SRM Institute of Science and Technology
Department of Mathematics
18MAB204T- Probability and Queueing Theory
Module – V
Tutorial Sheet – 13

Questions

1. If the initial state probability distribution of a Markov chain is $p^{(0)} = [5/6 \quad 1/6]$ and the tpm of the chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the probability of the chain after 2 steps.

2. The Markov chain with states 0, 1 and 2 is given by the tpm $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ and the initial state probability distribution is $p^{(0)} = (0.3 \quad 0.3 \quad 0.4)$. Find $p^{(3)}$.

3. Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that the probability of a rainy day following a dry day is 1/2. Given that May 1 is a dry day, find the probability that (i) May 3 is also a dry day and (ii) May 5 is also a dry day.

4. Suppose a communication system transmits the digits 0 and 1 through many stages. At each stage the probability that the same digit will be received by the next stage, as transmitted, is 0.75. What is the probability that a 0 is entered at the first stage is received as a 0 in the 2nd stage?

5. The tpm of a Markov chain with three states 0, 1, 2 is $P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$, and the initial distribution of the chain is $P(X_0 = i) = 1/3, \quad i = 0, 1, 2$. Find (i) $P(X_2 = 2)$ (ii) $P(X_3 = 2/X_2 = 1)$ and (ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.

6. A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability 1/2. He stops playing if he loses Rs. 2 or wins Rs. 4. (a) What is the tpm of the related Markov chain? (b) What is the probability that he lost his money at the end of 5 plays? (c) What is the probability that the game lasts more than 7 plays?

7. A gambler's luck follows a pattern. If he wins a game, the probability of his winning the next game is 0.6. However, if he loses a game, the probability of his losing the next game is 0.7. There is an even chance that the gambler wins the first game. What is the probability that he wins (i) the second game, (ii) the third game?

8. A Psychologist makes the following assumptions concerning the behaviour of mice subjected to a particular feeding schedule. For any particular trial, 80% of the mice that went right on the previous experiment will go right on this trial and 60% of those mice that went left in the previous experiment will go right on this trial. If 50% went right on the first trial, what would he predict for (i) the second trial and (ii) the third trial?