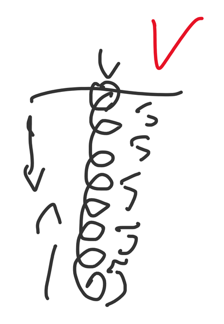
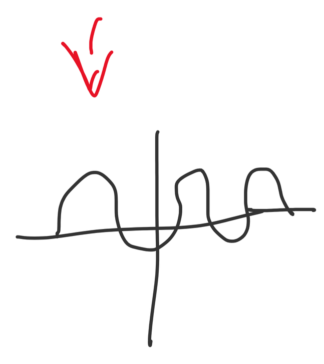
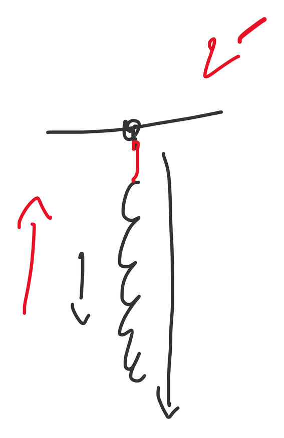
COLLISIONS MOMENTUM AND SPRINGS PART 4

Sunday, November 17, 2024 5:02 PM

IG: elevateiq_tutoring
Email: elevateiqtutor@gmail.com







Oscillatory motion:

- Simple Harmonic motion: A mass-spring system exhibits simple Harmonic motion when it oscillates about an equilibrium position
- motion is periodic; can be expressed as:

$$x(t) = A\cos(\omega t + \phi)$$

  - A = amplitude
  - ω = angular frequency
  - ϕ = phase constant

- Amplitude is the max displacement from the equilibrium position
- Period (T) is the time taken to complete one full cycle of motion
- Frequency is the number of cycles per second, related to period ($T = \frac{1}{f}$)

$$T = \frac{1}{f} \rightarrow f = \frac{1}{T}$$

- Energy is SHM:
  - Total ME in simple harmonic oscillator is conserved and is the sum of kinetic and potential energy.

$$ME = \underbrace{KE + PE + \underbrace{\tfrac{1}{2}kA^2}}$$

1 2 3

Damping:

- Affect of friction or resistance that reduces the amplitude of oscillation over time.
- Types of Damping:
  - underdamped: system oscillates with a gradually decreasing amplitude:

$$x(t) = Ae^{-\beta t}\cos(\omega_d t + \phi)$$

  - β = damping coefficient per unit mass $\beta \rightarrow b$
  - ω_d = damped angular frequency

  - Critically damped: system returns to equilibrium as quickly as possible without oscillating. This is the ideal case for systems that need to stabilize quickly
  - overdamped: system returns to equilibrium without oscillation but more slowly

- Energy loss: In damped systems, energy is lost to the environment (i.e as thermal energy) which causes amplitude to decrease

Resonance:

- occurs when system is driven by external periodic force at a frequency that matches the system's natural frequency, leading to a drastic increase in amplitude

$f_d = f_0$

$f_d \approx f_0$

- Characteristics:
  - At resonance, even a small periodic driving force can produce large oscillations
  - The amplitude of oscillation can become very large, potentially leading to system failure

- mathematical representation:

$$x(t) = A_0\cos(\omega_d t + \phi) + \frac{F_0}{m(\omega_0^2 - \omega^2)}\sin(\omega t)$$

  - F_0 = amplitude of driving force
  - m = mass of oscillator
  - ω = frequency of driving force

Example:

A mass-spring system consists of a mass of 0.5 kg attached to a spring with a spring constant of 200 N/m. The system is set into motion with an initial displacement of 0.1 m from its equilibrium position.

1. Calculate the period of oscillation for the mass-spring system.
2. If the system experiences a damping force proportional to the velocity with a damping coefficient of 5 N·s/m, determine the damped frequency of oscillation.
3. If the system is subjected to an external periodic driving force with a frequency of 1.5 Hz, discuss whether resonance will occur and explain your reasoning.

Solution:

Givens:

mass = 0.5 kg
spring constant, k = 200 N/m
initial displacement, x_0 = 0.1 m
damping coefficient b = 5 N·s/m
Driving frequency; f_d = 1.5 Hz

Unknowns:

$\rightarrow T = ?$
$\rightarrow f_d = ?$
$\beta = ?$

a) period formula:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.5}{200}}$$

$$= 2\pi(0.05)$$

$$= 0.314\,s$$

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f}$$

b) Determine damped frequency of oscillation:

$$\omega_0 = \frac{2\pi}{T}$$

$$\omega_0 = \frac{2\pi}{0.314}$$

$$= 20\ rad/s$$

Hz $\rightarrow f = \frac{1}{T}$
rad/s $\quad \omega_0 = \frac{2\pi}{T}$

f_d

$\frac{1}{f} \times 2\pi$

$\beta \rightarrow$ damping coefficient per unit mass

$$\beta = \frac{b}{2m}$$

$$= \frac{5}{2 \times 0.5}$$

$$= 5\,s^{-1}$$

$b \rightarrow$
$\beta \leftarrow$

$\beta = \frac{b}{2m}$

$$\omega_d = \sqrt{\omega_0^2 - \beta^2}$$

$$= \sqrt{20^2 - 5^2}$$

$$= 19.36\ rad/s \rightarrow Hz$$

$\frac{19.36}{2\pi} = Hz$

c) Discuss resonance:

$f_d = 1.5\,Hz$

$$f_0 = \frac{\omega_0}{2\pi}$$

$$= \frac{20}{2\pi}$$

$$= 3.18\,Hz$$

$f_d < f_0 \rightarrow 1.5\,Hz < 3.18\,Hz$

Resonance does not occur since f_d is not close to f_0.

$f_d = 3.45 \mid 3.16 \mid 3.17$

$f_d = 3\,Hz$

$f_d = f_0$?