

Common Integrals

Polyomials

$$\int x^k dx = \frac{1}{k+1} x^{k+1} + c$$

$$\int x^{-1} dx = \ln|x| + c$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int x^{-n} dx = \frac{1}{1-n} x^{1-n} + c, n \neq 1$$

$$\int x^{\frac{1}{2}} dx = \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + c$$

$$\int x^{\frac{1}{n}} dx = \frac{1}{\frac{n+1}{n}} x^{\frac{n+1}{n}} + c$$

Trig Functions

$$\int \cos(u) du = \sin(u) + c$$

$$\int \sin(u) du = -\cos(u) + c$$

$$\int \sec(u) \tan(u) du = \sec(u) + c$$

$$\int \csc(u) \cot(u) du = -\csc(u) + c$$

$$\int \tan(u) du = -\ln|\sec(u)| + c = \ln|\csc(u)| + c$$

$$\int \cot(u) du = \ln|\sin(u)| + c = -\ln|\csc(u)| + c$$

$$\int \sec(u) du = \ln|\sec(u)| + \ln|\tan(u)| + c$$

$$\int \csc(u) du = \ln|\csc(u)| - \cot(u) + c$$

$$\int \sec^2(u) du = \frac{1}{2} \sec(u) \tan(u) + \ln|\sec(u) + \tan(u)| + c$$

$$\int \csc^2(u) du = \frac{1}{2} (-\csc(u) \cot(u)) + \ln|\csc(u) - \cot(u)| + c$$

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Exponential & Logarithm Functions

$$\int e^u du = e^u + c$$

$$\int \sinh(u) du = \frac{e^u - e^{-u}}{2} + c$$

$$\int \cosh(u) du = \frac{e^u + e^{-u}}{2} + c$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\int u^{-1} du = \ln|u| + c$$

$$\int e^{au} du = \frac{e^{au}}{a} + c$$

$$\int e^{au} \sin(bu) du = \frac{e^{au}}{a^2+b^2} \left(a \sin(bu) - b \cos(bu) \right) + c$$

$$\int e^{au} \cos(bu) du = \frac{e^{au}}{a^2+b^2} \left(b \sin(bu) + a \cos(bu) \right) + c$$

Inverse Trig Functions

$$\int \frac{1}{\sqrt{1-u^2}} du = \arcsin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{\sqrt{1-u^2}} du = \arcsin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{u^2-1} du = \frac{1}{2} \ln\left(\frac{u-1}{u+1}\right) + c$$

$$\int \frac{1}{u^2-1} du = \frac{1}{2} \ln\left(\frac{u-1}{u+1}\right) - \frac{1}{2} \ln(1+u^2) + c$$

$$\int \frac{1}{\sqrt{u^2-1}} du = \frac{1}{2} \operatorname{sech}^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{\sqrt{u^2-1}} du = \frac{1}{2} \operatorname{sech}^{-1}\left(\frac{u}{a}\right) - \sqrt{1-u^2} + c$$

Hyperbolic Functions

$$\int \operatorname{sech}(u) du = \coth(u) + c$$

$$\int \operatorname{sech}(u) \tanh(u) du = \operatorname{sech}(u) + c$$

$$\int \operatorname{cosec}(u) \operatorname{coth}(u) du = -\operatorname{cosec}(u) + c$$

$$\int \operatorname{tanh}(u) du = -\ln|\cosh(u)| + c$$

$$\int \operatorname{sech}^2(u) du = \tanh^{-1}(\sinh(u)) + c$$

Miscellaneous

$$\int \frac{1}{\sqrt{u^2-a^2}} du = \frac{1}{2a} \ln\left(\frac{|u+a|}{|u-a|}\right) + c$$

$$\int \frac{1}{u^2-a^2} du = \frac{1}{2a} \ln\left(\frac{|u-a|}{|u+a|}\right) + c$$

$$\int \frac{1}{\sqrt{u^2-a^2}} du = \frac{u}{2} \sqrt{u^2-a^2} + \frac{a^2}{2} \ln\left|u+\sqrt{u^2-a^2}\right| + c$$

$$\int \sqrt{2ax-u^2} du = \frac{-u}{2} \sqrt{2ax-u^2} + \frac{a}{2} \cos^{-1}\left(\frac{u}{\sqrt{2a}}\right) + c$$

Standard Integration Techniques

Ex 6 and 7:

(1) $\int \frac{2x}{x^2+1} dx$

use substitution:

$$\begin{aligned} &\int \frac{2x}{x^2+1} dx \\ &\quad \text{Let } u = x^2+1 \\ &\quad \text{Then } \frac{du}{dx} = 2x \Rightarrow du = 2x dx \\ &\quad \text{Substitute: } \end{aligned}$$

$$\begin{aligned} &\int \frac{2x}{x^2+1} dx = \int \frac{2x}{u} du \\ &\quad \text{Integrate:} \\ &\quad \int \frac{2}{u} du = \ln|u| + C \rightarrow \ln|x^2+1| + C \end{aligned}$$

(2) $\int x^2 e^x dx$

use integration by parts:

$$\begin{aligned} &\int x^2 e^x dx \\ &\quad u = x^2 \quad u' = 2x \\ &\quad dv = e^x dx \quad v = \int e^x dx = e^x \\ &\quad \text{Substitute:} \end{aligned}$$

$$\begin{aligned} &uv - \int v du \\ &\quad \text{Substitute:} \end{aligned}$$

$$\begin{aligned} &2x e^x - \int 2x e^x dx \\ &\quad \text{Apply integration by parts again:} \end{aligned}$$

$$\begin{aligned} &2x e^x - \int 2x e^x dx \\ &\quad \text{Substitute:} \end{aligned}$$

$$\begin{aligned} &2x e^x - 2e^x \\ &\quad \text{Plug back into original:} \end{aligned}$$

$$\begin{aligned} &2x e^x - (2x e^x - 2e^x) \\ &\quad \text{Simplify:} \end{aligned}$$

(3) $\int \frac{6x^2 + 5x - 6}{x^3 - 6x} dx$

use partial fractions:

$$\begin{aligned} &\frac{x^3 - 6x}{x(x + \sqrt{6})(x - \sqrt{6})} \\ &\quad \text{Factor:} \end{aligned}$$

$$\begin{aligned} &\frac{6x^2 + 5x - 6}{x(x + \sqrt{6})(x - \sqrt{6})} \\ &\quad \text{Factor:} \end{aligned}$$

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