TRIG RATIOS PART 3

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Solving Trig Equations:

Example (1): $\sin\theta = -0.7$ $\quad 0 \le \theta \le 360$

Solution:

(1) Angle must be in quadrants where sin is negative

(2) Find Reference angle:

$\theta = \sin^{-1}(0.7) \approx 44.4°$

(3) Determining actual angles:

Q3: $180° + \theta_{ref} = 180 + 44.4 = 224.4°$

Q4: $360 - \theta_{ref} = 360 - 44.4 = 315.6°$

(4) within range $0 \le \theta \le 360°$

$\theta = 224.4°$
$\theta = 315.6°$

Example (2): $\tan\theta = 2.1$

Solution:

(1) θ must be in quadrants where tan is positive

(2) $\theta_{ref} = \tan^{-1}(2.1) \approx 64.6°$

(3) Q1: $64.6°$
Q3: $180 + \theta_{ref} = 180 + 64.6 = 244.6°$

Terminal arm calculation:

Example (1): For point $(-3, 14)$ which lies on terminal arm of angle θ.

- Find angle θ by calculating $\tan^{-1}(Y/X)$ and adjust based on quadrant
- Calculate related acute angle (this) by finding angle relative to x-axis

(X, Y)
$Y = 14$
$X = -3$
$\tan^{-1}\left(\frac{14}{-3}\right)$

Solution:

(1) P $(-3, 14)$ lies in Q2

(2) calculate the Radius

$r = \sqrt{x^2 + y^2}$
$= \sqrt{(-3)^2 + (14)^2}$
$= \sqrt{9 + 361}$
$= \sqrt{410}$
≈ 20.25

(3) find Principal angle:

$\tan\theta = \frac{Y}{X} = \frac{14}{-3}$

tan -> - due to placement in Q2

$\theta = \tan^{-1}\left(\left|\frac{14}{-3}\right|\right)$
$= \tan^{-1}\left(\frac{14}{3}\right) \approx 69.78°$

$180 - \theta = 180 - 69.78°$
$= 110.22°$

Drawing and calculation from coordinates:

Example (1): For the point $(5, 12)$ calculate θ

X, Y

Solution:

(1) Calculate the Radius

$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169}$

$r \approx 13.08$

(2) Determine trig ratios

$\sin\theta = \frac{Y}{r}$ $\quad \cos\theta = \frac{X}{r}$

$\tan\theta = \frac{Y}{X}$

(3) Calculate θ: can be determined by arctan - since the point is positive it is in Q1, so no adjustment needed

$\tan\theta = \frac{12}{5}$

$\theta = \tan^{-1}\left(\frac{12}{5}\right)$

$\approx 65.6°$

Solve for an equation:

Example (3): $3\cos\theta + 1 = 0$ $\quad 0 \le \theta \le 360$

Solution:

$3\cos\theta + 1 = 0$
$-1 \quad -1$
$\frac{3\cos\theta}{3} = \frac{-1}{3}$
$\cos\theta = -\frac{1}{3}$

(1) Find principal angle:

$\cos\theta = -\frac{1}{3}$ -> θ must be in Q2 and Q3

-> Find corresponding angles in these quadrants

$\theta_{ref} = \cos^{-1}\left(\left|-\frac{1}{3}\right|\right) = \cos^{-1}\left(\frac{1}{3}\right) = 70.53°$

(2) Calculate θ in Q2 and Q3:

Q2: $180 - 70.53° = 109.46°$

Q3: $180 + 70.53° = 250.53°$