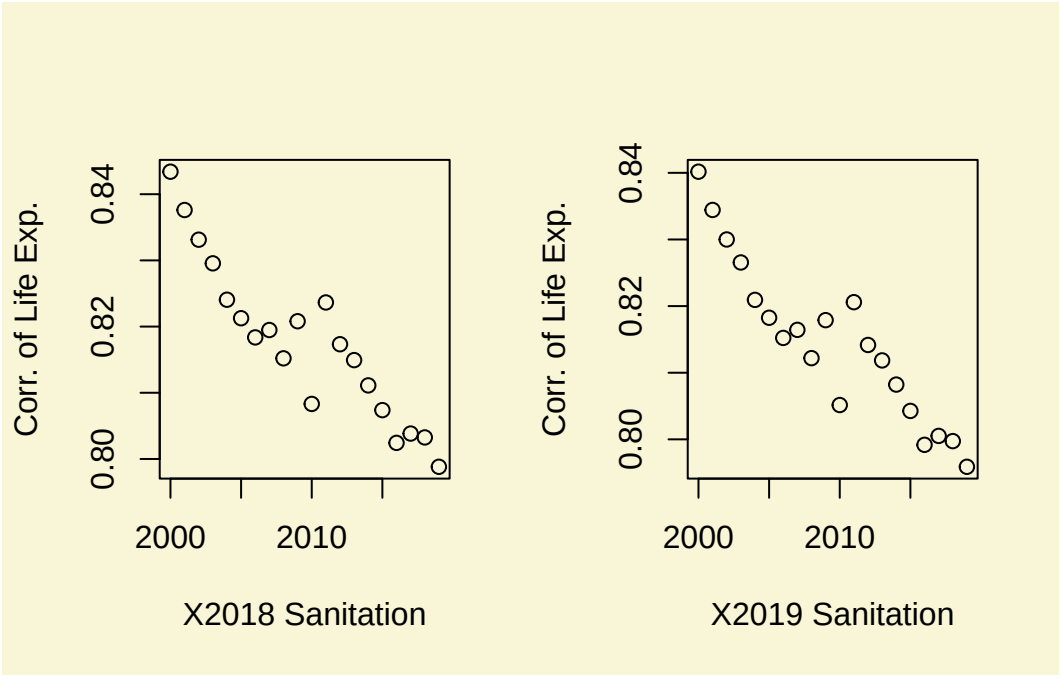

Corr. of Life Exp.
0.820
0.835
0.850
2000
2010
X2008 Sanitation
Corr. of Life Exp.
0.82
0.84
2000
2010
X2009 Sanitation


Corr. of Life Exp.
0.82
0.84
2000
2010
X2010 Sanitation
Corr. of Life Exp.
0.82
0.84
2000
2010
X2011 Sanitation

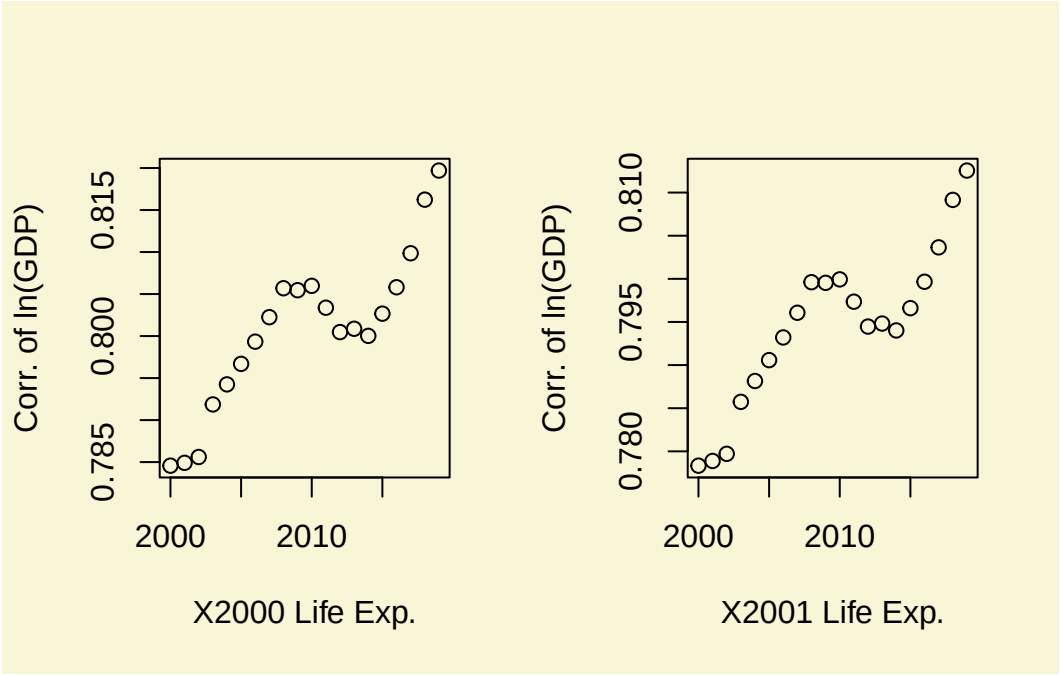

Corr. of Life Exp.
0.82
0.84
2000
2010
X2012 Sanitation
Corr. of Life Exp.
0.82
0.84
2000
2010
X2013 Sanitation

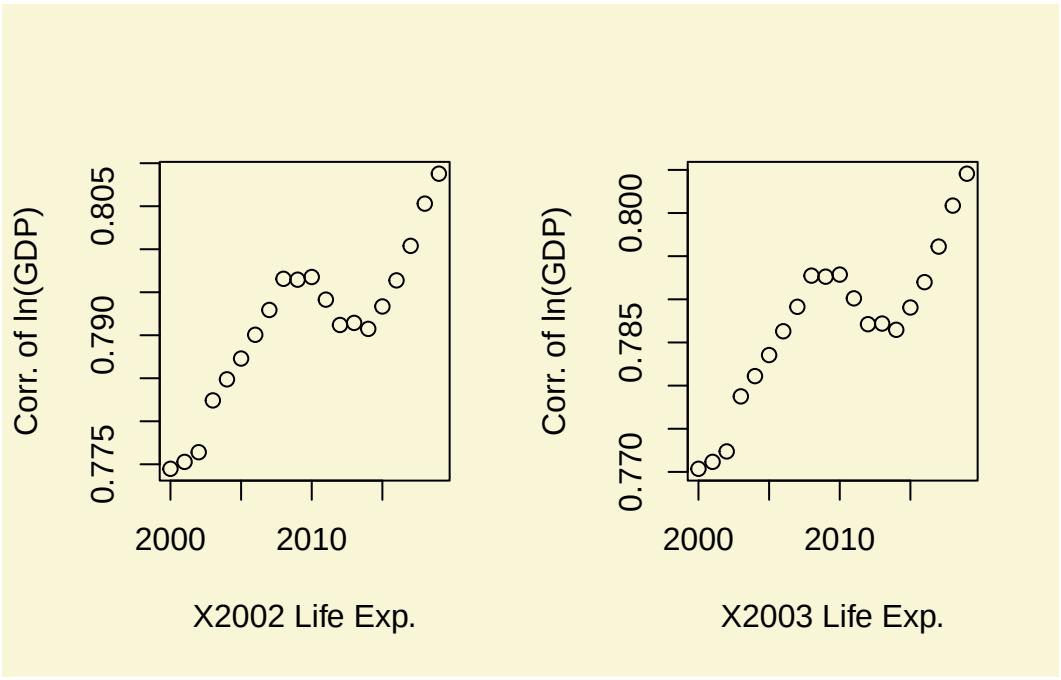

Corr. of Life Exp.
0.81
0.83
0.85
2000
2010
X2014 Sanitation
Corr. of Life Exp.
0.81
0.83
2000
2010
X2015 Sanitation

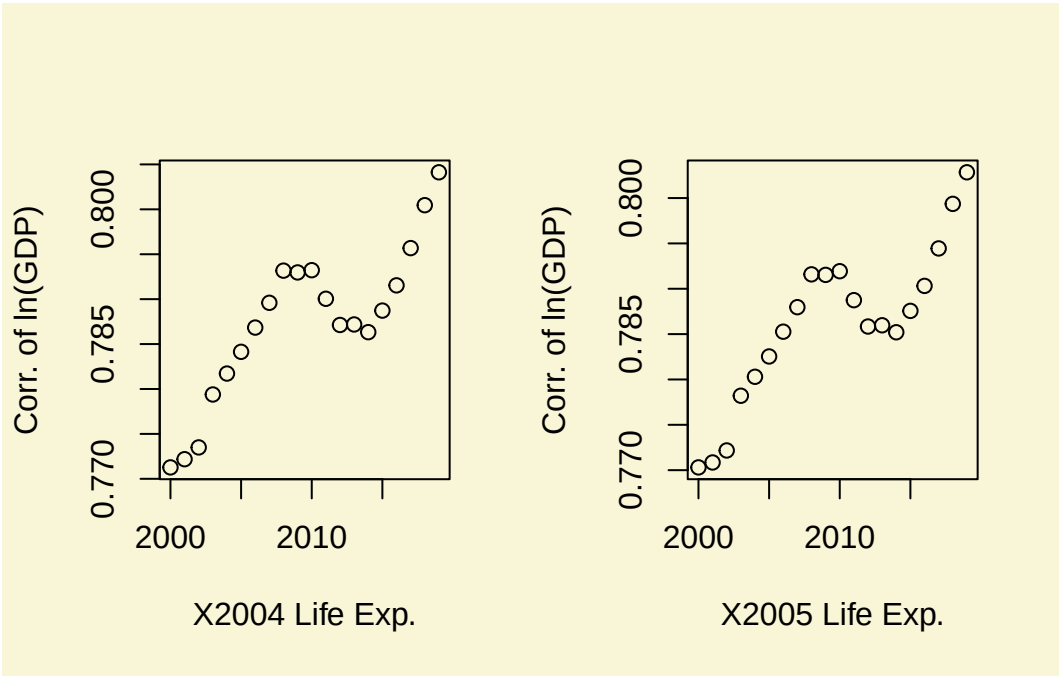

Corr. of Life Exp.
0.81
0.83
2000
2010
X2016 Sanitation
Corr. of Life Exp.
0.80
0.82
0.84
2000
2010
X2017 Sanitation

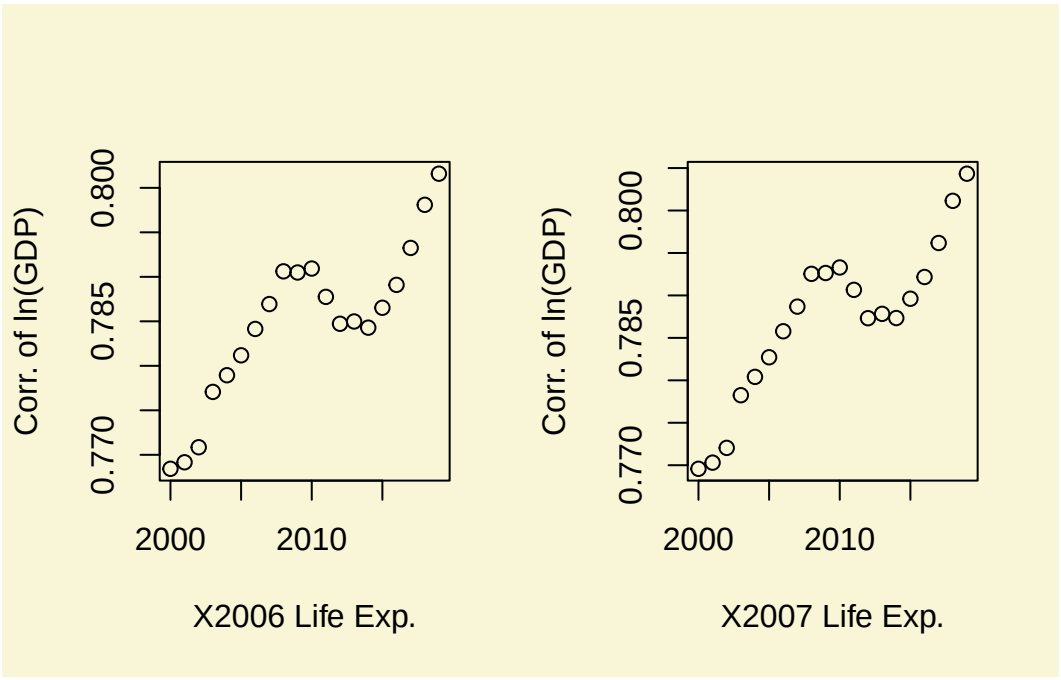


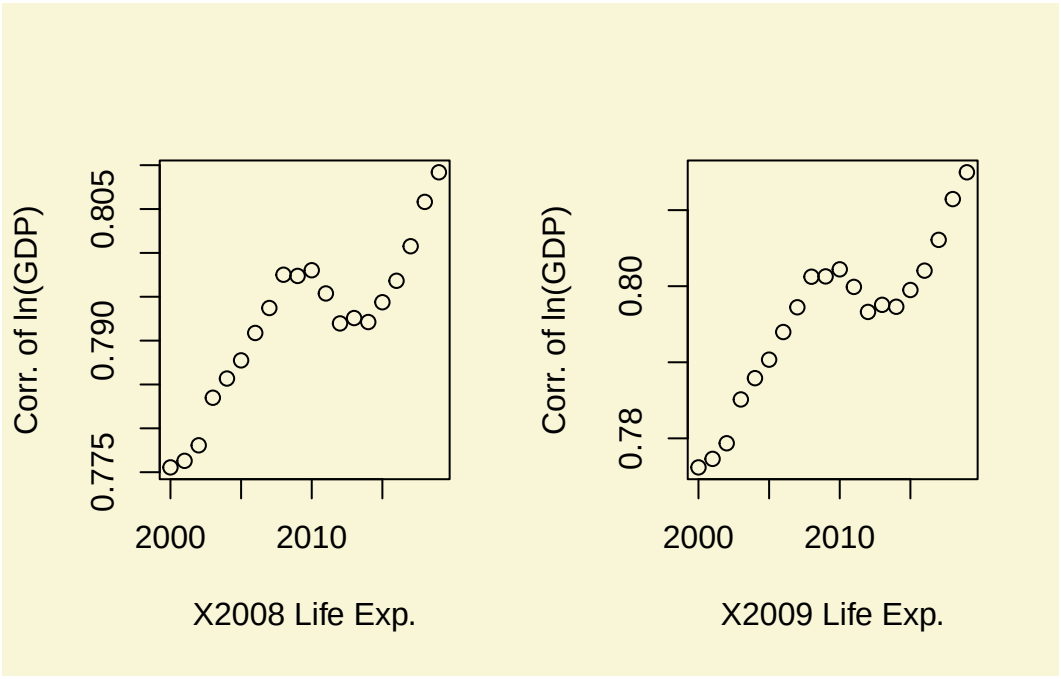
How an year's life expectancies correlate with the years' GDP.



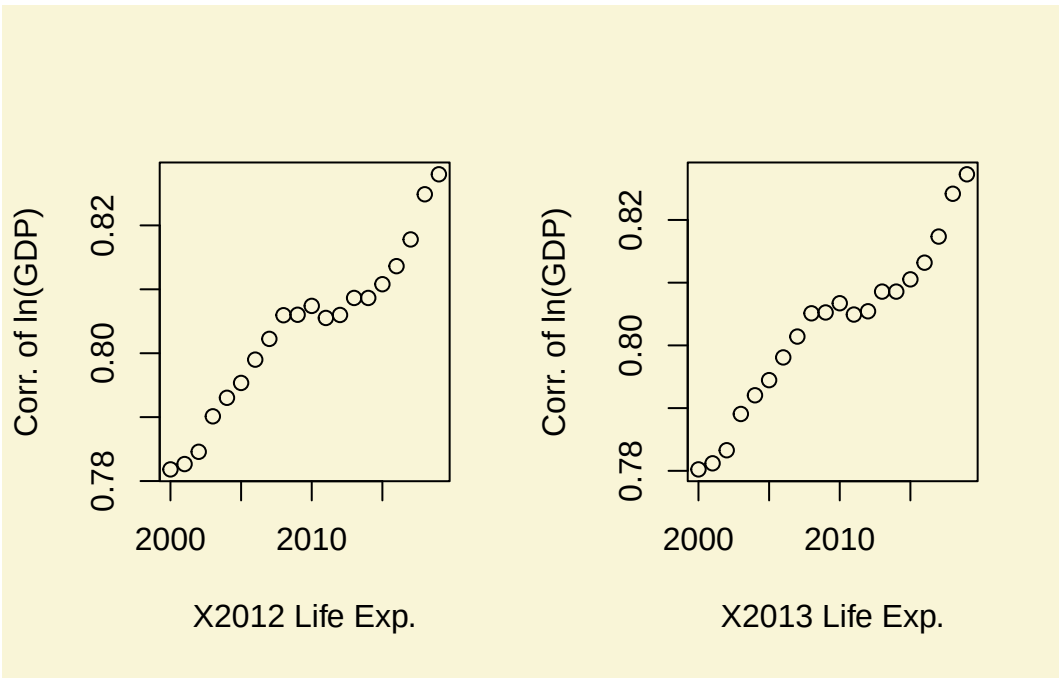

Corr. of ln(GDP)
0.775
0.790
0.805
2000
2010
X2002 Life Exp.
Corr. of ln(GDP)
0.770
0.785
0.800
2000
2010
X2003 Life Exp.

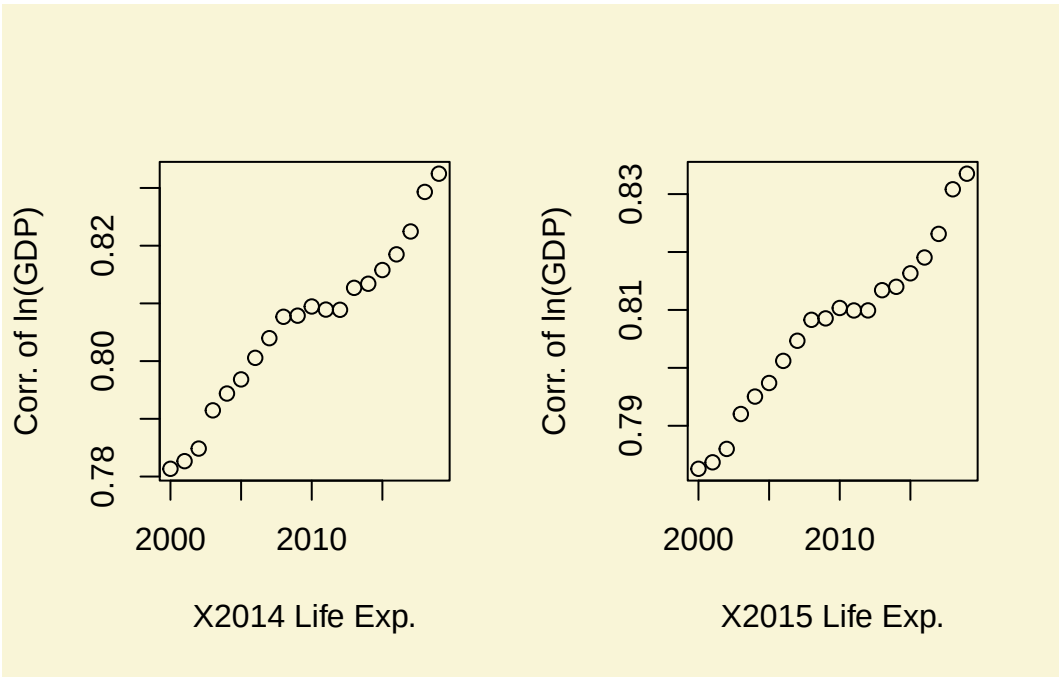

Corr. of ln(GDP)
0.770
0.785
0.800
2000
2010
X2004 Life Exp.
Corr. of ln(GDP)
0.770
0.785
0.800
2000
2010
X2005 Life Exp.

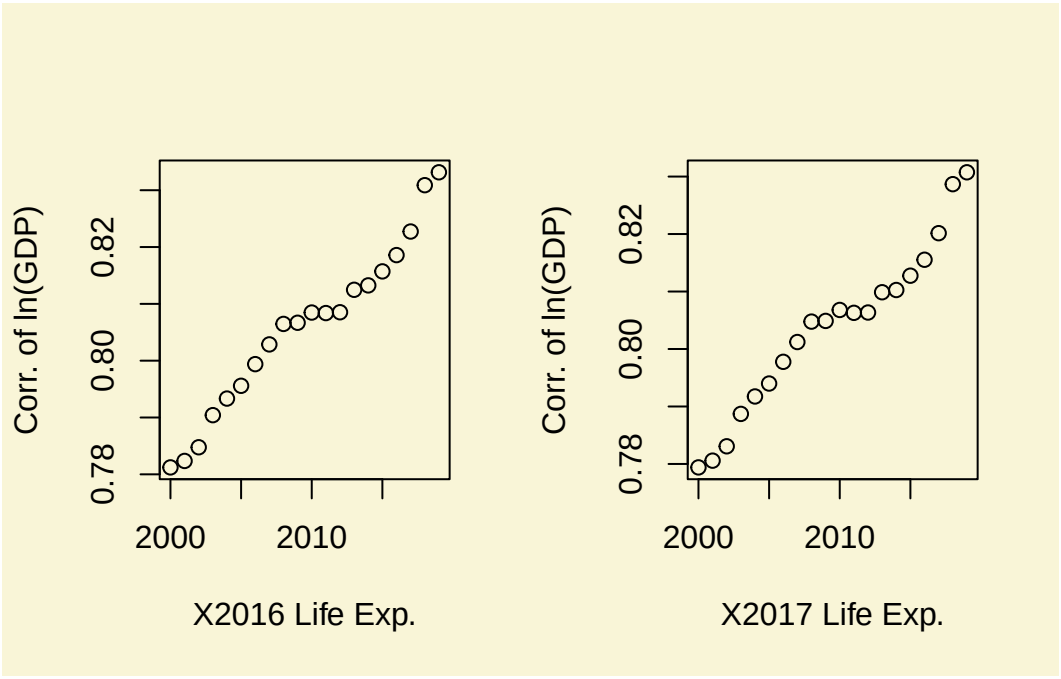

Corr. of ln(GDP)
0.770
0.785
0.800
2000
2010
X2006 Life Exp.
Corr. of ln(GDP)
0.770
0.785
0.800
2000
2010
X2007 Life Exp.


Corr. of ln(GDP)
0.775
0.790
0.805
2000
2010
X2008 Life Exp.
Corr. of ln(GDP)
0.78
0.80
2000
2010
X2009 Life Exp.

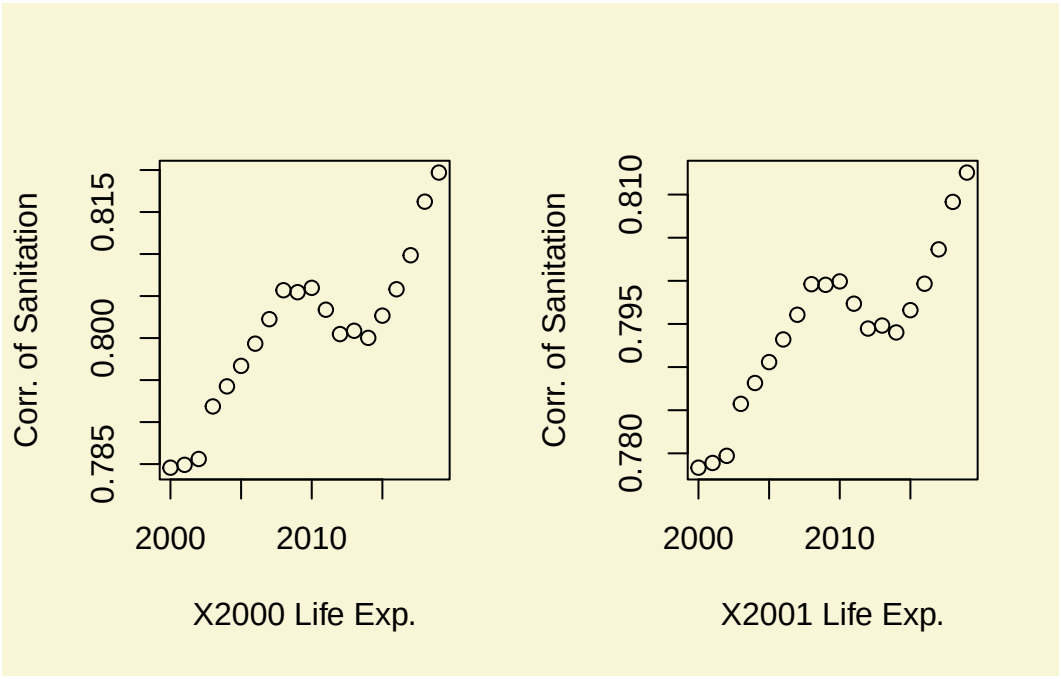

Corr. of ln(GDP)
0.76
0.78
2000
2010
X2010 Life Exp.
Corr. of ln(GDP)
0.78
0.80
0.82
2000
2010
X2011 Life Exp.

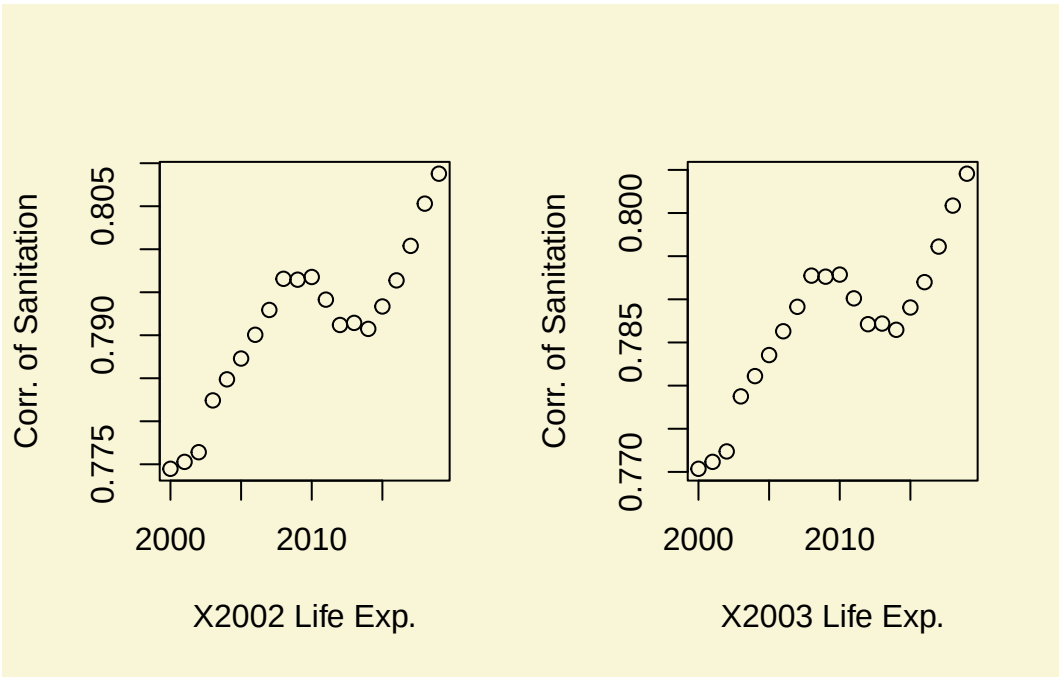

Corr. of ln(GDP)
0.78
0.80
0.82
2000
2010
X2012 Life Exp.
Corr. of ln(GDP)
0.78
0.80
0.82
2000
2010
X2013 Life Exp.

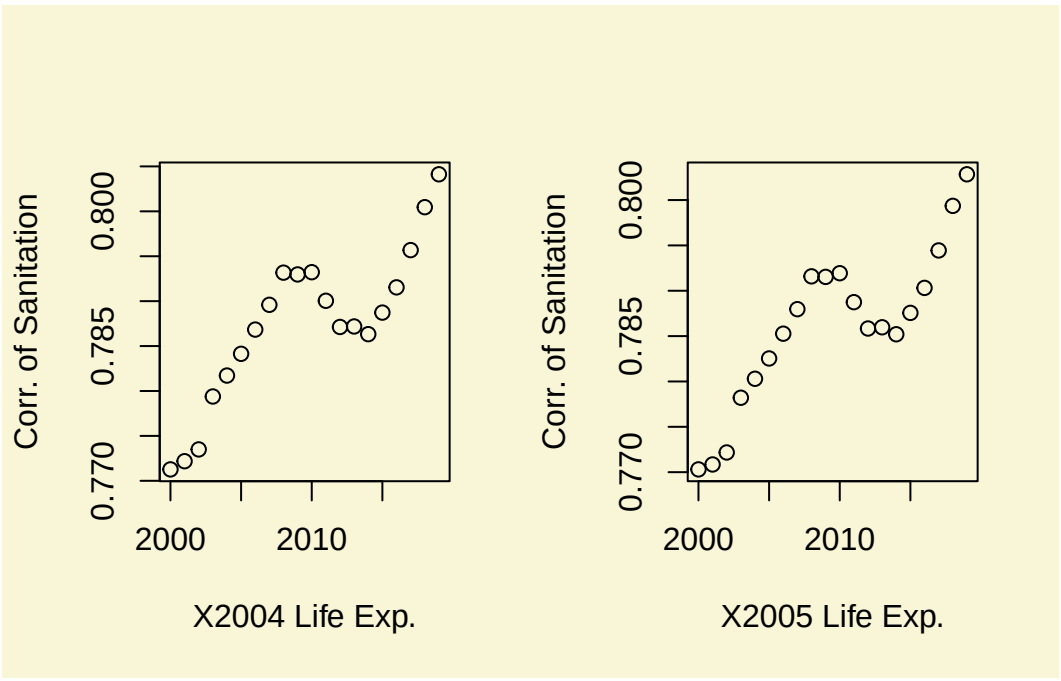

Corr. of ln(GDP)
0.78
0.80
0.82
2000
2010
X2014 Life Exp.
Corr. of ln(GDP)
0.79
0.81
0.83
2000
2010
X2015 Life Exp.

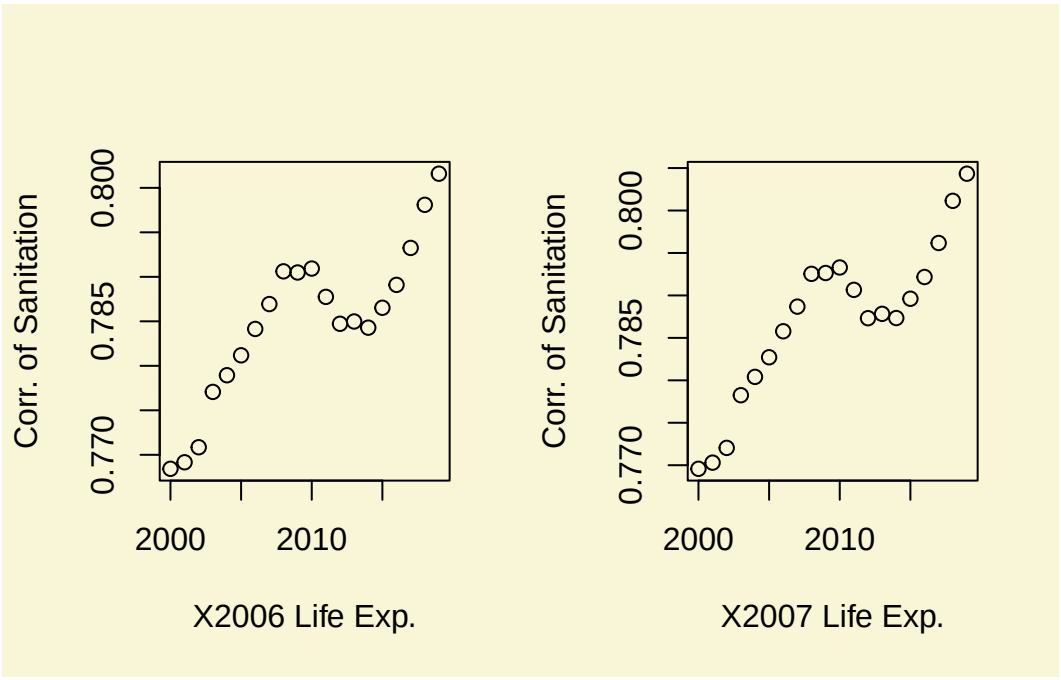

Corr. of ln(GDP)
0.78
0.80
0.82
2000
2010
X2016 Life Exp.
Corr. of ln(GDP)
0.78
0.80
0.82
2000
2010
X2017 Life Exp.

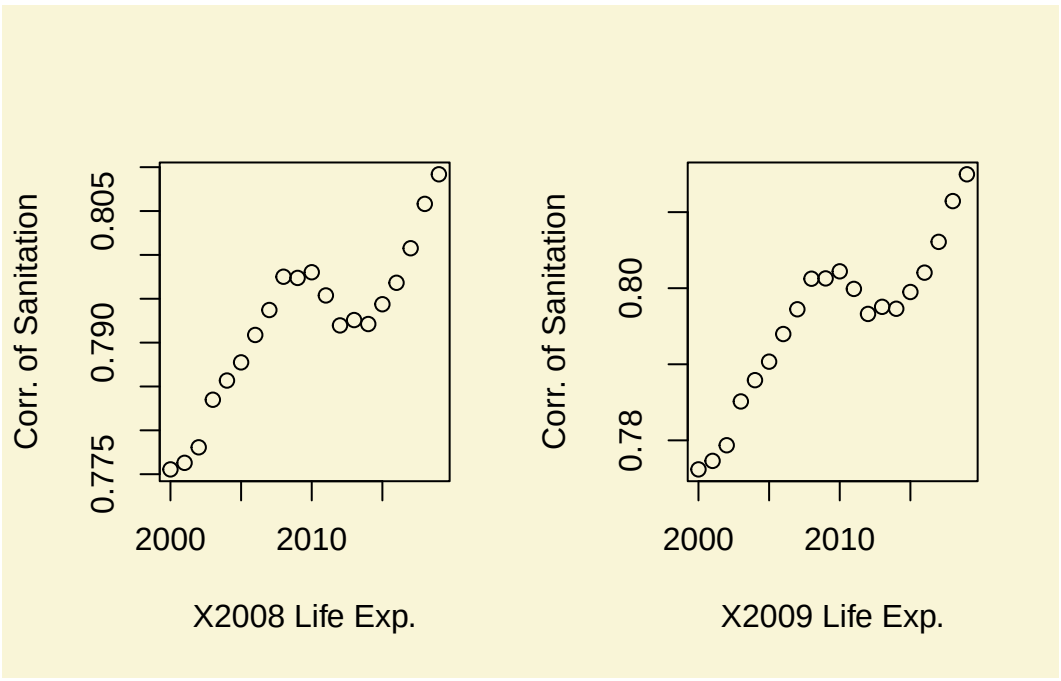
How an year's life expectancies correlate with the years' Sanitation.



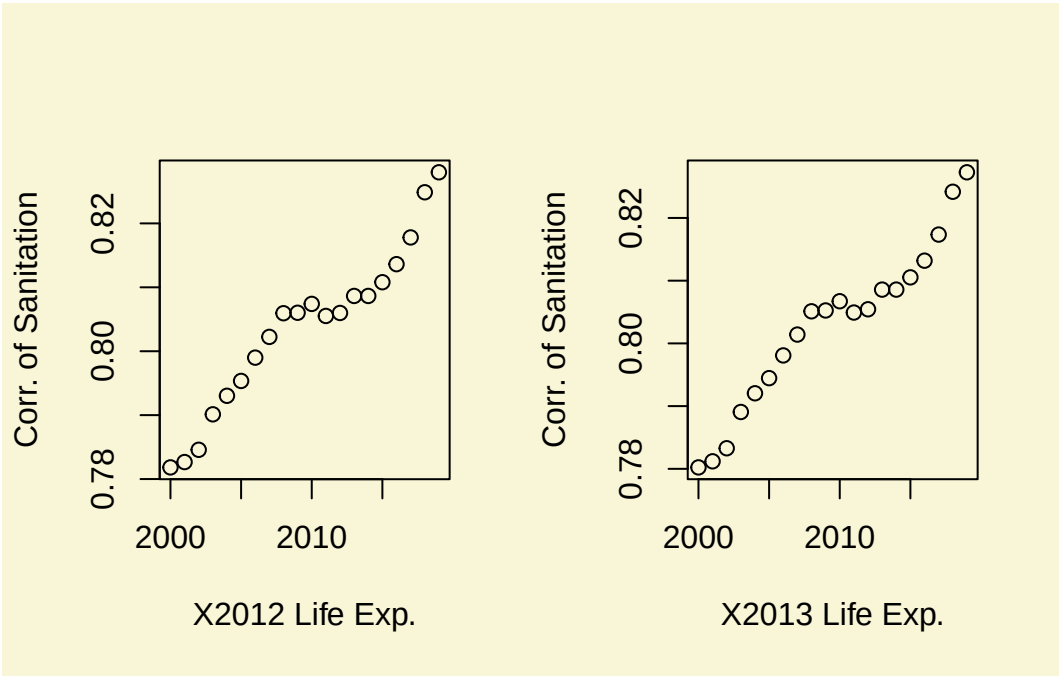

Corr. of Sanitation
0.775
0.790
0.805
2000
2010
X2002 Life Exp.
Corr. of Sanitation
0.770
0.785
0.800
2000
2010
X2003 Life Exp.


Corr. of Sanitation
0.770
0.785
0.800
2000
2010
X2004 Life Exp.
Corr. of Sanitation
0.770
0.785
0.800
2000
2010
X2005 Life Exp.


Corr. of Sanitation
0.770
0.785
0.800
2000
2010
X2006 Life Exp.
Corr. of Sanitation
0.770
0.785
0.800
2000
2010
X2007 Life Exp.


Corr. of Sanitation
0.775
0.790
0.805
2000
2010
X2008 Life Exp.
Corr. of Sanitation
0.78
0.80
2000
2010
X2009 Life Exp.


Corr. of Sanitation
0.76
0.78
2000
2010
X2010 Life Exp.
Corr. of Sanitation
0.78
0.80
0.82
2000
2010
X2011 Life Exp.


Corr. of Sanitation
0.78
0.80
0.82
2000
2010
X2012 Life Exp.
Corr. of Sanitation
0.78
0.80
0.82
2000
2010
X2013 Life Exp.

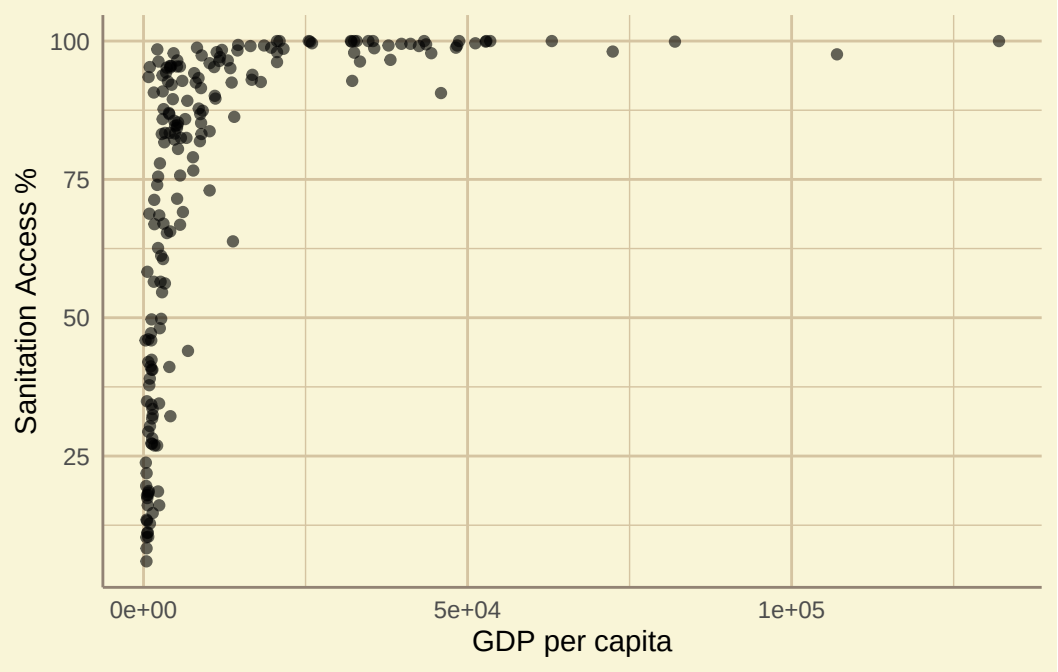

Corr. of Sanitation
0.78
0.80
0.82
2000
2010
X2014 Life Exp.
Corr. of Sanitation
0.79
0.81
0.83
2000
2010
X2015 Life Exp.


Corr. of Sanitation
0.78
0.80
0.82
2000
2010
X2016 Life Exp.
Corr. of Sanitation
0.78
0.80
0.82
2000
2010
X2017 Life Exp.

Thus our analysis is fairly robust with respect to variation in Time. Thus we only perform the analysis on the year 2010.

8.2 Raw GDP per capita

Sanitation vs GDP per capita



Life Expectancy vs GDP per capita

Scatter Plots of Sanitation or Life Exp. with respect to raw GDP are not linearly correlated. As this was beyond

our scope of knowledge, we instead considered the log (base e) of GDP.

Thus in reality, rise in GDP provides diminishing returns in citizen health the wealthier a country is.

9 Conclusion

Thus we have observed that Life expectancy is increases with rise in log of GDP per capita and Sanitation facility access. Life Expectancy is also more directly affected by increase in basic Sanitation access than with rise in log of GDP per capita.

9.1 Suggestions

Governments should consider that:

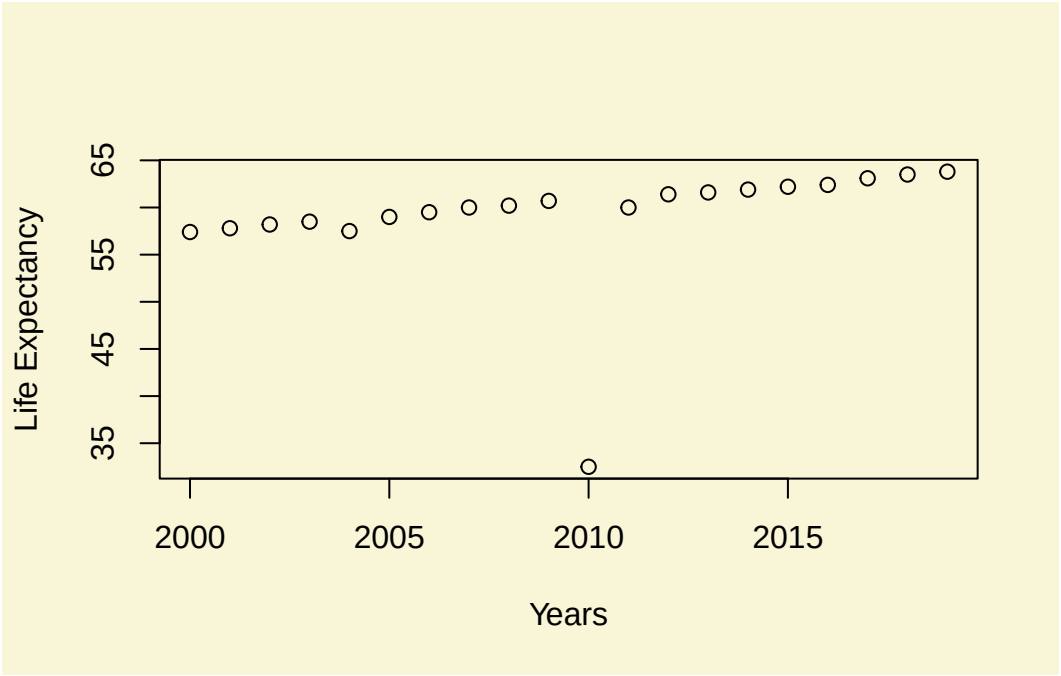
- Focusing on better sanitation services leads to visibly better lifespan of the citizens.
- Improvements in GDP per capita make practical improvements in sanitation and life expectancy only for poorer countries, due diminishing returns in the log scale.
- Improvements to life expectancies once made, are fairly stable.

9.2 Notable Countries

Some countries had alarmingly low Life Expectancies:

- The countries like the Central African Republic, Zambia, Zimbabwe have very low life expectancies due to endemic poverty and weak governance, contributing to a dire health situation.
- Haiti had a high child mortality rate in 2010 due to natural disasters and cholera outbreaks. This caused low Life expectancy for that year.

Time Series Plot



10 Credits

- Dr. Kiranmoy Das
- Gapminder
- Wikipedia
- Statology
- Quarto
- RStudio

11 Thank You