negligible resistance. No current will flow through $R_L$. So the circuit present after the short circuit will be omitted for analysis.

## 3.13 MESH AND NODAL ANALYSIS

### 3.13.1 Mesh Analysis

Mesh analysis is a useful technique to find the current in each branch and loop of the electric circuit. When the network has large number of voltage sources, mesh analysis is applied.

Consider the circuit given in fig 3.16 (a). Let us find the current in each loop using mesh method.

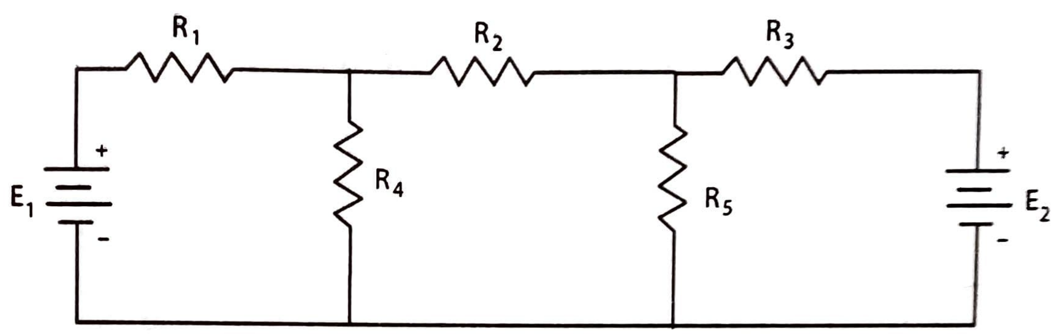



*Fig. 3.16 (a)*

**Steps:**

(i) The given circuit consist of three individual loops, since it does not have any crossovers

(ii) Current direction is marked in each loop.

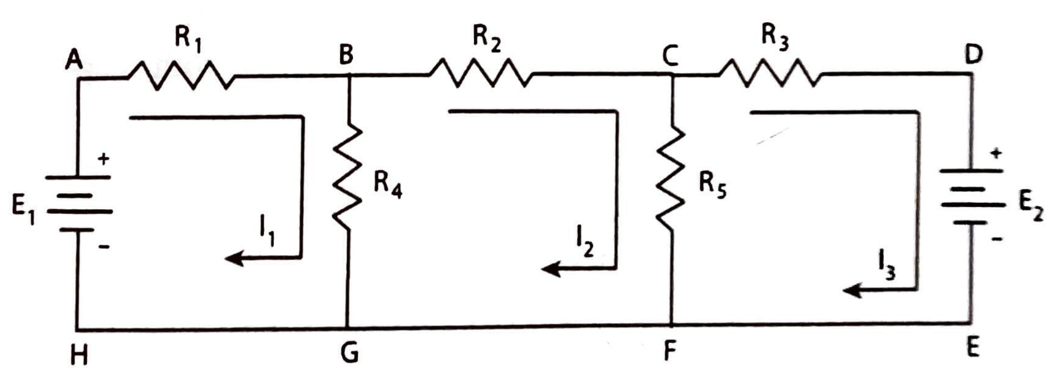



*Fig. 3.16 (b)*

Let us assume that current flow in each loop is one direction. Generally current direction is assumed from positive terminal of the battery. In the middle loop, we can assume any direction for current. Regarding polarity, the current entering point in the resistor is marked as +ve and current leaving point is marked as -ve.

(iii) Forming loop equations

By Kirchhoff's voltage law, in any loop

Sum of the voltage rise = sum of the voltage drops.

**First loop:** [A B G H]

$$E_1 - I_1R_1 + R_4(I_2 - I_1) = 0$$

$$E_1 = I_1R_1 + R_4(I_1 - I_2)$$

$$E_1 = (R_1 + R_4)I_1 - R_4I_2 \quad (1)$$

Note: While writing loop equation for the first loop, go along the direction of $I_1$. If any current comes opposite to $I_1$, take it as negative. Hence the current in $R_4$ is $(I_1 - I_2)$ with respect to first loop.

**Second loop:** [B C F G]

There is no voltage source in the second loop. Therefore the potential rise in the second loop is zero.

$$0 = I_2R_2 + R_5(I_2 - I_3) + R_4(I_2 - I_1)$$

$$0 = -I_1R_4 + (R_2 + R_4 + R_5)I_2 - R_5I_3 \quad (2)$$

Note: While writing loop equation for second loop, go along the direction of $I_2$. If any current comes opposite to $I_2$ take it as negative. If any current flows in the same direction of $I_2$ then takes it as positive. Hence current through $R_5$ is $(I_2 - I_3)$ and current through $R_4$ is $(I_2 - I_1)$

**Third loop:** [C D E F]

$$-E_2 = I_3R_3 + R_5(I_3 - I_2)$$

$$-E_2 = -R_5I_2 + (R_3 + R_5)I_3 \quad (3)$$

Note: While writing loop equation for third loop, go along the direction of $I_3$. If any current comes opposite to $I_3$, take it as negative. If current flows in the same direction $I_3$, take it as positive. The current through $R_5$ is $(I_2 + I_3)$ with respect to third loop.

(iv) Matrix method for solving the loop equations

Three loop equations (1), (2) and (3) can be written in matrix from as,

$$\begin{bmatrix} E_1 \\ 0 \\ -E_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_4 & -R_4 & 0 \\ -R_4 & R_2 + R_4 + R_5 & -R_5 \\ 0 & -R_5 & R_3 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} R_1 + R_4 & -R_4 & 0 \\ -R_4 & R_2 + R_4 + R_5 & -R_5 \\ 0 & -R_5 & R_3 + R_5 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} E_1 & -R_4 & 0 \\ 0 & R_2 + R_4 + R_5 & -R_5 \\ -E_2 & -R_5 & R_3 + R_5 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} R_1 + R_4 & E_1 & 0 \\ -R_4 & 0 & -R_5 \\ 0 & -E_2 & R_3 + R_5 \end{bmatrix}$$

$$\Delta_3 = \begin{bmatrix} R_1 + R_4 & -R_4 & E_1 \\ -R_4 & R_2 + R_4 + R_5 & 0 \\ 0 & -R_5 & -E_2 \end{bmatrix}$$

The loop current are given by

$$I_1 = \frac{\Delta_1}{\Delta}, I_2 = \frac{\Delta_2}{\Delta} \quad \text{and} \quad I_3 = \frac{\Delta_3}{\Delta}$$

### 3.13.2 Nodal Analysis

Nodal analysis is a useful technique to find the voltages at each node of the electric circuit. A node is the meeting point of two or more branches. When the network has large number of current sources, nodal analysis is applied.

(a) Consider the circuit given in fig 3.17 (a). Let us find the node voltages using nodal analysis

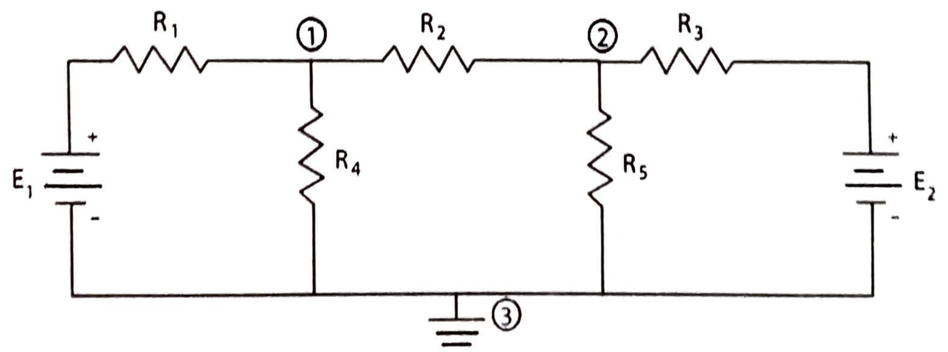



*Fig. 3.17 (a)*

**Steps**

(i) Mark the nodes. Take one of the nodes as reference node. Assume the voltages of other nodes as $V_1$, $V_2$, $V_3$, ... etc. Also mark the current directions in all the branches and name the currents as $I_1$, $I_2$ ... etc.

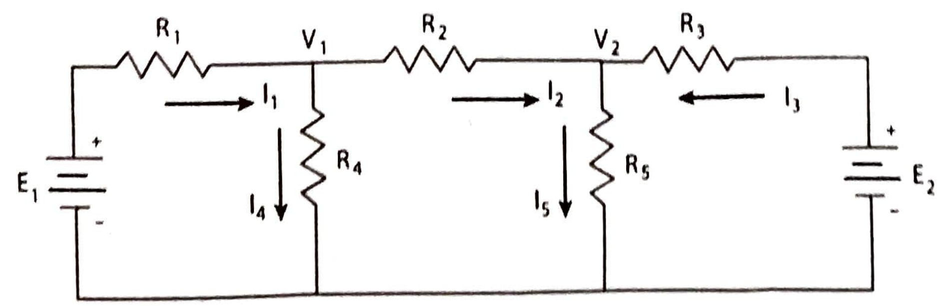



*Fig. 3.17 (b)*

(ii) At each node, write Kirchoff's current law equations. The circuit contains three nodes including the reference node (i.e. N = 3)

N - 1 = 3 - 1 = 2 equations have to be written

At node 1, by Kirchoff's current law

$$I_1 = I_2 + I_4$$

$$\frac{E_1 - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + \frac{V_1 - 0}{R_4}$$

$$\frac{E_1}{R_1} - \frac{V_1}{R_1} = \frac{V_1}{R_2} - \frac{V_2}{R_2} + \frac{V_1}{R_4}$$

$$\frac{E_1}{R_1} - \frac{V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1}{R_4} - \frac{V_2}{R_2}$$

$$\frac{E_1}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) V_1 - \frac{V_2}{R_2} \qquad (1$$

At node 2, by Kirchoff's current law

$$I_2 + I_3 = I_5$$

$$\frac{V_1 - V_2}{R_2} + \frac{E_2 - V_2}{R_3} = \frac{V_2 - 0}{R_5}$$

$$\frac{V_1}{R_2} - \frac{V_2}{R_2} + \frac{E_2}{R_3} - \frac{V_2}{R_3} = \frac{V_2}{R_5}$$

$$\frac{E_2}{R_3} = \frac{-V_1}{R_2} + \frac{V_2}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_5}$$

$$\frac{E_2}{R_3} = \frac{-V_1}{R_2} + V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) \qquad (2$$

(iii) Matrix method for solving node equations.

Equations (1) and (2) can be written in matrix form as

$$\begin{bmatrix} \frac{E_1}{R_1} \\ \frac{E_2}{R_3} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} & \frac{-1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} & \frac{-1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \dfrac{E_1}{R_1} & \dfrac{-1}{R_2} \\ \dfrac{E_2}{R_3} & \left(\dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_5}\right) \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} \left(\dfrac{1}{R_2} + \dfrac{1}{R_2} + \dfrac{1}{R_4}\right) & \dfrac{E_1}{R_1} \\ -\dfrac{1}{R_2} & \dfrac{E_2}{R_3} \end{bmatrix}$$

The node voltages are given by,

$$V_1 = \frac{\Delta_1}{\Delta}, V_2 = \frac{\Delta_2}{\Delta}$$

(b) Consider the circuit given in fig. 3.18

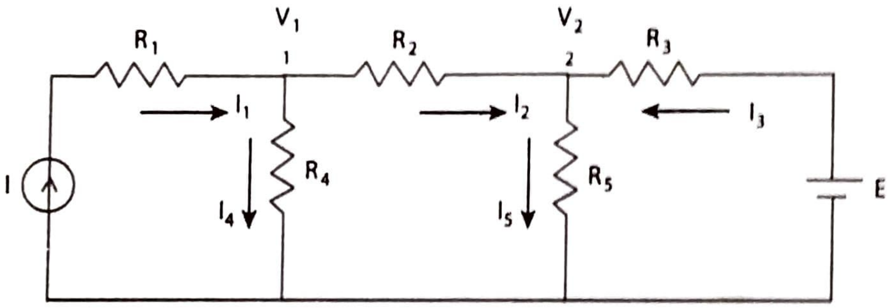



*Fig. 3.18*

Let $V_1$ and $V_2$ be the voltage at node 1 and 2,

At node 1, by Kirchhoff's current law

$$I = \frac{V_1}{R_4} + \frac{V_1 - V_2}{R_2} = \frac{V_1}{R_4} + \frac{V_1}{R_2} - \frac{V_2}{R_2}$$

$$I = \left(\frac{1}{R_2} + \frac{1}{R_4}\right) V_1 - \frac{V_2}{R_2} \qquad (1)$$

At node 2, by Kirchhoff's current law,

$$\frac{V_1 - V_2}{R_2} + \frac{E - V_2}{R_3} = \frac{V_2}{R_5}$$

$$\frac{E}{R_3} = \frac{V_2}{R_5} - \frac{V_1}{R_2} + \frac{V_2}{R_2} + \frac{V_2}{R_3}$$

$$\frac{E}{R_3} = \frac{-V_1}{R_2} + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) V_2 \qquad (2)$$

Equations (1) and (2) in matrix form,

$$\begin{bmatrix} I \\ \dfrac{E}{R_3} \end{bmatrix} \begin{bmatrix} \dfrac{1}{R_2}+\dfrac{1}{R_4} & \dfrac{-1}{R_2} \\ -\dfrac{1}{R_2} & \dfrac{1}{R_2}+\dfrac{1}{R_3}+\dfrac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \dfrac{1}{R_2}+\dfrac{1}{R_4} & \dfrac{-1}{R_2} \\ -\dfrac{1}{R_2} & \dfrac{1}{R_2}+\dfrac{1}{R_3}+\dfrac{1}{R_5} \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} I & \dfrac{-1}{R_2} \\ \dfrac{E}{R_3} & \dfrac{1}{R_2}+\dfrac{1}{R_3}+\dfrac{1}{R_5} \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} \dfrac{1}{R_2}+\dfrac{1}{R_4} & I \\ \dfrac{-1}{R_2} & \dfrac{E}{R_3} \end{bmatrix}$$

Now, $V_1 = \dfrac{\Delta_1}{\Delta}, V_2 = \dfrac{\Delta_2}{\Delta}$

## Solved Problems

### Problem 1

For the circuit in the given figure 3.19, calculate the load current $I_L$ across $R_L$, load voltage $V_L$ across $R_L$, and power consumed by the load ($R_L$).

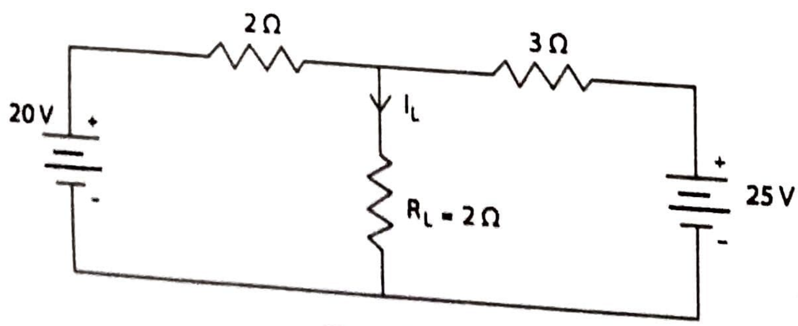



**Fig. 3.19**

***Solution:***

**Steps:**

(a) Assume current direction in each loop. Usually the current direction is taken from positive terminal of the battery.

**Problem 18**

Using mesh analysis, find mesh current in each loop given in fig 3.35.

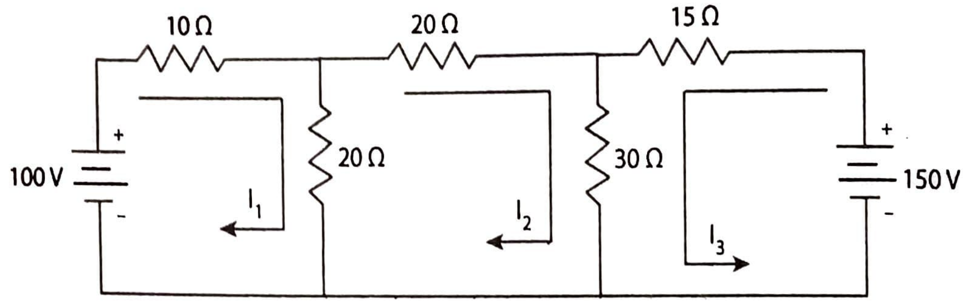



*Fig. 3.35*

***Solution:***

(i) Write loop equations.

For loop (1),

$$100 = 10\,I_1 + 20\,(I_1 - I_2)$$

$$100 = 10\,I_1 + 20\,I_1 - 20I_2$$

$$100 = 30\,I_1 - 20\,I_2 \qquad (1)$$

For loop (2),

$$0 = 20\,I_2 + 30\,(I_2 + I_3) + 20\,(I_2 - I_1)$$

$$0 = 20I_2 + 30I_2 + 30I_3 + 20I_2 - 20I_1$$

$$0 = -\,20I_1 + 70I_2 + 30I_3 \qquad (2)$$

For loop (3)

$$150 = 15\,I_3 + 30\,(I_2 + I_3)$$

$$150 = 30I_2 + 45\,I_3 \qquad (3)$$

(ii) Arranging equations in matrix form,

$$\begin{bmatrix} 100 \\ 0 \\ 150 \end{bmatrix} = \begin{bmatrix} 30 & -20 & 0 \\ -20 & 70 & 30 \\ 0 & 30 & 45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

(iii)

$$\Delta = \begin{bmatrix} 30 & -20 & 0 \\ -20 & 70 & 30 \\ 0 & 30 & 45 \end{bmatrix}$$

$$= 30\,(70 \times 45 - 30 \times 30) + 20(-20 \times 45)$$

$$= 30\,(2250) + 20\,(-900) = 49500$$

$$\Delta_1 = \begin{bmatrix} + & - & + \\ 100 & -20 & 0 \\ 0 & 70 & 30 \\ 150 & 30 & 45 \end{bmatrix}$$

$$= 100\ (70 \times 45 - 30 \times 30) + 20\ (-150 \times 30)$$

$$= 100\ (2250) + 20\ (-\ 4500) = 13500$$

$$\Delta_2 = \begin{bmatrix} + & - & + \\ 30 & 100 & 0 \\ -20 & 0 & 30 \\ 0 & 150 & 45 \end{bmatrix}$$

$$= 30\ (0 \times 45 - 150 \times 30) - 100\ (-20 \times 45)$$

$$= 30\ (-4500) - 100(-900) = -45000$$

$$\Delta_3 = \begin{bmatrix} + & - & + \\ 30 & -20 & 100 \\ -20 & 70 & 0 \\ 0 & 30 & 150 \end{bmatrix}$$

$$= 30\ (70 \times 150 - 30 \times 0) + 20\ (-20 \times 150) + 100\ (-20 \times 30)$$

$$= 30\ (10500) + 20\ (-3000) + 100\ (-600) = 195000$$

(iv) The current values are

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{135000}{49500} = 2.73\ \text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-45000}{49500} = -0.909\ \text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{195000}{49500} = 3.94\ \text{A}$$

(v) Verification of Answers:

Substitute the values of $I_1$, $I_2$ and $I_3$ in equation (2)

$$0 = -20\ I_1 + 70\ I_2 + 30\ I_3$$

$$= -20 \times 2.73 + 70 \times (0.909) + 30 \times 3.94 = 0$$

**Problem 19**

In the circuit given in fig 3.36, obtain the load current $I_l$, and power consumed by $R_L$.

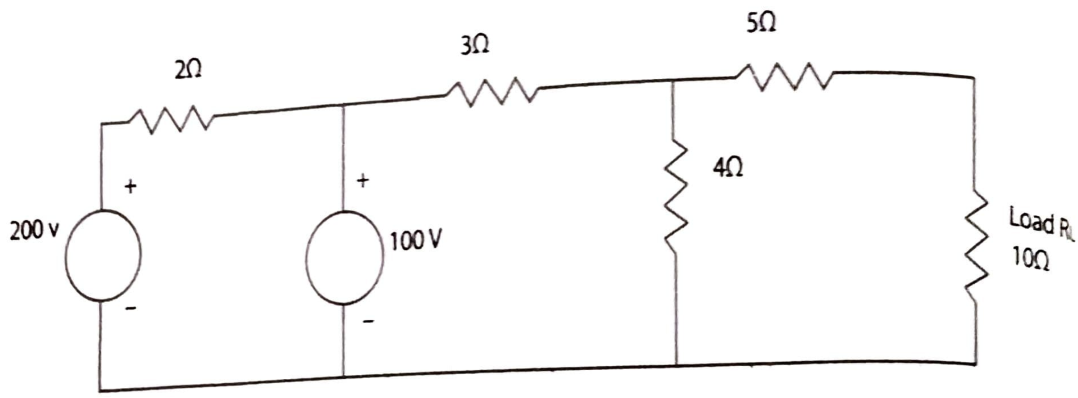



*Fig. 3.36 (a)*

***Solution:***

Load current $I_L$ can be found by using mesh method.

(i) Assume current direction

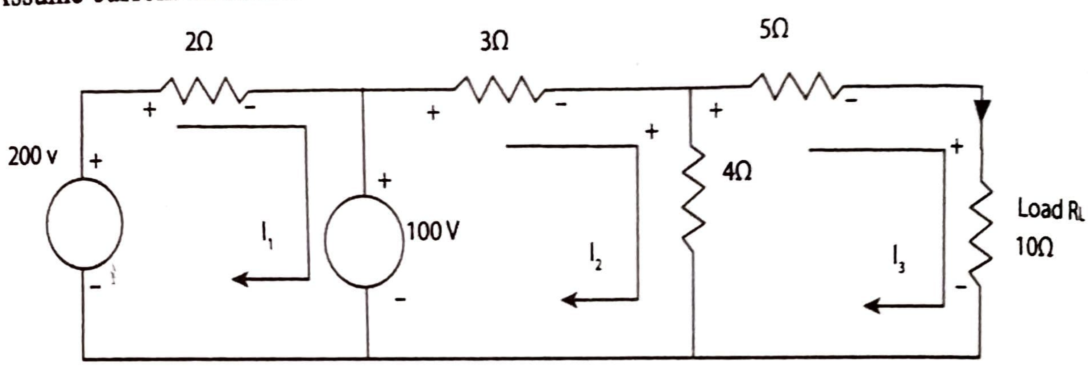



*Fig. 3.36 (b)*

(ii) Current $\quad I_L = I_3 = \dfrac{\Delta_3}{\Delta}$

(iii) Loop equations in matrix form

$$\begin{bmatrix} 200-100 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3+4 & -4 \\ 0 & -4 & 5+10+4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} 100 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & -4 \\ 0 & -4 & 19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

(iv) $$\Delta = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & -4 \\ 0 & -4 & 19 \end{bmatrix}$$

$$= 2\,(7 \times 19 - (-4 \times -4)) = 234$$

$$\Delta_3 = \begin{bmatrix} + & - & + \\ 2 & 0 & 100 \\ 0 & 7 & 100 \\ 0 & -4 & 0 \end{bmatrix}$$

$$= 2\,(7 \times 0 + 4 \times 100) + 100\,(0) = 800$$

(v) Load current $I_L = I_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{800}{234} = 3.418\ \text{A}$

Power consumed by $R_L = I_L^2 \times R_L$

$= 3.418^2 \times 10$

$= 116.9$ watts

**Problem 20**

In the following circuit, find the values of $E$ and $I$ for the circuit shown in fig 3.37(a).

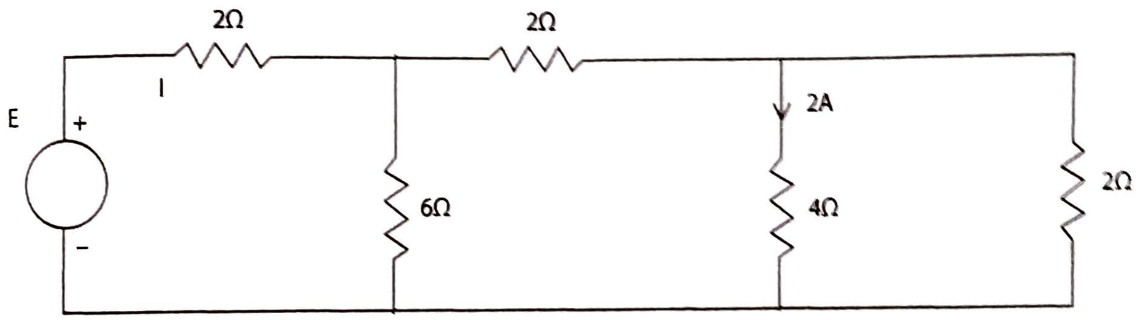



*Fig. 3.37(a)*

***Solution:***

This problem is solved by applying nodal analysis. The circuit consists of two nodes and one reference node. Assume node voltages as $V_1$ and $V_2$ and also assume the currents direction.

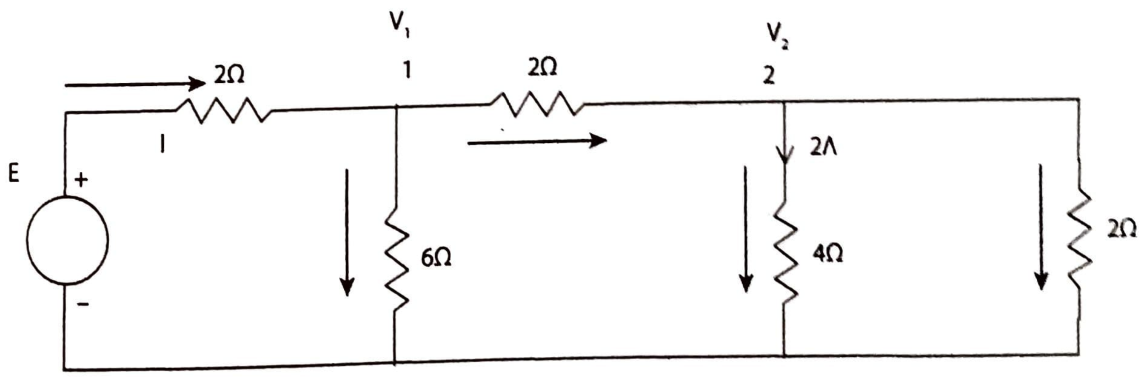



*Fig. 3.37 (b)*

From circuit, $V_2 = 4 \times 2 = 8$ V

By KCL,

At Node 1,

$$\frac{E-V_1}{2} = \frac{V_1}{6} + \frac{V_1-V_2}{2}$$

$$= \frac{V_1}{2} + \frac{V_1}{6} + \frac{V_1}{2} - \frac{V_2}{2}$$

$$\frac{E}{2} = V_1\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{2}\right] - \frac{V_2}{2}$$

$$0.5E = V_1\,(1.167) - 0.5\,V_2$$

At Node 2,

$$\frac{V_1-V_2}{2} = \frac{V_2}{4} + \frac{V_2}{2}$$

$$0 = \frac{-V_1}{2} + \frac{V_2}{2} + \frac{V_2}{4} + \frac{V_2}{2} = -\frac{V_1}{2} + V_2\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{2}\right]$$

$$0 = -0.5V_1 + 1.25\,V_2$$

Substituting $V_2 = 8V$ in equation (2),

$$0 = -0.5\,V_1 + 1.25 \times 8$$

$$0.5\,V_1 = 10$$

$$V_1 = 20V$$

From equation (1)

$$0.5E = 20 \times 1.167 - 0.5 \times 8$$

$$E = 38.68\text{ V}$$

$$\text{Current } I = \frac{E-V_1}{2} = \frac{38.68-20}{2} = 9.34\text{ A}$$

**Problem 21**

Using nodal analysis, find all node voltages in the given fig 3.38.

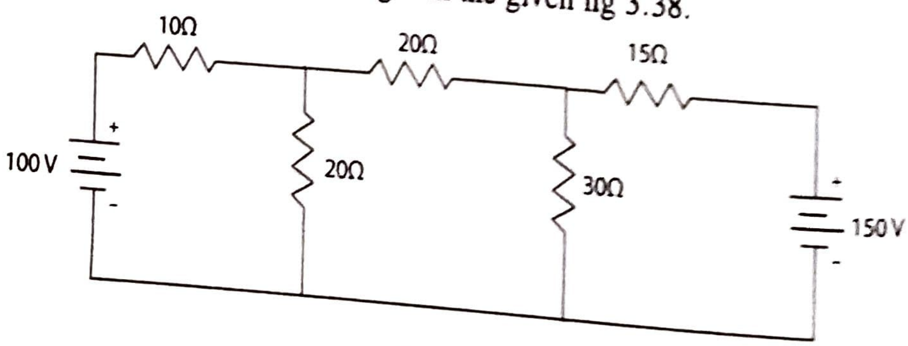
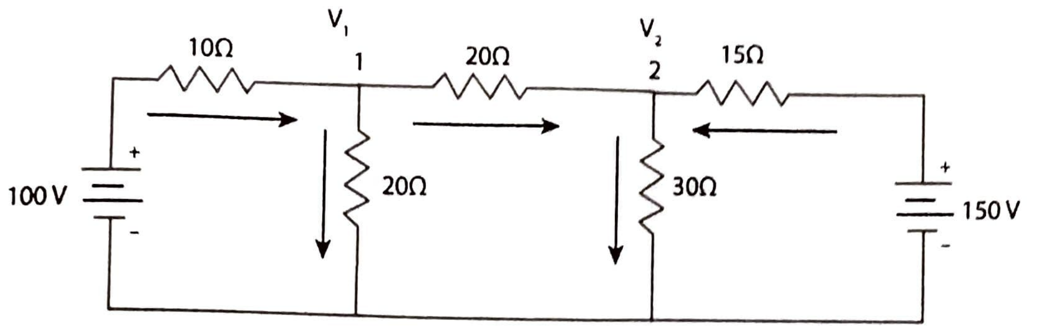



*Fig. 3.38*

***Solution:***

(i) No. of nodes in the given circuit is 3 including reference node. Nodes are marked as 1 and 2. Node voltages are assumed as $V_1$ and $V_2$. The directions of branch currents are also assumed.




(ii) Node equations

At node 1

$$\frac{100-V_1}{10} = \frac{V_1}{20} + \frac{V_1-V_2}{20}$$

$$\frac{100}{10} - \frac{V_1}{10} = \frac{V_1}{20} + \frac{V_1}{20} - \frac{V_2}{20}$$

$$10 - \frac{V_1}{10} = \frac{V_1}{20} + \frac{V_1}{20} - \frac{V_2}{20}$$

$$10 = 0.2V_1 - 0.05\,V_2 \tag{1}$$

At node 2

$$\frac{V_1-V_2}{20} + \frac{150-V_2}{15} - \frac{V_2}{30} = 0$$

$$\frac{V_1}{20} - \frac{V_2}{20} + \frac{150}{15} - \frac{V_2}{15} - \frac{V_2}{30} = 0$$

$$\frac{150}{15} = \frac{-V_1}{20} + \frac{V_2}{20} + \frac{V_2}{15} + \frac{V_2}{30}$$

$$10 = -0.05V_1 + 0.15\,V_2 \tag{2}$$

(iii) Equations in matrix form

$$\begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.05 \\ -0.05 & 0.15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(iv)

$$\Delta = \begin{bmatrix} 0.2 & -0.05 \\ -0.05 & 0.15 \end{bmatrix} = 0.2 \times 0.15 - 0.05 \times 0.05 = 0.0275$$

$$\Delta_1 = \begin{bmatrix} 10 & -0.05 \\ 10 & 0.15 \end{bmatrix} = 10 \times 0.15 + 10 \times 0.05 = 2$$

$$\Delta_2 = \begin{bmatrix} 0.2 & 10 \\ -0.05 & 10 \end{bmatrix} = 0.2 \times 10 + 10 \times 0.05 = 2.5$$

(v) The voltages are given as

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{2}{0.0275} = 72.73V, V_2 = \frac{\Delta_2}{\Delta} = \frac{2.5}{0.0275} = 91\text{ V}$$

**Problem 22**

Using nodal analysis, find all node voltages in the given fig 3.39.

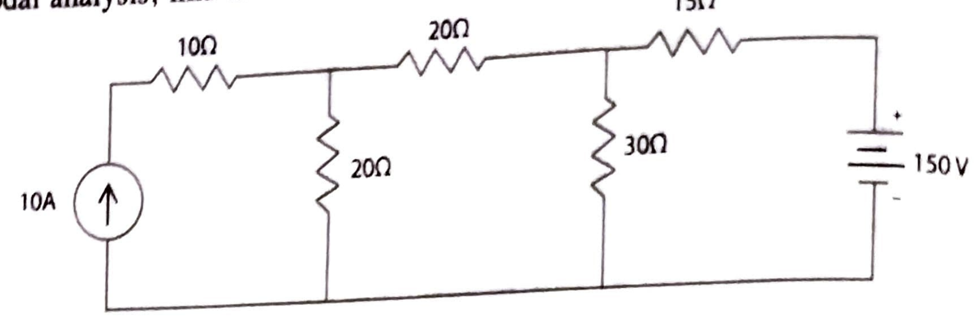
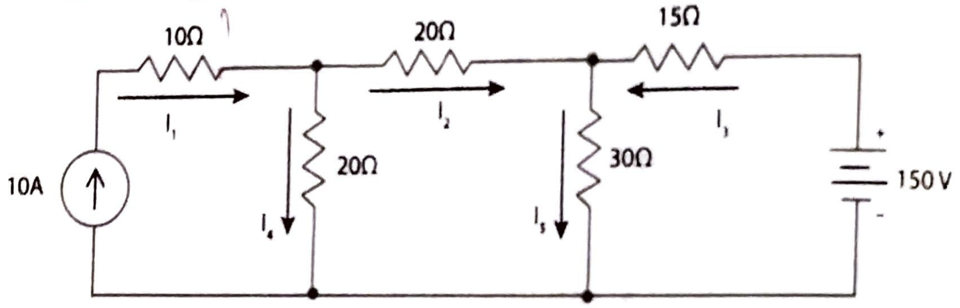



*Fig. 3.39*

***Solution:***

(i) Mark the node voltages and currents.




(ii) Directly write node equations in matrix form

[**Hint:** Element which is series with the current source should be ignored]

$$\begin{bmatrix}10\\10\end{bmatrix}=\begin{bmatrix}\frac{1}{20}+\frac{1}{20} & \frac{-1}{20}\\ \frac{-1}{20} & \frac{1}{20}+\frac{1}{30}+\frac{1}{15}\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

$$\begin{bmatrix}10\\10\end{bmatrix}=\begin{bmatrix}0.1 & -0.05\\ -0.05 & 0.15\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

(iii)

$$\Delta=\begin{bmatrix}0.1 & -0.05\\ -0.05 & 0.15\end{bmatrix}=0.1\times0.15-0.05\times0.05=0.0125$$

$$\Delta_1=\begin{bmatrix}10 & -0.05\\ 10 & 0.15\end{bmatrix}=10\times0.15+10\times0.05=2$$

$$\Delta_2=\begin{bmatrix}0.1 & 10\\ -0.05 & 10\end{bmatrix}=0.1\times10+10\times0.05=1.5$$

(iv) Voltages

$$V_1=\frac{\Delta_1}{\Delta}=\frac{2}{0.0125}=160V, V_2=\frac{\Delta_2}{\Delta}=\frac{1.5}{0.0125}=120\text{ V}$$