

Test: CLA-T1

Course Code & Title: 18CSC301T & Formal Language and Automata Theory
Year & Sem: III Year / V sem

Date: 16.08.2023

Duration: 50 mins
Max. Marks: 25

Course Articulation Matrix:

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PSO 1	PSO 2	PSO 3
CO-1	3														3
CO-2		3	2												3
CO-3		3	3												3
CO-4		3	3												3
CO-5			3	1									2		3

Part – A

Instructions: Answer all

Q. No	Question	Marks	BL	CO	PO	PI Code
1.a	<p>"The Enchanted Forest Maze"</p> <p>In a mystical land, there lies an Enchanted Forest Maze. The maze is said to have magical properties, and it changes its pathways whenever someone enters it. Inside the maze, explorers can find hidden treasures, but only if they can navigate the maze correctly.</p> <p>The maze is represented as a set of interconnected rooms, and each room has two doors: one labeled '0' and the other labeled '1'. When a brave explorer enters a room, they face a decision. They can choose to go through the door labeled '0', which leads to one or more other rooms, or they can go through the door labeled '1', which leads to a different set of rooms.</p> <p>The initial starting point for all explorers is a room called "The Foyer". From "The Foyer," explorers can choose either door '0' or door '1', and they may face different options in their journey.</p> <p>The transition rules for the Enchanted Forest Maze are as follows:</p> <ul style="list-style-type: none"> ○ From "The Foyer", if an explorer chooses door '0', they may find themselves in either "The Foyer" again or in "The Magic Garden". ○ From "The Foyer", if an explorer chooses door '1', they may end up in "The Dark Cave". ○ From "The Magic Garden", if an explorer chooses door '1', they will find themselves in "The Enchanted Lake". ○ From "The Dark Cave", regardless of the chosen door, there are no further transitions. <p>The explorers aim to ensure their successful reach of the treasures hidden in "The Enchanted Lake" by transforming the mystical maze into a deterministic model. They intend to avoid making any unambiguous decisions on which door to choose, considering the input they encounter (0 or 1) in each room. Kindly help the explorer to find their way by giving a DFA based on the NFA in the provided scenario.</p>					

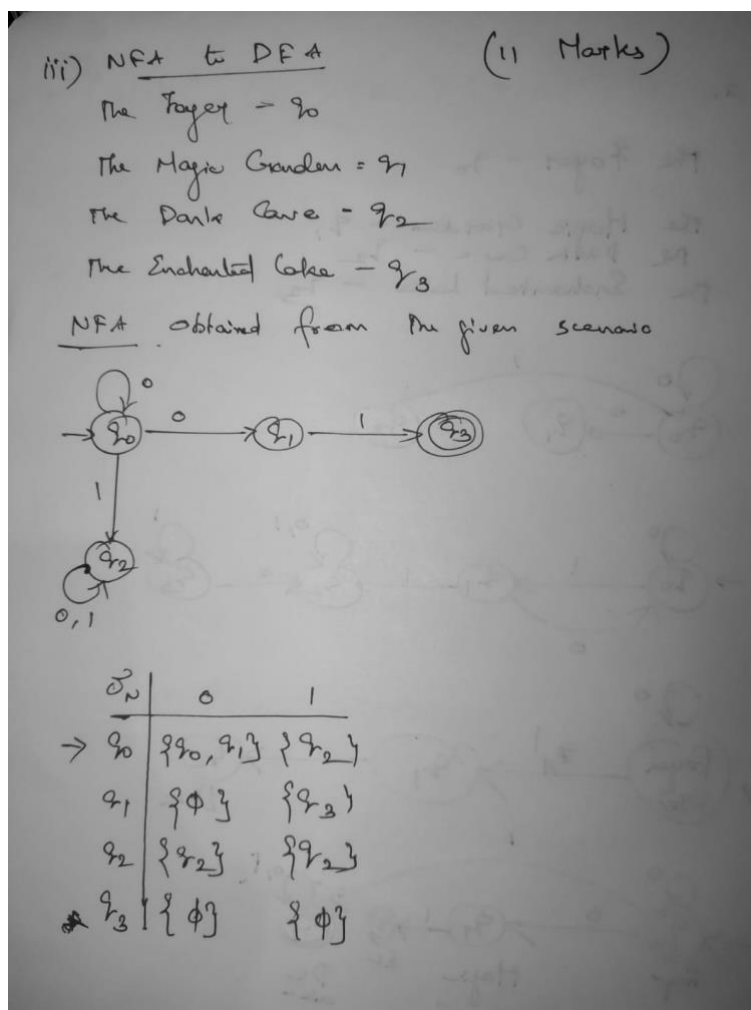
- i) In the mystical "Enchanted Forest Maze," explorers aim to transform the unpredictable maze into a deterministic model. This transformation mirrors the process of converting a Non-deterministic Finite Automaton (NFA) to a Deterministic Finite Automaton (DFA). What is the key motivation behind this conversion?
- To introduce epsilon (ϵ) transitions for added flexibility.
 - To reduce the number of states in the automaton.
 - To allow multiple transitions for the same input symbol.
 - To simplify the maze's interconnectedness.

Ans.: d. To simplify the maze's interconnectedness.

- ii) The process of converting the "Enchanted Forest Maze" from a Non-deterministic Finite Automaton (NFA) to a Deterministic Finite Automaton (DFA) follows the same principles as:
- Mapping a maze's pathways to a visual representation.
 - Creating a map legend for labyrinth navigation.
 - Transforming the maze's ambiguity into a clear navigation guide.
 - Introducing random choices into the maze's structure.

Ans.: c. Transforming the maze's ambiguity into a clear navigation guide.

- iii) Conversion of NFA to DFA from the given scenario



1	L1	1	1	1.3.1
1	L1	1	1	1.3.1
11	L3	1	1	1.3.1

$$\begin{aligned}\sigma_D([x_0], 0) &= \sigma_N(\{x_0\}, 0) \\ &= \{x_0, x_1\} \\ &= [x_0 \ x_1] \rightarrow \text{new state}\end{aligned}$$

$$\begin{aligned}\sigma_D([x_0], 1) &= \sigma_N(\{x_0\}, 1) \\ &= \{x_2\} \\ &= [x_2]\end{aligned}$$

$$\begin{aligned}\sigma_D([x_0 \ x_1], 0) &= \sigma_N(\{x_0, x_1\}, 0) \\ &= \sigma_N(x_0, 0) \cup \sigma(x_1, 0) \\ &= \{x_0, x_1\} \cup \{\emptyset\} \\ &= \{x_0, x_1\} \\ &= [x_0 \ x_1]\end{aligned}$$

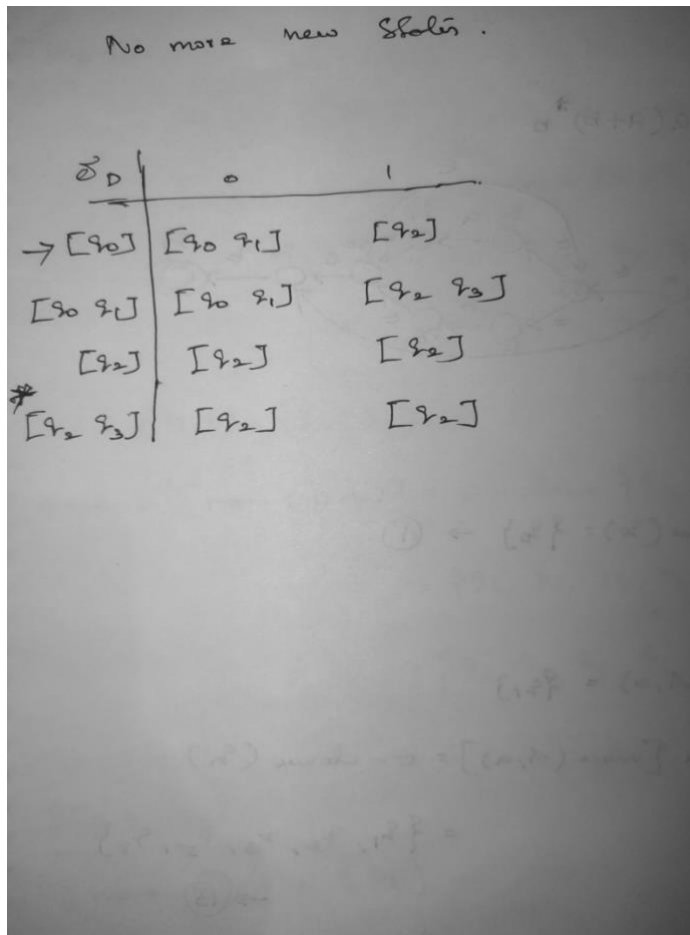
$$\begin{aligned}\sigma_D([x_0 \ x_1], 1) &= \sigma_N(\{x_0, x_1\}, 1) \\ &= \sigma_N(x_0, 1) \cup \sigma_N(x_1, 1) \\ &= \{x_2\} \cup \{x_3\} = \{x_2, x_3\} \\ &= [x_2 \ x_3]\end{aligned}$$

$$\begin{aligned}\sigma_D([x_2], 0) &= \sigma_N(\{x_2\}, 0) \\ &= \{x_2\} \\ &= [x_2]\end{aligned}$$

$$\begin{aligned}\sigma_D([x_2], 1) &= \sigma_N(\{x_2\}, 1) \\ &= \{x_2\} \\ &= [x_2]\end{aligned}$$

$$\begin{aligned}\sigma_D([x_2 \ x_3], 0) &= \sigma_N(\{x_2, x_3\}, 0) \\ &= \sigma_N(x_2, 0) \cup \sigma(x_3, 0) \\ &= \{x_2, \emptyset\} \\ &= \{x_2\} = [x_2]\end{aligned}$$

$$\begin{aligned}\sigma_D([x_2 \ x_3], 1) &= \sigma_N(\{x_2, x_3\}, 1) \\ &= \sigma_N(x_2, 1) \cup \sigma_N(x_3, 1) \\ &= \{x_2\} \cup \{\emptyset\} = [x_2]\end{aligned}$$



Or

1.b	<p>In a web application, users need to create usernames following specific criteria. The username should start with the character 'a', followed by any combination of 'a' and 'b' (including an empty string), and it must end with the character 'b'. To validate these usernames, design a deterministic model that accepts valid usernames and rejects invalid ones.</p>					
	<p>i) Which of the following usernames would be accepted by the deterministic model that enforces the specified criteria?</p> <p>a. ab b. abbba c. bba d. aaab e. aabbbbaa</p> <p>Ans.: Option a and d, both are correct</p>	1	L1	1	3	3.2.2
	<p>ii) The epsilon closure of a state in an Epsilon Nondeterministic Finite Automaton (ϵ-NFA) is:</p> <p>a. The set of all states reachable from the given state by following only epsilon (ϵ) transitions. b. The set of all states that have epsilon (ϵ) transitions to the given state. c. The set of all states reachable from the given state by following both epsilon (ϵ) transitions and input symbol transitions. d. The set of all states that share the same input symbol transitions with the given state.</p> <p>Ans.: a. The set of all states reachable from the given state by following only epsilon (ϵ) transitions.</p>	1	L1	1	3	3.2.2

iii) Write the proper regular expression for the above case and convert the same in to epsilon NFA.

5

L2

1

3

3.2.2

iv) Convert the obtained epsilon NFA to DFA.

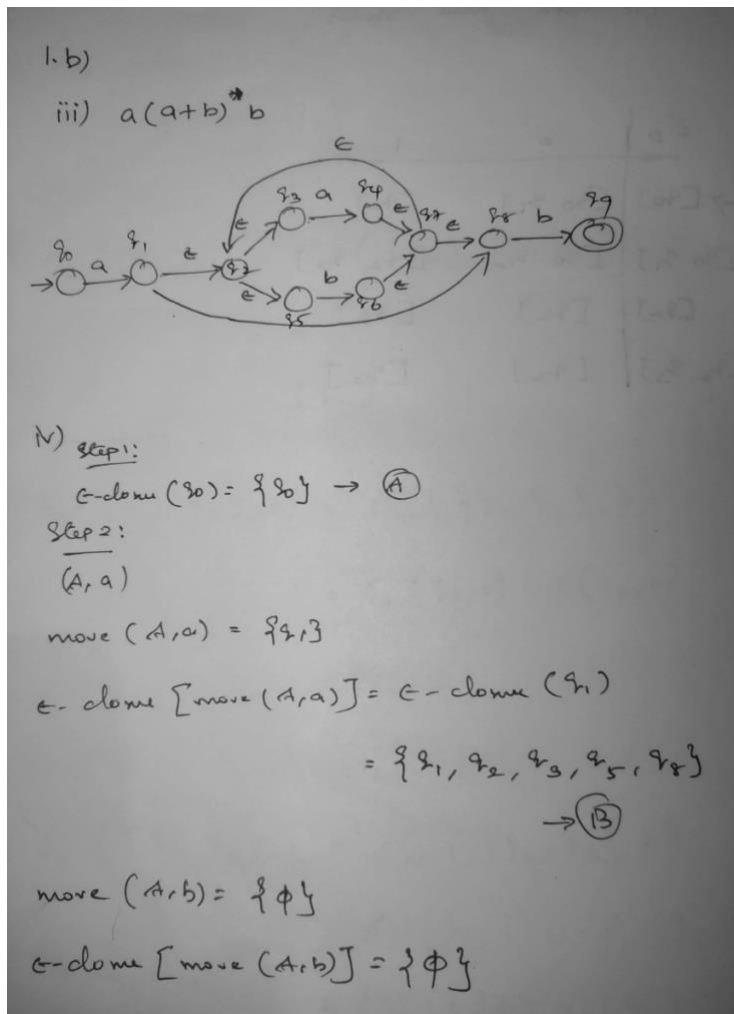
6

L3

1

3

3.2.2



Step 3:

$$\text{move}(B, a) = \{q_4\}$$

$$\begin{aligned} \leftarrow \text{clone} [\text{move}(B, a)] &= \leftarrow \text{clone} \{q_4\} \\ &= \{q_4, q_7, q_8, q_2, q_3, q_6\} \\ &= \{q_2, q_3, q_4, q_5, q_7, q_8\} \\ &\rightarrow \textcircled{C} \end{aligned}$$

$$\text{move}(B, b) = \{q_6, q_9\}$$

$$\begin{aligned} \leftarrow \text{clone} [\text{move}(B, b)] &= \leftarrow \text{clone} \{q_6, q_9\} \\ &= \{q_6, q_7, q_8, q_2, q_3, q_5, q_9\} \\ &= \{q_2, q_3, q_5, q_6, q_7, q_8, q_9\} \\ &\rightarrow \textcircled{D} \end{aligned}$$

Step 4:

$$\text{move}(C, a) = \{q_4\}$$

$$\leftarrow \text{clone} [\text{move}(C, a)] = \leftarrow \text{clone} \{q_4\} \rightarrow \textcircled{C}$$

$$\text{move}(C, b) = \{q_6, q_9\}$$

$$\leftarrow \text{clone} [\text{move}(C, b)] = \leftarrow \text{clone} \{q_6, q_9\} \rightarrow \textcircled{D}$$

Step 5:

$$\text{move}(D, a) = \{q_4\}$$

$$\leftarrow \text{clone} [\text{move}(D, a)] = \leftarrow \text{clone} \{q_4\} \rightarrow \textcircled{C}$$

$$\text{move}(D, b) = \{q_6, q_9\}$$

$$\leftarrow \text{clone} [\text{move}(D, b)] = \leftarrow \text{clone} \{q_6, q_9\} \rightarrow \textcircled{D}$$

δ_D	a	b
$\rightarrow A$	B	$\{\phi\}$
B	C	D
C	C	D
$\neq D$	C	D

2.a The air traffic control system involves three key regions: Alpha, Bravo, and Charlie. Aircraft are instructed to follow specific routes

	<p>through these regions. The system uses a finite state machine to ensure safe and coordinated movements.</p> <p>Here's how the finite state machine is defined:</p> <p>States: Alpha (A), Bravo (B), Charlie (C) Input Symbols: 0 (Maintain Current Altitude), 1 (Change Altitude) Initial State: Alpha (A) Accepting States: Alpha (A), Charlie (C)</p> <p>Transitions within the system:</p> <p>Aircraft in Alpha (A) that receive instruction 0 (Maintain Current Altitude) transition to Charlie (C). Aircraft in Alpha (A) that receive instruction 1 (Change Altitude) transition to Bravo (B). Aircraft in Bravo (B) that receive instruction 0 (Maintain Current Altitude) transition back to Alpha (A). Aircraft in Bravo (B) that receive instruction 1 (Change Altitude) stay in Bravo (B). Aircraft in Charlie (C) that receive instruction 0 (Maintain Current Altitude) remain in Charlie (C). Aircraft in Charlie (C) that receive instruction 1 (Change Altitude) transition to Bravo (B).</p> <p>Your mission is to apply DFA minimization techniques to optimize the air traffic control system. The goal is to simplify the state transitions while ensuring that the system still effectively manages aircraft movements.</p> <p>i) In the context of optimizing the air traffic control system's finite state machine, what is the main objective of DFA minimization?</p> <ol style="list-style-type: none"> To add more states to the system for increased flexibility. To introduce additional transitions to improve aircraft flow. To merge equivalent states and simplify the state machine. To create a more complex structure for advanced control. <p>Ans.: c. To merge equivalent states and simplify the state machine.</p> <p>ii) Considering the air traffic control system's original finite state machine, if two states are indistinguishable with respect to the language recognized by the DFA, what action should be taken during the minimization process?</p> <ol style="list-style-type: none"> Create new transitions between these states to distinguish them. Remove one of the indistinguishable states from the DFA. Merge these states into a single state. Add an additional input symbol to differentiate between them. <p>Ans.: b. Remove one of the indistinguishable states from the DFA.</p> <p>iii) Convert the given scenario's DFA in to minimized DFA</p>	1	L1	1	3	3.2.1
		1	L1	1	3	3.2.1
		10	L3	1	3	3.2.1

2. a) iii)

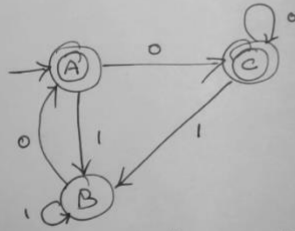
$$Q = \{A, B, C\}$$

$$\Sigma = \{0, 1\}$$

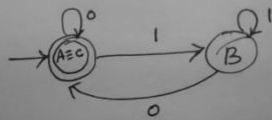
$$Q_0 = \{A\}$$

$$Q_F = \{A, C\}$$

As per the given scenario, the following DFA will be obtained,



[Any method can be used to bring the following minimized DFA]



Or

2.b

Suppose you have a staircase with 'n' steps, and you want to figure out how many ways you can reach the top by either taking one step or two steps at a time. Let's call this number of ways 'W(n)'. For example, for n=3, you can reach the top in three ways: (1+1+1), (1+2), or (2+1).

- i) The sum of the first 'n' odd natural numbers is given by the formula:
- $n(n+1)/2$
 - n^2
 - $n^2 + n$
 - $n^2 - n$

Ans.: b. n^2

- ii) Which of the following is an example of a base case in a mathematical induction proof?
- $P(0) = 0$
 - $P(1) = 1$
 - $P(2) = 4$
 - $P(3) = 9$

Ans.: b. $P(1)=1$

- iii) Using mathematical induction, prove that the number of ways 'W(n)' to reach the top of the staircase with 'n' steps is given by the formula:

$$W(n) = W(n-1) + W(n-2)$$

where $W(1) = 1$ and $W(2) = 2$.

Solution :

[Not necessary to solve exactly using the following approach]

1

L1

1

1

1.3.2

1

L1

1

1

1.3.2

10

L2

1

1

1.3.2

Step 1: Base Case

For $n=1$, there is only one way to reach the top: by taking one step. So, $W(1) = 1$, which matches our initial condition.

For $n=2$, there are two ways to reach the top: (1+1) or (2 steps at once). So, $W(2) = 2$, which also matches our initial condition.

Step 2: Inductive Hypothesis

Assume that the formula holds for some arbitrary positive integers 'k' and 'k+1', i.e., $W(k) = W(k-1) + W(k-2)$ and $W(k+1) = W(k) + W(k-1)$.

Step 3: Inductive Step

Now, we need to prove that the formula also holds for 'k+2'. That is, we need to show that $W(k+2) = W(k+1) + W(k)$.

Consider reaching step 'k+2'. To reach this step, you have two options: either take one step from step 'k+1' or take two steps from step 'k'.

If you take one step from step 'k+1', the number of ways to reach step 'k+2' is $W(k+1)$. (This is because the problem reduces to finding the number of ways to reach step 'k+1'.)

If you take two steps from step 'k', the number of ways to reach step 'k+2' is $W(k)$. (This is because the problem reduces to finding the number of ways to reach step 'k'.)

So, the total number of ways to reach step 'k+2' is the sum of the two possibilities above: $W(k+1) + W(k)$.

By our inductive hypothesis, we know that $W(k+1) = W(k) + W(k-1)$. Substituting this into the expression:

$$W(k+2) = W(k+1) + W(k)$$

But we also know that $W(k) = W(k-1) + W(k-2)$. So, we can replace $W(k)$ with $W(k-1) + W(k-2)$:

$$W(k+2) = (W(k) + W(k-1)) + W(k-1) + W(k-2)$$

$$W(k+2) = W(k-1) + W(k-1) + W(k-2) + W(k-2)$$

$$W(k+2) = 2W(k-1) + 2W(k-2)$$

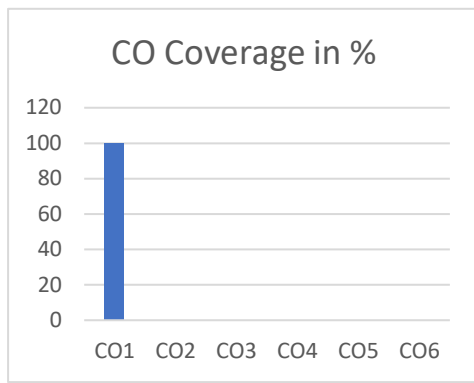
Now, we can factor out the common term of 2:

$$W(k+2) = 2(W(k-1) + W(k-2))$$

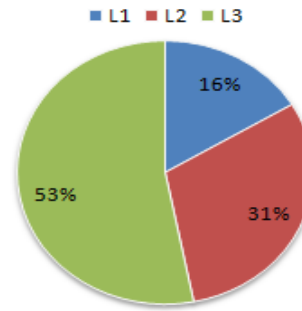
This confirms that the formula $W(n) = W(n-1) + W(n-2)$ holds for $n=k+2$.

***Program Indicators are available separately for Computer Science and Engineering in AICTE examination reforms policy.**

Course Outcome (CO) and Bloom's level (BL) Coverage in Questions



Bloom's Level Coverage



Approved by the Audit Professor/Course Coordinator