

Examples:

$$\textcircled{1} f(x) = \underbrace{(3x^2 + 2x)}_{g(x)} \underbrace{(2x+1)^2}_{h(x)}$$

$$g(x) = 3x^2 + 2x$$

$$h(x) = (2x+1)^2$$

$$g'(x) = 3 \cdot 2x^{2-1} + 2 \cdot 1x^{1-1} = 6x + 2$$

$$h'(x) = \underbrace{2}_{2 \cdot (2x+1)^{2-1}} \cdot (2x+1)^{2-1} = 2 \cdot (2x+1)^1 \cdot 2$$

$$\xrightarrow{2x+1 \rightarrow 2 \cdot 1x^{1-1} + 0}$$

 $\textcircled{2}$

$$= 4(2x+1) = 8x+4$$

$$f'(x) = \underbrace{(6x+2)}_{g'(x)} \underbrace{(2x+1)^2}_{h(x)} + \underbrace{(3x^2+2x)}_{g(x)} \underbrace{(8x+4)}_{h'(x)}$$

$$\text{Product Rule: } f(x) = g(x)h(x)$$

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

$$\text{Chain Rule: } f(x) = g(h(x))$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$\text{Sum Rule: } f(x) = g(x) + h(x)$$

$$f'(x) = g'(x) + h'(x)$$

$$\begin{array}{l} 3x^2 \rightarrow 3 \cdot 2x^{2-1} = 6x \\ 2x^1 \rightarrow 2 \cdot 1x^{1-1} = 2 \cdot 1x^0 = 2 \end{array}$$

$$\textcircled{2} f(x) = \frac{\sin(2x)}{x^2+1}$$

$$\text{Quotient Rule: } f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

$$g(x) = \sin(2x)$$

$$h(x) = x^2 + 1$$

$$g'(x) = \cos(2x) \cdot \underbrace{\frac{d}{dx} 2x}_{2}$$

$$\sin(2x) \xrightarrow{\cos}$$

$$2x \rightarrow 2 \cdot 1x^0 = 2$$

$$= \cos(2x) \cdot 2$$

$$= 2\cos(2x)$$

$$h'(x) = 2x + 0$$

$$= 2x$$

$$f'(x) = \frac{2\cos(2x)(x^2+1) - (\sin(2x))(2x)}{(x^2+1)^2} \rightarrow \frac{[h(x)]^2}{[h(x)]^2}$$

$$= \frac{[2\cos(2x)](x^2+1) - (2x\sin(2x))}{(x^2+1)^2}$$

$$\text{Chain rule: } f(x) = g(h(x))$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$\text{Sum rule: } f(x) = g(x) + h(x)$$

$$f'(x) = g'(x) + h'(x)$$

$$\textcircled{3} f(x) = \ln(\underbrace{x^2+1}_u)$$

$$\ln(u)$$

$$\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

$$u = x^2 + 1$$

$$u' = 2x + 0$$

$$u' = 2x$$

$$f'(x) = \frac{2x}{x^2+1}$$

$$\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

$$= \frac{u'}{u}$$

$$= \frac{2x}{x^2+1}$$

$$\text{Chain Rule: } f(x) = g(h(x))$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$\frac{d}{dx} u$$

$$\ln(u) \rightarrow \frac{1}{u} \cdot \frac{du}{dx}$$

$$x^2 \rightarrow 2 \cdot x^{2-1} = 2x$$

$$1 \rightarrow 0$$