

1.2a a)  $A_1 = \{0^n 1^n 2^n \mid n \geq 0\} = A$  (d. AS.1)

- Assume  $A$  is regular
- Assume  $A$  has a pumping length  $P$ .
- $|S| \geq P$
- $S = 0^P 1^P 2^P$
- Divide 'S' into  $x y z$  such that:

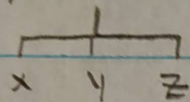
$$\begin{array}{c} \text{---} S \text{---} \\ | \\ x \ y \ z = 0^P 1^P 2^P \end{array}$$

CASE 1:

- let  $P=7$ , such that  $x y z = 0^7 1^7 2^7$
- $x y z = \underbrace{0000000}_{x \ y} \underbrace{1111111}_{z} \underbrace{2222222}_{z}$
- Pump  $y$ , such that  $y^i = y^2 = 000000$
- When combining the  $x y z$  using  $y^2$  we get:  
 $0000000000 1111111 2222222$
- This does not belong to  $A$ , since the # of 0, 1, 2's are not even.
- $|y| > 0$ ;  $|x y| \leq P$ ;  $0 \leq i < \infty$
- Therefore the language  $A_2$  is not regular by proof of contradiction.

1.29 b)  $A_2 = \{www \mid w \in \{a,b\}^*\}$

- Assume  $A_2$  is regular
- Assume  $A_2$  has a pumping length 'P'
- $S$  is a string such that  $|S| \geq P$
- $S = w^P w^P w^P$
- say,  $S = a^P b a^P b a^P b$



- let  $P = 5$ , so  $S = \underbrace{aaaaa}_{x} \underbrace{abaaaa}_{y} \underbrace{abaaaa}_{z} b$
- let  $|y| = y^2$  so  $y^2 = aaaa$  such that:  
 $S = aaaaaa \underbrace{abaaaa}_{y^2} abaaaa$
- $S = aaaaaa \underbrace{abaaaa}_{y^2} abaaaa$  which is not in  $A_2$
- This is not a member of language  $A_2$
- Since  $|y| > 0$ ;  $4 > 0$  and  $|xy| \leq P$ ;  $5 \leq 5$
- $A_2$  is not a regular language by proof of contradiction.



#### 1.4(6) a) Pumping Lemma

$$A_1 = \{ 0^n 1^m 0^n \mid m, n \geq 0 \}$$

- Assume  $A_1$  is a regular language
- Assume  $A_1$  has a pumping length 'P'
- Such that  $S = 0^P 1 0^P$

$$S = 0^P 1 0^P$$

$$\begin{array}{c} \overline{\overline{\overline{1}}} \\ x \quad y \quad z \end{array}$$

- Let  $P=8$ , such that  $S = 0^8 1 0^8 = x y z$

$$S = x y z = \underbrace{00000000}_x \underbrace{00000000}_y \underbrace{1000000000}_z$$

- Pump  $y' = y^2$ , such that  $y' = y^2 = 0000$
- If  $y' = y^2 = 0000$ , then  $S$  is:

$$S = 0000000000001000000000$$

- Since there are unequal amounts of 0's after and before '1' the language is not regular
- $|y| > 0$ ;  $|y| > 0$  and  $|x y^k z| < P$ ;  $6 < 8$
- By proof of contradiction  $A_1$  is not regular.

b)  $A = \{0^m 1^n \mid m \neq n\}$

- Assume  $A$  is a regular language
- Assume  $A$  has a pumping length
- Assume  $S = 0^p 1^{p+1}$
- Let  $p = 6$  such that  $S = 000000 111111$
- Let  $x y z = S$
- $\underbrace{000000}_x \underbrace{0}_y \underbrace{111111}_z$
- Pump  $y$  such that  $y^i = y^2 = 00$
- Combine this into  $S$ :  
 $S = 00000000 111111$
- The number of 0's equals the number of 1's
- This is a contradiction of  $m \neq n$
- Since  $|y| > 0$ ;  $|y| > 0$  and  $|xy| \leq p+1 \quad 7 \leq 7$
- By proof of contradiction,  $A$  is not regular.



1.46) c)  $A = \{w \mid w \in \{0,1\}^*$  is not a palindrome $\}$

•  $\bar{A} = \{w \mid w \in \{0,1\}^*$  is a palindrome $\}$

- Assume  $A$  is regular
- Then  $\bar{A}$  should also be regular

• Assume  $P$  is pumping length

•  $S = 0^P 1 0^{P+1} = x y z$

• Note

• let  $P=4$ , so  $S = 0^4 1 0^5$

$|x y| \leq P$  and

•  $S = \underbrace{0000}_x 1 \underbrace{00000}_z$

$y > 0$

• Pump  $y$  such that  $y^i = y^3 = 000$

• Combining  $x y^i z$

•  $S = 000000 1 00000$

•  $S$  is not a palindrome even though it belongs to  $\bar{A}$

•  $A$  is not a regular language by proof of contradiction.