LOGARITHMIC FUNCTIONS PART 3

Friday, November 1, 2024 12:00 PM

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Solving Exponential problems:

① Changing base to powers

- convert 8 to base 2: 2^3
- convert 16 to base 2: 2^4

$16 = 2 \times 2 \times 2 \times 2 = 16$

$8 \rightarrow 2 \times 2 \times 2 = 4 \times 2 = 8$

$2 \times 2 \times 2 = 8$

② Solving exponential equations using Logarithms

- When bases cannot easily be made the same, logarithms can be used to remove exponent by applying power law of logs.

Example: $4^{2x-1} = 8^{x+2}$

(1) take log of both sides

$\log(4^{2x-1}) = \log(8^{x+2})$

$2x-1 \cdot \log(4) = x+2 \cdot \log(8)$

$\rightarrow 2x\log(4) - \log(4) = x\log(8) + 2\log(8)$

$2x\log(4) - x\log(8) = 2\log(8) + \log(4)$

$x(2\log(4) - \log(8)) = 2\log(8) + \log(4)$

$x = \frac{2\log(8) + \log(4)}{2\log(4) - \log(8)}$ simplify =?

$x = \frac{2\log(2^3) + \log(2^2)}{2\log(2^2) - \log(2^3)}$

$x = \frac{6\log(2) + 2\log(2)}{4\log(2) - 3\log(2)}$

$x = 8$

③ Applying the quadratic formula

- Some exponential equations can be rearranged into a quadratic formula which can be solved using quadratic formula

Example: $2^{2x} - 2^x - 1 = 0$

(1) substitute $k = 2^x$

$2^{2x} - 2^x - 1 = 0$

$(2^x)^2 - 2^x - 1 = 0$

$k^2 - k - 1 = 0$

$2^{2x} = (2^x)^2$

$2^{2x} \div 2^x = 2^x$

$k^2 \div k = k$

(2) Apply the quadratic formula

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1, \quad b = -1, \quad c = -1$

$ax^2 + bx + c = 0$

$\frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$k = \frac{1 \pm \sqrt{5}}{2}$

(3) Substitute k back

$2^x = k$

$2^x = \frac{1 \pm \sqrt{5}}{2}$

$b^y = x$

$y = \log_b(x)$

(4) $\log_2\left(\frac{1 \pm \sqrt{5}}{2}\right)$ (calculator)

Solving Logarithmic equations:

① Solving simple logarithmic equations

- solved converting the logarithmic equations into equivalent exponential forms.

Example: $\log(x+5) = 2\log(x-1)$

(1) Apply power law

$\log x^n = \log(x^n)$

$\log(x+5) = \log((x-1)^2)$

(2) since both sides are log equals brackets no each other

$x+5 = (x-1)^2$

(3) Rearrange into a quadratic equation

$(x-1)^2 \rightarrow (x-1)(x-1)$

$x^2 - x - x + 1$

$x+5 = x^2 - 2x + 1$

$-x^2 + 2x - 1 + x + 5 = 0$

$\rightarrow -x^2 + 3x + 4 = 0$

$(x^2 - 3x - 4) = 0$

$x^2 - 3x - 4 = 0$

(4) solve using quadratic formula

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1, \quad b = -3, \quad c = -4$

$\frac{3 \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)}$

$\frac{3 \pm \sqrt{9 + 16}}{2}$

$\frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2}$

(5) Calculate Solutions

$x = \frac{3+5}{2} \rightarrow \frac{3+5}{2} = 4$

$\rightarrow \frac{3-5}{2} = -1$

$\log(-1)$ is undefined $\therefore x = 4$

$\log_{10}(4)$

Example 2: $\log(x+4) = 1$

(1) Rewrite to exponential form

$\rightarrow b^y = x$

$\log_{10}(x+4) = 1$

$\log_{10}(10^1) = \log_{10}(x+4)$

$10^1 = x+4$

$10 = x+4$

$x = 6$

$b = 10$

$x = x+4$

$y = 1$

$10^1 = x+4$