

Product Law of Logarithms:

states that log of product is equal to the sum of logs of the factors.

$\log_b(mn) = \log_b(m) + \log_b(n)$

$\log_{10}$

$\log(5)$

$\log_b(10) = \log_b(5) + \log_b(2)$

$\log_{10}(5)$

$\log(10) + \log(2)$

Quotient law of Logarithms

states that log of quotient is equal to the log of the numerator minus the log of the denominator.

$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$

$\log_{10}(m) - \log_{10}(n)$

$\log(5)$

$\log_{10}(5)$

Examples:

①  $\log_2(6) + \log_2(8) - \log_2(16)$

Step ① Apply product rule:  $\log_2(6) + \log_2(8) \rightarrow \log_2(6 \cdot 8)$

$mn = 6 \cdot 8$

$\log_b(m) + \log_b(n) = \log_b(mn)$

$\log_2(48) - \log_2(16)$

$\log_b(mn)$

Step ② Apply quotient law:  $\log_2(48) - \log_2(16) \rightarrow \log_2\left(\frac{48}{16}\right)$

$\log_2(3)$

②  $\log(x) + \log(y) + \log(3x) - \log(y)$

$\rightarrow \log_{10} 1 - \log_2 1 - \log_3 1 - \log_n 1 \rightarrow 0$

Step ① Apply product/quotient laws based on like terms

$\log(x) + \log(3x) \quad | \quad \log(y) - \log(y)$   
 $\log(x \cdot 3x) \quad | \quad \log(y/y)$   
 $\log(3x^2) \quad | \quad \log(1) = 0$   
 $\log(3x^2) + \log(1)$   
 $\log(3x^2)$   
 $2 \cdot \log(3x)$   
 $2 \log(3x)$

$\log_{10}(1) = 0$

$\log_{10}(10) = 1$

$\log_2(2) = 1$

$\log_3(3) = 1$

$\log(x) + \log(3x) + \log(y) - \log(y)$

Product rule

Practice:

①  $\log(12) - 3\log(2) + 2\log(2)$

②  $\log(100)$

①  $\log(12) - 3\log(2) + 2\log(2)$

$\log(12) - \log(2^3)$

$3\log(2)$   
 $\log(2^3)$

$\log_b(m) - \log_b(n)$

$m=12$   
 $n=2^3$

$\frac{m}{n} = \frac{12}{2^3}$

$\log(12/2^3)$   
 $\log(12/8)$   
 $\log(3/2) + \log(2^2)$   
 $\log(3/2 \cdot 4)$   
 $\log_2(6) = ?$

$2 \times 2 \times 2 = 8$

$2\log(2) \rightarrow \log(2^2)$

$\log_b(m) + \log_b(n) = \log_b(mn)$

②  $\log(100) = \log(10^2) + \log(10)$   
 $= \log(10^2) + \log(10)$   
 $= 2\log(10) + \log(10)$   
 $= 3\log_{10}(10) = ?$

$m=3/2, n=2^2=4$

$m \cdot n$

$\frac{3}{2} \times 2^2 \rightarrow \frac{3}{2} \times 4 = 6$

$\log(mn) \rightarrow \log(m) + \log(n)$

$\log(100)$   
 $\rightarrow x \cdot y$

$m=100$   
 $n=10$

$\log(100)$

$2\log(10)$

$\log_b(m) + \log_b(n)$

$2\log_b(m)$