Wednesday, November 24, 2021 1:09 PM

② Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos(\log(1+x))$

Sol

$$\left[(1+x)^2 D^2 + (1+x)D + 1\right] y = 4\cos(\log(1+x))$$

Put $(1+x) = e^z$, $z = \log(1+x)$ | $(1+x)D = D'$, $(1+x)^2 D^2 = D'(D'-1)$

$$\left(D'(D'-1) + D' + 1\right) y = 4\cos z$$

$$\left(D'^2 - D' + D' + 1\right) y = 4\cos z$$

$$\left(D'^2 + 1\right) y = 4\cos z \quad \text{TYPE 2}$$

A.E D' by m

$m^2 + 1 = 0$

$m = \pm i$

$CF = A\cos z + B\sin z$

$$PI = 4 \cdot \frac{1}{D'^2+1}\cos z \qquad D'^2 = -1$$

$$= 4\,\frac{1}{0}$$

$$= 4 \cdot z \cdot \frac{1}{2D'}\cos z$$

$$= 2z\,\frac{1}{D'}\cos z \qquad D' = \frac{d}{dz}$$

$$= 2z \int \cos z\, dz$$

$$= 2z \sin z$$

$$y = A\cos z + B\sin z + 2z\sin z$$

$$y = A\cos(\log(1+x)) + B\sin(\log(1+x)) + 2\log(1+x)\sin(\log(1+x))$$

Homework

① Solve $x^2 \frac{d^2y}{dx^2} - 3y\frac{dy}{dx} = 0$

② Solve $(2x-1)^2 \frac{d^2y}{dx^2} - 4(2x-1)\frac{dy}{dx} + 8y = 8x$

Answer

① $y = A + Bx^4$

② $y = A(2x-1) + B(2x-1)^2 - (2x-1)\log(2x-1) + \frac{1}{2}$