FORCES PART 4

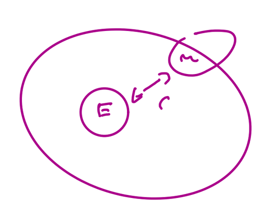
Sunday, October 20, 2024 11:26 PM

Gravitational force:
- Attractive force that acts between two masses] 4

Centripetal force:
- Force that is needed to move object in circular motion
- Directed towards centre of circle] 5

Tension force:
- Force that is transmitted through a string, cable, rope
- When forces are applied to either end
- Always pulls along direction of cable and away from object it is applied to] 6

(1) Gravitational Force:



Newton's law of gravitational force:

$g \neq F_g$

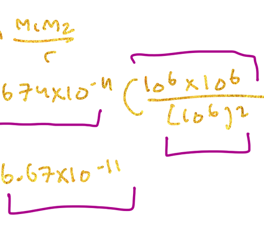
$$\left[F_g = G \frac{m_1 m_2}{r^2} \right]$$

$G = 6.674 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

Question: 3 asteroids are floating in space at the vertices of equilateral triangle. Each asteroid has a mass of 10^6 kg and side length of the triangle is 10^3 km. Calculate net gravitational force acting on one of the asteroids due to the other two.

Solution:

Approach:
-> Calculate gravitational force exerted on one asteroid by each of the other two, then find vector sum of all forces



Choosing point A / asteroid A as object of forces

Givens/ unknowns:
Side length = 10^3 km -> 10^6 m
$m = 10^6$ kg
$F_g = G \frac{m_1 m_2}{r^2}$ $G = 6.674 \times 10^{-11}$
$F_{net\,g} = ?$

Step (1) Calculate F_g from A -> B

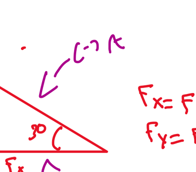
$F_g = G \frac{m_1 m_2}{r}$

$= 6.674 \times 10^{-11} \left(\frac{10^6 \times 10^6}{(10^6)^2} \right)$ -> $\left(\frac{(10^6)^2}{(10^6)^2} \right)$

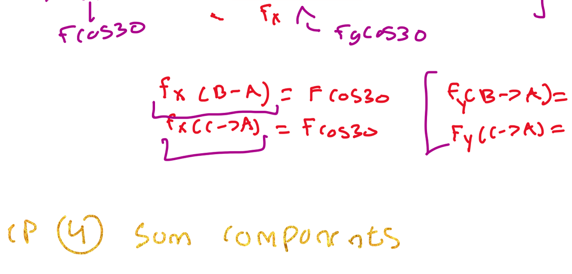
$= 6.67 \times 10^{-11}$

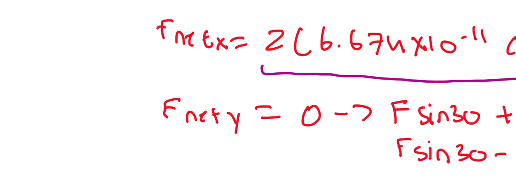
Step (2) Determine Direction of each Force

-> Forces will be directed along lines connecting centers of asteroids



Step (3) Resolve each force into force components





$F_x = F\cos30$
$F_y = F\sin30$

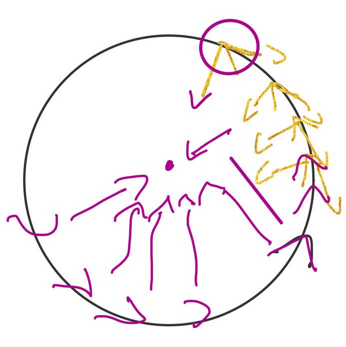
$F_x(B \to A) = F\cos30$ $F_y(B \to A) = F\sin30$
$F_x(C \to A) = F\cos30$ $F_y(C \to A) = -F\sin\theta$

Step (4) Sum components

$F_{net\,x} = 2(6.674 \times 10^{-11} \cos30°) = 1.156 \times 10^{-10}$ N

$F_{net\,y} = 0 \to F\sin30 + (-F\sin30)$
$F\sin30 - F\sin30 \to 0$

(2) Centripetal Force -> Resultant force acting towards center of curve
~~-> keeps~~ object moving on circular path



-> Continuous change in direction of velocity vector
↳ acceleration

-> centripetal acceleration points toward center of the circle around which the object is moving

-> force that provides acceleration is centripetal force

Formula:

$$\left[F_c = \frac{mv^2}{r} \right]$$

Types of questions:

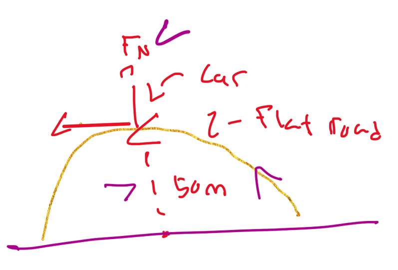
(1) Vehicles on curved Roads

-> Flat curve: vehicle travelling around flat curve, frictional force between tires and the road provides centripetal force. Without sufficient friction, vehicle would slide outward

-> Example: Calculate max speed at which car can travel around a curve of radius 50m if $\mu_s = 0.3$ between the road and tires.

Solutions:

Givens/Unknowns:
$\mu_s = 0.3$ $F_f = ?$
$\to v = ?$ $F_c = ?$
$r = 50m$



$[F_s = \mu_s mg$ ↳ F_N

$F_c = \frac{mv^2}{r}$

$F_s = F_c$ -> friction provides centripetal force

$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{r}$ $\div m$

$\mu_s g = \frac{v^2}{r}$

$\sqrt{v^2} = (\mu_s g) r$
$v = \sqrt{(\mu_s g) r}$

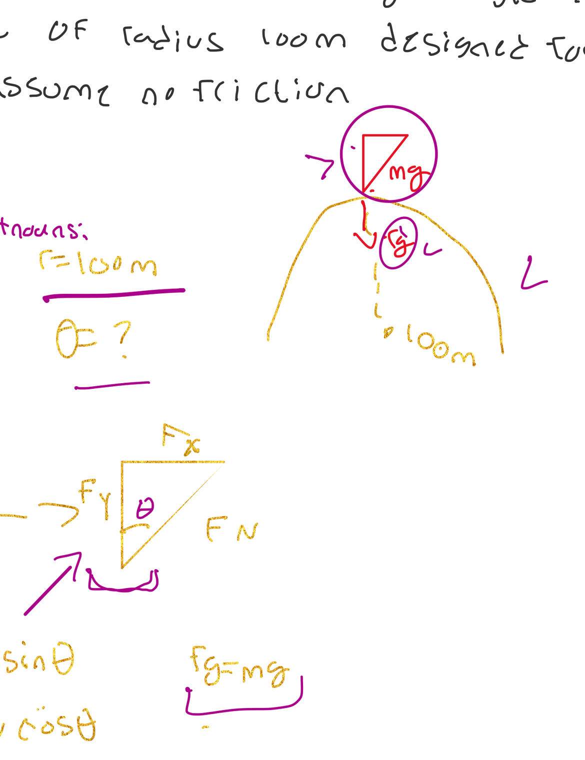
$\sqrt[2]{v^2}$ $\sqrt[3]{v^3}$

$v = \sqrt{(0.3 \times 9.81) \cdot 50}$
$= 12.13$ m/s

-> Banked curves: when on a road that is banked, the normal force can be resolved into two components where one component is the centripetal force. Higher speeds -> less reliance on friction

Example: Determine ideal banking angle for a road curve of radius 100m designed for a speed of 20 m/s. Assume no friction

Solutions:
Givens/ Unknowns:
$v = 20$ m/s $r = 100m$
$F_f = 0$ N $\theta = ?$
$F_c = \frac{mv^2}{r}$
F_N ->



$\{ F_x = F_N \sin\theta$
$\ \ F_y = F_N \cos\theta$

$F_g = mg$

$F_y = F_g$
$F_N \cos\theta = mg$

$F_x = F_c$
$\left(F_N \sin\theta = \frac{mv^2}{r} \right]$

θ: $\tan\theta = \frac{F_N \sin\theta}{F_N \cos\theta} = \frac{(mv^2/r)}{(mg)} = \frac{v^2}{gr}$

$\frac{mv^2}{r} \div mg \to \frac{mv^2}{r} \times \frac{1}{mg} = \frac{v^2}{rg}$

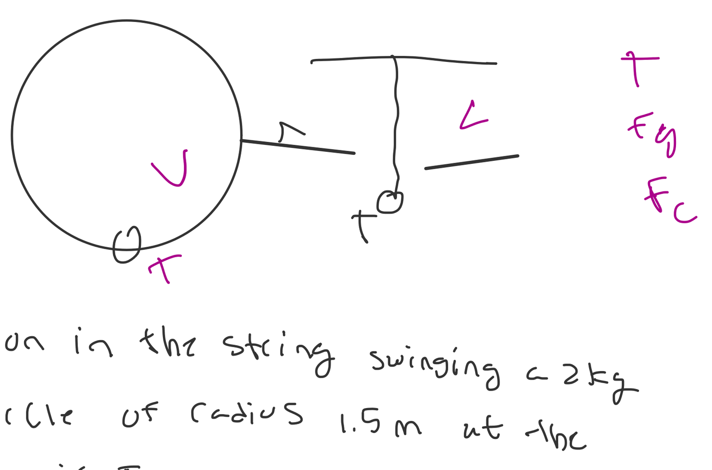
$\tan\theta = \frac{opp}{adj}$

$\theta = \tan^{-1}\left(\frac{20^2}{(9.81)(100)} \right)$

$\theta = 22.18°$

(2) Motion in Vertical Circle

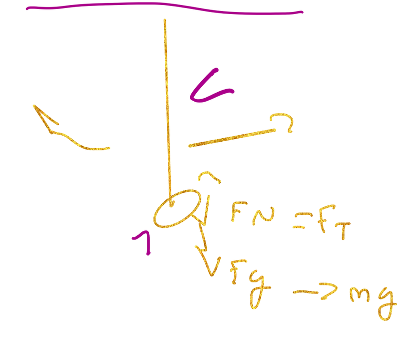
-> When an object moves in a vertical circle, such as a roller coaster or pendulum, gravity and tension play roles at different points. At Bottom of circle, tension counteracts gravitational force pulling downwards and provide centripetal force to continue circular motion



Example: Calculate tension in the string swinging a 2kg mass in a vertical circle of radius 1.5m at the bottom, if the speed is 5m/s.

Solution:

Givens/ unknowns:
$\to T = ?$ $v = 5$ m/s
$F_{net} = ?$ $r = 1.5m$
$m = 2kg$



$F_N = T$

$F_{total} \to T - mg = \frac{mv^2}{r}$ (T: by string)
↳ $F_{net} = F_c$

$T = mg + \frac{mv^2}{r}$

$T = 2(9.81) + \frac{(2 \cdot 5^2)}{1.5}$

$T = 19.62 + 33.33$

$T = 52.95$ N