

Envelopes

Def:- $f(x, y, \alpha) = 0$

Method 1 $f(x, y, \alpha + \delta\alpha) = 0$

Step 1 :- $\frac{\partial f}{\partial \alpha} = 0$

Method 2

$$A\alpha^2 + B\alpha + C = 0$$

Then envelope

$$B^2 - 4AC = 0$$

① Find the envelope of the family of the straight lines

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

Sol

$$y - mx = \pm \sqrt{a^2 m^2 + b^2}$$

$$(y - mx)^2 = a^2 m^2 + b^2$$

$$y^2 - 2ymx + m^2 x^2 - a^2 m^2 - b^2 = 0$$

$$(\underline{x^2 - a^2})m^2 - 2ymx + (\underline{y^2 - b^2}) = 0$$

envelope $B^2 - 4AC = 0$

$$A = (x^2 - a^2), B = -2yx$$

$$C = y^2 - b^2$$

$$4x^2 y^2 - 4(x^2 - a^2)(y^2 - b^2) = 0$$

$\div 4$

$$x^2 y^2 - x^2 y^2 + a^2 y^2 + b^2 x^2 - a^2 b^2 = 0$$

envelope $\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$