Signal Sampling and Reconstruction

Analog to digital (A/D) conversion:

The analog-to-digital conversion is basically a 2 step process:

- Sampling
  - Converts continuous-time analog signal $x_a(t)$ to discrete-time continuous value signal $x(n)$.
  - It is obtained by taking the "samples" of $x_a(t)$ at discrete-time intervals, T_s
- Quantization
  - Converts discrete-time continuous valued signal to discrete time discrete valued signal.

These steps are shown in Fig. 8.

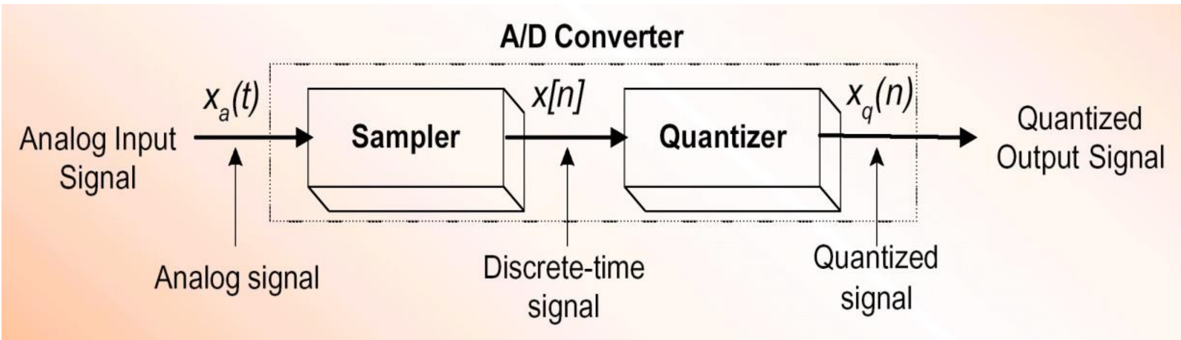

Fig. 8. Basic steps of ADC

Sampling of continuous signal

Sampling is the processes of converting continuous-time analog signal, $x_a(t)$, into a discrete-time signal by taking the "samples" at discrete-time intervals.

- Sampling analog signals makes them discrete in time but still continuous valued.
- If done properly (**Nyquist theorem** is satisfied), sampling does not introduce distortion.

Fig. 9 shows an analog (continuous-time) signal (solid line) defined at every point over the time axis and amplitude axis. Hence, the analog signal contains an infinite number of points.

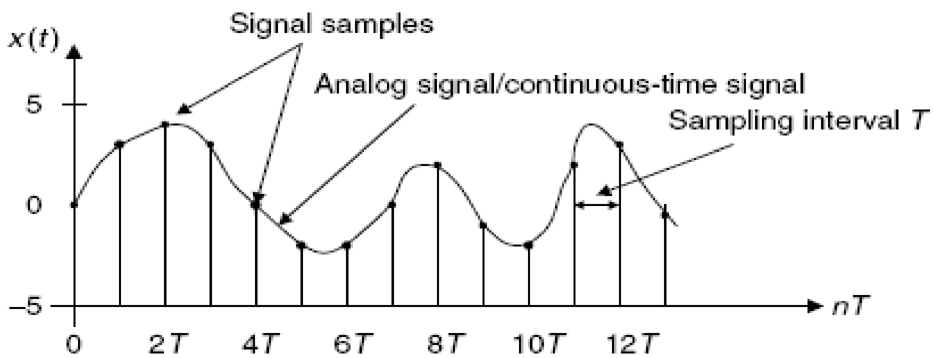

Fig. 9. Display of analog (continuous) signal and digital samples versus the sampling time instants

It is impossible to digitize an infinite number of points. Furthermore, the infinite points are not appropriate to be processed by the digital signal processor or computer, since they require an infinite amount of memory and infinite amount of processing power for computations. Sampling can solve such a problem by taking samples at the fixed time interval, as shown in Fig. 9 and Fig. 10, where the time ***T*** represents the **sampling interval** or **sampling period** in seconds. As shown in Fig. 10, each sample maintains its voltage level during the sampling interval *T* to give the ADC enough time to convert it. This process is called **sample and hold**.

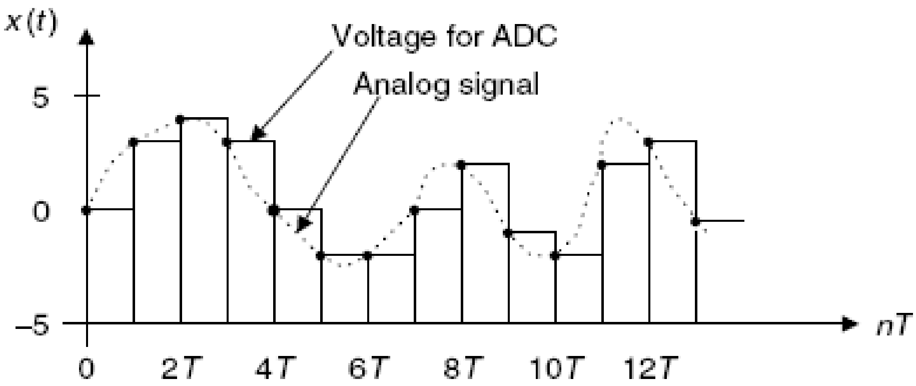

Fig. 10. Sample-and-hold analog voltage for ADC

For a given sampling interval *T*, which is defined as the time span between two sample points, the **sampling rate** or **sampling frequency** is the rate at which the signal is sampled, expressed as the number of samples per second (reciprocal of the sampling interval).

$f_S = 1/T_S$ Samples per second (Hz)

- If the signal is slowly varying, then fewer samples per second will be required than if the waveform is rapidly varying. So, the optimum sampling rate depends on the *maximum frequency* component present in the signal.

Nyquist sampling theorem or Nyquist criterion:

If an analog signal is not appropriately sampled, **aliasing** will occur, which causes unwanted signals in the desired frequency band (i.e. if the sampling is performed at a proper rate, no info is lost about the original signal and it can be properly reconstructed later).

"If a signal is sampled at a rate at least, but not exactly equal to twice the max frequency component of the waveform, then the waveform can be exactly reconstructed from the samples without any distortion". The condition is described as

$$f_S \geq 2 f_{\max}$$

Where, $f_{\max}$ is the maximum-frequency component of the analog signal to be sampled.

$\boldsymbol{T_s}$ is called the **Nyquist interval**: It is the longest time interval that can be used for sampling a band limited signal and still allow reconstruction of the signal at the receiver without distortion.

Example: Find the Nyquist frequency and Nyquist interval of the following signals:

a) speech signal containing frequencies up to 4 kHz
b) audio signal possessing frequencies up to 20 kHz

sol.

a) to sample a speech signal containing frequencies up to 4 kHz, the Nyquist rate (minimum sampling rate fs) is chosen to be at least 8 kHz, or 8,000 samples per second (fs=2fm) and Nyquist interval (maximum time interval T_s) is $1/f_s$ = 1/8 kHz = 0.125 ms.
b) to sample an audio signal possessing frequencies up to 20 kHz, at least 40,000 samples per second, or 40 kHz, of the audio signal are required and Nyquist interval (maximum time interval T_s) is $1/f_s$ = 1/40 kHz = 25 μs.

Sampled signal spectrum:

Fig. 11 depicts the sampled signal $\boldsymbol{x_s(t)}$ obtained by sampling the continuous signal x(t) at a sampling rate of f_s samples per second. Mathematically, this process can be written as the product of the continuous signal and the sampling pulses (pulse train):

$$x_s(t) = x(t)\, p(t)$$

Where, p(t) is the pulse train with a period $T = 1/ f_s$.

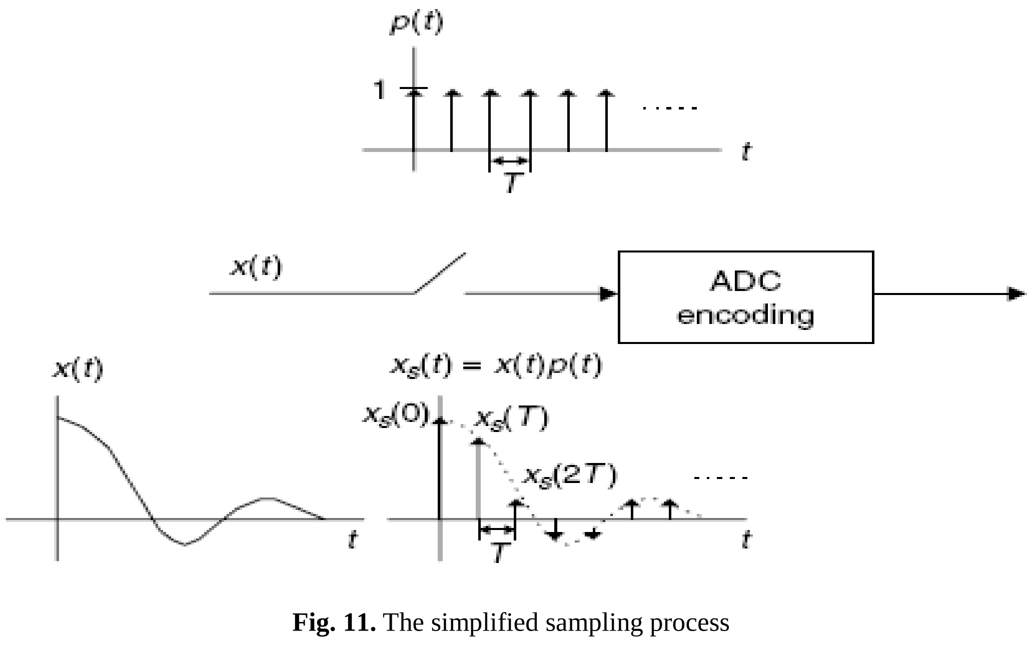

Fig. 11. The simplified sampling process

From the spectral analysis shown in Fig. 12, it is clear that the sampled signal spectrum consists of the scaled baseband spectrum centered at the origin and its replicas centered at the frequencies of $\pm nf_s$ ($\pm n/T_s$) (multiples of the sampling rate) for each of n = 1,2,3, . . .

In Fig. 12, three possible sketches are classified. Given the original signal spectrum $X(f)$ plotted in Fig. 12(a), the sampled signal spectrum is plotted in Fig. 12(b), where, the replicas have separations between them. In Fig. 12(c), the baseband spectrum and its replicas are just connected. In Fig. 12(d), the original spectrum and its replicas are overlapped; that is, there are many overlapping portions in the sampled signal spectrum.

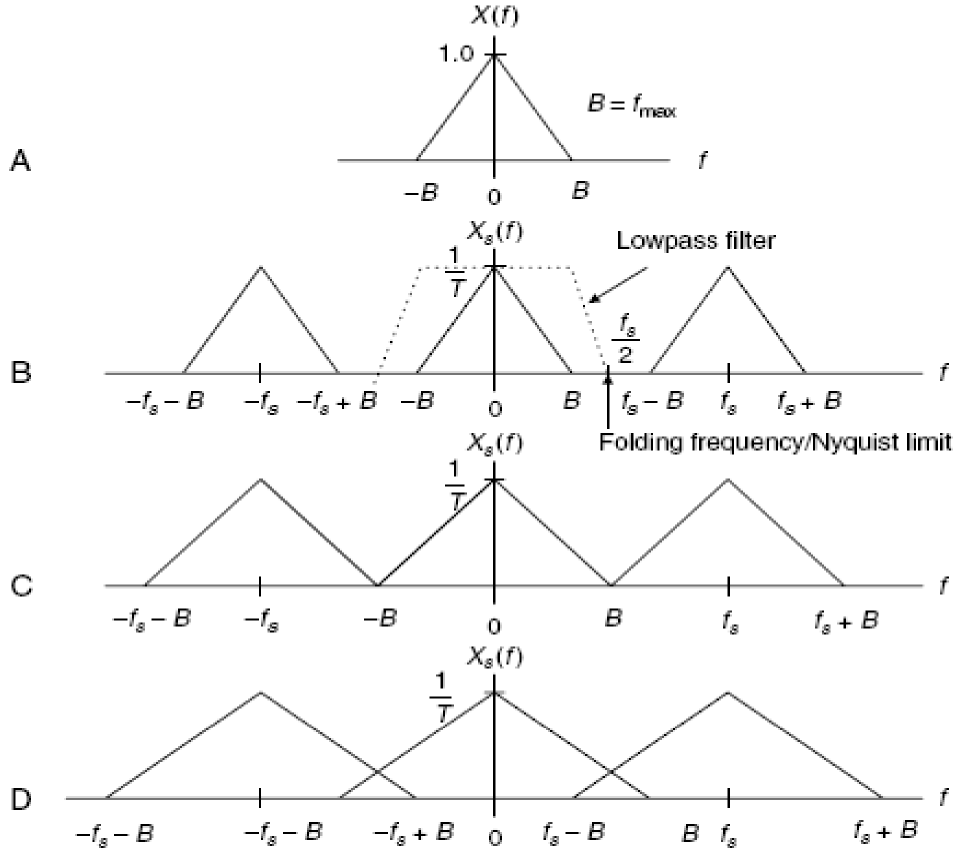

Fig. 12. Plots of the sampled signal spectrum

If applying a lowpass reconstruction filter to obtain exact reconstruction of the original signal spectrum,

- As long as $\boldsymbol{f_s > 2B}$, no overlap of repeated replicas $X(f - n/T_s)$ will occur in $X_s(f)$. Hence, the signal at the output of the filter will be the original signal spectrum without distortion as shown in Fig. 13.
- If the waveform is undersampled (i.e. $\boldsymbol{f_s < 2B}$), then there will be spectral overlap in the sampled signal. Hence, the signal at the output of the filter will be different from the original signal spectrum as shown in Fig. 14. [This is the outcome of aliasing].
- This implies that whenever the sampling condition is not met, an irreversible overlap of the spectral replicas is produced.