Multiplexing and Practice problems and Solutions

Electrical and Electronic Engineering (Ahsanullah University of Science and Technology)

Multiplexing:

Multiplexing is the set of techniques that allows the simultaneous transmission of multiple signals across a single data link. In a multiplexed system, n lines share the bandwidth of one link. Figure 6.1 shows the basic format of a multiplexed system. The lines on the left direct their transmission streams to a multiplexer (MUX), which combines them into a single stream (many to-one). At the receiving end, that stream is fed into a demultiplexer (DEMUX), which separates the stream back into its component transmissions (one-to-many) and directs them to their corresponding lines.

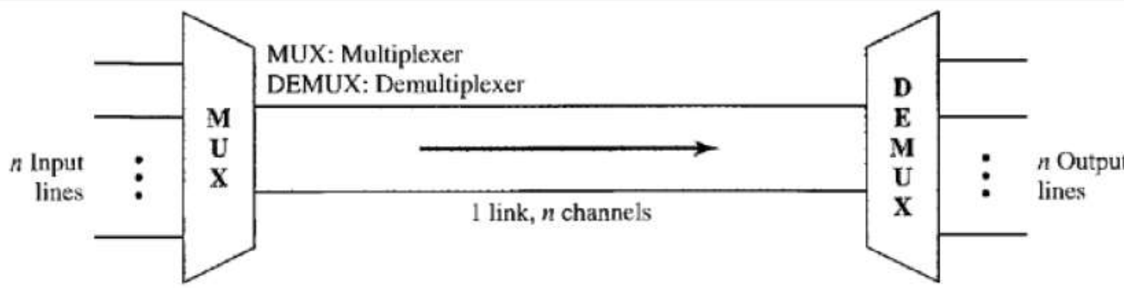
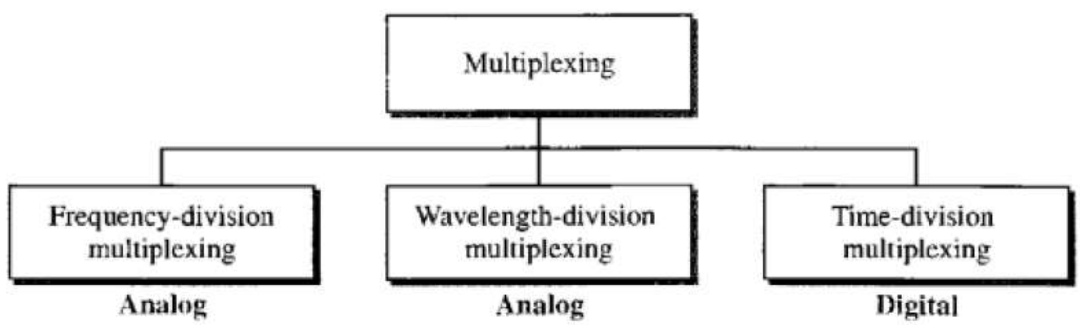


Figure 6.1 Dividing a link into channels

Categories of multiplexing:



Time Division Multiplexing (TDM):

An important feature of the sampling process is a conservation of time. That is, the transmission of the message samples engages the communication channel for only a fraction of the sampling interval on a periodic basis, and in this way some of the time interval between adjacent samples is cleared for use by other independent message sources on a time-shared basis.

TDM enables the joint utilization of a common communication channel by a plurality of independent message sources on a time shared basis without mutual interference.

TDM is a digital multiplexing technique for combining several low-rate digital channels into one high-rate one. In synchronous TDM, the data rate of the link is n times faster, and the unit duration is n times shorter.

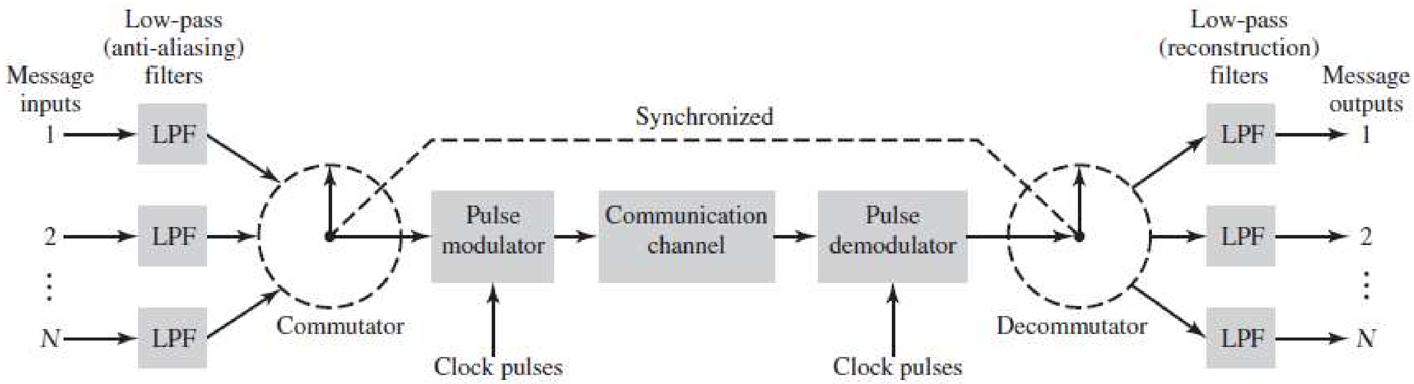
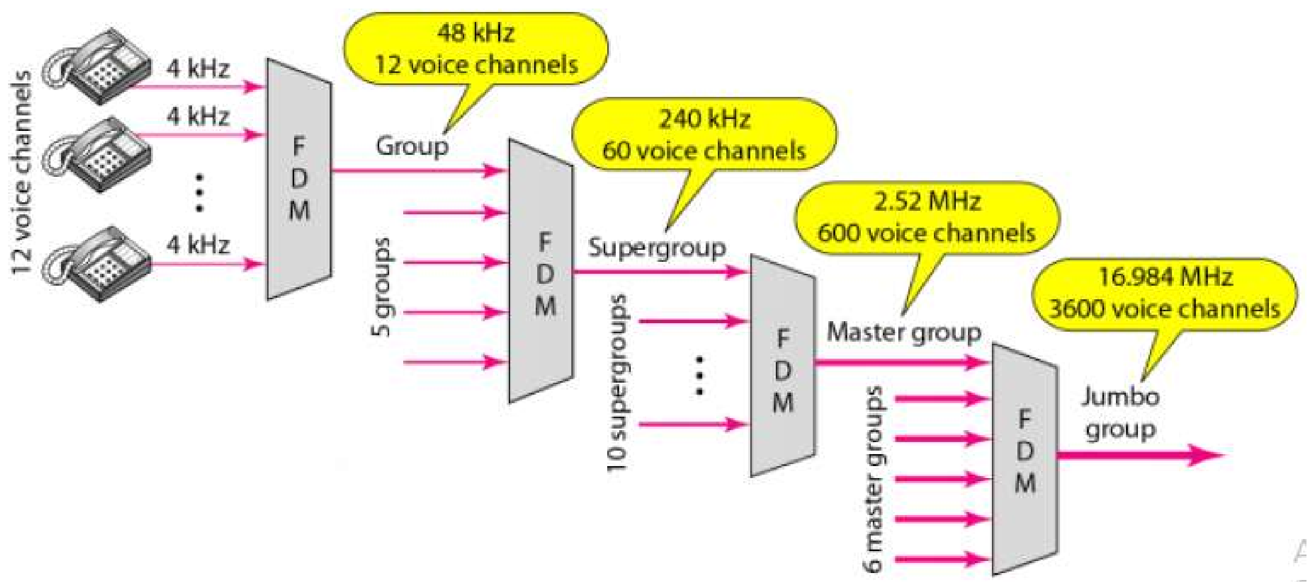


FIGURE 5.21 Block diagram of TDM system.

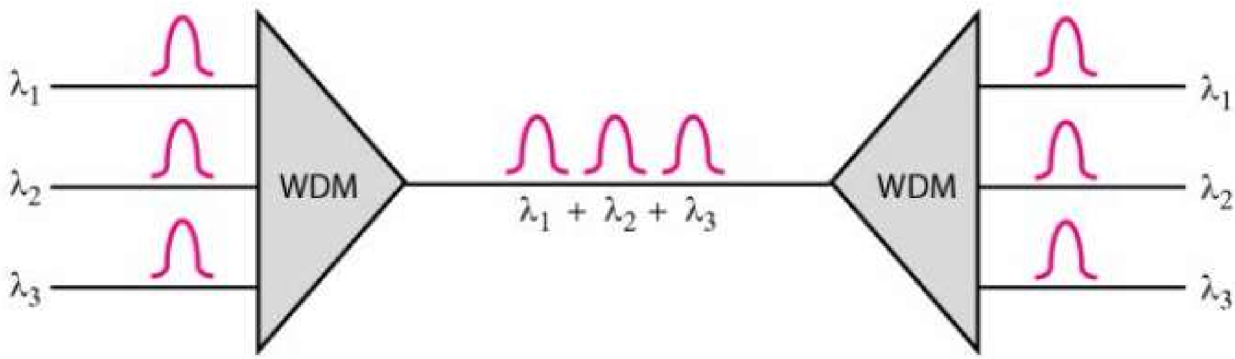
FDM:

FDM enables joint utilization (simultaneous transmission) of a common channel by a number of independent signals (or messages) on a frequency shared basis without mutual interference.



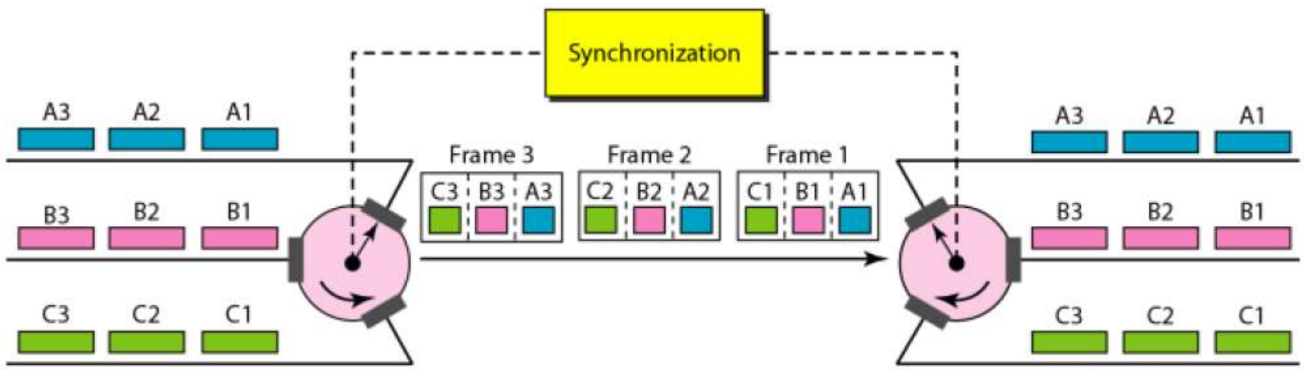
WDM:

WDM enables joint utilization (simultaneous transmission) of a common channel by a number of independent signals (or messages) on a wavelength shared basis without mutual interference.



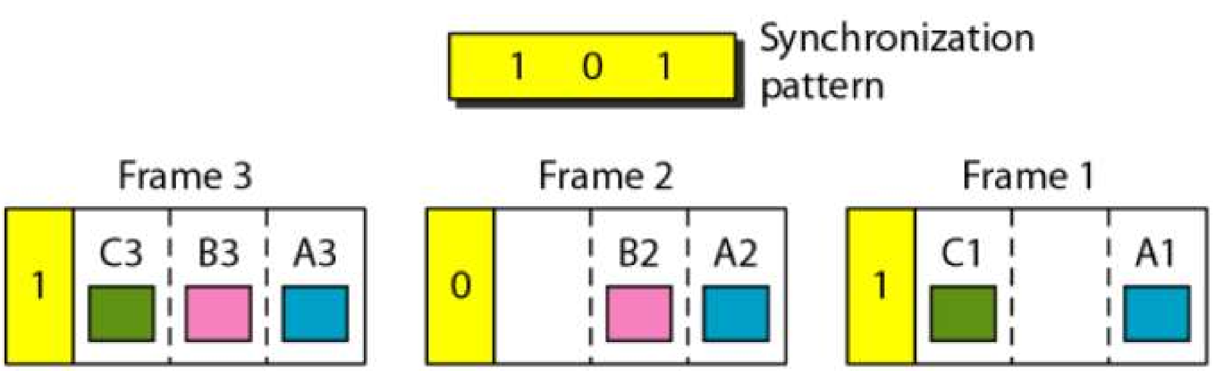
Interleaving:

The process of taking a group of bits from each input line for multiplexing is called interleaving.



Synchronization:

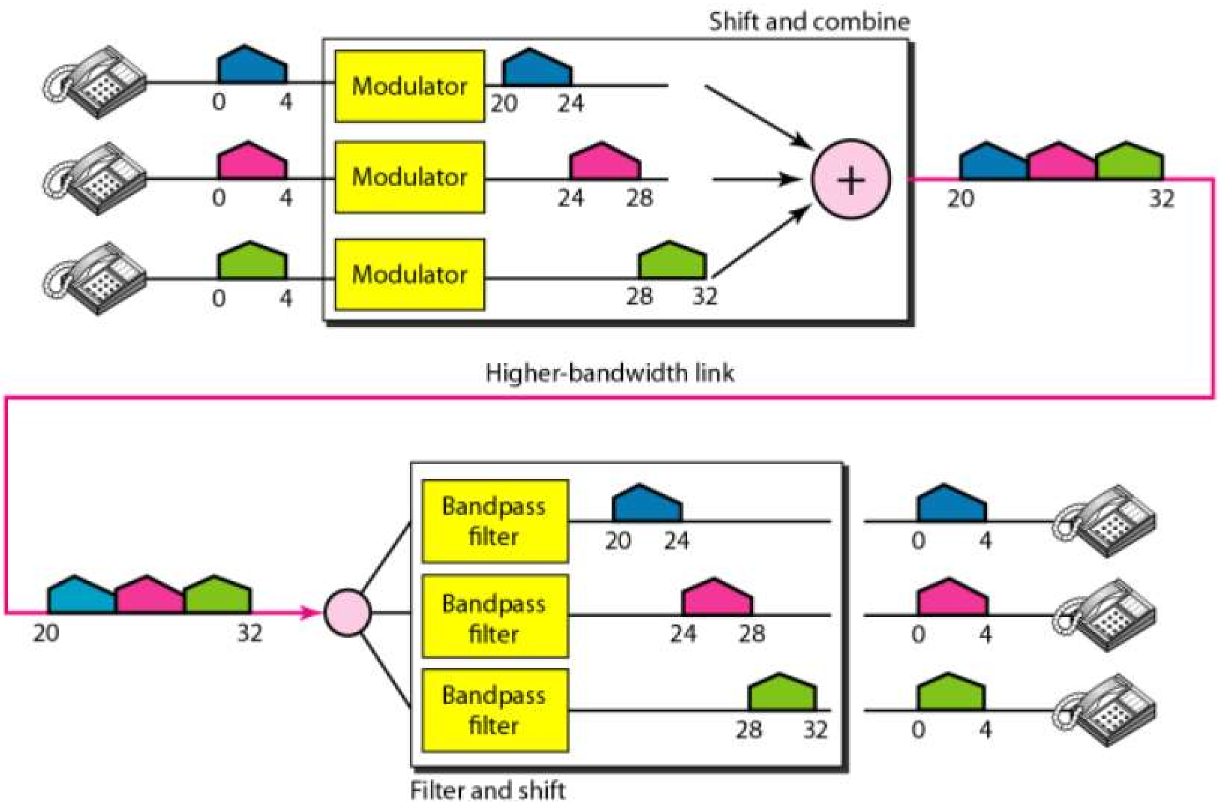
1. To ensure that the receiver correctly reads the incoming bits, i.e., knows the incoming bit boundaries to interpret a "1" and a "0", a known bit pattern is used between the frames.
2. The receiver looks for the anticipated bit and starts counting bits till the end of the frame.
3. Then it starts over again with the reception of another known bit.
4. These bits (or bit patterns) are called synchronization bit(s).
5. They are part of the overhead of transmission.



Practice Problems (FDM):

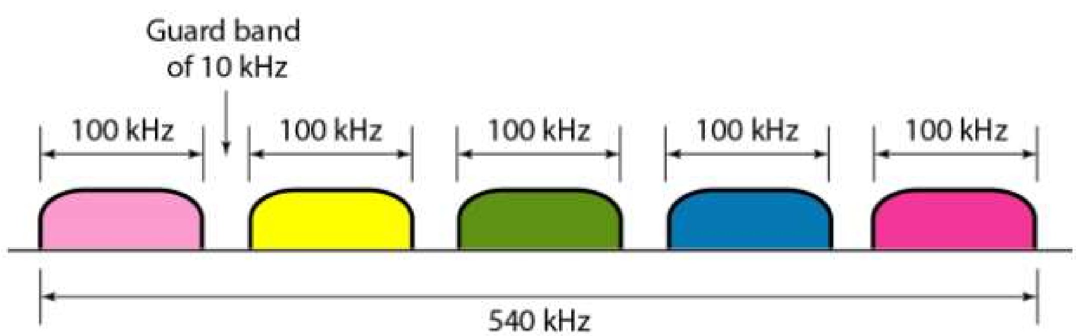
Example 01: Assume that a voice channel occupies a bandwidth of 4 kHz. We need to combine three voice channels into a link with a bandwidth of 12 kHz, from 20 to 32 kHz. Show the configuration, using the frequency domain. Assume there are no guard bands.

Sol: We use the 20- to 24-kHz bandwidth for the first channel, the 24- to 28-kHz bandwidth for the second channel, and the 28- to 32-kHz bandwidth for the third one. Then we combine them as shown in the Figure



Example 02: Five channels, each with a 100-kHz bandwidth, are to be multiplexed together. What is the minimum bandwidth of the link if there is a need for a guard band of 10 kHz between the channels to prevent interference?

Sol:



Practice Problems (TDM):

Example 03: In Figure, the data rate for each one of the 3 input connections is 1 kbps. If 1 bit at a time is multiplexed (a unit is 1 bit), what is the duration of (a) each input slot, (b) each output slot, and (c) each frame?

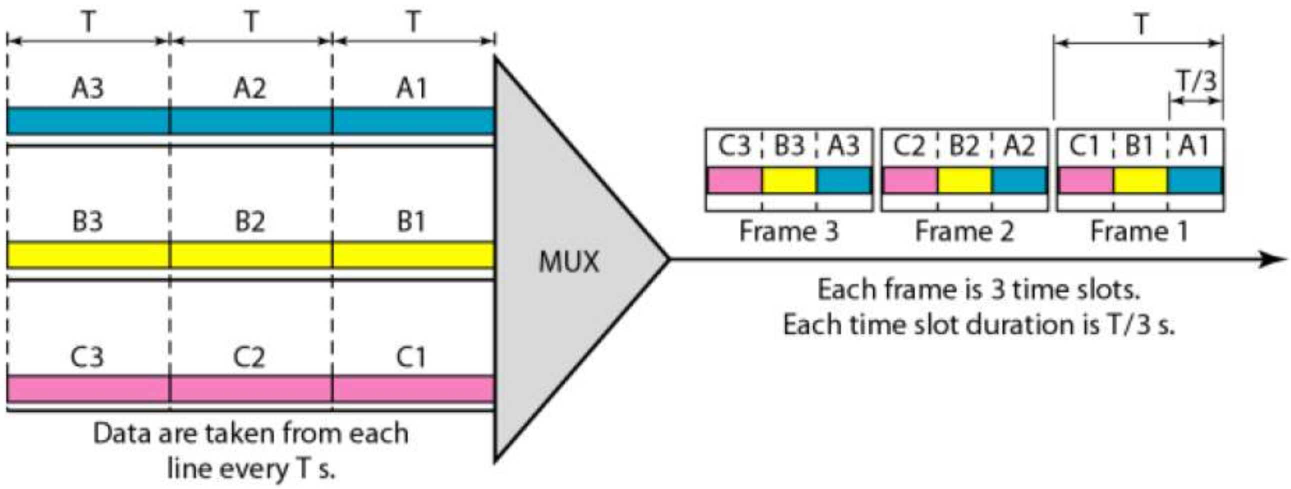
Solution:

a. The data rate of each input connection is 1 kbps. This means that the bit duration is 1/1000 s or 1 ms. The duration of the input time slot is 1 ms (same as bit duration).

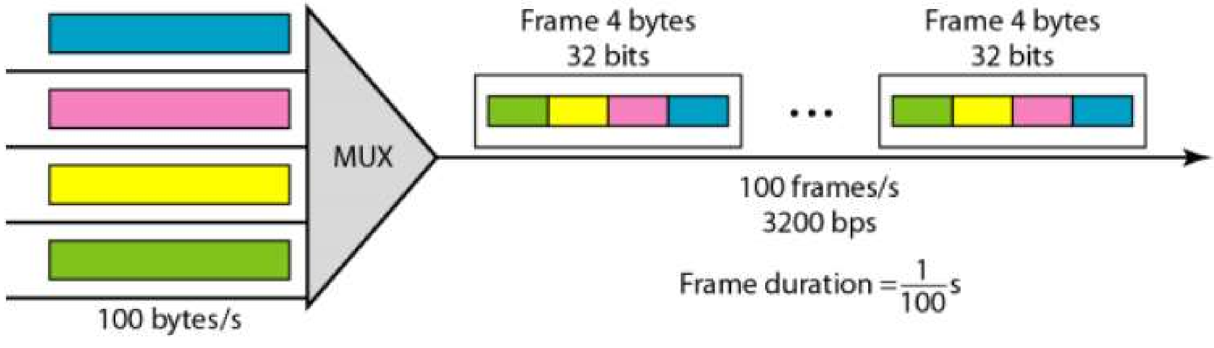
b. The duration of each output time slot is one-third of the input time slot. This means that the duration of the output time slot is 1/3 ms.

c. Each frame carries three output time slots. So the duration of a frame is 3 × 1/3 ms, or 1 ms.

Note: The duration of a frame is the same as the duration of an input unit.



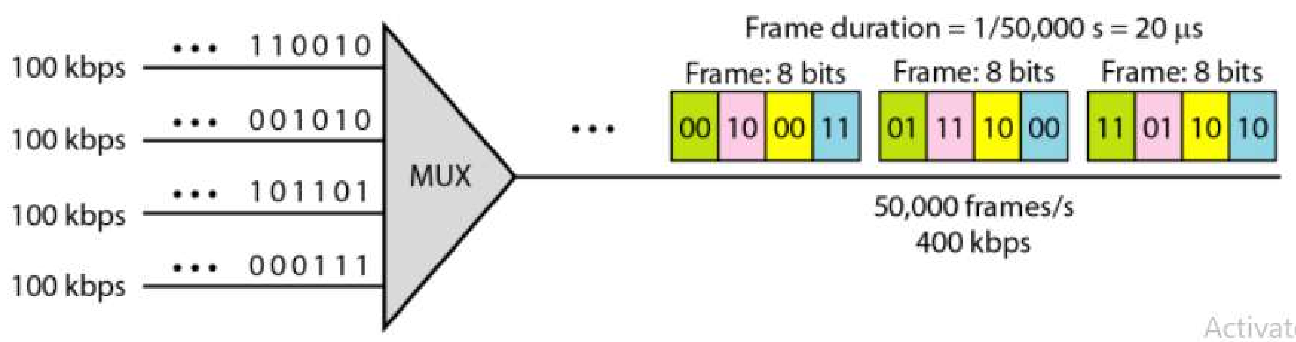
Example 04: Four channels are multiplexed using TDM. If each channel sends 100 bytes /s and we multiplex 1 byte per channel, show the frame traveling on the link, the size of the frame, the duration of a frame, the frame rate, and the bit rate for the link.



Solution:

The multiplexer is shown in Figure 6.16. Each frame carries 1 byte from each channel; the size of each frame, therefore, is 4 bytes, or 32 bits. Because each channel is sending 100 bytes/s and a frame carries 1 byte from each channel, the frame rate must be 100 frames per second. The bit rate is 100 × 32, or 3200 bps.

Example 05: A multiplexer combines four 100-kbps channels using a time slot of 2 bits. Show the output with four arbitrary inputs. What is the frame rate? What is the frame duration? What is the bit rate? What is the bit duration?

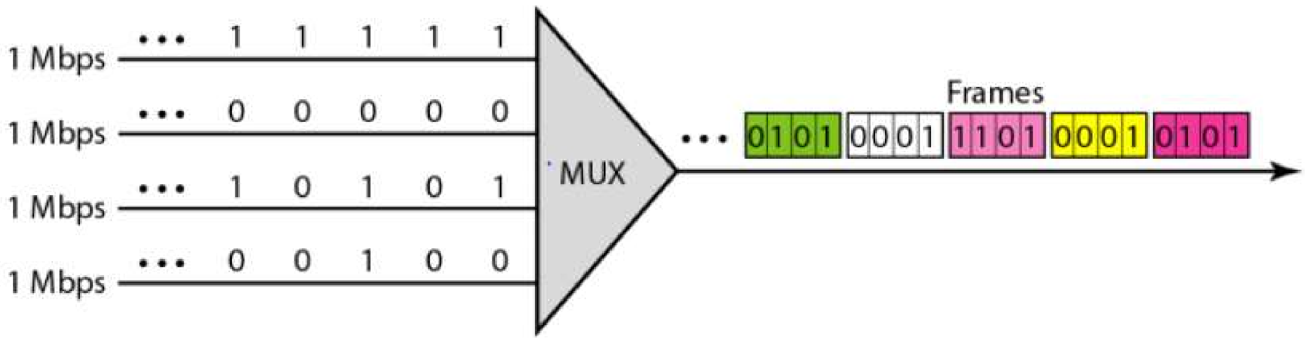


Solution:

Figure 6.17 shows the output (4x100kbps) for four arbitrary inputs. The link carries 400K/(2x4)=50,000 2x4=8bit frames per second. The frame duration is therefore 1/50,000 s or 20µs. The bit duration on the output link is 1/400,000s, or 2.5 µs.

Home work:

Example 06: Figure shows synchronous TDM with 4 1Mbps data stream inputs and one data stream for the output. The unit of data is 1 bit. Find (a) the input bit duration, (b)the output bit duration, (c) the output bit rate, and (d) the output frame rate.



Solution:

a. The input bit duration is the inverse of the bit rate:

1/1 Mbps = 1 µs.

b. The output bit duration is one-fourth of the input bit duration, or ¼ µs.

c. The output bit rate is the inverse of the output bit duration or 1/(4µs) or 4 Mbps. This can also be deduced from the fact that the output rate is 4 times as fast as any input rate; so the output rate = 4 × 1 Mbps = 4 Mbps.

d. The frame rate is always the same as any input rate. So the frame rate is 1,000,000 frames per second. Because we are sending 4 bits in each frame, we can verify the result of the previous question by multiplying the frame rate by the number of bits per frame.

Example 07: Four 1-kbps connections are multiplexed together. A unit is 1 bit. Find (a) the duration of 1 bit before multiplexing, (b) the transmission rate of the link, (c) the duration of a time slot, and (d) the duration of a frame.

Solution:

a. The duration of 1 bit before multiplexing is 1 / 1kbps, or 0.001 s (1 ms).

b. The rate of the link is 4 times the rate of a connection, or 4 kbps.

c. The duration of each time slot is one-fourth of the duration of each bit before multiplexing, or 1/4 ms or 250 µs. Note that we can also calculate this from the data rate of the link, 4kbps. The bit duration is the inverse of the data rate, or 1/4 kbps or 250 µs.

d. The duration of a frame is always the same as the duration of a unit before multiplexing, or 1ms. We can also calculate this in another way. Each frame in this case has four time slots. So the duration of a frame is 4 times 250 µs, or 1

Example 08: We have four sources, each creating 250 8-bit characters per second. If the interleaved unit is a character and 1 synchronizing bit is added to each frame, find (a) the data rate of each source,
(b) the duration of each character in each source, (c) the frame rate, (d) the duration of each frame, (e) the number of bits in each frame, and (f) the data rate of the link.

Solution:

a. The data rate of each source is 250 × 8 = 2000bps = 2 kbps.
b. Each source sends 250 characters per second; therefore, the duration of a character is 1/250 s, or 4ms.
c. Each frame has one character from each source, which means the link needs to send 250 frames per second to keep the transmission rate of each source.
d. The duration of each frame is 1/250 s, or 4 ms. Note that the duration of each frame is the same as the duration of each character coming from each source.
e. Each frame carries 4 characters and 1 extra synchronizing bit. This means that each frame is

   4 × 8 + 1 = 33 bits.

Example 09: Two channels, one with a bit rate of 100 kbps and another with a bit rate of 200 kbps, are to be multiplexed. How this can be achieved? What is the frame rate? What is the frame duration? What is the bit rate of the link?

Solution:
We can allocate one slot to the first channel and two slots to the second channel. Each frame carries 3 bits. The frame rate is 100,000 frames per second because it carries 1 bit from the first channel. The bit rate is 100,000 frames/s × 3 bits per frame, or 300 kbps.

<u>**Practice Problems (PCM and TDM): Next page**</u> (Ref: Lathi)

Practice Problems (PCM and TDM): (Ref: Lathi)

Example 6.2:

A signal $m(t)$ band-limited to 3 kHz is sampled at a rate $33\frac{1}{3}\%$ higher than the Nyquist rate. The maximum acceptable error in the sample amplitude (the maximum quantization error) is 0.5% of the peak amplitude m_p. The quantized samples are binary coded. Find the minimum bandwidth of a channel required to transmit the encoded binary signal. If 24 such signals are time-division-multiplexed, determine the minimum transmission bandwidth required to transmit the multiplexed signal.

Sol:

The Nyquist sampling rate is $R_N = 2 \times 3000 = 6000$ Hz (samples per second). The actual sampling rate is $R_A = 6000 \times (1\frac{1}{3}) = 8000$ Hz.

The quantization step is Δv, and the maximum quantization error is $\pm\Delta v/2$. Therefore, from Eq. (6.14),

$$\frac{\Delta v}{2} = \frac{m_p}{L} = \frac{0.5}{100}m_p \Longrightarrow L = 200$$

For binary coding, L must be a power of 2. Hence, the next higher value of L that is a power of 2 is $L = 256$.

From Eq. (6.19), we need $n = \log_2 256 = 8$ bits per sample. We require to transmit a total of $C = 8 \times 8000 = 64,000$ bit/s. Because we can transmit up to 2 bit/s per hertz of bandwidth, we require a minimum transmission bandwidth $B_T = C/2 = 32$ kHz.

The multiplexed signal has a total of $C_M = 24 \times 64,000 = 1.536$ Mbit/s, which requires a minimum of $1.536/2 = 0.768$ MHz of transmission bandwidth.

Example 6.3:

A signal $m(t)$ of bandwidth $B = 4$ kHz is transmitted using a binary companded PCM with $\mu = 100$. Compare the case of $L = 64$ with the case of $L = 256$ from the point of view of transmission bandwidth and the output SNR.

Sol: For a companded PCM, SNR in dB = $10 \log \mathbf{c} + 10 \log (L^2) = C + 6n$

$$\frac{S_o}{N_o} \simeq \frac{3L^2}{[\ln(1+\mu)]^2}$$

For $L = 64,\ n = 6$, and the transmission bandwidth is $nB = 24$ kHz,

$$\frac{S_o}{N_o} = (\alpha + 36) \text{ dB}$$

$$\alpha = 10 \log \frac{3}{[\ln (101)]^2} = -8.51$$

Hence,

$$\frac{S_o}{N_o} = 27.49 \text{ dB}$$

For $L = 256,\ n = 8$, and the transmission bandwidth is 32 kHz,

$$\frac{S_o}{N_o} = \alpha + 6n = 39.49 \text{ dB}$$

The difference between the two SNRs is 12 dB, which is a ratio of 16. Thus, the SNR for $L = 256$ is 16 times the SNR for $L = 64$. The former requires just about 33% more bandwidth compared to the latter.

Problem 6.2-6:

A message signal $m(t)$ is transmitted by binary PCM without compression. If the SNR (signal-to-quantization-noise ratio) is required to be at least 47 dB, determine the minimum value of L required, assuming that $m(t)$ is sinusoidal. Determine the SNR obtained with this minimum L.

Sol:

For a sinusoid, $\frac{\widetilde{m^2(t)}}{m_p^2} = 0.5$. The SNR = 47 dB =50119. From Eq. (6.16)

$$\frac{S_o}{N_o} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2} = 3L^2(0.5) = 50119 \quad \Longrightarrow L = 182.8$$

Because L is a power of 2, we select $L = 256 = 2^8$. The SNR for this value of L is

$$\frac{S_o}{N_o} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2} = 3(256)^2(0.5) = 98304 = 49.43 \text{ dB}$$