b. Solve $\left(D^2-6DD'+5D'^2\right)z=e^x\sinh y+xy$

29. a. Find the Fourier series to represent $\left(x-x^2\right)$ in the interval $(-\pi,\pi)$. Deduce the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-......\infty$.

(OR)

b. Find the Fourier series as far as the second harmonic to represent the function given by the following data.

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

30. a. A tightly stretched string with fixed end points $x=0$ *and* $x=l$ is initially at rest in its equilibrium position. If it is set vibrating by giving each of its points a velocity $\mu x(l-x)$, find $y(x,t)$.

(OR)

b. Find the solution of the equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions (i) $u(0,t)=0$ (ii) $u(l,t)=0$ for $t>0$ and (iii) $u(x,0)=\begin{cases}x, & 0\le x<l/2\\ l-x & l/2<x<l\end{cases}$

31. a. Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$. Hence evaluate $\int_0^\infty \frac{x\cos x-\sin x}{x^3}dx$.

(OR)

b. Evaluate $\int_0^\infty \frac{dx}{\left(x^2+9\right)\left(x^2+16\right)}$ using transforms techniques.

32. a.i. Find $Z\left[\cos^3 t\right]$

ii. Find the inverse Z-transform of $\frac{3z}{(z-1)(z-2)}$ using residues.

(OR)

b. Solve the difference equation $y(n+2)-7y(n+1)+12y(n)=2^n$ where $y(0)=0,\ y(1)=0$ by using Z-transform.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third Semester

18MAB201T – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The general integral of $z=xp+yq$ is
(A) $\phi(x+y,\ y+z)=0$ (B) $\phi(x-y,\ x/z)=0$
(C) $\phi\left(\frac{x}{y},\frac{y}{z}\right)=0$ (D) $\phi(x/y,\ y+z)=0$

2. The solution of $pq=x$ is
(A) $z=\frac{x^2}{2a}+ay+c$ (B) $z=\frac{y^2}{2a}+ax+c$
(C) $z=x+y+1$ (D) $z=x-ay$

3. The complementary function of $\left(D^3-3D^2D'\right)z=0$
(A) $z=f_1(y-x)+f_2(y-2x)+f_3(y+2x)$ (B) $z=f_1(y)+f_2(y)+f_3(y+3x)$
(C) $z=f_1(y)+f_2(y)+f_3(y-3x)$ (D) $z=f_1(y)+xf_2(y)+f_3(y+3x)$

4. The particular integral of $\left(D^2\right)z=x^3y$ is
(A) $\frac{x^5y}{20}$ (B) x^3y
(C) x^4y^2 (D) x^2y^2

5. The constant a_0 of the Fourier series for the function $f(x)=x$ is $0\le x\le 2\pi$
(A) π (B) 2π
(C) 3π (D) 0

6. If $f(x)$ is an even function in $(-\pi,\ \pi)$ then the value of b_n in the Fourier series expansion of $f(x)$ is
(A) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$ (B) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\, dx$
(C) 0 (D) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\, dx$

7. The value of fourier series of $f(x)$ in $0<x<2\pi$ at $x=0$ is
(A) $f(0)$ (B) $f(2\pi)$
(C) 0 (D) $\frac{f(0)+f(2\pi)}{2}$

8. For half-range cosine series of $f(x)=\cos x$ in $(0, \pi)$ the value of a_0 is
(A) 4 (B) $2/\pi$
(C) $4/\pi$ (D) 0

9. The proper solution of $u_t=\alpha^2 u_{xx}$ is
(A) $u=(Ax+B)C$ (B) $u=(A\cos\lambda x+B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$
(C) $u=\left(Ae^{\lambda x}+Be^{-\lambda x}\right)e^{\alpha^2\lambda^2 t}$ (D) $u=(At+B)$

10. The one dimensional heat equation in steady state is
(A) $\frac{\partial u}{\partial t}=0$ (B) $\frac{\partial^2 u}{\partial x^2}=0$
(C) $\frac{\partial^2 u}{\partial t^2}=0$ (D) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial t}=0$

11. One dimensional heat equation is used to find
(A) Density (B) Time
(C) Temperature distribution (D) Displacement

12. How many initial and boundary conditions are required to solve wave equation
(A) Two (B) Three
(C) Five (D) Four

13. Under Fourier cosine transform $f(x)=\frac{1}{\sqrt{x}}$ is
(A) Self-reciprocal function (B) Inverse function
(C) Cosine function (D) Complex function

14. $F[f(x-a)]=$ where $F\{f(x)\}=F(s)$
(A) $e^{ias}F(a)$ (B) $e^{ias}F(x)$
(C) $e^{ias}F(s)$ (D) $e^{iax}F(a)$

15. $F[f(x)*g(x)]=$
(A) $F(s)+G(s)$ (B) $F(s)G(s)$
(C) $F(s)-G(s)$ (D) $F(s)G(s)$

16. If $F(s)=F\{f(x)\}$ then $\int_{-\infty}^{\infty}|f(x)|^2\,dx$
(A) $\int_{-\infty}^{\infty}|F(s)|^2\,ds$ (B) $\int_{-\infty}^{\infty}|F(x)|^2\,dx$
(C) $\int_{0}^{\infty}|F(x)|^2\,dx$ (D) $\int_{0}^{\infty}|F(s)|^2\,ds$

17. What is $Z(6)$
(A) $\frac{z}{z-1}$ (B) $6.\frac{z}{z-1}$
(C) $\frac{1}{6}\frac{z}{z-1}$ (D) $\frac{z-1}{z}$

18. What is Z-transform of na^n?
(A) $\frac{z}{(z-a)^2}$ (B) $\frac{z}{(z-a)^3}$
(C) $\frac{a}{(z-a)^2}$ (D) $\frac{az}{(z-a)^2}$

19. Find $Z^{-1}\left[\frac{z}{z-a}\right]$
(A) a^{n+1} (B) a^{n-1}
(C) a^n (D) a

20. Poles of $\phi(z)=\frac{z^n}{(z-3)(z-4)}$
(A) $z=3,0$ (B) $z=3,4$
(C) $z=4,0$ (D) $z=0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the solution of $px^2+qy^2=z^2$.

22. Form a partial differential equation by eliminating arbitrary constants a and b from $(x-a)^2+(y-b)^2+z^2=c^2$.

23. Find the half range sine series for $f(x)=x(l-x)$ in $(0,l)$.

24. Find the Fourier sine transform of $f(x)=e^{-4x}$.

25. State various possible solutions of one dimensional heat equation.

26. Find $Z[1/n]$.

27. Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Solve by finding SI (i) $z=px+qy+2\sqrt{pq}$ (ii) Solve: $\frac{y^2 z}{x}p+xzq=y^2$.

(OR)