Wednesday, November 24, 2021 12:39 PM

② Solve $(x^2D^2+4xD+2)y = x\log x$

Sol

Put $x = e^z$, $z = \log x$

$\because xD = D'$

$\because x^2D^2 = D'(D'-1)$

$$(D'(D'-1)+4D'+2)y = e^z z$$

$$(D'^2 - D' + 4D' + 2)y = e^z z$$

$$(D'^2+3D'+2)y = e^z z$$

→ Type 5

A.E D' by m

$$m^2+3m+2 = 0$$

$$m = -1, -2$$

$$CF = Ae^{-z} + Be^{-2z}$$

$$PI = \frac{1}{D'^2+3D'+2} e^z z$$

↓ D' by $D'+1$

$$= e^z \left(\frac{1}{(D'+1)^2+3(D'+1)+2} z \right)$$

↓ Type 3

$$= e^z \left(\frac{1}{D'^2+5D'+6} z \right)$$

$$= e^z \left(\frac{1}{6\left(1+\frac{D'^2+5D'}{6}\right)} \right) z$$

$$= \frac{e^z}{6}\left[1+\left(\frac{D'^2+5D'}{6}\right)\right]^{-1} z$$

$(1+x)^{-1} = 1 - x \dots$

$$= \frac{e^z}{6}\left[1 - \frac{D'^2}{6} - \frac{5D'}{6}\right] z$$

$$= \frac{e^z}{6}\left[z - \frac{D'^2(z)}{6} - \frac{5D'(z)}{6}\right]$$

$D' = \frac{d}{dz}$

$D'(z) = \frac{d}{dz}(z) = 1$

$D'^2(z) = 0$

$$PI = \frac{e^z}{6}(z - 5/6)$$

$$y = Ae^{-z} + Be^{-2z} + \frac{e^z}{6}(z - 5/6)$$

$$\boxed{y = \frac{A}{x} + \frac{B}{x^2} + \frac{x}{6}(\log x - 5/6)}$$

$e^z = x$, $e^{-z} = \frac{1}{x}$