

3.16) b) - Let L_1 be Turing recognizable
- Let L_2 be Turing recognizable

- Let M_1 ^{recognize} decide L_1
- Let M_2 ^{recognize} decide L_2
- Let M recognize $L_1 \circ L_2$

- 1) w is a ^{word} string from $L_1 \circ L_2$
- 2) Divide each string of $L_1 \circ L_2$ into w_1 and w_2 nondeterministically
- 3) Run w_1 on L_1
 - If halting, then reject
- 4) Run w_2 on L_2
 - If halting then reject
 - If accept, then accept

Therefore Turing recognizable languages are closed under concatenation.

3.16) c) Let L_1 be a Turing recognizable language

Let L_1^* be obtained by $*$

Let M_1 be a machine that recognizes L_1^*

1) For an input string w divide it into n parts:

$w_1, w_2 \dots w_n$

2) For each part run M_1 on all divided parts.

- If all parts are accepted, then accept

- ~~If~~ Else, then reject

Therefore the star is clo

Therefore Turing recognizable languages are closed under $*$.

3.16) 2)

- Let L_1 and L_2 be Turing recognizable
- Let M_1 and M_2 recognize L_1 and L_2

1) Assume s is an input string of ~~L_1~~ $L_1 \cup L_2$

2) run s on M_1 ,

- If it accepts run s on L_2
- Else reject

3) If M_2 accepts L_2 then accept

- Else then reject

Turing recognizable languages are closed under intersection.