

## Integration to integrals:

-> One of the two operations that defines calculus with differentiation

-> Primary purposes:

-> Find areas under curves, volumes, centroid points

-> Two main types:

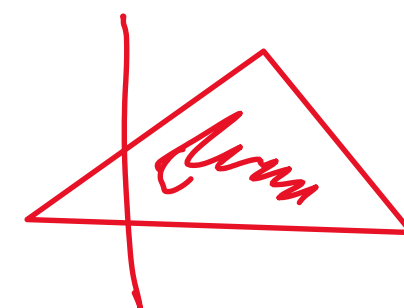
-> Indefinite integrals: Represent a family of functions and include an arbitrary constant  $C$

↳ Known as constant of integration

-> Essentially the antiderivative of a function

-> Definite integrals: have upper and lower limits; used to calculate area under a curve from one point to another  
-> gives specific value

$$\int \underbrace{x}_{u} \underbrace{dx}_{du} \rightarrow du$$



$C$

## Basic integration rules:

① Constant Rule:  $\int a \, dx = ax + C$ , where  $a$  is a constant

② Power Rule:  $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$  for all  $n \neq -1$

③ Sum Rule:  $\int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx$

Examples:

①  $\int 5 \, dx \rightarrow 5x + C$

②  $\int 3x^2 \, dx \rightarrow \frac{3x^{2+1}}{2+1} + C = \frac{3x^3}{3} + C = x^3 + C$

③  $\int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx$

$\int (2x^2 + 3x + 4) \, dx \rightarrow \int 2x^2 \, dx + \int 3x \, dx + \int 4 \, dx$

$\rightarrow \left[ \frac{2x^{2+1}}{2+1} + C_1 \right] + \left[ \frac{3x^{1+1}}{1+1} + C_2 \right] + \left[ \frac{4x + C_3}{1} \right]$   
 $= \frac{2x^3}{3} + \frac{3x^2}{2} + 4x + C$   
 $C = C_1 + C_2 + C_3$

## Techniques of integration:

① Substitution: useful when you spot a function and its derivative in an integral

Ex:  $\int x \cos(x^2 + 1) \, dx \rightarrow \text{let } u = x^2 + 1$

② Integration by Parts: method derived from Product Rule of differentiation

Useful formula:  $\int u \, dv = uv - \int v \, du$

③ Partial Fractions: used for integrating rational functions, where you express the fraction as a sum of simple fractions

$\frac{A}{x} + \frac{B}{x} + \frac{C}{x}$