

### Solving Trig Equations:

Example (1):  $\sin \theta = -0.7$   $0 \leq \theta \leq 360$

Solution:

- Angle must be in quadrants where sin is negative

- Find reference angle:

$$\theta = \sin^{-1}(0.7) \approx 44.4^\circ$$

- Determining actual angles:

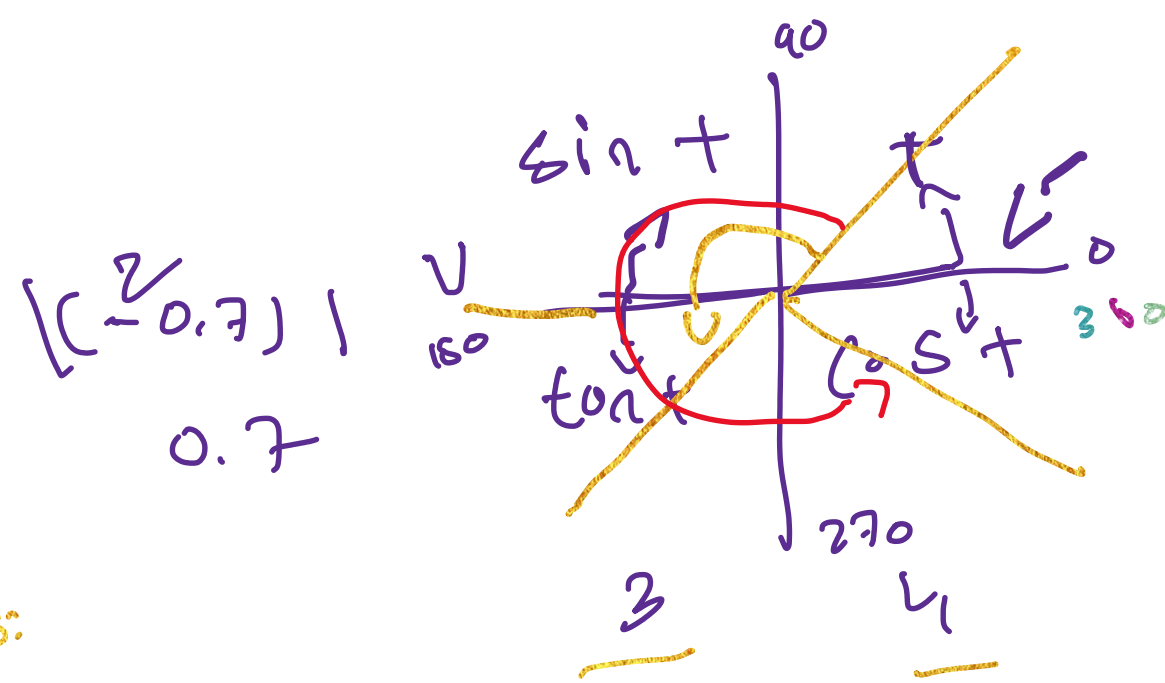
$$Q3: 180^\circ + \theta_{ref} = 180 + 44.4 = 224.4^\circ$$

$$Q4: 360 - \theta_{ref} = 360 - 44.4 = 315.6^\circ$$

- within range  $0 \leq \theta \leq 360^\circ$

$$\theta = 224.4^\circ$$

$$\theta = 315.6^\circ$$



start  $\theta$

$$0 \leq \theta \leq 420^\circ \rightarrow 720^\circ$$

Example (2):  $\tan \theta = 2.1$

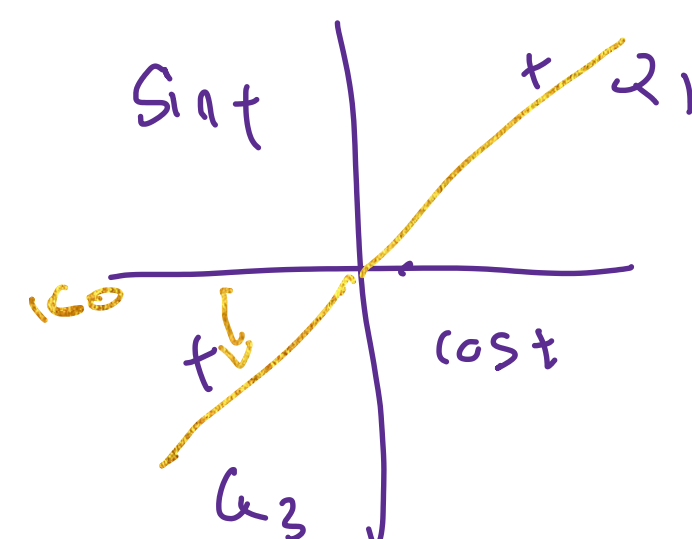
Solution:

- $\theta$  must be in quadrants where tan is positive

$$\theta_{ref} = \tan^{-1}(2.1) \approx 64.6^\circ$$

$$Q1: 64.6^\circ$$

$$Q3: 180 + \theta_{ref} = 180 + 64.6 = 244.6^\circ$$



### Terminal arm calculation:

Example (1): For point  $(-7, 19)$  which lies on terminal arm of angle.

- Find angle  $\theta$  by calculating  $\tan^{-1}(y/x)$  and adjust based on quadrant

- Calculate related acute angle ( $\theta_{ref}$ ) by finding angle relative to x-axis

Solution:

- P  $(-7, 19)$  lies in  $Q2$

- Calculate the Radius

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-7)^2 + (19)^2}$$

$$= \sqrt{49 + 361}$$

$$= \sqrt{410}$$

$$\approx 20.25$$

- Find principal angle:

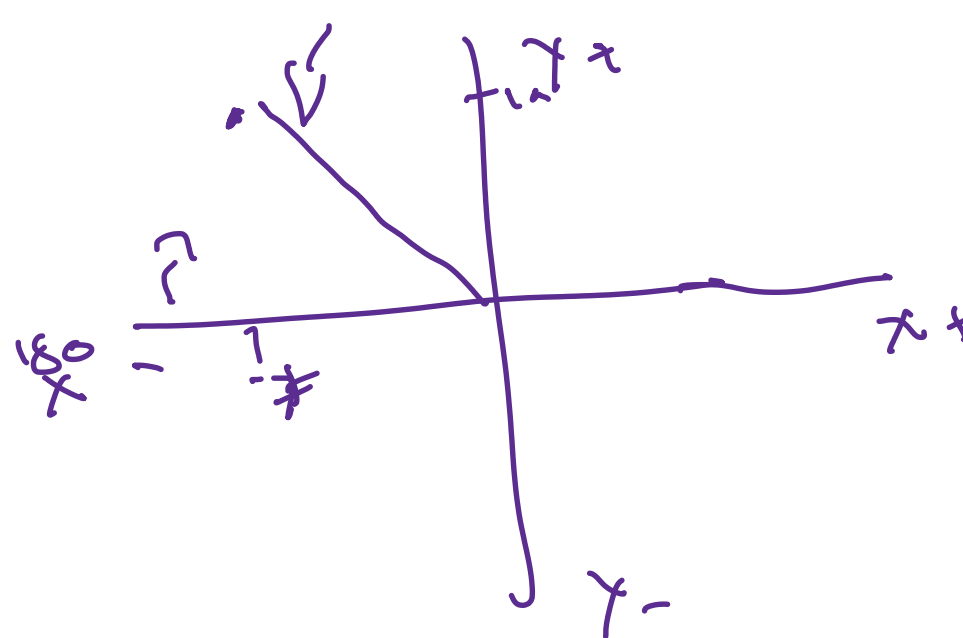
$$\tan \theta = \frac{y}{x} = \frac{19}{-7}$$

$\tan^{-1}$  - due to placement in  $Q2$

$$\theta = \tan^{-1}\left(\left|\frac{19}{-7}\right|\right)$$

$$= \tan^{-1}\left(\frac{19}{7}\right) \approx 69.78^\circ$$

$$180 - \theta = 180 - 69.78^\circ = 110.22^\circ$$



### Drawing and Calculation from Coordinates:

Example (1): For the point  $(5, 11)$  calculate  $\theta$

Solution:

- Calculate the Radius

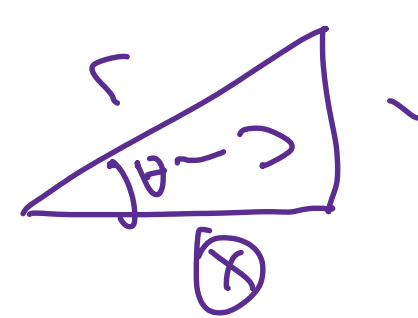
$$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + 11^2} = \sqrt{25 + 121} = \sqrt{146}$$

$$r \approx 12.08$$

- Determine trig ratios

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

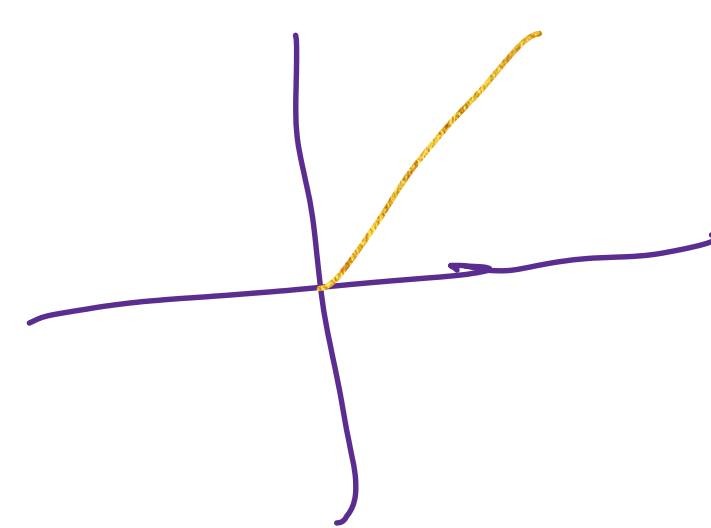


- Calculate  $\theta$ : can be determined by arctan - since the point is positive it is in  $Q1$ , so no adjustment needed

$$\tan \theta = \frac{11}{5}$$

$$\theta = \tan^{-1}\left(\frac{11}{5}\right)$$

$$\approx 65.6^\circ$$



### Solve for an equation:

Example (1):  $3 \cos \theta + 1 = 0$   $0 \leq \theta \leq 360$

Solution:

$$3 \cos \theta + 1 = 0$$

$$3 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{3}$$

- Find principal angle:

$$\cos \theta = -\frac{1}{3} \rightarrow \theta \text{ must be in } Q2 \text{ and } Q3$$

$\rightarrow$  Find corresponding angles in these quadrants

$$\theta_{ref} = \cos^{-1}\left(\left|-\frac{1}{3}\right|\right) = \cos^{-1}\left(\frac{1}{3}\right) = 70.52^\circ$$

- Calculate  $\theta$  in  $Q2, Q3$ :

$$Q2: 180 - 70.52^\circ$$

$$= 109.48^\circ$$

$$Q3: 180 + 70.52^\circ$$

$$= 250.52^\circ$$

