COLLISIONS MOMENTUM AND SPRINGS PART 1

Wednesday, November 6, 2024 10:53 AM

IG: elevateiq_tutoring
Email: elevateiqtutor@gmail.com

Collisions:

$P = m \cdot v$

- A collision occurs when two or more objects come into contact with each other.
- Results in a transfer of energy and momentum
- Elastic/inelastic collisions

- Types:

① Elastic: Both momentum and kinetic energy are conserved

- Characteristics:
  - objects bounce off each other without any loss of kinetic energy

    $v_i \neq v_f$ $\quad KE = \frac{1}{2}mv^2$

  - Total kinetic energy before collision equals the total kinetic energy after the collision

    Example: Two billiard balls colliding on a pool table

    KE: 5J

  - Equations:

    ① Conservation of momentum: $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

    ② Conservation of kinetic energy: $\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

② Inelastic collisions: momentum is conserved but kinetic energy is not conserved

Characteristics:

- Objects may stick together after the collision (completely inelastic) or bounce apart (partially inelastic)

  $\boxed{1} \rightarrow \boxed{2}$

- Some kinetic energy is transformed into other forms of energy

  $KE_i \neq KE_f$

Example: a car crash where cars crumple together

Equations:

① Conservation of momentum (kinetic energy is not conserved)

② Completely inelastic collision:

- if the objects stick together after the collision $\rightarrow$ $v_f = \dfrac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}$

Feature	Elastic	Inelastic
Kinetic energy	Conserved	Not conserved
Momentum	Conserved	Conserved
Deformation	No perm deformation	Perm deformation
Example	Billiard balls	Car crash

Key Concepts:

① Coefficient of restitution (e):

- Measure of how elastic a collision is; defined as the ratio of the relative speed after the collision to the relative speed before the collision
- Formula: $e = \dfrac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$
- Values defined:
  - $e = 1 \rightarrow$ perfectly elastic collision
  - $e = 0 \rightarrow$ perfectly inelastic collision
  - $0 < e < 1 \rightarrow$ partially inelastic collision

Problem solving strategy:

① Identify type of collision

② write down conservation equations

③ substitute known values

④ solve for unknowns

⑤ check work

Example: A 3 kg object moving at 4 m/s collides elastically with a 2 kg object at rest. What are their velocities after the collision?

Givens:

$m_1 = 3\,kg$

$m_2 = 2\,kg$

$v_{1i} = 4\,m/s$

$v_{2i} = 0\,m/s$

Unknowns:

$v_{f1} = ?$

$v_{f2} = ?$

Solution:

① Set up conservation eq:

① $m_1v_{1i} + m_1v_{2i} = m_1v_{1f} + m_2v_{2f}$

$3(4) + 2(0) = 3v_{1f} + 2v_{2f}$

② $\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

$\frac{1}{2}(3)(4)^2 + 0 = \frac{1}{2}(3)v_{1f}^2 + \frac{1}{2}(2)v_{2f}^2$

②

Eq ① $12 = 3v_{1f} + 2v_{2f}$

Eq ② $24 = \frac{3}{2}v_{1f}^2 + v_{2f}^2$

③ solve for system of eqs:

① $12 = 3v_1 + 2v_2$

② $24 = \frac{3}{2}v_1^2 + v_2^2$

Eq ①: solve for v_2

$2v_2 = 12 - 3v_1$

$v_2 = \dfrac{12 - 3v_1}{2}$

$v_2 = 6 - \frac{3}{2}v_1$

use substitution into eq 2

$24 = \frac{3}{2}v_1^2 + (6 - \frac{3}{2}v_1)^2 \rightarrow (-\frac{3}{2}v_1 + 6)(-\frac{3}{2}v_1 + 6)$

$\frac{9}{4}v_1^2 - 9v_1 - 9v_1 + 36$

$\frac{9}{4}v_1^2 - 18v_1 + 36$

$24 = \frac{3}{2}v_1^2 + \frac{9}{4}v_1^2 - 18v_1 + 36$

$24 = \frac{15}{4}v_1^2 - 18v_1 + 36$

$\frac{15}{4}v_1^2 - 18v_1 + 36 - 24 = 0$

$4\left(\frac{15}{4}v_1^2 - 18v_1 + 12\right) = (0)4$

$\dfrac{15v_1^2 - 72v_1 + 48}{3} = \dfrac{0}{3}$

$5v_1^2 - 24v_1 + 18 = 0$

$v_1 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow ax^2 + bx + c$: 5, -24, 18

$v_1 = \dfrac{24 \pm \sqrt{(-24)^2 - 4(5)(18)}}{2(5)}$

$v_1 = \dfrac{24 \pm \sqrt{256}}{10}$

$v_1 = \dfrac{24 \pm 16}{10} \rightarrow \dfrac{24 + 16}{10} = \dfrac{40}{10} = 4\,m/s$

$v_f \neq v_i$ $\quad v_{1i} = 4\,m/s$

$\rightarrow \dfrac{24 - 16}{10} = 0.8\,m/s$

$v_1 = 4\,m/s$ cannot be the answer due to the v_{1i} being $4\,m/s$ $\therefore$ $v_1 = 0.8\,m/s$ is v_{1f}

substitute v_1 into eq ①

$v_2 = 6 - \frac{3}{2}(0.8)$

$v_2 = 6 - \frac{3}{2}\left(\frac{8}{10}\right)$

$v_2 = 6 - \frac{12}{10}$

$v_2 = \frac{60}{10} - \frac{12}{10}$

$v_2 = \frac{48}{10}$

$v_2 = 4.8\,m/s$

$v_1 = 0.8\,m/s \quad v_2 = 4.8\,m/s$