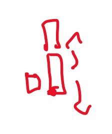
COLLISIONS MOMENTUM AND SPRINGS PART 3

Sunday, November 10, 2024 2:51 PM

IG: elevateiq_tutoring
Email: elevateiqtutor@gmail.com

Hooke's Law:

- States that force is required to stretch or compress a spring by a distance x is proportional to that distance.

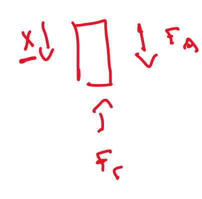


$$F = kx$$

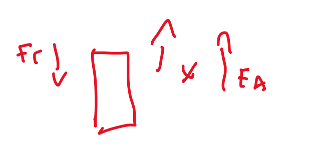
- x -> displacement from spring's natural length
- k -> spring constant

- Spring constant (k): indicates how stiff the spring is. A higher k value means a stiffer spring that requires more force to stretch or compress.

- Displacement (x) is always in the opposite direction as force (restoring force) exerted by the spring



- Direction of Forces:
  - when spring is stretched (apply force to pull away from its natural length), the x is positive in the direction of the applied force. However, the restoring force F exerted by the spring acts in the opposite direction, trying to return the spring to its original position.



$$\underbrace{F = -kx}$$

$F = kx$

$F = -kx$

  - when spring is compressed (push towards its natural length), the x is negative, F is positive

Elastic potential energy:

- Elastic potential energy (U) stored in a spring when it is stretched or compressed.

$$U = \frac{1}{2}kx^2 \to \text{displacement}$$

- Potential energy stored in a spring increases with the square of the displacement.

- Relationship between KE and U: $U = \frac{1}{2}kx^2 \to KE = \frac{1}{2}mv^2$
  - In a mass-spring system, the total ME is conserved if no non-conservative forces (like friction) are acting on it. The total ME is the sum of the KE and U

$$ME = KE + U$$

  - As spring oscillates, energy is converted between KE and U
    - when spring is at its max displacement (stretched/compressed) the KE = 0 and all energy is stored in U
    - As the spring returns to equilibrium position, potential energy is converted to KE, reaching its max when spring passes through equilibrium position

- Elastic gravitational potential energy:
  - Energy an object possesses due to its position in a gravitational field:

$$E_g = mgh$$

Work done on a spring:

- Work and energy: work done on a spring when it is stretched/compressed is equal to change in elastic potential energy. When force is applied to stretch a spring work is done against the restoring force

$W = \Delta KE$

$W = \Delta U$

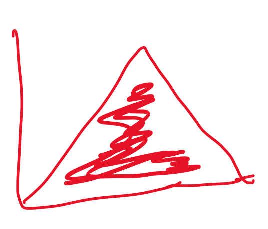
$W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

- Work calculation:
  - work done to stretch a spring from its natural length to a displacement (x) can be found using the area under the force vs displacement graph, which is a triangle in this case:

$$W = \frac{1}{2}Fx \to F = kx$$

$$W = \frac{1}{2}(kx)x$$

$$\underbrace{W = \frac{1}{2}kx^2} \to U = \frac{1}{2}kx^2$$



$\frac{1}{2}bh$

Examples:

$W = U$

(1) A 3kg block is placed on a frictionless surface and attached to a spring with a spring constant of 200 N/m. The spring is initially at its natural length. The block is pushed, compressing the spring by 0.4 meters. After the block is released, it moves and compresses the spring further by an additional 0.2 meters before coming to a stop momentarily.

a) Calculate work done on the spring when it is compressed 0.4m

b) Determine the total elastic potential energy stored in the spring when it is compressed by 0.6m → [0.6 → 0.2 + 0.4

c) Find speed of the block just after it is released from the initial compression at 0.4m.

0.4m

Solution:

Givens:

k = 200 N/m

m_1 = 3kg

x_1 = 0.4m

x_2 = 0.2m x_t = 0.6m

Unknowns:

$W_{0.4m}$ = ?

E_u = ?

v = ?

a) $W = \frac{1}{2}kx^2$

$= \frac{1}{2}(200)(0.4)^2$

$= 16J$ ✓

b) $U = \frac{1}{2}kx^2$

$= \frac{1}{2}(200)(0.6)^2 \to x = 0.2 + 0.4$

$= 36J$

c) $KE = \frac{1}{2}mv^2$

$W = KE$

$W = \frac{1}{2}mv^2$

$16 = \frac{1}{2}(3)v^2$

$v^2 = \frac{16}{1.5}$

$\sqrt{v} = \sqrt{\frac{16}{1.5}}$ | $v = 3.27$ m/s

(2) A spring with a spring constant of 250 N/m is compressed by 0.3 meters and then released. A 2kg block is attached to the spring and is placed on frictionless horizontal surface

a) Calculate the max speed of the block as it moves away from the spring after being released

b) The block then travels a distance of 1.5m before colliding with a second spring (same k value) that is initially at its natural length. Calculate how much this second spring compresses when the block collides with it

c) If the second spring is compressed to its max extent, determine total ME of the system at that point, considering both springs and the block

Solution:

Givens:

k = 250 N/m

x_i = 0.3m

m = 2kg

Δd = 1.5m

Unknowns:

v = ?

x_2 = ?

ME = ?

a) $U = \frac{1}{2}kx^2$

$= \frac{1}{2}(250)(0.3)^2$

$= 11.25J$

$KE = \frac{1}{2}mv^2$

PE = KE → 11.25 J

$11.25 = \frac{1}{2}(2)v^2$

$v^2 = \frac{11.25}{\frac{1}{2}(2)}$ | $\sqrt{v} = \sqrt{11.25}$

$v = 3.35$ m/s

b) $KE = U_f$

$U_f = \frac{1}{2}kx^2$

$11.25 = \frac{1}{2}(250)x^2$

$11.25 = 125x^2$

$x^2 = \frac{11.25}{125}$

$\sqrt{x} = \sqrt{\frac{11.25}{125}}$

$x = 0.3m$

c) $ME = PE + KE \to 0$

$= U_f + KE \to 0$

$= 11.25J$