3.11 Let I be a doubly infinate tape Turing Machine

Assume $I \longleftrightarrow M$ for languages of M

$I \rightarrow M$

1) Mark the very left side of I
2) Prevent I from moving left of the mark and this simulates M

$M \rightarrow I$

1) Use a two tape Machine M
2) Make/spit I into two parts
3) Make the first part of I represent the first tape of M. and secend part of I represent the second tape of M

Thus D is equivelent to I

3.15) c)

Let L be a Turing decidable language
Let M be Turing Machine that decides L

1) On input string w, split w into n different ways

2) Run the M machine on each different way
   - If M accept, accept

3) If all ways have been tried and no successes, reject.

Therefore L is closed on under "*"

3.15) d) Let L be Turing decidable
Let M be Machine decides L

1) on input string w, accept if M reject

2) If rejects, then accept

Therefore decidable languages are closed under complementation.

3.15) c) Let L_1 be Turing decidable
Let L_2 be Turing decidable
Let M_1 decide L_1.
Let M_2 decide L_2

1) Run M_1 on input string w.
   - If reject the reject

2) Else run M_2 on w. ~~reject, then~~
   - If reject, then reject

3) Else Accept

Therefore $L_1 \cap L_2$ is closed under intersection.

3.16) b) - Let L_1 be Turing recognizable
- Let L_2 be Turing recognizable

- Let M_1 ~~decide~~ recognize L_1
- Let M_2 decide L_2
- Let M recognize $L_1 \circ L_2$

1) w is a ~~string~~ word from $L_1 \circ L_2$

2) Divide each string of $L_1 \circ L_2$ into w_1 and w_2 nondeterministicly

3) run w_1 on L_1
   - If halting, then reject

4) run w_2 on L_2
   - If halting then reject
   - If accept, then accept

Therefore Turing recognizable languages are closed under concatenation

3.16) c) Let L_1 be a Turing Recognizable language

Let L_1^* be obtained by $*$

Let M_1 be a machine that recognizes L_1^*

1) For an input string w divide it into n parts:
   $w_1, w_2 \ldots w_n$

2) For each part run M_1 on all divided parts.
   - If all parts parts are accepted, then accept
   - ~~If~~ Else, then reject

~~Therefore the star is clo~~

Therefore Turing recognizable languages are closed under $*$.

3.16) d)

- Let L_1 and L_2 be Turing recognizable
- Let M_1 and M_2 recognize L_1 and L_2

1) Assume S is an input string of $L_1 \cup L_2$

2) run S on M_1:
   - If it accepts run S on L_2
   - Else reject

3) If M_2 accepts L_2 then accept
   - Else then reject

Turing recognizable languages are closed under intersection.