PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Solve (i) $z = px + qy + \sqrt{1+p^2+q^2}$ (ii) $x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2)$. Find also singular integral.

(OR)

b. Solve (i) $(D^2 - 2DD' + D'^2)z = \cos(x-3y)$ (ii) $(D^2 - DD'^2)z = e^{x+2y}$.

29. a. Find the Fourier series of $f(x) = x + x^2 \text{ in} (-\pi, \pi)$ of periodicity 2π. Hence deduce $\Sigma \frac{1}{n^2} = \frac{\pi^2}{6}$.

(OR)

b. Compute the first two harmonics of the fourier series f(x) given by the following table.

x	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1

30. a. A tightly stretched string with fixed end point $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$. Find the displacement.

(OR)

b. A rod of length l has its ends A and B kept at 0°C and 100°C respectively unit steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature u(x, t).

31. a. Find the Fourier transform of f(x) if $f(x) = \begin{cases} 1-|x| & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$ hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$.

(OR)

b. Use transform method to evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$

32. a.i. Find $Z(a^n)$ and $Z(n^2)$.

ii. Using residues find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$.

(OR)

b. Solve the equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$ by using Z-transform.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
3rd to 7th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The partial differential equation formed by eliminating arbitrary constant a, b is $z = (x+a)(y+b)$
(A) $z = p+q$ (B) $z = p-q$
(C) $z = p/q$ (D) $z = pq$

2. The complementary function of $(D^2 + 2DD' + D'^2)z = 0$ is
(A) $\phi_1(y-x) + \phi_2(y-x)$ (B) $\phi_1(y-x) + x\phi_2(y-x)$
(C) $\phi_1(y-x) + \phi_2(y+x)$ (D) $\phi_1(y-x) + x\phi_2(y+x)$

3. The particular integral of $(D^2 - 2DD')z = e^{2x}$
(A) $e^{2x}/4$ (B) $e^{2x+y}/4$
(C) e^{2x} (D) $e^{2x}/2$

4. The complete solution of $z = px + qy + p^2q^2$ is
(A) $z = ax + by^2 + ab^2$ (B) $z = ax^2 + by + ab^2$
(C) $z = ax + by + a^2b^2$ (D) $z = ax + by + c$

5. sin$x$ is a periodic function with period
(A) π (B) π/2
(C) 2π (D) 4π

6. The constant a_0 of the Fourier series for the function $f(x) = k,\ 0 \le x \le 2\pi$ is
(A) k (B) 2k
(C) 0 (D) k/2

7. The RMS value of $f(x) = x$ in $-1 \le x \le 1$ is
(A) 1 (B) 0
(C) $1/\sqrt{3}$ (D) –1

8. Half range cosine series for f(x) is (0, π) is

(A) $\sum_{n=1}^{\infty} a_n \cos nx$
(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$
(C) $\sum_{n=1}^{\infty} b_n \sin nx$
(D) $\frac{a_0}{2} - \sum a_n \cos nx$

9. The proper solution of the problems of vibration of string is

(A) $y(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})(ce^{\lambda at} + De^{\lambda at}$
(B) $y(x,t) = (Ax+B)(ct+1)$
(C) $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$
(D) $y(x,t) = Ax + B$

10. The one dimensional wave equation is

(A) $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$
(B) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$
(C) $\frac{\partial y}{\partial t} = a \frac{\partial^2 y}{\partial x^2}$
(D) $\frac{\partial^2 y}{\partial x^2} = a \frac{\partial^2 y}{\partial t^2}$

11. One dimensional heat equation is used to find

(A) Density
(B) Temperature distribution
(C) Time
(D) Displacement

12. A rod of length l has its ends A and B kept at 0° and 100° respectively, until steady state conditions prevail. Then the initial condition is given by

(A) $u(x,0) = ax + b + 100l$
(B) $u(x,0) = \frac{100x}{l}$
(C) $u(x,0) = 100xl$
(D) $u(x,0) = (x+l)100$

13. $F\left[e^{iax} f(x)\right]$

(A) $F(s+a)$
(B) $F(s-a)$
(C) $F(sa)$
(D) $F(s/a)$

14. $F[xf'(x)] =$

(A) $\frac{dF(s)}{ds}$
(B) $i\frac{dF(s)}{ds}$
(C) $-i\frac{dF(s)}{ds}$
(D) $-\frac{dF(s)}{ds}$

15. The fourier cosine transform of $Fc\left[e^{-4x}\right]$

(A) $\sqrt{\frac{2}{\pi}} \frac{4}{16+s^2}$
(B) $\sqrt{\frac{2}{\pi}} \frac{4}{4+s^2}$
(C) $\sqrt{\frac{\pi}{2}} \frac{4}{16+s^2}$
(D) $\sqrt{\frac{\pi}{2}} \frac{4}{4+s^2}$

16. $F[f(x) * g(x)] =$

(A) $F(s)+G(s)$
(B) $F(s)-G(s)$
(C) $F(s)G(s)$
(D) $F(s)/G(s)$

17. What is Z(7)

(A) $\frac{z}{z-1}$
(B) $7\frac{z}{z-1}$
(C) $\frac{1}{7}\frac{z}{z-1}$
(D) $\frac{z-1}{z}$

18. What is $Z\left[na^n\right]$

(A) $\frac{az}{(z-a)^2}$
(B) $\frac{z}{(z-a)^2}$
(C) $\frac{a}{(z-a)^2}$
(D) $\frac{z}{(z-a)^3}$

19. If $z[f(t)] = F(z)$ then $\lim_{z\to\infty} F(z) =$

(A) $f(0)$
(B) $f(1)$
(C) $\lim_{x\to\infty} f(t)$
(D) $f(\infty)$

20. $\phi(z) = \frac{z^n(2z+4)}{(z-2)^3}$ has a pole of order

(A) 2
(B) 1
(C) 3
(D) 4

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Form the Partial differential equation by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$.

22. Find the half range Fourier sine series for $f(x) = x$ in $0 < x < \pi$.

23. Write the one dimensional heat flow equation and all the possible solutions.

24. Find the Fourier sine transform of e^{-ax} $a > 0$.

25. Find Z-transform of $r^n\cos n\theta$.

26. Find $z^{-1}\left(\frac{1}{(z-1)(z-2)}\right)$ by convolution.

27. Solve $p^2 + q^2 = x + y$.