1.29 a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

- Assume A is regular
- Assume A has a pumping length 'P'.
- $|S| \geq P$
- $S = 0^P 1^P 2^P$
- Divide 'S' into xyz such that:

$$\overbrace{x\,y\,z}^{S} = 0^P 1^P 2^P$$

CASE 1:

- let $P=7$, such that $xyz = 0^7 1^7 2^7$
- $xyz = \underbrace{00}_{x}\underbrace{00}_{y}\underbrace{000\,1111111\,2222222}_{z}$

- Pump $\underbrace{y}_{00}$, such that $y^i = y^2 = 0000$

- when combining the xyz using y^2 we get:
  000000000 1111111 2222222

- This does not belong to A, since the # of 0, 1, 2's are not even.

- $|y| > 0; 4 > 0$ and $|xy| \leq P; 6 \leq 7$

- Therefore the language A_2 is not regular by proof of contradiction

1.29 b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

- Assume A_2 is regular
- Assume A_2 has a pumping length 'p'
- S is a string such that $|S| \geq p$
- $S = w^p w^p w^p$
- say, $S = a^p b a^p b a^p b$

  x y z

- let $p = 5$, so $S = \underbrace{aa}_{x}\underbrace{aa}_{y}\underbrace{abaaaaabaaaaab}_{z}$

- let $y^i = y^2$ so $y^2 = aaaa$ such that:
  $S = aaaaaaabaaaaabaaaaab$

- This is not a member of language A_2

- Since $|y| > 0$; $4 > 0$ and $|xy| \leq p$; $5 \leq 5$

- A_2 is not a regular language by proof of contradiction.

1.46) a) Pumping Lemma

$A_1 = \{0^n 1^m 0^n \mid m, n \geq 0\}$

- Assume A_1 is aregealur language
- Assume A_1 has a pumping length 'P'
- Such that $S = 0^P 1 0^P$
- $\underbrace{S}_{x\,y\,z} = 0^P 1 0^P$
- Let $P = 8$, such that $S = 0^8 1 0^8 = xyz$
- $S = xyz = \underbrace{00}_{x}\underbrace{0000}_{y}\underbrace{00\,1\,00000000}_{z}$
- Pump $y^i = y^2$, such that $y^i = y^2 = 0000$
- If $y^i = y^2 = 0000$, then S is:

  $S = 000000000000100000000$
- Since there are unequal amounts of 0's after and before '1' the language is not regular
- $|y| > 0$; $4 > 0$ and $|xy| \leq P$; $6 < 8$
- By proof of contradiction A_1 is not reagular.

b) $A = \{0^m 1^n \mid m \neq n\}$

- Assume A is a reuguiar language
- Assume A has a pumping length

- Assume $S = 0^p 1^{p+1}$
- let $P = 6$ such that $S = 000000\,1111111$
- let $xyz = S$
- $\underbrace{00000}_{x}\underbrace{0}_{y}\underbrace{1111111}_{z}$
- Pump y such that $y^i = y^2 = 00$
- Combine this into S:

$S = 0000000\,1111111$
- The number of 0's equals the number of 1's
- This is a contradiction of $m \neq n$
- Since $|y| > 0$; $1 > 0$ and $|xy| \leq p+1$ $7 \leq 7$

- By proof of contradiction, A is not reagular.

1.46) c) $A = \{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$

- $\overline{A} = \{w \mid w \in \{0,1\}^*$ is a palindrome$\}$
- Assume A is regular
- Then $\overline{A}$ should also be regular
- Assume P is pumping length
- $S = 0^P 1 0^{P+1} = xyz$
- let $P = 4$, so $S = 0^4 1 0^5$
- $S = \underbrace{000}_{x}\underbrace{0}_{y}\underbrace{100000}_{z}$

Note
$|xy| \leq P$ and
$|y| > 0$

- Pump y such that $y^i = y^3 = 000$
- Combining xy^iz
- $S = 000000\,100000$
- S is not a palindrome even though it belongs to $\overline{A}$
- A is not a regular language by proof of contradiction.