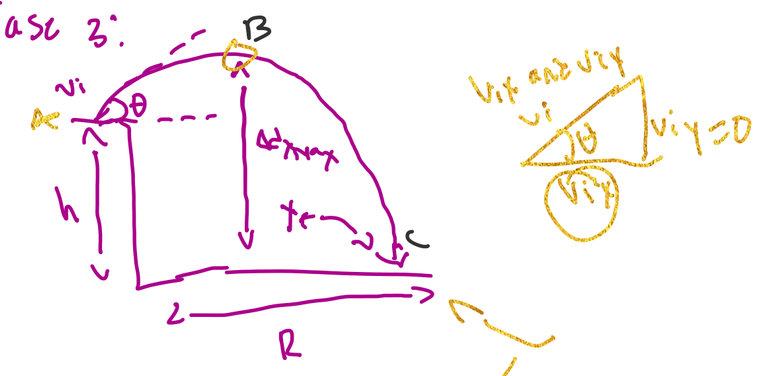
Projectile motion p3

Wednesday, October 16, 2024 4:03 PM

Case 3:



Case based rules:

→ $\Delta d_{yf} = h + v_{iy}t + \frac{1}{2}a_y t^2$

Quadratic → ax^2+bx+c

$t = \frac{-b \pm \sqrt{b^2-4ac}}{a^2}$ → total

$ax^2 = \frac{1}{2}a\Delta t^2$

$bx = v_{iy}\Delta t$

$c = h$

$a = \frac{1}{2}(-9.81)$

$b = v_{iy}$

$c = h$

$x = \Delta t$

→ $t_{A\to B}: \frac{v\sin\theta}{g}$

→ $t_{B\to C}: h + \Delta d_{yh} = \frac{1}{2}a\Delta t^2$

$\Delta t^2 = \frac{2(h+\Delta d_{yh})}{a}$

$\Delta t = \sqrt{\frac{2(h+\Delta d_{yh})}{a}}$

Add two Δt values to get total time of flight

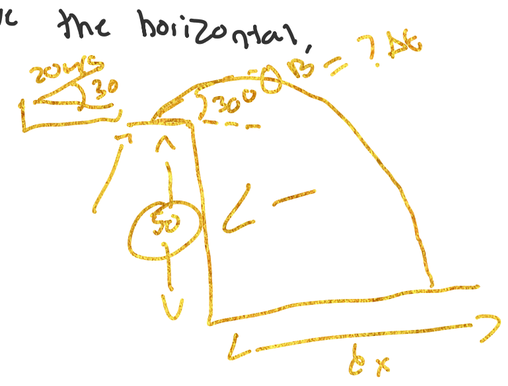
→ $h = \frac{v^2\sin^2\theta}{2g}$

→ $\Delta d_{ymax} = h + \frac{(v_{iy})^2}{2g}$ ← $g = -$

Example: A projectile is launched from the top of a 50m cliff with an initial velocity of 20m/s at an angle of 30° above the horizontal,

a) Find time at max height ✓

b) Find Range



(1) **Given/Unknowns:**

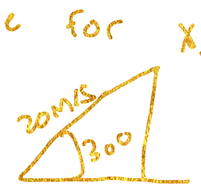
Given: $v_i = 20\,m/s\ [30°]$

$h = 50\,m$

Unknowns: $\Delta t_{max} = ?$ ← Δt = max height

$R\ (\Delta d_x) = ?$

(2) Solve for X, Y components of v_i



$v_{ix} = v_i\cos 30$

$v_{ix} = 20\cos 30$

$= 10\sqrt{3}\,m/s$

$v_{iy} = v_i\sin 30$

$= 20\sin 30$

$= 10\,m/s$

SohCahToa

(3) Use $\Delta t_{a\to B}$ for Δt_{max}

$\Delta t_{max} = \frac{v\sin\theta}{g}$

$= \frac{20\sin 30}{9.81} \Big| = 1.025$ ✓

(4) Calculate total time:

v_{iy} $y(t): h + v_i\Delta t + \frac{1}{2}a\Delta t^2$ ← $\frac{1}{2}(-9.81)$

$0 = 50 + 10\Delta t - 4.905\Delta t^2$

$t = \frac{-10 \pm \sqrt{10^2-4(-4.905)(50)}}{2(-4.905)}$

$t = \frac{-10 \pm \sqrt{1081}}{-9.81}$

$t = -2.33$, $t = 4.37\,s$ ✓ Δt ≠ −

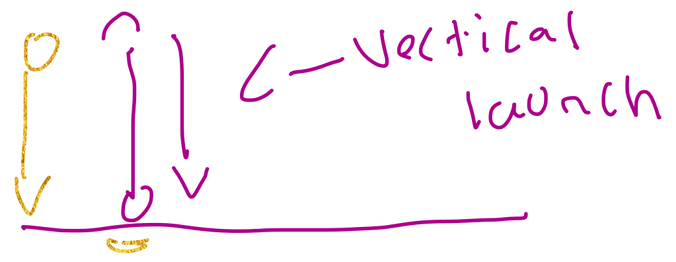
(5) Calculate Range (Δd_x)

$\Delta d_x = v_{ix}\Delta t$

$= 10\sqrt{3}\ (4.37)$

$= 75.69\,m$

Case 4:



Case based rules:

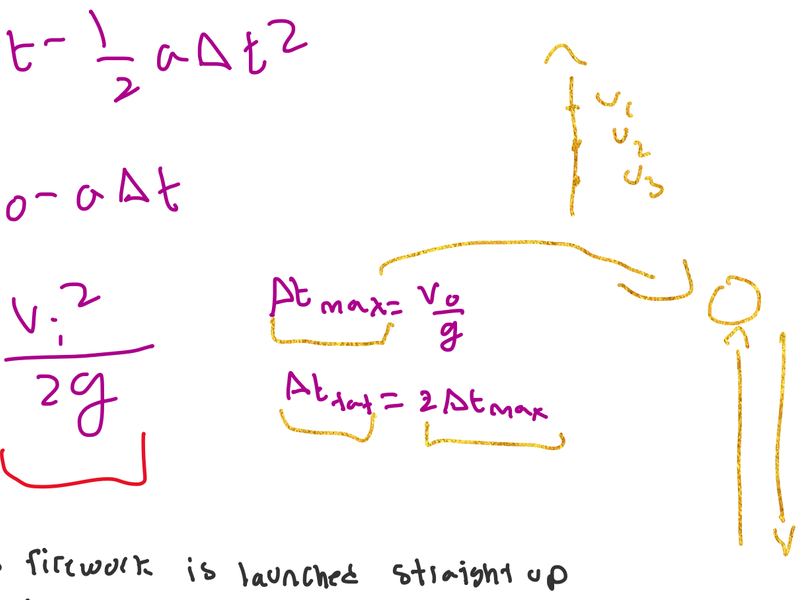
$\Delta d_y = v_i\Delta t - \frac{1}{2}a\Delta t^2$

$v(t) = v_0 - a\Delta t$

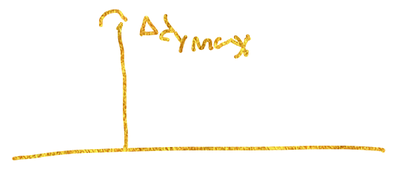
$\Delta d_{ymax} = \frac{v_i^2}{2g}$

$\Delta t_{max} = \frac{v_0}{g}$

$\Delta t_{total} = 2\Delta t_{max}$



Example: A firework is launched straight up into the air with initial velocity of 60m/s. Calculate Δd_{ymax} and Δt at Δd_{ymax}.



(1) **Given/Unknown:**

Given: $v_i = 60\,m/s$

Unknown: $\Delta d_{ymax} = ?$

$\Delta t_{max} = ?$

(2) Calculate Δd_{max}:

$\Delta d_{ymax} = \frac{v_i^2}{2g} = \frac{60^2}{2(9.81)}$

$= 183.4\,m$

(3) Calculate Δt_{max}:

$\Delta t_{max} = \frac{v_i}{g}$

$= \frac{60}{9.81} \Big| = 6.125$ ✓