b. In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60% between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class. (Assume normal distribution of marks).

30.a. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant decrease in the consumption of tea after the increase in duty.

(OR)

b. The following table gives for a sample of married women, the level of education and the marriage adjustment score:

		Marriage adjustment			
		Very low	Low	High	Very high
Level of education	College	24	97	62	58
	High school	22	28	30	41
	Middle school	32	10	11	20

Can you conclude from the above data that the higher the level of education, the greater is the degree of adjustment in marriage?

31.a. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and if it takes exactly 1.5 min to reach the correct seat after purchasing the ticket.

(i) Can he expect to be seated for the start of the picture?
(ii) What is the probability that he will be seated for the start of the picture?
(iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture?

(OR)

b. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic
(ii) What is the probability that an arriving patient will not wait?
(iii) What is the expected waiting time until a patient is discharged from the clinic?

32.a. Three boys A, B and C are throwing a ball to each other. A always throw the ball to B and B always throw the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the tpm and classify the states.

(OR)

b. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find

(i) The probability that he takes a train on the third day and
(ii) The probability that he drives to work in the long run

* * * * *

Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third to Seventh Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If $E(X)=3$ and $E\left(X^2\right)=12$, then $Var(X)$ is
(A) 0 (B) 2
(C) 1 (D) 3

2. Family size can be represented by the random variable x. Determine the average family size for

x:	2	3	4	5
$P(x)$:	0.17	0.47	0.26	0.10

(A) 2.94 (B) 3.00
(C) 3.29 (D) 3.86

3. If the random variable x has the p.d.f $f(x)=\begin{cases} ax^3; 0<x<1 \\ 0 \quad otherwise \end{cases}$ then the value of a is
(A) 3 (B) 4
(C) 1/2 (D) 3/4

4. The expectation of the number, on a die when thrown, is
(A) 3 (B) 2
(C) 1 (D) 7/2

5. The mean and variance of a binomial distribution is
(A) $\mu=np, \sigma^2=npq$ (B) $\mu=npq, \sigma^2=np$
(C) $\mu=nq, \sigma^2=npq$ (D) $\mu=np, \sigma^2=pq$

6. If the probability of success on each trail is $\frac{1}{3}$, what is the expected no. of trials required for the first success?
(A) 2 (B) 3
(C) 4 (D) 5

7. A RV X has a uniform distribution over (–3, 3). The value of K for which $P(X>K)=\frac{1}{3}$ is
(A) 3 (B) 2
(C) 1 (D) –2

8. If X is exponentially distributed with mean 10 then the pdf is
   (A) $10e^{-10x}; x \geq 0$ (B) $\frac{1}{10}e^{-10x}; x \geq 0$
   (C) $\frac{1}{10}e^{x/10}; x \geq 0$ (D) $\frac{1}{10}e^{-x/10}; x \geq 0$

9. A type II error occurs when:
   (A) The null hypothesis is incorrectly accepted when it is false (B) The null hypothesis is incorrectly rejected when it is true
   (C) The sample mean differs from the population mean (D) The test is biased

10. The degrees of freedom for testing a sample mean of a sample of size 'n' is
   (A) n (B) n–1
   (C) –n (D) n+1

11. Area of the rejection region depends on
   (A) Size of α (B) Size of β
   (C) Test – statistic (D) Number of values

12. Which of the following values is not typically used for α?
   (A) 0.01 (B) 0.05
   (C) 0.10 (D) 0.25

13. The overall effective arrival rate is $\lambda' =$
   (A) $\mu(1-P_0)$ (B) $\lambda(1-P_0)$
   (C) μP_0 (D) λP_0

14. In queueing theory, generally the service rate is denoted by
   (A) λ (B) μ
   (C) $\lambda\mu$ (D) $\lambda+\mu$

15. The average number of customers in the system in (M/M/1:∞/FIFO) model is
   (A) $\frac{\lambda}{\mu-\lambda}$ (B) $\frac{\mu}{\lambda-\mu}$
   (C) $\frac{\lambda}{\mu+\lambda}$ (D) $\frac{\mu}{\lambda+\mu}$

16. The probability that the system is idle is denoted by
   (A) P_0 (B) P_1
   (C) P_2 (D) P_n

17. Chapman – Kolmogorov theorem states that
   (A) $\left[P_{ij}^{(n)}\right] = \left[P_{ij}\right]^n$ (B) $\left[P_{(n)}\right] = \left[P_{ij}\right]^n$
   (C) $\left[nP_{ij}\right] = \left[P_{ij}\right]^n$ (D) $P_{ij}^{(n)} = P_{ij}$

18. The period d_i of a Markov chain is given by
   (A) $GCD\left\{m; P_{ii}^{(m)} \geq 0\right\}$ (B) $GCD\left\{m; P_{ij}^{(m)} \geq 0\right\}$
   (C) $GCD\left\{m; P_{ii}^{(m)} > 0\right\}$ (D) $GCD\left\{m; P_{ij}^{(m)} > 0\right\}$

19. Markov process is one in which the future value is independent of _____ values.
   (A) Present (B) Past
   (C) Future (D) None

20. In a transition probability matrix, the sum of all elements of any row is
   (A) 0 (B) 1
   (C) 2 (D) –1

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A continuous RV X has a pdf $f(x) = Kx^2e^{-x}$, $0 < x < \infty$. Find (i) K (ii) Mean of x.

22. The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month
   (i) Without a breakdown (ii) With atleast one breakdown.

23. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?

24. What do the letters with symbolic representation (a/b/c):(d/e) of a queueing model represent?

25. A gambler has ₹2. He bets ₹1, with probability 1/2. He stops playing if he loses ₹2 (or) wins ₹4. What is the tpm of the related Markov chain?

26. If the tpm of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the steady state distribution of the chain?

27. A mechanist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter of 1.85 cm with a S.D of 0.1 cm on the sample. Would you say that the work of the mechanist is inferior?

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28.a.i. If X denotes the sum of the numbers obtained when 2 dice are thrown, obtain an upper bound for $P\{|X-7| \geq 4\}$. Compare with the exact probability. (8 Marks)

ii. Given the RV X with density function. (4 Marks)

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find the pdf of $Y = 8X^3$.

(OR)

b. A random variable X has the following distribution.

x	–2	–1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

(i) Find k (ii) Evaluate P(X < 2) (iii) P(-2 < X <2) (iv) CDF of X (v) Mean of X (v) Variance of X.

29.a.i. Find the moment generating function of exponential distribution. Hence find its mean and variance of exponential distribution.

ii. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test?
   (a) On the fourth trial and (b) In fewer than 4 trials?

(OR)