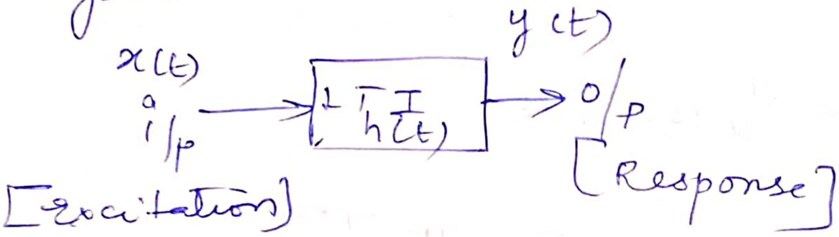
LTI System :-

→ Linear
→ Time Invariant

<u>Convolution</u> :- To provide the response of an LTI System.



h(t) → Impulse response of the S/m

→ Mathematical operation that provides relationship b/w x(t), y(t) and h(t)

→ By using Convolution we can find Zero state response of the S/m.

o/p of LTI S/m

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Re

$x(t)$ $\quad h(t)$

$\downarrow t=\tau \quad \downarrow t=\tau$ ①

$x(\tau) \quad h(\tau)$ ②

$\downarrow$

Time reversal

$\downarrow$

$h(-\tau)$ ③

$\downarrow$ Time shift

$h(-(\tau-t)) = h(t-\tau)$ ④

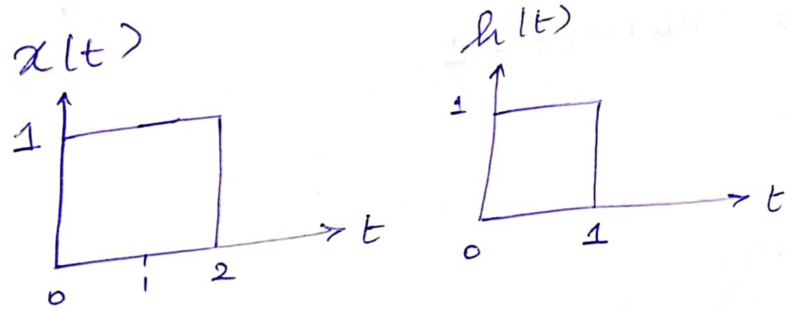
$\downarrow$ ⑤

$x(\tau)\, h(t-\tau)$

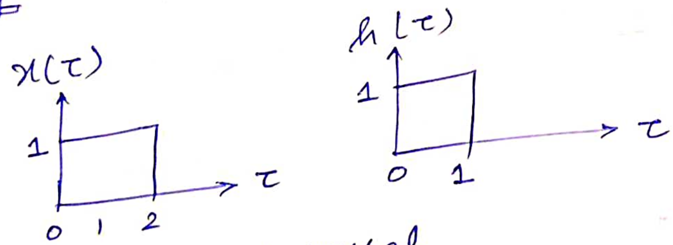
$\downarrow$ ⑥

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)$$

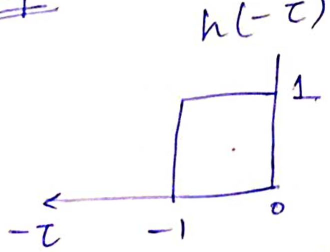
1)



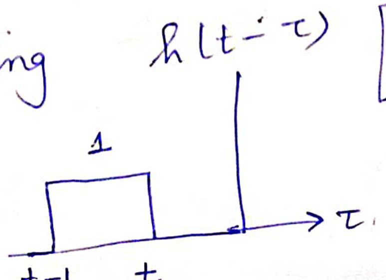
Step 1 : Replace t by τ



Step 2 :- Time reversal



Step 3: Time Shifting



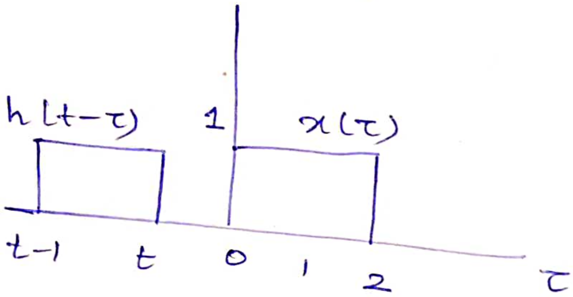
[t is unknown it may shift to right also but S/I want to move from −∞ to right so only shifting to left]

Step 4 : Calculate Integration

$x(\tau)\,h(t-\tau)$

~~$t\tau = 1$~~

$t-\tau = t-1$



Case 1 : Consider $t < 0$

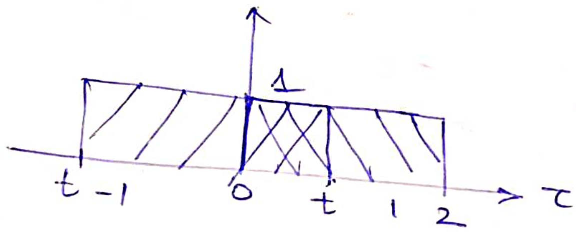
No overlap b/w $x(\tau)\,h(t-\tau)$

$x(\tau) \cdot h(t-\tau) = 0$

$\Rightarrow \boxed{y(t) = 0;\ t<0}$

Case 2 :- $t > 0$ and $t<1$ (ie) $0<t<1$

$x(\tau)\,h(t-\tau)$



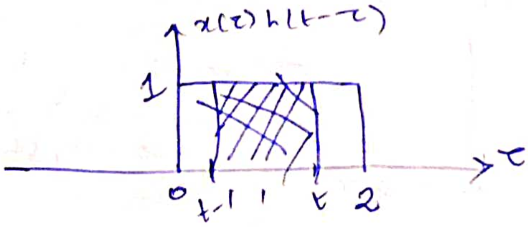
$0 - t \Rightarrow$ amplitude is 1

$$y(t) = \int_0^t x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_0^t 1 \cdot d\tau = [\tau]_0^t = t$$

$\boxed{y(t) = t}$

Case 3 : $t > 1$ (ie) Move $h(t-\tau)$ further to right and $t<2 \Rightarrow 1<t<2$

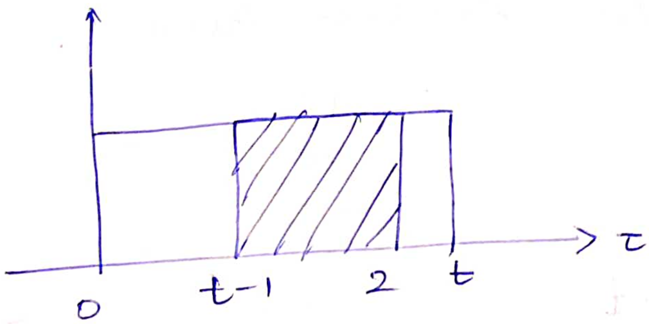


$$y(t) = \int_{t-1}^{t} 1\,d\tau$$

$\boxed{y(t) = 1}$

Case 4 :- $2 < t < 3$

$x(\tau)h(t-\tau)$



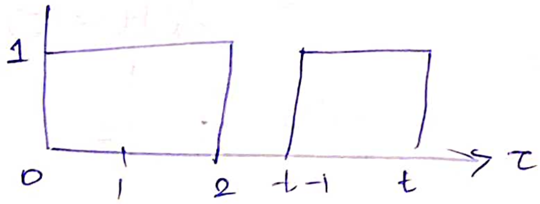
$$y(t) = \int_{t-1}^{2} 1 \, d\tau = [\tau]_{t-1}^{2}$$

$$= -t + 3$$

$$\boxed{y(t) = 3 - t}$$

Case 5 :- $t > 3$

$x(\tau)\, h(t-\tau)$

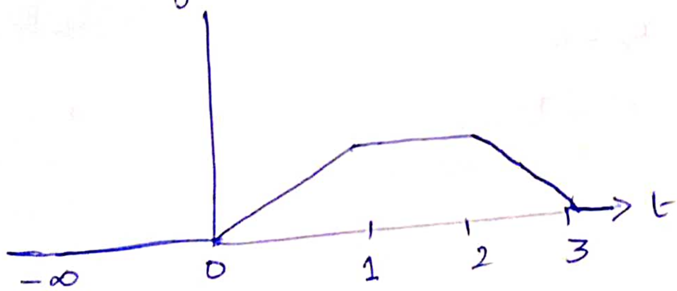


$$y(t) = 0$$

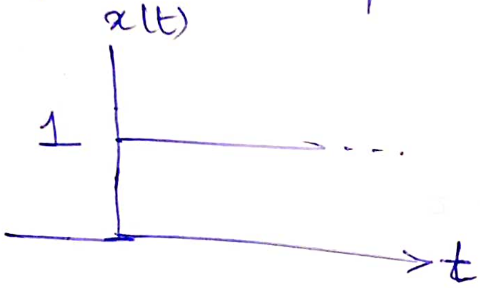
$$y(t) = \begin{cases} 0 \, ; & t < 0 \\ t \, ; & 0 < t < 1 \\ 1 \, ; & 1 < t < 2 \\ 3-t \, ; & 2 < t < 3 \\ 0 \, ; & t > 3 \end{cases}$$

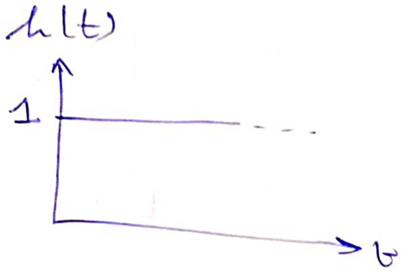
~~y(2)=0.4~~

$y(t)$

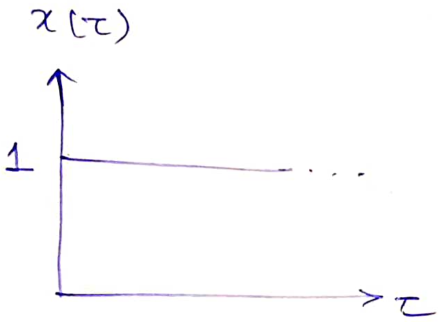


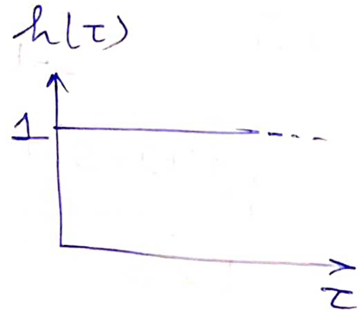
2) Find output $y(t)$ of an LTI system for the input and impulse response given.



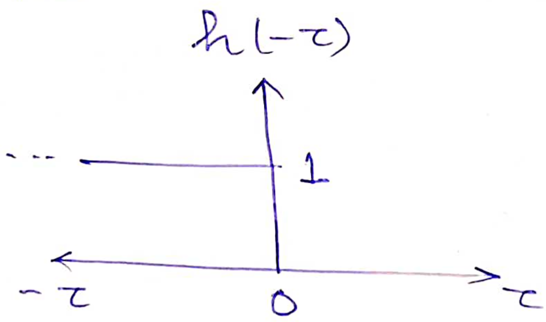


Step 1:-

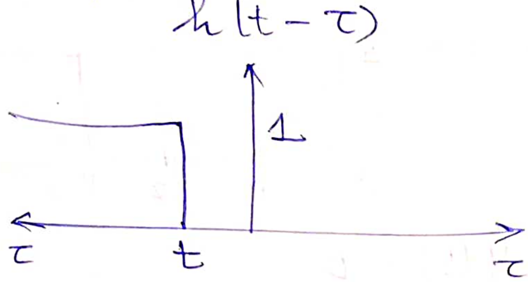




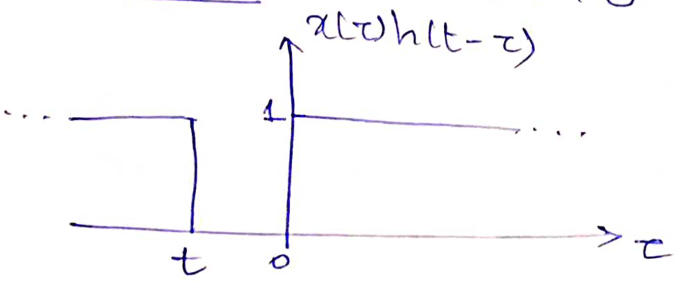
Step 2:-



Step 3



Step 4:- Case(i):- when $t < 0$

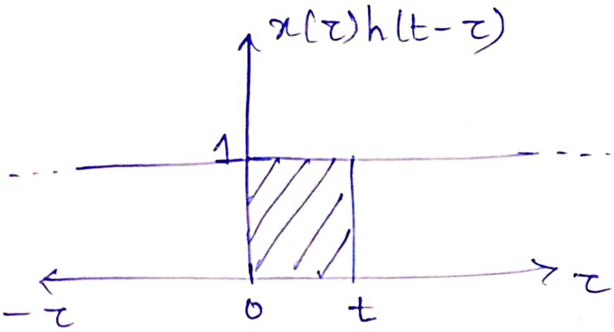


$\Rightarrow$ No overlap

$x(\tau)\, h(t-\tau) = 0$

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = 0$$

Case(ii) $t > 0$



$$x(\tau)\,h(t-\tau) = 1 \cdot 1$$

$$\int_0^t x(\tau)\,h(t-\tau)\,d\tau = \int_0^t 1 \cdot d\tau$$

$$= [\tau]_0^t \Rightarrow t$$

$$y(t) = \begin{cases} 0, & t<0 \\ t, & t>0 \end{cases} \Rightarrow r(t)$$

Convolution Integral

1) $x(t) = e^{-2t}u(t)$

$h(t) = u(t+2)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{-3\tau}u(\tau)\,u(t-\tau+2)\,d\tau$$

$$= \int_0^{\infty} e^{-3\tau}u(t-\tau+2)\,d\tau$$

$u(t-\tau+2) = 1$ for $t+2 \geq \tau$

i.e. $\tau \leq t+2$

$$= \int_0^{t+2} e^{-3\tau}\,d\tau = \left[\frac{e^{-3\tau}}{-3}\right]_0^{t+2}$$

$$= \frac{-1}{3}\left[e^{-3(t+2)} - e^0\right] = \frac{1-e^{-3(t+2)}}{3}$$

2) $x(t) = t \cdot u(t)$

$h(t) = u(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \tau \cdot u(\tau)\, u(t-\tau)\, d\tau$$

$u(\tau) = 1$ for $\tau > 0$

$u(t-\tau) = 1$ for $t > \tau$

$$= \int_{0}^{t} d\tau$$

$$\boxed{y(t) = t}$$

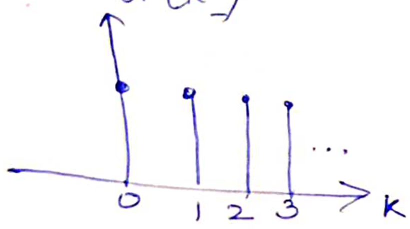
Convolution Sum:-

1) Calculate $y[n] = u[n] * u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, u[n-k]$$

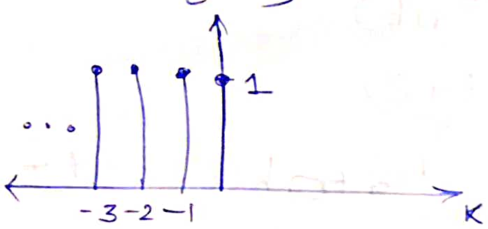
Step 1

$u[k]$



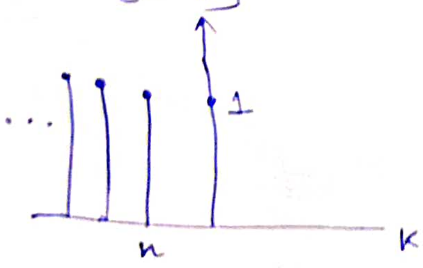
Step 2

$u[-k]$

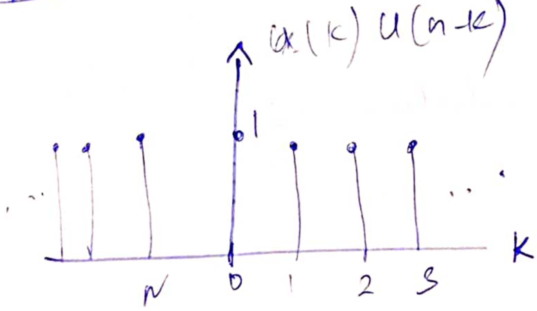


Step 3

$u[n-k]$



Step 4 :- Case (i) $n < 0$

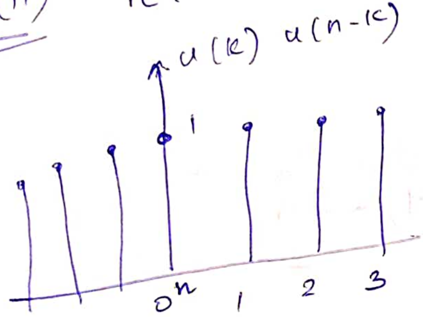


no overlap.

$x(K)\, u(n-K) = 0$

$$y(n) = 0 \; ; \; n < 0$$

Case (ii) $n > 0$



$u(K)\, u(n-K) = \underset{n=-\infty}{0} + \dots + \underset{n=1}{1} + \underset{n=2}{1} + 1$

$= n+1 \; ; \; n \geq 0$

$$y(n) = \cancel{\begin{cases} 0 \; ; & n < 0 \\ \sum_{k=0}^{n} 1 \; ; & n \end{cases}}$$

$$\boxed{y(n) = \begin{cases} 0 \; ; & n < 0 \\ \sum_{K=0}^{N} 1 \; ; & n \geq 0 \end{cases}}$$ (or)

$$\boxed{y(n) = \begin{cases} 0 \; ; & n < 0 \\ n+1 \; ; & n \geq 0 \end{cases}}$$ (or)

$$\boxed{y(n) = (n+1)\, u(n)}$$

H.W Calculate convolution of

$x(n) = a^n u[n]$

$h[n] = u[n]$

Linear Convolution

1) What is the linear Convolution of

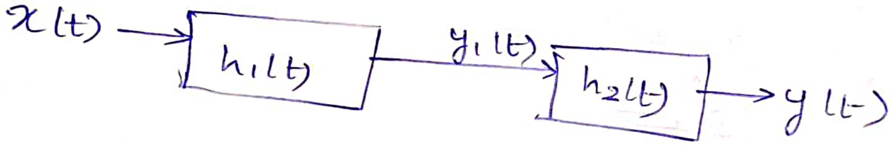
$x_1(n) = \{1, -2, 3, 1\}$

$x_2(n) = \{2, -3, -2\}$

	1	-2	3	1
2	2	-4	6	2
-3	-3	6	-9	-3
-2	-2	4	-6	-2

$y(n) = \{2, -7, 10, -3, -9, -2\}$

Cascade Connection of Systems



$y(t) = y_1(t) * h_2(t)$

$= \int_{-\infty}^{\infty} y_1(\tau)\, h_2(t-\tau)\, d\tau$

$y_1(\tau) = x(\tau) * h_1(\tau)$

$= \int_{-\infty}^{\infty} x(m)\, h_1(\tau - m)\, dm$

$y(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(m)\, h_1(\tau - m)\, h_2(t-\tau)\, dm\, d\tau$

Here put $\tau - m = n$, then we get

$y(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(m)\, h_1(n)\, h_2(t-m-n)\, dm\, dn$

$= \int_{-\infty}^{\infty} x(m) \left[\int_{-\infty}^{\infty} h_1(n)\, h_2((t-m)-n)\, dn \right] dm$

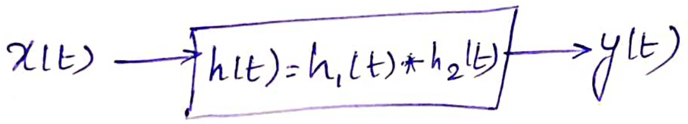
The integration square brackets indicate convolution of $h_1(t)$ and $h_2(t)$ evaluated at $t-m$ ie,

$$\int_{-\infty}^{\infty} h_1(n)\, h_2((t-m)-n)\,dn = h(t-m)$$

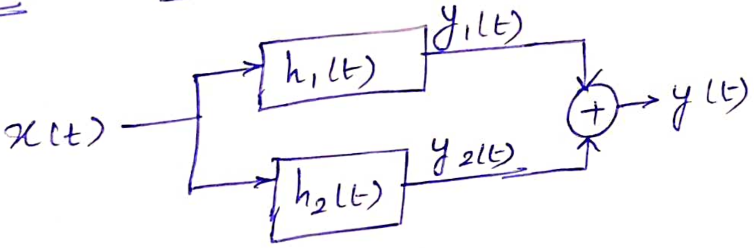
Hence,

$$\Rightarrow y(t) = \int_{-\infty}^{\infty} x(m)\, h(t-m)\,dm$$

$$= x(t) * h(t)$$



Parallel connection of systems :-



$$y(t) = y_1(t) + y_2(t)$$

$$= x(t) * h_1(t) + x(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h_1(t-\tau)\,d\tau + \int_{-\infty}^{\infty} x(\tau)\, h_2(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\left[h_1(t-\tau) + h_2(t-\tau)\right]d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\,d\tau$$

$$= x(t) * h(t)$$