B.Tech. DEGREE EXAMINATION, NOVEMBER 2019

Fourth Semester

MA1014 - PROBABILITY AND QUEUING THEORY

(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

(Statistical table, normal distribution table, t-table, f-table chi-square table to be provided)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours   Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. If a random variable X has the pdf $f(x) = \begin{cases} 3x, 0 < x < 1 \\ 0, \text{ otherwise} \end{cases}$ then the pdf of $y = 4x + 3$ is

   (A) $\frac{3}{4}(y-3)$ (B) $\frac{3}{16}(y-3)$

   (C) $\frac{1}{4}(y-3)$ (D) $\frac{3}{2}(y-3)$

2. $Var(4x+8)$ is
   (A) $12 \cdot Var(x)$ (B) $4 \cdot Var(x) + 8$
   (C) $16 \cdot Var(x)$ (D) $16 \cdot Var(x) + 8$

3. A random variable X has mean $\mu = 12$ and variance $\sigma^2 = 9$ and an unknown probability distribution, then $P(6 < x < 18)$ is
   (A) 1/2   (B) 3/4
   (C) 1/4   (D) 1/8

4. The expectation of the number on a die when thrown is
   (A) 1   (B) 7/2
   (C) 3   (D) 2

5. Mean of the Poisson distribution is
   (A) λ (B) $\lambda + 1$
   (C) λ^2 (D) $\lambda - 1$

6. If X is uniformly distributed in (0, 10), then $P(X > 8)$ is
   (A) 1/5   (B) 1/10
   (C) 3/5   (D) 1/3

7. If X is exponentially distributed with mean 10, then the pdf is
   (A) $10e^{-10x}, x \geq 0$ (B) $\frac{1}{10}e^{-10x}, x \geq 0$

   (C) $\frac{1}{10}e^{x/10}, x \geq 0$ (D) $\frac{1}{10}e^{-x/10}, x \geq 0$

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30. a. The following data relate to the marks obtained by 11 students in 2 tests, one held at the beginning of a year and the other at the end of the year after intensive coaching.

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

Do the data indicate that the students have benefited by coaching?

(OR)

b. The nicotine contents in two random samples of tobacco are given below.

Sample 1	21	24	25	26	27	
Sample 2	22	27	28	30	31	36

Can you say that the two samples came from the same population?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 10 min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 min.
   (i)   Find the average number of persons waiting in the system
   (ii) What is the probability that a person arriving at the booth will have to wait in the queue?
   (iii) What is the probability that it will take him more than 10 min altogether to wait for phone and complete his call?
   (iv) The telephone department will install a second booth when convinced that an arrival has to wait on the average for atlest 3 min for phone. By how much the flow of arrivals should increase in order to justify a second booth?

(OR)

b. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room cannot accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.
   (i)   Find the effective arrival rate at the clinic?
   (ii) What is the probability that an arriving patient will not wait?
   (iii) What is the expected waiting time until a patient is discharged from the clinic?

32. a. The three state Makov chain is given by the tpm $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

   (i) Classify the states of the Markov chain,
   (ii) Find the steady state distribution of the chain.

(OR)

b. There are two white marbles in urn A and 3 red marbles in urn B. At each step of the process a marble is selected from each urn and the marbles selected are interchanged. Let a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps? In the long run, what is the probability that there are two red marbles in urn A?

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8. For a standard normal variable, the mean and variance are respectively
   (A) 1 and 0　　　　　　　　　　　(B) μ and σ^2
   (C) 0 and 1　　　　　　　　　　　(D) μ and σ

9. The form of the alternative hypothesis can be
   (A) One-tailed　　　　　　　　　　(B) Two-tailed
   (C) Neither one nor two tailed　　　　(D) One or two tailed

10. The value set for α is known as
    (A) The rejection level　　　　　　　(B) The acceptance level
    (C) The significance level　　　　　　(D) The error in the hypothesis test

11. The Chi-square test is not very effective if the sample is
    (A) Small　　　　　　　　　　　　(B) Large
    (C) Irregular　　　　　　　　　　　(D) Heterogeneous

12. When the researcher rejects a true null hypothesis, a _____ error occurs.
    (A) Type I　　　　　　　　　　　(B) Type A
    (C) Type II　　　　　　　　　　　(D) Type B

13. In which basis the service is provided in queueing theory
    (A) LCFO　　　　　　　　　　　　(B) LIFO
    (C) FCFS　　　　　　　　　　　　(D) FCLS

14. The symbolic notations of queueing model is represented by
    (A) Kendall　　　　　　　　　　　(B) Euler
    (C) Fisher　　　　　　　　　　　　(D) Neumann

15. The probability that the system is idle is denoted by
    (A) P_0　　　　　　　　　　　　(B) P_1
    (C) P_2　　　　　　　　　　　　(D) P_n

16. the traffic intensity of a queueing system is
    (A) λ　　　　　　　　　　　(B) μ
    (C) λ/μ　　　　　　　　　　(D) μ/λ

17. Markov process is one in which the future value is independent of _____ value.
    (A) Present　　　　　　　　　　　(B) Past
    (C) Future　　　　　　　　　　　　(D) State

18. Chapman-Kolmogorov theorem states that
    (A) $\left[P_{ij}^{(n)} \right] = \left[P_{ij} \right]^n$　　　　　　(B) $\left[P_{(n)} \right] = \left[P_{ij} \right]^n$
    (C) $\left[nP_{ij} \right] = \left[P_{ij} \right]^n$　　　　　　(D) $\left[\frac{1}{n} P_{ij} \right] = \left[P_{ij} \right]^n$

19. Transition matrix is a _____ with sum of the row as 1.
    (A) Zero matrix　　　　　　　　　(B) Square matrix
    (C) Rectangular matrix　　　　　　　(D) Triangular matrix

20. The limiting probability $\lim_{n\to\infty} p^{(n)}$
    (A) 0　　　　　　　　　　　　　　(B) 1
    (C) π　　　　　　　　　　　　　(D) P

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. If the random variable 'X' takes the values 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, find the probability distribution.

22. If a random variable X has the MGF $M_X(t) = \dfrac{3}{3-t}$, find the mean and variance of X.

23. It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the probability of atmost 2 defectives using Poisson distribution.

24. Define Type I error and type II error.

25. The mean life time of a sample of 125 bulbs is found as 1550 hrs. The company manufacturing bulbs claims that the average life of their bulbs is 1600 hrs. Is the claim acceptable at 5% LOS?

26. Explain the symbolic representation of kendals notation in queueing model.

27. A student's study habits are as follows: If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night as well. Find the transition probability matrix of a Markov chain.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28.a. A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
p(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) the value of k (ii) $P(1.5 < X < 4.5 / X > 2)$ and (ii) the smallest value of c for which $P(X \le c) > 1/2$.

(OR)

b. A fair die is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

29. a. Fit a Poisson distribution to the following data and calculate the theoretical frequencies.

Death	0	1	2	3	4	Total
Frequency	122	60	15	2	1	200

(OR)

b. In a normal distribution, 70% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution? What percentage of items are under 49?