2.6 a) Set of strings of $\Sigma = \{a, b\}$ where more a's than b's

$S \to TaT$

$T \to TT \mid aTb \mid bTa \mid a \mid \varepsilon$

b) complement of $\{a^n b^n \mid n \geq 0\}$

★ Break up into several languages

$S \to S_1 \mid S_2 \mid S_3$

$S_1 \to aS_1b \mid aS_1 \mid a$

$S_2 \to aS_2b \mid S_2b \mid b$

$S_3 \to TbTaT$

$T \to aT \mid bT \mid \varepsilon$

2.9) $A = \{a^i b^j c^k \mid i=j \mid j=k \text{ for } i,j,k \geq 0\}$

$A = L_1 \cup L_2$

$L_1 = \{a^i b^j c^k \mid i,j,k \geq 0 \; i=j\}$
$L_2 = \{a^i b^j c^k \mid i,j,k \geq 0 \; j=k\}$

$S \to S_1 \mid S_2$

$S_1 \to S_1 c \mid D \mid \varepsilon$
$D \to aDb \mid \varepsilon$

$S_2 \to aS_2 \mid E \mid \varepsilon$
$E \to bEc \mid \varepsilon$

The language is ambigoous because a string $s = a^n b^n c^n$ can be used for eitrier S_1 or S_2

2.14) $A \to BAB \mid B \mid \varepsilon$
$B \to 00 \mid \varepsilon$

- Add A_0 *

$A_0 \to A$
$A \to BAB \mid B \mid \varepsilon$
$B \to 00 \mid \varepsilon$

- Remove $A \to \varepsilon$

$A_0 \to A \mid \varepsilon$
$A \to BAB \mid BB \mid B$
$B \to 00 \mid \varepsilon$

- Remove $B \to \varepsilon$

$A_0 \to A \mid \varepsilon$
$A \to BAB \mid BB \mid B \mid BA \mid AB \mid A$
$B \to 00$

- Unit Rules, remove $A \to A$ and $A \to B$

$A_0 \to A \mid \varepsilon$
$A \to BAB \mid BB \mid 00 \mid BA \mid AB$
$B \to 00$

- Change $A_0 \to A$

$A_0 \to BAB \mid BA \mid AB \mid 00 \mid BB \mid \varepsilon$
$A \to BAB \mid BA \mid AB \mid 00 \mid BB$
$B \to 00$

- Replace double terminals 00

$A_0 \to BAB \mid BA \mid AB \mid CC \mid BB \mid \varepsilon$
$A \to BAB \mid BA \mid AB \mid CC \mid BB$
$B \to CC$
$C \to 0$

- Get rid of 3 variables

$A_0 \to BD \mid BA \mid AB \mid CC \mid BB \mid \varepsilon$
$A \to BD \mid BA \mid AB \mid CC \mid BB$
$B \to CC$
$C \to 0$
$D \to AB$

2.2(b) Proof by induction

Base case: $n=1$

- let $s = a$, where "a" is of length 1 and in Chomsky Normal Form such that

$S \to a$

$n = 1$

$2n - 1 =$

$2(1) - 1$

$2 - 1$

$= 1$

So it is true that $2n-1$ is the number of derrived steps for $n=1$.

Induction Step: $n = k+1$

- $2(k+1) + 1 =$

$2k + 2 - 1$

$2k + 1$

we can assume $n = |x| + |y|$ since for $n = 2k+1$

$S \to BC$

$B \xrightarrow{*} x$

$C \xrightarrow{*} y$

It requiresat least 2n-1 steps to derive a string into CNF by Induction

2.31 L = { Palindrome over {0,1} eq 0's & 1's }

Proof by Contradiction

Assume L is reagular and has pumping length
Assume a string, $s = uvxyz = 0^P 1^{2P} 0^P$.
Such that:

1) $uv^ixy^iz \in L$
2) $|vy| > 0$
3) $|vxy| \leq P$

CASE 1 : Contains middle and end, of s:

since $uv^ixy^iz \in L$, let $i=2$
Pump... uv^2xy^2z = $\underbrace{00....}_{u}\underbrace{111111\ 10101...}_{vxy}\underbrace{00}_{z}$

Since their will be more characters at end of string than the begginning, s cannot be a palindrome - Contradiction

CASE 2: Contains middle and end of s.
since $uv^ixy^iz \in L$, let $i=2$
Pump... uv^2xy^2z = $\underbrace{000...}_{u}\underbrace{1111...}_{v}\underbrace{1}_{x}\underbrace{1010010}_{y}\underbrace{0}_{z}$

Since the pumping occurred at beggining the begging contains a different sequence of numbers than the end of the string.

Proof by Contradiction. L is not context free