Aman Hogan

Hmwrk 1

- Kin
- invkin
- Jacobian

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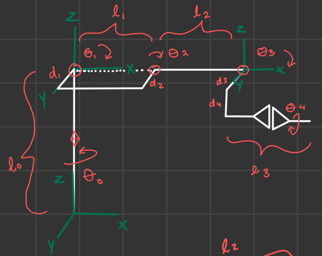
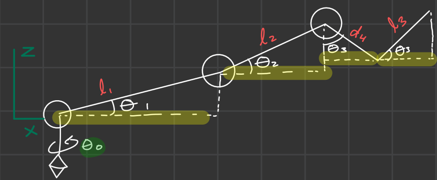
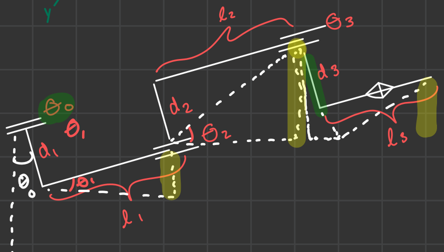
Forward Kinematics
Trigonometric Solution



Find x and y

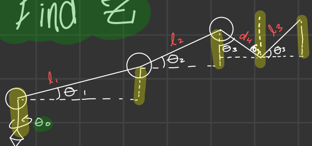


$$\boxed{x = \cos(\theta_0)\left[(l_1\cos\theta_1) + (l_2\cos(\theta_1+\theta_2)) + (l_3\cos(\theta_1+\theta_2+\theta_3)) + (d_4\sin(\theta_1+\theta_2+\theta_3))\right] - d_3\sin\theta_0}$$



$$\boxed{y = \sin\theta_0\left[(l_1\sin\theta_1) + (l_2\cos(\theta_1+\theta_2)) + (d_4\sin(\theta_1+\theta_2+\theta_3)) + (l_3\cos(\theta_1+\theta_2+\theta_3))\right] - d_3\sin\theta_0}$$

Find Z

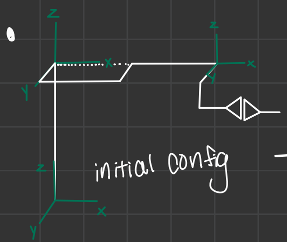


$$\boxed{z = d_0 - (l_1\sin\theta_0) - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)}$$

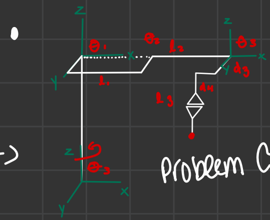
General explanation :

Given joints along the same axis, use trig fonctions to find their respective lengths. Then (for x and y) you have to compensate for the initial θ_0 rotation. For X-Y, you also have to subtract d_3 since its static from the total length. Also, after trial and error, I figured out you have to take signs into account for some joint. Given these facts, I was able to derive the Forward Kinematics using trigonometry.

Inverse Kinematics



initial config →

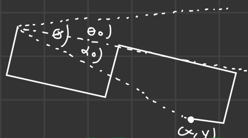


Problem Config

Must first determine θ_0. θ_0 can be found using trig in the top down view. Also, we must take into account d, which is the offset of θ°

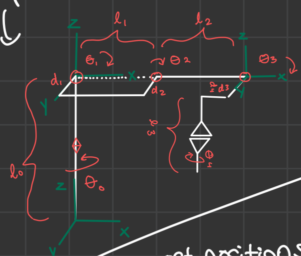
Top View

$d = \sin^{-1}\left(\frac{d_3}{\sqrt{x^2+y^2}}\right)$



$$\boxed{\theta_0 = \tan^{-1}(y/x) - d}$$

- Now we must try to get all the links aligned in the same plane to perform the 3DOF reduction.



Transform endeffector coordinates

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x - d_4\cos\theta_0 + d_3\sin\theta_0 \\ y - d_3\cos\theta_0 - d_4\sin\theta_0 \\ z + l_3 - l_0 \end{pmatrix}$$

get positions aligned in x-y Plane

$$x'' = \sqrt{(x')^2 + (y')^2}$$

$$y'' - (z')$$

Get hypotonuse

$$hyp = \sqrt{(x'')^2 + (y'')^2}$$

$$\beta = \arctan\left(\frac{y''}{x''}\right)$$

$$\boxed{\theta_1 = \beta + \gamma}$$

★ law of cosines

$$\gamma = \frac{hyp^2 + d_1^2 - d_2^2}{2 \cdot hyp \cdot d_1}$$

$$\boxed{\theta_2 = \frac{hyp^2 - d_1^2 - d_2^2}{2 \cdot l_1 \cdot l_2}}$$

$$\boxed{\theta_3 = \frac{\pi}{2} - \theta_1 - \theta_2}$$

Since the end effector rotated about z you need to accomidate for this.

Jacobian

Use $x, y, \ \& \ z$ from part 1 $\Longrightarrow$ Take Partial Derivatives

$$x = \cos\theta_0 \left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) - d_3\sin\theta_0$$

$$y = \sin\theta_0 \left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) + d_3\cos\theta_0$$

$$z = l_0 - l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)$$

$$J(\vec{\theta}) = \begin{pmatrix} \frac{dx}{d\theta_0} & \frac{dx}{d\theta_1} & \frac{\partial x}{\partial\theta_2} & \frac{\partial x}{\partial\theta_3} \\ \frac{dy}{d\theta_0} & \frac{dy}{d\theta_1} & \frac{\partial y}{\partial\theta_2} & \frac{\partial y}{\partial\theta_3} \\ \frac{\partial z}{\partial\theta_0} & \frac{\partial z}{\partial\theta_1} & \frac{\partial z}{\partial\theta_2} & \frac{\partial z}{\partial\theta_3} \end{pmatrix}$$

$$\frac{dx}{d\theta_0} = -\sin\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) - d_3\cos\theta_0$$

$$\frac{dx}{d\theta_1} = \cos\theta_0\left(-l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dx}{d\theta_2} = \cos\theta_0\left(l_1\cos\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dx}{d\theta_3} = \cos\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dy}{d\theta_0} = \cos\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\sin(\theta_1+\theta_2+\theta_3) + l_3\cos(\theta_1+\theta_2+\theta_3)\right) - d_3\sin\theta_0$$

$$\frac{dy}{d\theta_1} = \sin\theta_0\left(-l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dy}{d\theta_2} = \sin\theta_0\left(l_1\cos\theta_1 - l_2\sin(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dy}{d\theta_3} = \sin\theta_0\left(l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + d_4\cos(\theta_1+\theta_2+\theta_3) - l_3\sin(\theta_1+\theta_2+\theta_3)\right)$$

$$\frac{dz}{d\theta_0} = 0$$

$$\frac{dz}{d\theta_1} = +l_1\cos\theta_1 - l_2\cos(\theta_1+\theta_2) - d_4\sin(\theta_1+\theta_2+\theta_3) - l_3\cos(\theta_1+\theta_2+\theta_3)$$

$$\frac{dz}{d\theta_2} = -l_2\cos(\theta_1+\theta_2) - d_4\sin(\theta_1+\theta_2+\theta_3) - l_3\cos(\theta_1+\theta_2+\theta_3)$$

$$\frac{dz}{d\theta_3} = -d_4\sin(\theta_1+\theta_2+\theta_3) - l_3\cos(\theta_1+\theta_2+\theta_3)$$