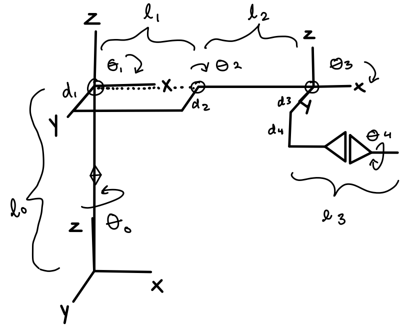
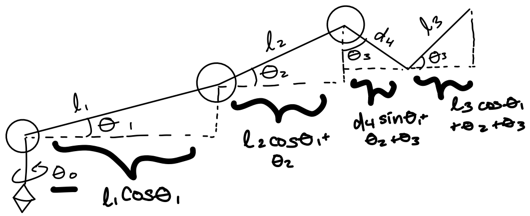
Forward Kinematics

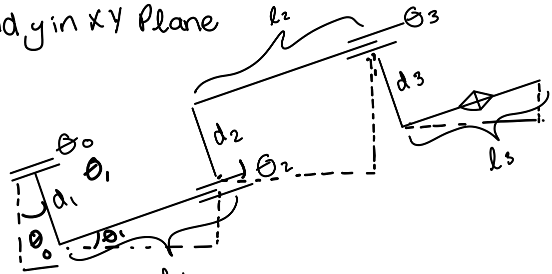


- Find x in xz plane



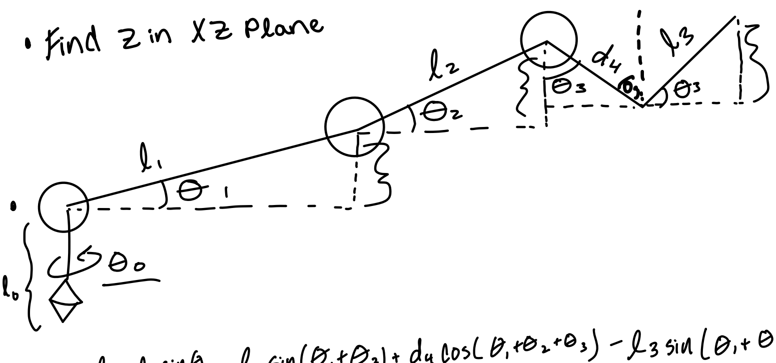
- $x = \cos\theta_0 \left(l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + d_4 \sin(\theta_1+\theta_2+\theta_3) + l_3 \cos(\theta_1+\theta_2+\theta_3)\right) - d_3 \sin\theta_0$

- Find y in xy Plane



- $y = \sin\theta_0 \left(l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + d_4 \sin(\theta_1+\theta_2+\theta_3) + l_3 \cos(\theta_1+\theta_2+\theta_3)\right) + d_3 \cos\theta_0$

- Find z in xz Plane



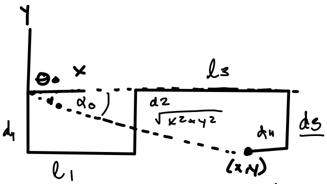
- $z = l_0 - l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) + d_4 \cos(\theta_1+\theta_2+\theta_3) - l_3 \sin(\theta_1+\theta_2+\theta_3)$

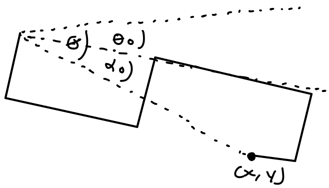
General explanation :

Given joints along the same axis, use trig fonctions to find their respective lengths. Then (for x and y) you have to compensate for the initial θ_0 rotation. For X-Y, you also have to subtract d_3 since its static from the total length. Also, after trial and error, I figured out you have to take signs into account for some joint. Given these facts, I was able to derive the Forward Kinematics using trigonometry.

Inverse Kinematics

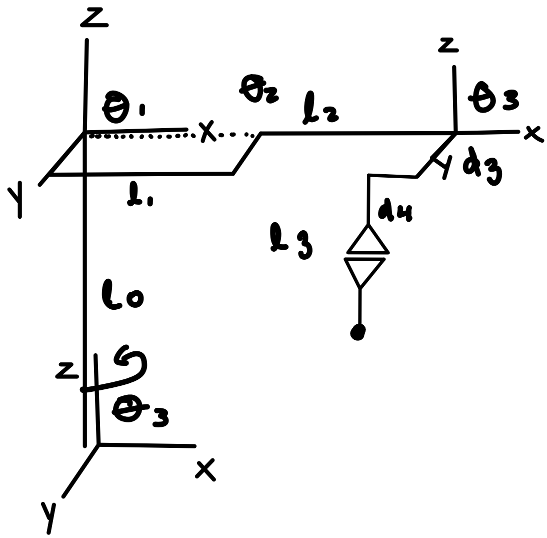
- Get initial configuration
- Get θ_0
- Get top down View
- Find θ_0's offset, (d) to get θ_0





- offset $= d = \sin^{-1}\left(\frac{d_3}{\sqrt{x^2+y^2}}\right)$
- $\theta_0 = \tan^{-1}\left(\frac{y}{x}\right) - d$

- Now we must try to get all the links aligned in the same plane to perform the 3DOF Reduction



- Transform endeffector coordinates

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x - d_4\cos\theta_0 + d_3\sin\theta_0 \\ y - d_3\cos\theta_0 - d_4\sin\theta_0 \\ z + l_3 - l_0 \end{pmatrix}$$

- get positions aligned in X-Y Plane
  - $X'' = \sqrt{(x')^2 + (y')^2}$
  - $y'' - (z')$

- Get hypotonuse
- $hyp = \sqrt{(x'')^2 + (y'')^2}$ $\boxed{\theta_1 = \beta + \gamma}$
- $\beta = \arctan^{-1}\left(\frac{y''}{x''}\right)$

- $\gamma = \frac{hyp^2 + l_1^2 - l_2^2}{2 \cdot hyp \cdot l_1}$ (law of cosines)
- $\boxed{\theta_2 = \frac{hyp^2 - l_1^2 - l_2^2}{2 \cdot l_1 \cdot l_2}}$ → • $\boxed{\theta_3 = \frac{\pi}{2} - \theta_1 - \theta_2}$

Since the end effector rotated about z you need to accomidate for this.

Jacobian

Use $x, y, \& z$ from part 1 $\longrightarrow$ Take Partial Derivatives

$$x = \cos\theta_0 \left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) - d_3\sin\theta_0$$

$$y = \sin\theta_0 \left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) + d_3\cos\theta_0$$

$$z = l_0 - l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)$$

$$J(\vec{\theta}) = \begin{pmatrix} \frac{dx}{d\theta_0} & \frac{dx}{d\theta_1} & \frac{dx}{d\theta_2} & \frac{\partial x}{\partial\theta_3} \\ \frac{dy}{d\theta_0} & \frac{dy}{d\theta_1} & \frac{dy}{d\theta_2} & \frac{dy}{\partial\theta_3} \\ \frac{dz}{d\theta_0} & \frac{dz}{d\theta_1} & \frac{\partial z}{d\theta_2} & \frac{\partial z}{\partial\theta_3} \end{pmatrix}$$

$$\frac{dx}{d\theta_0} = -\sin\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) - d_3\cos\theta_0$$

$$\frac{dx}{d\theta_1} = \cos\theta_0\left(-l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dx}{d\theta_2} = \cos\theta_0\left(l_1\cos\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dx}{d\theta_3} = \cos\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dy}{d\theta_0} = \cos\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) - d_3\sin\theta_0$$

$$\frac{dy}{d\theta_1} = \sin\theta_0\left(-l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dy}{d\theta_2} = \sin\theta_0\left(l_1\cos\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dy}{d\theta_3} = \sin\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dz}{d\theta_0} = 0$$

$$\frac{dz}{d\theta_1} = +l_1\cos\theta_1 - l_2\cos(\theta_1+\theta_2) - d_4\sin(\theta_1+\theta_2+\theta_3) - l_3\cos(\theta_1+\theta_2+\theta_3)$$

$$\frac{dz}{d\theta_2} = -l_2\cos(\theta_1+\theta_2) - d_4\sin(\theta_1+\theta_2+\theta_3) - l_3\cos(\theta_1+\theta_2+\theta_3)$$

$$\frac{dz}{d\theta_3} = -d_4\sin(\theta_1+\theta_2+\theta_3) - l_3\cos(\theta_1+\theta_2+\theta_3)$$