# Team Assignment #2

**Course:** Quantum Field Theory on a Quantum Computer **Assigned:** Sep 19, 2025 **Due:** Sep 26, 2025 (23:59 IST)

**Teams:** Work in teams. Submit one PDF report (max 20 pages, including appendix) and a zip with code/data. List all team members on the first page.
**Tools:** Preferably use QISKIT and Python. Acknowledge AI whenever used. However, we are going to look at what inputs were given as a team and how you went beyond AI.
**Reproducibility:** Submit commented nbs.

---

## Question 1

Controlled phase rotations are $R_k = \mathrm{diag}(1, e^{i\pi/2^{k-1}})$ for $k \geq 2$. Unless stated otherwise, take $n = 3$ ($N = 8$).

**Input vector and padding.** Use the complex vector

$$v = (1,\, i,\, 2.5,\, 4+i,\, 5,\, 7) \in \mathbb{C}^6.$$

Pad to length $N = 8$ with zeros: $v' = (1,\, i,\, 2.5,\, 4+i,\, 5,\, 7,\, 0,\, 0)$. When comparing with classical FFT, use the *unitary* normalization $\frac{1}{\sqrt{N}}$ so magnitudes/phases align with the QFT unitary. Clearly state any alternate padding/normalization you test.

**Erroneous QFT.** Define $\widetilde{\mathrm{QFT}}_N$ as the usual QFT circuit *except* that every gate that should be $R_3$ (i.e. phase $e^{i\pi/4}$) is replaced by $R_2$ (phase $e^{i\pi/2}$). All other gates (Hadamards, $R_2$, $R_4$, ..., final swaps) remain correct.

**Q1.1** Implement the *erroneous* QFT $\widetilde{\mathrm{QFT}}_8$ where every intended $R_3$ is replaced by $R_2$. Provide a minimal diagram or gate list to document exactly which locations are affected.

**Q1.2** Prepare the (padded) input state $|v'\rangle$ by amplitude encoding (normalize appropriately). Using the Hadamard test with a single ancilla, estimate the complex *output* amplitudes

$$a_j^{(\mathrm{err})} = \langle j|\, \widetilde{\mathrm{QFT}}_8\, |v'\rangle \quad (j = 0, \ldots, 7).$$

Report the estimated $\mathrm{Re}\, a_j^{(\mathrm{err})}$ and $\mathrm{Im}\, a_j^{(\mathrm{err})}$ with error bars.

**Q1.3** Baselines and comparisons:

- Compute the *classical* DFT $\hat{v}'$ using NumPy/FFT with $1/\sqrt{N}$ normalization.
- Compute the *correct* QFT output amplitudes $a_j^{(\mathrm{true})} = \langle j|\, \mathrm{QFT}_8\, |v'\rangle$.

Produce a figure with magnitudes and phases vs. $j$ for: Hadamard-test (erroneous), correct QFT, and classical FFT.

**Q1.4** Now imagine that you are dealing with the correct Quantum Fourier Transform circuit for the same problem above. However, now the careless experimenter has introduced a noisy $R_3$. This noise can be modeled by the unitary

$$U_{noise} = \begin{pmatrix} 1-\epsilon & \delta \\ \delta & 1-\epsilon \end{pmatrix} \tag{1}$$

Choose $\delta$ such that matrix is unitary. Let $\epsilon$ be some random real number between 0 and 0.1. In front of every $R_3$ (control is with $U_{noise}R_3$) this number changes.

Rerun this and report your findings comparing with correct classical FFT.

## Question 2

Consider the nearest-neighbour transverse-field Ising model (TFIM) with open boundary conditions on $N = 10$ qubits:

$$H \;=\; J\sum_{j=1}^{N-1} Z_j Z_{j+1} \;+\; h\sum_{j=1}^{N} X_j, \quad \text{with} \quad J = 1,\; h = 0.5.$$

The computational basis ordering is $|q_{10}\, q_9 \cdots q_1\rangle$, and the initial state is

$$|\psi(0)\rangle = |00\cdots 0\rangle\,.$$

We study real-time evolution $|\psi(t)\rangle = e^{-iHt}\,|\psi(0)\rangle$ at $t \in \{0.1,\,0.5,\,1.0\}$ using even-order Suzuki–Trotter formulas $S_{2k}$ with $k = 1, 2, 3$. The number of Trotter steps is fixed to $r = 50$ for all cases.

**Target states.** For each $t$, define

$$|\psi_k(t)\rangle \;:=\; U_{2k}(t)\;|\psi(0)\rangle\,, \qquad k \in \{1, 2, 3\},$$

where $U_{2k}(t)$ is the order-$2k$ Suzuki–Trotter approximation with $r = 50$ steps. The *comparison* state is $|\psi_{\text{comp}}(t)\rangle := |\psi_3(t)\rangle$.

**Q2.1** For each $t$ and $k \in \{1, 2\}$, estimate the complex inner product

$$\langle\psi_k(t)\,|\,\psi_{\text{comp}}(t)\rangle$$

using a single-ancilla Hadamard test (measure $X$ for the real part and $Y$ for the imaginary part). Report point estimates with error bars, and clearly state the number of measurement shots used.

### Tasks

**T1.** Implement $U_2, U_4, U_6$ with $r = 50$ slices each. Explain briefly how you decompose $e^{-iZ_jZ_{j+1}\alpha}$ and $e^{-iX_j\beta}$ into native gates.

**T2.** For each $t \in \{0.1, 0.5, 1.0\}$, estimate Re and Im of $\langle\psi_k(t)|\psi_{\text{comp}}(t)|\psi_k(t)|\psi_{\text{comp}}(t)\rangle$ for $k = 1, 2$. Present a table of results with error bars, and discuss the trend with $t$ and with the formula order $2k$.

**T3.** Compare your overlaps with statevector-simulated overlaps (no Trotter error). Check that $k = 2$ is uniformly closer to $k = 3$ than $k = 1$; if deviations occur, comment on possible reasons such as finite $r$, accumulation, or sampling error.

**Q2.2** Consider the state

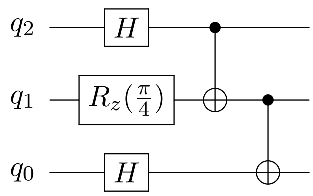
$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|00\cdots 0\rangle + |11\cdots 1\rangle\,. \tag{2}$$

Starting with the computational basis where all input-qubits are $|0\rangle$ design a quantum circuit to time evolve this state. Repeat the exercise in Q2.1 for this state and report your findings.

## Question 3

In this exercise you will apply the Quantum Phase Estimation (QPE) algorithm to a small unitary operator defined by a short circuit. The QPE procedure was discussed in lecture.

**The unitary.** Consider a three-qubit system (qubits labelled top to bottom as $q_2, q_1, q_0$). Define the unitary $U$ by the following circuit:




**Tasks.**

**T1.** Determine at least one eigenstate $|\phi\rangle$ of $U$ and its corresponding eigenvalue $e^{2\pi i\varphi}$, with $\varphi \in [0, 1)$. Show your reasoning.

**T2.** Using $|\phi\rangle$ as input to QPE, implement the algorithm with $t = 2, 4, 6, 8$ ancilla (phase) qubits. In each case, run the full QPE procedure and record the measured bit strings. Convert them to decimal fractions and compare with the exact $\varphi$ from part (1).

**T3.** For each $t$, discuss the accuracy and spread of your estimates. How does the distribution concentrate as $t$ increases? Present your results as a table or histogram, and include a short paragraph interpreting the convergence of the estimates.

**T4.** Reflect briefly on the role of the initial eigenstate: what happens if you mistakenly supply a superposition that is not an eigenstate of $U$? Give a suitable example.