LOGARITHMIC FUNCTIONS PART 1

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Review of exponential functions:

Form: $y = a(b)^x$

a -> initial amount
b -> growth factor ($b > 1$)
decay factor ($0 < b < 1$)

x -> number of times a has increased/decreases

y -> Future amount after x periods

x -> y

Example: Suppose an insect colony starts with 50 insects and the population doubles every 3 days.

a) Find population after 12 days:

Given: $a = 50$ insects
$b = 2$ -> doubles
$x = 3$

12 days -> 4 groups of 3 days

$\therefore 1213 \to 4$

$y = a(b)^x$
$= 50(2)^4$
$= 50 \times 16$
$= 800$ insects ✓

(1)	(2)	(3)	(4)
3	6	9	12
100	200	400	800
	x2	x2	x2

b) Time to reach 25600 insects

$y = 25600$

$y = a(b)^x$
$\frac{25600}{50} = \frac{50(2^x)}{50}$
$512 = 2^x$ -> $2^9 = 512$ -> $9 \times 3 = 27$ days

$2^5 = 32$
$2^8 = 256$

Inverse functions:

- Replaces x and y and solves for y
- $F(x) = y \to F^{-1}(y) = x$

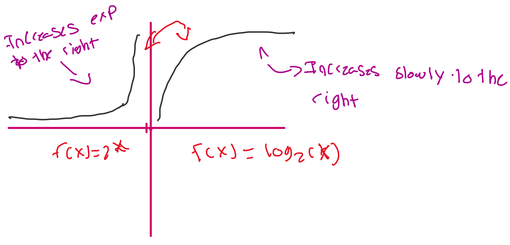
$f(x) = mx + b$
$y = mx + b$
$x = my + b \to x - b = my \to y = \frac{x-b}{m}$

$F(x) = mx + b$
$f^{-1}(x) = \frac{x-b}{m}$

- Inverse of the exponential function:

$y = 2^x$ -> Rearrange for x: $\log_2(y) = x$
$y = 2^x$

$x = 2^y \to y = \log_2(x)$



Logarithmic functions:

- solves the exp given the base and result

$y = \log_b(x) \to b^y = x$

Examples:

(1) $16 = 2^4 = y$ -> $\log_2(16) = 4$

(2) $\log_3(81) \to 3^x = 81$ -> $3^4 = 81$

$b^y = x$

$x = \log_b(x)$
$4 = \log_2(16)$

$b^y = x$

Power law of logarithms:

- $\log_b(x^n) \to n \cdot \log_b(x)$

$b \neq 1$
$b > 0$
$x > 0$

Change of base:

$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$ <- 8
<- 4

$\log_4(8)$ $\{ 8 = 2^3, 4 = 2^2 \}$ $x = 2$

$3 = \log_2(8) \to 2^3$
$2 = \log_2(4) \to 2^2$

$\log_3(9)$
$9 = 3^2$
$3 = 3^1$

$\frac{\log_3(9)}{\log_3(3)} = \frac{2}{1} = 2$

Practice:

(1) Eval $\log_3(9^2) \to 2 \cdot \log_3(9) \to 2 \cdot 2 = 4$

$9 = 3^2$

(2) Eval $\log_4(8^3) \to 3 \cdot \log_4(8)$

$\log_4(8) = \frac{\log_2(8)}{\log_2(4)} = \frac{3}{2}$

$8 = 2^3 \to \log_2(8) = 3$
$4 = 2^2 \to \log_2(4) = 2$

$3 \cdot \frac{3}{2} = 4.5$