LOGARITHMIC FUNCTIONS PART 5

Monday, November 4, 2024 1:00 PM

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Natural Logarithm:

(1) what is e?

-> e is an irrational number

-> non-terminating and non-repeating

-> approx equal to 2.71828

-> limit of $(1+\frac{1}{n})^n$ as $n \to \infty$

-> constant

ln -> $\log_e$

$\log_{10}$

log

Example: compound interest at various compounding levels

(1) when you invest \$1 at 100% interest for 1 year with different compounding frequencies, the formula used is $A: (1+\frac{1}{n})^n$ where n is the number of times interest is compounded annually

-> Annually: $A=(1+\frac{1}{1})^1=2$ $n=1$

-> Semi-annually: $A=(1+\frac{1}{2})^2=2.25$ $n=2$

-> Quarterly: $A=(1+\frac{1}{4})^4 \approx 2.441$ $n=4$

-> Monthly: $A=(1+\frac{1}{12})^{12}=2.613$ $n=12$

-> Daily: $A=(1+\frac{1}{365})^{365}=2.715$ $n=365$

-> Continuously: $A=e \approx 2.71828$

compounded interest approaches e, showing how e arises naturally

Review of Rules:

(1) $\log_b(x^n) = n\log_b(x)$

(2) $\log_b(mn) = \log_b(m) + \log_b(n)$

(3) $\log_b(m/n) = \log_b(m) - \log_b(n)$

(4) $\log_b(m) = \frac{\log_c(m)}{\log_c(b)}$ c = common base

For natural log (base e):

- $\ln(e)=1$

$\ln_e$

$\log_{10}(10)=1$

Solving problems involving e:

- $e^1=e$
- $\ln(e)=1$

- $\frac{20}{3} = \cancel{3}e^x / \cancel{3}$

$\frac{20}{3}=e^x$

$b^y=x$

$\log_b(x)=y$

$\ln(\frac{20}{3})=x$

$b^y=x$

$e^x=\frac{20}{3}$

- $e^{2x}=55$

$2x=\ln(55)$

$x=\frac{\ln(55)}{2}$

$b^y=x$

$e^{2x}=55$

$\ln_e$

- $2\ln(x-3) = 7 = 3$

$\frac{\cancel{2}\ln(x-3)}{\cancel{2}} = \frac{10}{2}$

$\ln(x-3) = 5$

$e^5=x-3$

$x=e^5+3$

$\ln(x-3)=5$

$\log_e(x)=y$

$b^y=x$

b=e

$\ln_e$

- $\ln(4e^x)=2$

$\ln(4)+\ln(e^x)=2$

$\ln(4)+x\ln(e)=2$

$\ln(4)+x=2$

$x=2-\ln(4)$

$4 \cdot e^x$

$\log_b(mn)=\log_b(m)+\log_b(n)$

$\log_b(e^x)$ $x\log_b(e)$

$\ln_e(e)=1$