PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Solve (i) $x^2p - y^2q = (x-y)z$ (ii) $p^2+q^2=z$.

(OR)

b. Solve the equation $\left(D^2+4DD'-5D'^2\right)z = xy+\sin(2x+3y)$.

29. a. Find the Fourier series expansion of period 2 for the function $f(x)=\begin{cases}\pi x & in\ 0\le x\le 1\\ \pi(2-x) & in\ 1\le x\le 2\end{cases}$. Deduce the sum $\sum\limits_{n=1,3,\ldots}^{\infty}\frac{1}{n^2}$.

(OR)

b. Find the Fourier series upto the second harmonic from the data.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

30. a. The ends of an uniform string of length $2l$ are fixed. The initial displacement is $y(x,0)=kx(2l-x)$, $0<x<2l$ while the initial velocity is zero. Find the displacement at any distance x from the end $x=0$ at any time t.

(OR)

b. Find the solution of the equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ that satisfies the condition $u(0,t)=0$ and $u(l,t)=0$ for $t\ge 0$ and $u(x,0)=\begin{cases}x, & for\ 0<x<\frac{l}{2}\\ l-x & for\ \frac{l}{2}<x<l\end{cases}$.

31. a. Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1-|x|, & for\ |x|<1\\ 0 & for\ |x|>1\end{cases}$ and hence prove that $\int_0^{\infty}\frac{\sin^4 x}{x^4}dx=\frac{\pi}{3}$.

(OR)

b. Show that $e^{-x^2/2}$ is self-reciprocal under Fourier transform by finding the Fourier transform of $e^{-a^2x^2}$, $a>0$.

32. a. Find (i) $Z\left(2^n\cos\frac{n\pi}{2}\right)$ (ii) $Z^{-1}\left(\frac{z(z+1)}{(z-1)^3}\right)$ using the method of residues.

(OR)

b. Solve using Z-transform $y_{n+2}-3y_{n+1}-10y_n=0$, given that $y_0=y_1=0$.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2019
1st to 7th Semester

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15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The partial differential equation formed by eliminating the arbitrary function 'f' from $z=f\left(x^2-y^2\right)$ is
   (A) $qx+py=0$ (B) $qx=py$
   (C) $qx=p$ (D) $py=q$

2. The complete integral of $z=px+qy+\sqrt{1+p^2+q^2}$ is
   (A) $z=ax-by$ (B) $z=ax+by$
   (C) $z=ax+by+\sqrt{1+a^2+b^2}$ (D) $z=ax+by-\sqrt{1+a^2+b^2}$

3. General solution of $\left(D^2+4DD'-5D'\right)^2 z=0$
   (A) $z=f_1(y-5x)+f_2(y+x)$ (B) $z=f_1(y+5x)+f_2(y+x)$
   (C) $z=f_1(y-5x)+f_2(y-x)$ (D) $z=f_1(y)+f_2(y-x)$

4. The particular integral of $\left(D^2+2DD'+D'^2\right)=e^{x-y}$ is
   (A) e^{x-y} (B) $\frac{x^2}{2}e^{x-y}$
   (C) $\frac{x}{2}e^{x-y}$ (D) $\frac{x^2}{2}e^{x+y}$

5. If $f(x)=|x|$ in $(-\pi,\pi)$ then the constant term a_0 of the Fourier series is
   (A) 2π (B) 0
   (C) $\pi/2$ (D) π

6. If $f(x)=|\sin x|$ then its period is
   (A) π (B) 2π
   (C) 0 (D) $\pi/2$

7. The root mean square value of $f(x)=x\ in\ -1\le x\le 1$ is
   (A) 1 (B) 0
   (C) $\frac{1}{\sqrt{3}}$ (D) -1

8. Half-Range sine series for $f(x)$ in $(0,\pi)$ is

(A) $\sum_{n=1}^{\infty} b_n \sin nx$ (B) $\sum_{n=1}^{\infty} a_n \cos nx$

(C) $\frac{a_0}{2}+\sum_{1}^{\infty} a_n \cos nx$ (D) $\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right)$

9. In wave equation $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) $\frac{m}{T}$ (B) $\frac{T}{m}$

(C) Tm (D) T^m

10. The steady state solution of $u_t=\alpha^2 u_{xx}$ is

(A) $u=c_1x$ (B) $u=c_1+c_2t$

(C) $u=c_1x+c_2$ (D) $u=zero$

11. Classify $u_{xx}+2u_{xy}+u_{yy}=0$

(A) Parabolic (B) Elliptic

(C) Hyperbolic (D) Geodesic

12. The number of initial and boundary conditions to solve $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ are

(A) Three (B) Two

(C) Four (D) One

13. If $F\{f(x)\}=F(s)$, then $F\{f(ax)\}=$

(A) $\frac{1}{|a|}F\left(\frac{s}{a}\right)$ (B) $aF(s)$

(C) $aF\left(\frac{s}{a}\right)$ (D) $F(as)$

14. The Fourier sine transform of $e^{-ax}\,(a>0)$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+a^2}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2+a^2}\right)$

(C) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2-a^2}\right)$ (D) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2-a^2}\right)$

15. $F^{-1}[F(s)G(s)]=$

(A) $f(x)g(x)$ (B) $f(x)*g(x)$

(C) $f(x)+g(x)$ (D) $f(x)-g(x)$

16. If $F\{f(x)\}=F(s)$ then $\int_{-\infty}^{\infty}|f(x)|^2\,dx=$

(A) $\int_{0}^{\infty}|F(s)|^2\,ds$ (B) $\int_{-\infty}^{\infty}|F(x)|^2\,dx$

(C) $\int_{-\infty}^{\infty}|F(s)|^2\,ds$ (D) $\int_{0}^{\infty}|F(x)|^2\,dx$

17. $Z\{(-1)^n\}=$

(A) $\frac{z}{z+1}$ (B) $\frac{z}{z-1}$

(C) $z(z+1)$ (D) $\frac{1}{z+1}$

18. $Z\left[na^n\right]=$

(A) $\frac{z}{(z-a)^3}$ (B) $\frac{a}{(z-a)^2}$

(C) $\frac{z}{(z-a)^2}$ (D) $\frac{az}{(z-a)^2}$

19. $z\left(\sin\frac{n\pi}{2}\right)=$

(A) $\frac{z}{z^2+1}$ (B) $\frac{z}{z^2-1}$

(C) $\frac{z^2}{z^2-4}$ (D) $\frac{z^2}{z^2+1}$

20. Poles of $f(z)=\frac{z^n}{(z+1)(z+2)}$ are

(A) $z=1,2$ (B) $z=-1,-2$

(C) $z=1,-2$ (D) $z=-1,2$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Form a partial differential equation by eliminating arbitrary constants a, b, from $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$.

22. Find half-range sine series of $f(x)=a$ in $(0,l)$.

23. Write down the three mathematically possible solutions of one dimensional heat flow equation.

24. Find the Fourier transform of $f(x)$ defined as $f(x)=\begin{cases}1 & for\ |x|<a\\ 0 & for\ |x|>a\end{cases}$.

25. Find the Z-transform of $\frac{1}{n(n-1)}$.

26. Solve the equation $pq+p+q=0$.

27. Find the Fourier sine transform of $f(x)$ defined as $f(x)=\begin{cases}\sin x & when\ 0<x<a\\ 0 & when\ x>a\end{cases}$.