MA-102
B. Tech. II Sem (2021-2022)
Tutorial sheet-01

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1. Solve the following systems by Gauss elimination method:

   (a)
$$\begin{aligned} x+y+z&=4 \\ 2x+5y-2z&=3 \\ x+7y-7z&=5 \end{aligned}$$

Solution- The augmented matrix of the given system is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{array}\right]$$

Apply operation $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$ we get,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{array}\right]$$

Again apply operation $R_3 \to R_3 - 2R_2$ we get,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 0 & 0 & 11 \end{array}\right]$$

The above matrix can be written as

$$\begin{aligned} x+y+z&=4 \\ 3y-4z&=-5 \\ 0&=11 \end{aligned}$$

Therefore this system has no solution.

(b)

$$\begin{aligned}
&x - 2y + 3z = 9 \\
&-x + 3y = -4 \\
&2x - 5y + 5z = 17
\end{aligned}$$

Solution- The augmented matrix of the given system is:

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

Apply operation $R_2 \to R_2 + R_1$ and $R_3 \to R_3 - 2R_1$ we get,

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array}\right]$$

Again apply operation $R_3 \to R_3 + R_2$ we get,

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

The above matrix can be written as

$$\begin{aligned}
&x - 2y + 3z = 9 \\
&y + 3z = 5 \\
&2z = 4 \\
&z = 2
\end{aligned}$$

Therefore by backward substitution we get,

$$x = 1, y = -1, z = 2$$

(c)

$$
\begin{aligned}
&2x + 3y + z = 25 \\
&-x - 2y + 4z = -25 \\
&3x - y + 2z = -2
\end{aligned}
$$

Solution- The augmented matrix of the given system is:

$$
\left[\begin{array}{ccc|c}
2 & 3 & 1 & 25 \\
-1 & -2 & 4 & -25 \\
3 & -1 & 2 & -2
\end{array}\right]
$$

Apply operation $R_2 \to R_2 + 1/2R_1$ and $R_3 \to R_3 - 3/2R_1$ we get,

$$
\left[\begin{array}{ccc|c}
2 & 3 & 1 & 25 \\
0 & -1/2 & 9/2 & -25/2 \\
0 & -11/2 & 1/2 & -79/2
\end{array}\right]
$$

Again apply operation $R_3 \to R_3 - 11R_2$ we get,

$$
\left[\begin{array}{ccc|c}
2 & 3 & 1 & 25 \\
0 & -1/2 & 9/2 & -25/2 \\
0 & 0 & -49 & 98
\end{array}\right]
$$

The above matrix can be written as

$$
\begin{aligned}
&2x + 3y + z = 25 \\
&-y/2 + 9/2z = -25/2 \\
&-49z = 98 \\
&z = -2
\end{aligned}
$$

Therefore by backward substitution we get,

$$
x = 3, y = 7, z = -2
$$

(d)

$$
\begin{aligned}
&2x - y + 2z = 5 \\
&x + 3y - z = 2 \\
&4x + 4y + z = -2
\end{aligned}
$$

Solution:

Let us write the given System of equations as,

$$Ax = b \quad \text{where} \quad A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -1 \\ 4 & 4 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 2 \\ -2 \end{bmatrix} \quad .$$

The augmented matrix is,

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & 5 \\ 1 & 3 & -1 & 2 \\ 4 & 4 & 1 & -2 \end{array}\right]$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 2 & -1 & 2 & 5 \\ 4 & 4 & 1 & -2 \end{array}\right]$$

Apply operations $R_2 \to R_2 - 2R_1$and $\mathrm{R}_3 \to R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & -7 & 4 & 1 \\ 0 & -8 & 5 & -10 \end{array}\right]$$

$$R_3 \to R_3 - \frac{8}{7}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & -7 & 4 & 1 \\ 0 & 0 & \frac{3}{7} & \frac{-78}{7} \end{array}\right]$$

$$x + 3y - z = 2$$
$$-7y + 4z = 1$$
$$\frac{3}{7}z = \frac{-78}{7}$$

Therefore by backward substitution we get, $x = 21, y = -15, z = -26.$

(e)

$$x + 4y - z = 4$$
$$x + y - 6z = -4$$
$$3x - y - z = 1$$

Solution- The augmented matrix of the given system is:

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 1 & 1 & -6 & -4 \\ 3 & -1 & -1 & 1 \end{array}\right]$$

Apply operation $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 3R_1$ we get,

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 0 & -3 & -5 & -8 \\ 0 & -13 & 2 & -11 \end{array}\right]$$

Again apply operation $R_3 \to R_3 - \frac{13}{3}R_2$ we get,

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 4 \\ 0 & -3 & -5 & -8 \\ 0 & 0 & \frac{71}{3} & \frac{71}{3} \end{array}\right]$$

The above matrix can be written as

$$\begin{aligned} & x + 4y - z = 4 \\ & -3y - 5z = -8 \\ & \frac{71}{3}z = \frac{71}{3} \\ & z = 1 \end{aligned}$$

Therefore by backward substitution we get,

$$x = 1, y = 1, z = 1$$

2. Use Gauss elimination method to show that following system has no solution:

$$\begin{aligned} & 2\sin x - \cos y + 3\tan z = 3 \\ & 4\sin x + 2\cos y - 2\tan z = 10 \\ & 6\sin x - 3\cos y + \tan z = 9 \end{aligned}$$

Solution: Let us first consider,

$$\begin{aligned} X &= \sin x, \\ Y &= \cos y, \\ Z &= \tan z. \end{aligned}$$

Then the given system of linear equations reduces to the following system of linear equations:

$$\begin{aligned} 2X - Y + 3Z &= 3 \\ 4X + 2Y - 2Z &= 10 \\ 6X - 3Y + Z &= 9 \end{aligned}$$

Now let us first solve this system of linear equations with the help of Gauss elimination method. The augmented matrix of the given system is:

$$\left[\begin{array}{rrr|r} 2 & -1 & 3 & 3 \\ 4 & 2 & -2 & 10 \\ 6 & -3 & 1 & 9 \end{array}\right]$$

By the operations $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{rrr|r} 2 & -1 & 3 & 3 \\ 0 & 4 & -2 & 4 \\ 0 & 0 & -8 & 0 \end{array}\right]$$

So we get,

$$\begin{aligned} 2X - Y + 3Z &= 3 \\ 4Y - 8Z &= 4 \\ -8Z &= 0 \end{aligned}$$

Therefore by backward substitution we get,

$$\begin{aligned} X &= 2, \\ Y &= 1, \\ Z &= 0. \end{aligned}$$

This implies $\sin x = 2$, which is impossible. Hence the given system of equations do not admit any solution.
Hence proved.

3. Show that every elementary matrix is invertible.
Solution: Let E be an elementary matrix corresponding to the elementary row operation e which means $E = e(I)$. If e_1 is the inverse operation of e and $E_1 = e_1(I)$, then

$$EE_1 = e(E_1) = e(e_1(I)) = I$$

and

$$E_1E = e_1(E) = e_1(e(I)) = I$$

Hence, E is invertible and $E_1 = E^{-1}$.

$\Big($ **Note:** Let e be an elementary row operation and let E be the $m \times m$ elementary matrix $E = e(I)$. Then, for every $m \times n$ matrix A, $e(A) = EA\Big)$

4. Find LU or PLU for following matrices and hence find solution for $Ax = b$ for given vector b :

   (a) $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$

   Solution: As we know LU decomposition is possible only when the leading minors are non-zero.
   Here,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

   So leading submatrices are,

$$A_1 = 1,\ A_2 = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}\ and\ A_3 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

   with $|A_1| = 1,\ |A_2| = 2,\ |A_3| = 4$.
   Since all of them are nonzero so matrix A can be decomposed into LU form.
   Now considering $\qquad IA = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

   By using elementary operation $R_2 \longrightarrow R_2 - 2R_1$,we have

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix} = A_1(say)$$

$$or, \quad E_1A = A_1$$
$$or, \quad IE_1A = A_1$$

Now using elementry operation $R_3 \longrightarrow R_3 - 3R_1$, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} E_1 A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 2 \end{bmatrix} = A_2(say)$$

again $\quad IE_2E_1A = A_2$
Applying $R_3 \rightarrow R_3 - 2R_2$ we have,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} E_2 E_1 A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = U$$

$$\Rightarrow \quad E_3E_2E_1A = U \ or \ A = E_1^{-1}E_2^{-1}E_3^{-1}U$$

$$\Rightarrow L = E_1^{-1}E_2^{-1}E_3^{-1}$$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

Now, $Ax = b$

$\Rightarrow LUx = b.$

$$\text{let } Ux = Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\Rightarrow LY = b$$

first we will solve $LY = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

$$y_1 = 1$$

$$2y_1 + y_2 = 4$$

$$3y_1 + 2y_2 + y_3 = 7$$

using forward substitution

$$y_1 = 1, \quad y_2 = 2, \quad y_3 = 0$$

Now, we will solve, $Ux = Y$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$x_1 + x_3 = 1$$

$$2x_2 = 2$$

$$2x_3 = 0$$

So the required solution is

$$x_1 = 1,\ x_2 = 1 \ and \ x_3 = 0.$$

(b) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Solution: First we see principal minors of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

As we can see $A_1 = 0$, so it can not be decomposed in LU form, so we use PLU method.
Now, $\quad IA = A$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_2 \longleftrightarrow R_1$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A_1(say)$$

$$PA = A_1.$$

again $IPA = A_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_3 \longrightarrow R_3 - R_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} PA = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = U$$

$$\Rightarrow \quad E_1 PA = U$$

$$\text{Then} \quad PA = E^{-1}U \implies L = E_1^{-1}$$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Now,

$$Ax = b$$

$$\Rightarrow PAx = Pb$$

$$\Rightarrow LUx = b' \qquad (b' = Pb)$$

Let $Ux = Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \Rightarrow \quad LY = b'$

first we will solve $LY = b'$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y_1 = 0, \quad y_2 = 0, \quad y_1 + y_3 = 1 \Rightarrow y_3 = 1$$

Now, we will solve $Ux = Y$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

$$x_2 + x_3 = 0$$

$$x_3 = 1$$

$$\text{So, } x_2 = -1 \quad \& \quad x_1 = 1$$

Therefore required solution

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

(c) $A = \begin{bmatrix} 1 & 4 & 2 \\ -2 & -8 & 3 \\ 0 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} -2 \\ 32 \\ 1 \end{bmatrix}$

Solution: Leading submatrices are,

$$A_1 = 1,\ A_2 = \begin{bmatrix} 1 & 4 \\ -2 & -8 \end{bmatrix},\ A_3 = \begin{bmatrix} 1 & 4 & 2 \\ -2 & -8 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Here $|A_2| = 0$, so we use PLU method.
As $\quad IA = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 4 & 2 \\ -2 & -8 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Applying elementery row operation $R_2 \longleftrightarrow R_3$, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ -2 & -8 & 3 \end{bmatrix} = B(say)$$

$$PA = B$$
$$or,\ IPA = B$$
applying $R_3 \longrightarrow R_3 + 2R_1$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} PA = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$E_1PA = U \implies L = E_1^{-1}$$

$$\begin{aligned} Ax &= b \\ PAx &= Pb = b' \text{ (say)} \\ \Rightarrow \quad LUx &= b' \end{aligned}$$

let $\quad Ux = Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \Rightarrow LY = b'$

first we will solve $Ly = b'$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 32 \end{bmatrix}$$

$$y_1 = -2, \quad y_2 = 1$$

$$-2y_1 + y_3 = 32 \Rightarrow y_3 = 28$$

Now. we will solve $Ux = Y$

$$\begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 28 \end{bmatrix}$$

$$7x_3 = 28 \Rightarrow x_3 = 4$$

$$x_3 + x_2 = 1 \Rightarrow x_2 = -3$$

$$x_1 + 4x_2 + 2x_3 = -2$$

$$\Rightarrow x_1 = 2$$

So, required solution

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}.$$

Use Gauss-Jordan method to find the solution of following system:

$$\begin{aligned} 2x + y + z &= 1 \\ 4x - 6y &= 1 \\ -2x + 7y + 2z &= 1 \end{aligned}$$

Solution- The augmented matrix of the given system is:

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 4 & -6 & 0 & 1 \\ -2 & 7 & 2 & 1 \end{array}\right]$$

$R_1 \to R_1 \div 2$

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0.5 \\ 4 & -6 & 0 & 1 \\ -2 & 7 & 2 & 1 \end{array}\right]$$

$R_2 \to R_2 - 4 \times R_1$

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0.5 \\ 0 & -8 & -2 & -1 \\ -2 & 7 & 2 & 1 \end{array}\right]$$

$R_3 \to R_3 + 2 \times R_1$

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0.5 \\ 0 & -8 & -2 & -1 \\ 0 & 8 & 3 & 2 \end{array}\right]$$

$R_2 \to R_2 \div (-8)$

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0.5 \\ 0 & 1 & 0.25 & 0.125 \\ 0 & 8 & 3 & 2 \end{array}\right]$$

$R_1 \to R_1 - 0.5 \times R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.375 & 0.4375 \\ 0 & 1 & 0.25 & 0.125 \\ 0 & 8 & 3 & 2 \end{array}\right]$$

$R_3 \to R_3 - 8 \times R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.375 & 0.4375 \\ 0 & 1 & 0.25 & 0.125 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \to R_1 - 0.375 \times R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.0625 \\ 0 & 1 & 0.25 & 0.125 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_2 \to R_2 - 0.25 \times R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.0625 \\ 0 & 1 & 0 & -0.125 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Hence, solution by Gauss Jordan method is $x = 0.0625, y = -0.125$ and $z = 1$.

Find the inverse of the following matrices using Gauss-Jordan method.

1. $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 4 & 7 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 3 & 2 \\ 2 & 4 & 7 \end{bmatrix}$

3. $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 3 & -2 \\ 2 & 4 & 1 \end{bmatrix}$

Solution: **(a)** Augmented matrix is given by:

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{array}\right]$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2 \times R_1$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$$

Applying $R_1 \to R_1 - 2 \times R_2$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 3 & -2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$$

Applying $R_2 \to R_2 + R_3$ and $R_1 \to R_1 - 5 \times R_3$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 13 & -2 & -5 \\ 0 & 1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right] = [I \mid A^{-1}].$$

(b) Similarly, we find A^{-1}.

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{array}\right]$$

Applying, $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 2R_1$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -3 & -4 & -2 & 1 & 0 \\ 0 & -2 & 1 & -2 & 0 & 1 \end{array}\right]$$

Applying, $R_2 \to -\frac{1}{3}R_2$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{4}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -2 & 1 & -2 & 0 & 1 \end{array}\right]$$

Applying, $R_1 \to R_1 - 3R_2$ and $R_3 \to R_3 + 2R_2$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 1 & \frac{4}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{11}{3} & -\frac{2}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

Applying, $R_3 \to \frac{3}{11}R_3$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 1 & \frac{4}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{2}{11} & -\frac{2}{11} & \frac{3}{11} \end{array}\right]$$

$R_2 \to R_2 - \frac{4}{3}R_3$ and $R_1 \to R_1 + R_3$,

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{13}{11} & \frac{9}{11} & \frac{3}{11} \\ 0 & 1 & 0 & \frac{10}{11} & -\frac{1}{11} & -\frac{4}{11} \\ 0 & 0 & 1 & -\frac{2}{11} & -\frac{2}{11} & \frac{3}{11} \end{array}\right] = [I \mid A^{-1}]$$

(c) The augmented matrix is given by:

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ -1 & 3 & -2 & 0 & 1 & 0 \\ 2 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

Applying $R_1 \to R_1 + 2R_2$ and $R_3 \to R_3 + 2R_2$,

$$\to \left[\begin{array}{ccc|ccc} 0 & 5 & -1 & 1 & 2 & 0 \\ -1 & 3 & -2 & 0 & 1 & 0 \\ 0 & 10 & -3 & 0 & 2 & 1 \end{array}\right]$$

Interchanging $(-1)R_1$ with R_2,

$$\to \left[\begin{array}{ccc|ccc} 1 & -3 & 2 & 0 & -1 & 0 \\ 0 & 5 & -1 & 1 & 2 & 0 \\ 0 & 10 & -3 & 0 & 2 & 1 \end{array}\right]$$

Applying $R_2 \to (\frac{1}{5})R_2$

$$\to \left[\begin{array}{ccc|ccc} 1 & -3 & 2 & 0 & -1 & 0 \\ 0 & 1 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 10 & -3 & 0 & 2 & 1 \end{array}\right].$$

Applying $R_3 \to R_3 + (-10)R_2$ and $R_1 \to R_1 + 3R_2$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{7}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & -1 & -2 & -2 & 1 \end{array}\right].$$

Applying $R_3 \to (-1)R_3$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{7}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & 1 & 2 & 2 & -1 \end{array}\right].$$

Applying $R_1 \to R_1 + (-\frac{7}{5})R_3$ and $R_2 \to R_2 + (\frac{1}{5})R_3$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{11}{5} & -\frac{13}{5} & \frac{7}{5} \\ 0 & 1 & 0 & \frac{3}{5} & \frac{4}{5} & -\frac{1}{5} \\ 0 & 0 & 1 & 2 & 2 & -1 \end{array}\right] = [I \mid A^{-1}].$$