SRM INSTITUTE OF SCIENCE AND TECHNOLOGY DEPARTMENT OF MATHEMATICS 18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS (2021 -2022 EVEN)

MODULE - 2 : VECTOR CALCULUS

TUTORIAL SHEET – I

S.No.	Questions	Answers
Part - A		
1	Find the gradient of ϕ , where ϕ is $(x^2 + y^2 + z^2)^{\frac{3}{2}}$	$3\vec{r}$
2	Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, 2, 3)$ in the direction of $2\vec{\imath} + \vec{\jmath} - \vec{k}$	$\frac{86}{\sqrt{6}}$
3	If \vec{r} is the position vector of the point (x, y, z) , find $\nabla \left(\frac{1}{r}\right)$	$-\frac{\vec{r}}{r^3}$
4	Find a unit vector normal to the surface $x^2 + y^2 - z = 10$ at $(1, 1, 1)$	$\hat{n} = \frac{2\vec{i} + 2\vec{j} - \vec{k}}{3}$
5	Find 'a' such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.	a = -5
Part - B		
6	Prove that $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential.	$\phi = y^2 \sin x + xz^3 - 4y + c$
7	Find the angle between the surfaces $x \log z = y^2 - 1$ and $x^2y = 2 - z$ at the point $(1, 1, 1)$	$\theta = \cos^{-1}\left(\frac{1}{\sqrt{30}}\right)$
8	Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = \vec{r} $ and hence deduce $\nabla^2\left(\frac{1}{r}\right)$	
9	Prove that $\vec{F} = (2x + yz)\vec{i} + (4y + zx)\vec{j} - (6z - xy)\vec{k}$ is solenoidal and find its scalar potential.	$\phi = x^2 + 2y^2 - 3z^2 + xyz + c$
10	Find the constants a, b, c so that $\vec{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.	a = 4, b = 2, c = -1