Wednesday, November 24, 2021 1:20 PM

Method of Variation of Parameters

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = F(x)$$

$CF = A f_1 + B f_2$

$PI = P f_1 + Q f_2$

$$P = -\int \frac{f_2}{f_1 f_2' - f_2 f_1'} F(x)\,dx$$

$$Q = \int \frac{f_1}{f_1 f_2' - f_2 f_1'} F(x)\,dx$$

$\boxed{y = CF + PI}$

① Solve $\frac{d^2y}{dx^2} + y = \sec x$ by the method of variations of parameters

Sol

$$(D^2+1)y = \underbrace{\sec x}_{F(x)}$$

A.E $m^2+1=0$

$m = \pm i$

$CF = A\cos x + B\sin x$

$CF = A f_1 + B f_2$

$f_1 = \cos x$	$f_2 = \sin x$
$f_1' = -\sin x$	$f_2' = \cos x$

$\text{Dem} = f_1 f_2' - f_2 f_1'$

$= \cos^2 x + \sin^2 x = 1$

$$P = -\int \frac{f_2}{f_1 f_2' - f_2 f_1'} F(x)\,dx$$

$$= -\int \frac{\sin x}{1} \sec x\,dx$$

$$= -\int \frac{\sin x}{\cos x}\,dx = -\int \tan x\,dx$$

$$= -(-\log(\cos x))$$

$$\boxed{P = \log(\cos x)}$$

$$Q = \int \frac{f_1}{f_1 f_2' - f_2 f_1'} F(x)\,dx$$

$$= \int \frac{\cos x}{1} \sec x\,dx$$

$$= \int \cancel{\cos x}\,\frac{1}{\cancel{\cos x}}\,dx$$

$\boxed{Q = x}$

$PI = P f_1 + Q f_2$

$= \log(\cos x)\cdot\cos x + x\sin x$

$$\boxed{y = A\cos x + B\sin x + \log(\cos x)\cos x + x\sin x}$$