Name → Soham Ghosh ID → 221001001034 • Graph Assignment Batch → BCS 2A, B.Tech → CSE

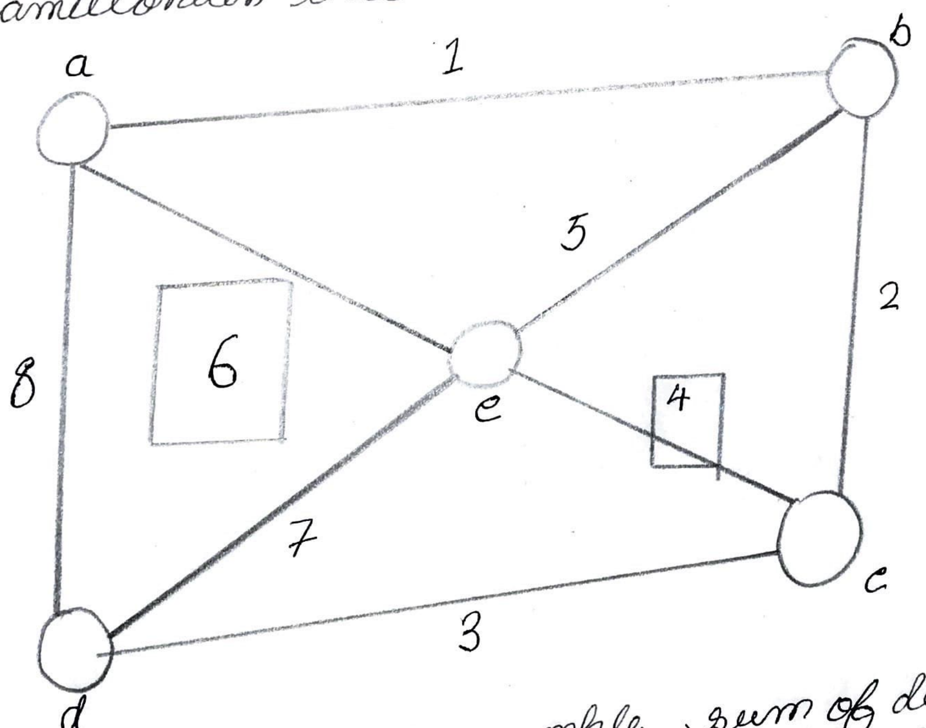
Q) Give 2 examples to show that a graph satisfying the theorem should be a hamiltonian.

Q) Give 2 counter-examples to prove the sufficiency requirement.

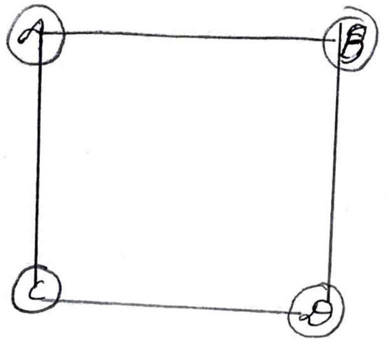
Ans) Hamiltonian graph - A connected graph G is called hamiltonian graph if there is a cycle which includes every vertex of G and the cycle is called Hamiltonian cycle. Hamiltonian walk in graph G is a walk that passes through each vertex exactly once.

Dirac's Theorem :- If G is a simple graph with n vertices, where $n \geq 3$ if $deg(v) \geq \{n\}/\{2\}$ for each vertex V, then the graph G is hamiltonian graph.

Ore's Theorem :- If G is a simple graph with n vertices, where $n \geq 2$ if $deg(x) + deg(y) \geq n$ for each pair of non-adjacent vertices x and y, then the graph G is hamiltonian theorem.



In the above example, sum of degree of a b c vertices is 6 and is greater than total vertices, 5 using Ore's Theorem, it is an hamiltonian theorem graph



Example :- 2

deg (A) = deg (B) = deg (C) = deg (D) = 2

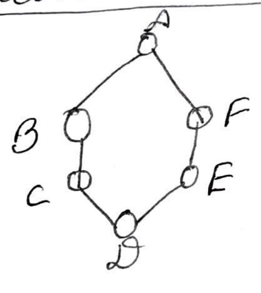
∴ Total no of vertices is 4 (n) and 2 = 4/2 ⇒ 2 = 2

∴ This graph also satisfies dirac's theorem and hence is a

Eg: Hamiltonian theorem

Q2) The suffeciency requirement tells us that if the dirac's theorem is satisfied, means that the graph is a hamiltoneian circuit but the non satisfaction of the dirac's theorem does not mean that that graph is not hamiltonian ckt.

<u>Counter example 1</u>



deg (A) = deg (B) = deg (C) = deg (D) = deg (E) = deg (F)

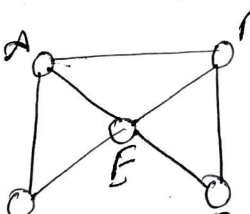
and total no of vertices (n) = 6

n/2 = 3

2 < 3, so dirac's theorem not possible

But this is a hamiltonean ckt as (A → B → C → D → E → F → A)

<u>Counter example 2</u>



No of vertices (n) = 5 ; n/2 = 2.5

deg (A) = 3 ≥ n/2

deg (B) = 3 ≥ n/2

deg (C) = 2 < 2.5

deg (D) = 2 < 2.5

deg (E) = 4 ≥ n/2

But the degree of C and D are than (n/2) so dirac's theorem does not satisfy.

But it is a hamiltonian ckt (A → B → E → D → E → A)