SCHOOL OF DISTANCE EDUCATION

M. Sc. MATHEMATICS

MTH3C13: FUNCTIONAL ANALYSIS

(Core Course)

THIRD SEMESTER

MCQ's in Functional Analysis

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UNIVERSITY OF CALICUT

M.Sc. MATHEMATICS

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SELF LEARNING MATERIAL

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Multiple Choice Questions

1. The linear span of empty set equals:

   (a) Empty set

   (b) Zero subspace

   (c) The whole space

   (d) None of these

   Answer : (b)

2. Which of the following is not a linear space over $\mathbb{R}$?

   (a) $\mathbb{C}$

   (b) $\mathbb{R}$

   (c) $\mathbb{Q}$

   (d) None of these

   Answer : (c)

3. Which of the following is not a linear space ?

   (a) $\mathbb{C}$ over $\mathbb{R}$

   (b) $\mathbb{Q}$ over $\mathbb{R}$

(c) $\mathbb{R}$ over $\mathbb{Q}$

(d) $\mathbb{C}$ over $\mathbb{Q}$

Answer : (b)

4. Dimension of $\mathbb{C}^n$ as a linear space over $\mathbb{C}$ is :

(a) n

(b) $n+1$

(c) n^2

(d) $2n$

Answer : (a)

5. Dimension of $\mathbb{C}^n$ as a linear space over $\mathbb{R}$ is :

(a) n

(b) $n+1$

(c) $2(n+1)$

(d) $2n$

Answer : (d)

6. If E_1 and E_2 are subspaces of a linear space E, then which of the following is false?

(a) $E_1 \cap E_2$ is always a subspace of E.

(b) $E_1 + E_2$ is always a subspace of E.

(c) $E_1 \cup E_2$ is always a subspace of E.

(d) $E_1 \cup E_2$ is never a subspace of E.

Answer : (c)

7. If E is finite dimensional linear space of dimension n, and F is a subset of E with m elements, where $m < n$, then which of the following is true?

(a) F can span E.

(b) F is linearly independent in E.

(c) F is linearly dependent in E.

(d) F can not be a basis of E.

Answer : (d)

8. Which of the following is not a linear space over $\mathbb{C}$?

(a) The set of all convergent sequences in $\mathbb{C}$.

(b) The set of all bounded sequences in $\mathbb{C}$.

(c) The set of all sequences in $\mathbb{C}$ that converges to 0.

(d) The set of all sequences in $\mathbb{C}$ that converges to a real number.

Answer : (d)

9. Which of the following linear space is infinite dimensional?

(a) $\mathbb{R}$ over $\mathbb{Q}$

(b) $\mathbb{Q}$ over $\mathbb{Q}$

(c) $\mathbb{C}$ over $\mathbb{C}$

(d) $\mathbb{C}$ over $\mathbb{R}$

Answer : (a)

10. Pick the incorrect statement:

(a) If S spans the linear space E and if $S \subset T$, then T also spans E.

(b) Any single vector in E is linearly independent.

(c) Any set of vectors in E that includes the zero vector is linearly dependent.

(d) If S is a linearly independent set in a linear space E and if $T \subset S$, then T is also linearly independent.

Answer : (b)

11. Consider $f : \mathbb{R} \to \mathbb{R}$. Which of the following is not a linear map?

    (a) $f(x) = x$

    (b) $f(x) = x^2$

    (c) $f(x) = 3x$

    (d) $f(x) = 0$

    Answer : (b)

12. A linear map $A : E_1 \to E_2$ between two linear spaces is an isomorphism if:

    (a) $\ker A = \{0\}$ and $\mathrm{Im} A = E_2$.

    (b) $\ker A \neq \{0\}$ and $\mathrm{Im} A = E_2$.

    (c) $\ker A = \mathrm{Im} A$.

    (d) $\ker A = E_1$ and $\mathrm{Im} A = E_2$.

    Answer : (a)

13. Which of the following denotes the space of all bounded scalar sequences?

    (a) c

(b) ℓ_∞

(c) ℓ_p

(d) s

Answer : (b)

14. Which of the following is not a property of norm in general?

(a) $\|x\| \geq 0$

(b) $\|x+y\| \leq \|x\| + \|y\|$

(c) $\|kx\| = k\|x\|$

(d) $\|x\| = 0$ iff $x = 0$

Answer : (c)

15. If $\|\cdot\|_1$ and $\|\cdot\|_2$ are two norms on a linear space E, then $\|\cdot\|_1$ is stronger than $\|\cdot\|_2$ if and only if :

(a) $\exists\ C > 0$ such that $\|x\|_2 \leq C\|x\|_1$, for all $x \in E$.

(b) $\exists\ C > 0$ such that $\|x\|_1 \leq C\|x\|_2$, for all $x \in E$.

(c) $\exists\ 0 < C < 1$ such that $\|x\|_2 \leq C\|x\|_1$, for all $x \in E$.

(d) $\exists\ 0 < C < 1$ such that $\|x\|_1 \leq C\|x\|_2$, for all $x \in E$.

Answer : (a)

16. The Minkowski's inequality for scalar sequences $a = (a_i)$ and $b = (b_i)$ states that:

(a) $\|ab\| \leq \|a\| \ \|b\|$

(b) $\|ab\| \geq \|a\| \ \|b\|$

(c) $\|a+b\| \geq \|a\| \ + \ \|b\|$

(d) $\|a+b\| \leq \|a\| \ + \ \|b\|$

Answer : (d)

17. Let $(E, \| \cdot \|)$ be a normed space and let d be the metric induced by the norm on E. If $x, y \in E$ and if $d(x, y) = r$, then which of the following is false?

(a) $d(x+z, y+z) = r$, for any $z \in E$.

(b) $d(rx, ry) = r^2$

(c) $d(ax, ay) = |a|r$, for any scalar a.

(d) $d(rx+y, ry+x) = (r-1)r$.

Answer : (d)

18. Let $C[a, b]$ be the space of all complex valued continuous functions on $[a, b]$. Under which of the following norms, $C[a, b]$ is a Banach space?

(a) $\|f\| = (\int_a^b |f(t)|^2 dt)^{1/2}$

(b) $\|f\| = \int_a^b |f(t)| dt$

(c) $\|f\| = (\int_a^b |f(t)|^3 dt)^{1/3}$

(d) None of these.

Answer : (d)

19. A complete normed space is known as a :

(a) Hilbert space

(b) Compact space

(c) Banach space

(d) Euclidean space

Answer : (c)

20. Which of the following is a Banach space?

(a) Space of all polynomial functions on $[a, b]$ with the supremum norm

(b) Space of all continuous functions on $[a, b]$ with the supremum norm

(c) Space of all polynomial functions on $[a, b]$ with the p-norm

(d) Space of all continuous functions on $[a, b]$ with the p-norm

Answer : (b)

21. The term Hilbert space stands for a :

(a) Complete inner product space

(b) Compact linear space

(c) Complete normed space

(d) Complete metric space

Answer : (a)

22. Consider the statements.

(i) Every finite dimensional normed linear space is a Banach space.

(ii) Every Banach space is finite dimensional linear space.

(a) Only (i) is true

(b) Only (ii) is true

(c) Both (i) and (ii) are true

(d) Neither (i) nor (ii) is true.

Answer : (a)

23. Let H be a Hilbert space and L be a subspace of H. Then which of the following is false?

    (a) $L^\perp$ is a subspace of H.

    (b) $L^\perp$ is a closed subspace of H.

    (c) $L \cap L^\perp = \{0\}$

    (d) $L \cap L^\perp = \phi$

    Answer : (d)

24. Which of the following subspaces of ℓ_∞ is not a Banach space?

    (a) c

    (b) c_0

    (c) s^*

    (d) ℓ_p

    Answer : (c)

25. Let $X = C([0,1], \mathbb{R})$ be equipped with the supremum norm. Let Y be the subspace of polynomial functions, then :

    (a) Y is a dense subspace of X.

(b) Y is a closed subspace of X.

(c) Y is an open subspace of X.

(d) None of these.

Answer : (a)

26. Which of the following is not a Banach space?

(a) Linear space of all n-tuples $x = (a_1, a_2, ..., a_n)$ with $\|x\| = \max\limits_i |a_i|$.

(b) Linear space of all 2-summable sequences $x = (a_1, a_2, ...)$ with $\|x\| = (\sum\limits_{i=1}^{\infty} |a_i|^2)^{1/2}$.

(c) Linear space of all bounded sequences $x = (a_1, a_2, ...)$ with $\|x\| = \sup\limits_i |a_i|$.

(d) Linear space of all continuous functions on $[0, 1]$ with $\|f\| = \int_0^1 |f(t)|dt$.

Answer : (d)

27. The distance between any two orthonormal vectors in an inner product space is:

(a) 1

(b) $\sqrt{2}$

(c) 1

(d) 2

Answer : (b)

28. Pick the INCORRECT statement:

(a) Every Hilbert space is a normed space

(b) Every Banach space is a topological space

(c) Every normed space is a metric space

(d) Every Banach space is a Hilbert space

Answer : (d)

29. Which of the following is a Banach space?

(a) $P[a,b]$ with supremum norm

(b) $C[a,b]$ with supremum norm

(c) s^* with supremum norm

(d) $C[a,b]$ with p-norm

Answer : (b)

30. Consider the statements:

(i) Every normed space is complete.

(ii) Every normed space can be identified as a dense subspace of a complete normed space.

(a) Only (i) is true

(b) Only (ii) is true

(c) Both (i) and (ii) are true

(d) Neither (i) nor (ii) are true.

Answer : (b)

31. Which of the following is true in a normed space?

(a) Union of any family of open sets is open.

(b) Intersection of any family of open sets is open.

(c) Union of any family of closed sets is closed.

(d) Intersection of any family of closed sets is open.

Answer : (a)

32. If $p \geq q \geq 1$, which of the following is true?

(a) $\ell_p \subset \ell_q$

(b) $\ell_p \supset \ell_q$

(c) $\ell_p = \ell_q$

(d) None of these.

Answer : (b)

33. Which of the following is Cauchy-Schwartz inequality?

(a) $|\langle x, y\rangle| \leq \langle x, x\rangle^{1/2} \cdot \langle y, y\rangle^{1/2}$

(b) $|\langle x, y\rangle| \geq \langle x, x\rangle^{1/2} \cdot \langle y, y\rangle^{1/2}$

(c) $|\langle x, y\rangle| \leq \langle x, y\rangle^{1/2} \cdot \langle y, x\rangle^{1/2}$

(d) $|\langle x, y\rangle| \leq \langle x, x\rangle \cdot \langle y, y\rangle$

Answer : (a)

34. Which of the following is known as the parallelogram law?

(a) $\|x + y\|^2 = 2\|x\|^2 + 2\|y\|^2$

(b) $\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + \|y\|^2$

(c) $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$

(d) $\|x + y\|^2 - \|x - y\|^2 = 2\|x\|^2 + \|y\|^2$

Answer : (c)

35. Two vectors x, y in an inner product space are orthogonal if :

(a) $\langle x, y\rangle = 0$

(b) $\|x\| = \|y\| = 1$

(c) $\langle x, y\rangle \neq 0$

(d) None of these.

Answer : (a)

36. If two vectors x, y in an inner product space are orthogonal, then:

(a) $\|x + y\|^2 = 2\|x\|^2 + 2\|y\|^2$

(b) $\|x + y\|^2 = \|x\|^2 + \|y\|^2$

(c) $\|x + y\| = 0$

(d) None of these.

Answer : (b)

37. If $\{f_i\}$ is a complete system in a Hilbert space H and if $x \perp f_i$ for all i, then:

(a) $\|x\| = 1$

(b) $\{x\}$ is linearly independent.

(c) $x = 0$

(d) None of these.

Answer : (c)

38. Consider the following statements about a Hilbert space H:

(i) H is separable if it has a countable dense subset.

(ii) H is separable if it has a complete orthonormal system.

(a) Only (i) is true

(b) Only (ii) is true

(c) Both (i) and (ii) are true

(d) Neither (i) nor (ii) are true.

Answer : (c)

39. If L_1 and L_2 are two subspaces of a Hilbert space H, then which of the following is true?

(a) $(L_1 + L_2)^{\perp} = L_1^{\perp} \cap L_2^{\perp}$

(b) $(L_1 + L_2)^{\perp} = L_1^{\perp} + L_2^{\perp}$

(c) $(L_1 \cap L_2)^{\perp} = L_1^{\perp} + L_2^{\perp}$

(d) $(L_1 + L_2)^\perp = L_1^\perp \cup L_2^\perp$

Answer : (a)

40. If L is a closed subspace of a Hilbert space H, then which of the following is true?

(a) $L \oplus L^\perp \neq H$

(b) $L \cup L^\perp = H$

(c) $L \oplus L^\perp = H$

(d) $(L^\perp)^\perp = L^\perp$

Answer : (c)

41. If E is a closed subspace of a Hilbert space H and $\mathrm{codim} E = 1$, then which of the following is true?

(a) $\mathrm{dim} E^\perp = 1$

(b) $\mathrm{codim} E^\perp = 1$

(c) $E^\perp = \{0\}$

(d) None of these

Answer : (a)

42. If X, Y are normed spaces and if $A : X \to Y$ is a bijective, bounded linear map, then:

(a) A is always an open map.

(b) A is an open map if X is a Banach space.

(c) A is an open map if Y is a Banach space.

(d) A is an open map if both X and Y are Banach spaces.

Answer : (d)

43. Which of the following is true?

(a) If A, B are invertible linear operators on X, then $A+B$ is invertible.

(b) If A, B are invertible linear operators on X, then $A-B$ is invertible.

(c) If A, B are invertible linear operators on X, then AB is invertible.

(d) If A is invertible linear operator on X, and k is any scalar, then kA is invertible.

Answer : (c)

44. If X and Y are normed spaces, then the space of bounded linear operators $L(X, Y)$ is a Banach space if and only if:

(a) X is a Banach space.

(b) Y is a Banach space.

(c) Both X and Y are Banach spaces.

(d) Both X and Y are finite dimensional spaces.

Answer : (b)

45. If X and Y are normed spaces, and if $T : X \to Y$ is a linear operator, then T is bounded if and only if:

(a) T maps bounded subsets of X into bounded subsets of Y.

(b) T maps open subsets of X into open subsets of Y.

(c) T maps closed subsets of X into closed subsets of Y.

(d) T is invertible.

Answer : (a)

46. If $A : H \to H$ is a bounded linear operators on a Hilbert space H, then:

(a) $\|A\| = \sup\{\langle Ax, y\rangle; \|x\| \leq 1, \|y\| \leq 1\}$

(b) $\|A\| = \sup\{|\langle Ax, y\rangle|; \|x\| \leq 1, \|y\| \leq 1\}$

(c) $\|A\| = \sup\{\langle Ax, y\rangle; x, y \in H\}$

(d) $\|A\| = \inf\{|\langle Ax, y\rangle|; \|x\| \leq 1, \|y\| \leq 1\}$

Answer : (b)

47. For any bounded linear operator $A : X \to Y$, $\ker A$ is:

(a) a closed subspace of Y.

(b) an open subspace of Y.

(c) a closed subspace of X.

(d) an open subspace of X.

Answer : (c)

48. For any normed space X, the dual space X^* is:

(a) Always a Banach space.

(b) Always a compact set.

(c) Always finite dimensional.

(d) Always an infinite dimensional.

Answer : (a)

49. If T is the shift operator on ℓ_2, then:

(a) $\|T\| = \frac{1}{\sqrt{2}}$

(b) $\|T\| = \sqrt{2}$

(c) $\|T\| = 1$

(d) $\|T\| = \infty$

Answer : (c)

50. Any bounded subset in $\mathbb{R}^n$ is :

(a) compact

(b) relatively compact

(c) open

(d) closed

Answer : (b)

51. Consider the statements:

(i) Every compact operator is bounded.

(ii) Every bounded operator is compact. Then:

(a) Only (i) is true.

(b) Only (ii) is true.

(c) Both (i) and (ii) are true.

(d) Neither (i) nor (ii) is true.

Answer : (a)

52. $M \subset C[a,b]$ is relatively compact if and only if :

(a) M is uniformly bounded.

(b) M is equicontinuous.

(c) M is closed and bounded.

(d) M is uniformly bounded and equicontinuous.

Answer : (d)

53. If $A : X \to Y$ is a bounded operator, then:

(a) $A^* : Y^* \to X^*$ is bounded.

(b) $A^* : X^* \to Y^*$ is bounded.

(c) $A^* : Y^* \to X^*$ is linear, but need not be bounded.

(d) $A^* : X^* \to Y^*$ is linear, but need not be bounded.

Answer : (a)

54. If $A : X \to Y$, $x \in X$ and $f \in Y^*$, then $\langle Ax, f\rangle$ equals:

(a) $\langle A^*x, f\rangle$

(b) $\langle x, A^*f\rangle$

(c) $\langle A^*f, x\rangle$

(d) $\langle A^*x, A^*f\rangle$

Answer : (b)

55. Every bounded operator of finite rank is :

(a) compact

(b) open

(c) has a non zero adjoint.

(d) None of these.

Answer : (a)

56. Rank of a linear operator A equals:

(a) $\dim(\ker A)$

(b) $\dim(\mathrm{Im} A)$

(c) $\dim(\mathrm{Im} A^*)$

(d) $\dim(\ker A^*)$

Answer : (b)

57. Norm convergence is also known as :

(a) Uniform convergence

(b) Strong convergence

(c) Weak convergence

(d) None of these.

Answer : (a)

58. Consider the statements:

(i) Strong convergence is weaker than norm convergence.

(ii) Weak convergence is weaker than strong convergence.

Then:

(a) Only (i) is true.

(b) Only (ii) is true.

(c) Both (i) and (ii) are true.

(d) Neither (i) nor (ii) is true.

Answer : (c)

59. If T is the right shift operator in ℓ_2, then :

(a) T is one to one.

(b) T is onto.

(c) T is invertible.

(d) None of these.

Answer : (a)

60. If $\mathcal{U}$ is the left shift operator in ℓ_2, then :

(a) $\mathcal{U}$ is one to one.

(b) $\mathcal{U}$ is onto.

(c) $\mathcal{U}$ is invertible.

(d) None of these.

Answer : (b)

61. If T is the right shift operator and $\mathcal{U}$ is the left shift operator in ℓ_2, then :

(a) $\mathcal{U}T = Id$.

(b) $T\mathcal{U} = id$.

(c) $T\mathcal{U} = \mathcal{U}T$.

(d) None of these.

Answer : (a)

62. If T is the right shift operator and $\mathcal{U}$ is the left shift operator in ℓ_2, then which of the following is false?

(a) $\mathcal{U}T = Id$.

(b) $T\mathcal{U} \neq id$.

(c) $\ker T\mathcal{U} \neq 0$.

(d) $\ker \mathcal{U}T \neq 0$.

Answer : (d)

63. A bijective map $A : X \to Y$ is open if and only if :

(a) $A : X \to Y$ is invertible.

(b) $A : X \to Y$ is bounded.

(c) $A^{-1} : Y \to X$ is bounded.

(d) $A^{-1} : Y \to X$ is open.

Answer : (c)

64. If $\{A_n\}$ is a sequence of operators on a normed space X, then $A_n \to A$ strongly if and only if:

(a) $A_n x \to Ax$ for all $x \in X$.

(b) $\|A_n - A\| \to 0$ as $n \in \infty$.

(c) $f(A_n x) \to f(Ax)$ for all $x \in X$ and for all $f \in X^*$.

(d) None of these.

Answer : (a)

65. If T is a bounded linear operator, then:

    (a) $\|Tx\| \leq \|T\| \cdot \|x\|$
    (b) $\|Tx\| \geq \|T\| \cdot \|x\|$
    (c) $\|Tx\| = \|T\| \cdot \|x\|$
    (d) None of these.
    Answer : (a)

66. Which of the following function do not define a norm in $\mathbb{R}^2$?

    (a) $f(x, y) = \sup\{|x|, |y|\}$
    (b) $f(x, y) = (|x|^2 + |y|^2)^{1/2}$
    (c) $f(x, y) = |x| + |y|$
    (d) $f(x, y) = (|x|^{1/2} + |y|^{1/2})^2$
    Answer : (d)

67. Which of the following is not a complete normed space?

    (a) ℓ_∞ / c_0
    (b) ℓ_∞ / c
    (c) ℓ_∞ / s^*

(d) ℓ_∞/Y, where Y =span $\{(1,1,1,...)\}$.

Answer : (c)

68. Every complete subspace of a normed space is:

(a) closed.

(b) open

(c) finite

(d) None of these.

Answer : (a)

69. Let X be the normed space of all continuous functions on $[0,1]$ with the norm $\|f\| = \int_0^1 |f(t)|dt$. Then:

(a) X is a proper closed subspace of $L_1[0,1]$.

(b) X is a proper dense subspace of $L_1[0,1]$.

(c) X is a Banach space.

(d) None of these.

Answer : (b)

70. For x, y in a normed space X, which of the following is not necessarily true?

(a) $\|x+y\| \leq \|x\| + \|y\|$

(b) $|\|x\| - \|y\|| \leq \|x-y\|$

(c) $|\|x\| - \|y\|| \leq \|x\| + \|y\|$

(d) $\|x-y\| \leq \|x\| - \|y\|$

Answer : (d)

71. Let M be a closed subspace of a normed space N. Then the quotient space N/M is a Banach space if and only if:

(a) M is a Banach space.

(b) N is a Banach space.

(c) $N = M$

(d) None of these.

Answer : (b)

72. Which of the following normed space is not separable?

(a) $(\ell_\infty, \|\cdot\|_\infty)$

(b) $(\ell_p, \|\cdot\|_p)$, $1 \leq p < \infty$

(c) $(\mathbb{C}^n, \|\cdot\|_p)$, $1 \leq p < \infty$

(d) $(\mathbb{C}^n, \|\cdot\|_\infty)$

Answer : (a)

73. If E is a normed space and if d is the metric induced by the norm, then for any scalar k, $d(kx, ky)$ equals:

(a) $d(x, y)$

(b) $|k|d(x, y)$

(c) $kd(x, y)$

(d) $k^2d(x, y)$

Answer : (b)

74. Let $e = (1, 1, 1, ...) \in \ell_\infty$, then c_0+ span$\{e\}$ equals:

(a) c_0

(b) c

(c) ℓ_∞

(d) None of these.

Answer : (b)

75. For x, y in a normed space X, $|\|x+y\|-\|x-y\|| \leq$ ....

(a) $2\|y\|$

(b) $2(\|x\| + \|y\|)$

(c) $2\|x\|$

(d) $\|x\| + \|y\|$

Answer : (a)

76. Let $X = (\ell_\infty, | \cdot \|_\infty)$ and Y be a finite dimensional subspace of X. Then which of the following is not a Banach space?

(a) X/c

(b) X/Y

(c) X/c_0

(d) X/s^*

Answer : (d)

77. Pick the incorrect statement:

(a) Every linear subspace of a normed space is convex.

(b) Every ball in a normed space is convex.

(c) Intersection of two convex sets is convex.

(d) Union of two convex sets is convex.

Answer : (d)

78. Which of the following is non-separable normed space?

(a) $L_1[0,1]$

(b) $L_\infty[0,1]$

(c) $L_2[0,1]$

(d) $C[0,1]$

Answer : (b)

79. Let X be the space of differentiable functions on $[0,1]$, $Y = C[0,1]$ both with the supremum norm and $A : X \to Y$ be the map defined by $Af = f'$, the derivative of f. Then A is:

(a) Linear and bounded.

(b) Bounded but not linear.

(c) Linear and continuous.

(d) Linear but not continuous.

Answer : (d)

80. Let X be a normed space and f be a bounded, non-zero linear functional on X. Then, which of the following is not true?

(a) f is onto.

(b) f is continuous.

(c) ker$f$ is a closed subspace of X.

(d) f is an open map.

Answer : (d)

81. If f is a linear functional on a normed space X, then ker$f$ is:

(a) closed in X.

(b) dense in X.

(c) either closed or dense in X.

(d) None of these.

Answer : (c)

82. Which of the following is true?

(a) Every metric space is a normed space.

(b) Every complete normed space is finite dimensional.

(c) Every finite dimensional normed space has a unique norm.

(d) The dual space of a normed space is a complete normed space.

Answer : (d)

83. Let x, y be elements of a Hilbert space H, such that $\|x\| = 3, \|y\| = 4$ and $\|x+y\| = 7$. Then $\|x-y\|$ equals:

(a) 1

(b) 2

(c) 3

(d) $\sqrt{2}$

Answer : (a)

84. Pick out the correct statement.

(a) ℓ_1 is not reflexive.

(b) ℓ_1 is not separable.

(c) ℓ_2 is not reflexive.

(d) ℓ_2 is not separable.

Answer : (a)

85. Dual space of $(\ell_2, \|\cdot\|_2)$ is:

(a) $(\ell_2, \|\cdot\|_1)$

(b) $(\ell_\infty, \|\cdot\|_\infty)$

(c) $(\ell_2, \|\cdot\|_2)$

(d) $(\ell_2, \|\cdot\|_\infty)$

Answer : (c)

86. Which of the following is not a normed space?

(a) ℓ_∞ with $\|x\| = sup|x_i|$.

(b) c with $\|x\| = sup|x_i|$.

(c) c_0 with $\|x\| = sup|x_i|$.

(d) c with $\|x\| = \lim_{i\to\infty} |x_i|$.

Answer : (d)

87. Dual space of $(c_0, \|\cdot\|_\infty)$ is:

(a) $(c_0, \|\cdot\|_1)$

(b) $(c_0, \|\cdot\|_\infty)$

(c) $(\ell_1, \|\cdot\|_1)$

(d) $(\ell_\infty, \|\cdot\|_\infty)$

Answer : (c)

88. In a normed space$E$, which of the following need not true?

(a) The mapping $(x, y) \to x + y$ is continuous.

(b) The mapping $(k, y) \to k \cdot x$ is continuous.

(c) The mapping $x \to \|x\|$ is continuous.

(d) None of these.

Answer : (d)

89. With the usual inner product on $\mathbb{R}^3$, the vectors x, y, z forms an orthonormal basis. If $x = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$, $y = (0, 0, 1)$, then z can choose to be :

(a) $(0, 1, 0)$

(b) $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

(c) $(0, 0, 1)$

(d) $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0)$

Answer : (d)

90. Let E be a normed space and let d be the metric induced by the norm. Then for $x, y \in E$, $d(x - y, 0)$ equals:

(a) $d(x, 0) - d(y, 0)$

(b) $d(x, x - y)$

(c) $d(x, y)$

(d) None of these.

Answer : (c)

91. Which of the following is not true?

(a) The space c_0 is a closed subspace of ℓ_∞.

(b) The space s^* is a closed subspace of ℓ_∞.

(c) The space c is a closed subspace of ℓ_∞.

(d) The space $P[0,1]$ is not closed in $C[0,1]$.

Answer : (b)

92. Let X be an inner product space. Then the orthogonal complement of $\{0\}$ is:

(a) X

(b) $\{0\}$

(c) $X \backslash \{0\}$

(d) $X^\perp$

Answer : (a)

93. Let $X = \mathbb{R}^2$ with usual inner product, and $A : X \to X$ be defined by $A(x, y) = (x, x)$. Then $A^*(x, y)$ equals:

(a) (y, y)

(b) $(x, -x)$

(c) $(x + y, 0)$

(d) $(0, x + y)$

Answer : (c)

94. Let φ be the bounded linear functional on $\mathbb{R}^2$ defined by $\varphi(x, y) = 2x$. Then the unique element of $\mathbb{R}^2$ representing φ given by the Riesz representation theorem is:

(a) $(0, 1)$

(b) $(2, 0)$

(c) $(1, 0)$

(d) $(0, 2)$

Answer : (b)

95. Let $H = L_2[-\pi, \pi]$ and $x, y \in H$ be defined as $x(t) = e^{i5t}$ and $y(t) = e^{i10t}$. Then $\|x + y\|$ equals:

(a) $2\sqrt{\pi}$

(b) $\sqrt{2}$

(c) $\sqrt{2\pi}$

(d) $\pi\sqrt{2}$

Answer : (a)

96. In a Hilbert space, which of the following may not be true?

(a) $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

(b) $|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$

(c) If $x_n \to x, y_n \to y$, then $\langle x_n, y_n\rangle \to \langle x, y\rangle$.

(d) None of these.

Answer : (d)

97. Let E be a normed space and A, B be bounded linear operators on E. Then which of the following is true?

(a) $\|AB\| \leq \|A\| \cdot \|B\|$

(b) $\|AB\| \geq \|A\| \cdot \|B\|$

(c) $\|AB\| = \|A\| \cdot \|B\|$

(d) None of these.

Answer : (a)

98. Let M be a nonempty subset of an inner product space X. Which of the following is not true?

(a) $M^{\perp} = M^{\perp\perp\perp}$

(b) $M \subset M^{\perp\perp}$

(c) $M = M^{\perp\perp}$

(d) If $\overline{M} = X$, then $M^{\perp} = \{0\}$

Answer : (c)

99. Let H be a Hilbert space over $\mathbb{R}$ and $x, y \in H$, be such that $\|x\| = 4$, $\|y\| = 3$, and $\|x - y\| = 3$. Then $\langle x, y \rangle$ equals:

(a) 6

(b) 8

(c) 10

(d) None of these.

Answer : (b)

100. Let $x \in \ell_{\infty}$ be defined by $x = (x_n)$, where $x_n = \sin(\pi/n)$. Then $\|x\|_{\infty}$ equals:

(a) 2

(b) 0

(c) 1

(d) $\frac{1}{\sqrt{2}}$

Answer : (c)

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