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Section: T2

18MAB204T– Probability and Queuing Theory
Tutorial Sheet - 4

PART–A

Q1.) i) $M_X(t) = (0.4e^t + 0.6)^8$

The M.G.F of a binomial distribution is given by,

$M_X(t) = (q+pe^t)^n$

on comparing, $p=0.4$, $q=0.6$, $n=8$

$E(X) = \text{Mean} = \mu_1' = np = 8\times 0.4 = 3.2$

ii) $M_X(t) = E(e^{tx})$

$M_Y(t) = E(e^{ty})$

$= E(e^{t(3X+2)})$

$= E(e^{3xtX} \cdot e^{2t})$

$= e^{2t}E(e^{tX}\cdot e^{3})$

$= e^{2t}(0.4e^{3t}+0.6)^8$

Q2.) $n=10$

$P(\text{getting a head}) = \frac{1}{2} = p$

$P(\text{not getting a head}) = 1-\frac{1}{2} = \frac{1}{2} = q(\text{tail})$

i) $P(X=7) = {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{10-7}$

$= \frac{120}{1024}$

In hundred sets $= \frac{100\times 120}{1024} \approx 11.72 \approx 12$ cases

Q2

ii) $P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^{10} + {}^{10}C_8 \left(\frac{1}{2}\right)^{10} + {}^{10}C_9 \left(\frac{1}{2}\right)^{10} + \ldots$

$= 0.172$

For 100 sets $= 0.172 \times 100 \approx 17$ cases

Q3.) $P(X=3) = P(X=4) \Rightarrow \frac{\cancel{e^{-\lambda}} \lambda^3}{3!} = \frac{\cancel{e^{-\lambda}} \lambda^4}{4!}$

i) $P(X=0) = ?$

ii) $P(X=2) = ?$

$\Rightarrow \lambda = \frac{4!}{3!} = 4$

i) $P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-4} = 0.0183$

ii) $P(X=2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{(4)^2 e^{-4}}{2} = 8e^{-4} = 0.1465$

Q4.) $P(X=2) = \frac{2}{3} P(X=1) \Rightarrow$

$\frac{\cancel{e^{-\lambda}} \lambda^2}{2} = \frac{2}{3}\left(\frac{\cancel{e^{-\lambda}} \lambda}{1}\right)$

i) $P(X=0)$

ii) $P(X=3)$

$\boxed{\lambda = \frac{4}{3}}$

i) $P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!}$

$= e^{-4/3}$

ii) $P(X=3) = \frac{e^{-\lambda}\lambda^3}{3!}$

$= \frac{e^{-4/3}\left(4/3\right)^3}{6}$

$= e^{-4/3} \times \frac{\cancel{64}^{32}}{27} \times \frac{1}{\cancel{6}_3} = \frac{32}{81} e^{-4/3}$

(x_i) No. of heads	(f_i) Frequencies	$x_i \cdot f_i$	$P(X=x) = {}^5C_x (0.4)^x (0.6)^{5-x}$	$N \times P(X=x)$
0	12	0	${}^5C_0 (0.4)^0 (0.6)^5 = 0.0778$	15.552 ≈ 16
1	56	56	${}^5C_1 (0.4)^1 (0.6)^4 = 0.2592$	51.840 ≈ 52
2	74	148	${}^5C_2 (0.4)^2 (0.6)^3 = 0.3456$	69.120 ≈ 69
3	39	117	${}^5C_3 (0.4)^3 (0.6)^2 = 0.2304$	46.080 ≈ 46
4	18	72	${}^5C_4 (0.4)^4 (0.6)^1 = 0.0768$	15.360 ≈ 15
5	1	5	${}^5C_5 (0.4)^5 (0.6)^0 = 0.0102$	2.048 ≈ 2
	200	398	verified	200

$$\text{mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{398}{200} = 1.99 \approx 2$$

$np = 2$

$5p = 2$

$\boxed{p = 0.4}$

$\boxed{q = 1 - 0.4 = 0.6}$

Q6.) $n = 5$

$P(\text{man of this age will be alive 30 yrs hence}) = \frac{2}{3} = p$

$P(\text{man of this age will not be alive 30 yrs hence}) = 1 - \frac{2}{3} = \frac{1}{3}\ (q)$

i) $P(\text{all men will be alive}) = P(X=5)$

$= {}^5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^0 = \frac{32}{243}$

ii) $P(\text{atleast one man will be alive}) = P(X \geq 1)$

$= 1 - P(X<1) = 1 - [P(X=0)]$

$= 1 - \left[{}^5C_0 \left(2/3\right)^0 \left(\frac{1}{5}\right)^3\right]$

$= \frac{242}{243}$

$\Rightarrow P(X=0)+P(X=1)+P(X=2)+P(X=\ldots$

$\Rightarrow {}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5+{}^5C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4+{}^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3+{}^5C_3\left(\ldots\right)\left(\frac{1}{3}\right)\ldots$

$\Rightarrow \frac{131}{243}$

Q7.) 10 am – 11 am $\Rightarrow \lambda_1 = 2\ (X)$

11 am – 12 noon $\Rightarrow \lambda_2 = 6\ (Y)$

$Z = X+Y$

$\lambda'(Z) = \lambda_1 + \lambda_2$

$\boxed{\lambda' = 8}$

(As X and Y are independent)

Required probability $\Rightarrow P(Z>4)$

$\Rightarrow 1 - P(Z \leq 4)$

$\Rightarrow 1 - P[(Z=0)+(Z=1)+(Z=2)+(Z=3)+(Z=4)\ldots$

$\Rightarrow 1 - \left[\frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda^1}{1!}+\frac{e^{-\lambda}\lambda^2}{2!}+\frac{e^{-\lambda}\lambda^3}{3!}+\frac{e^{-\lambda}\lambda^4}{4!}\ldots\right.$

$\Rightarrow 0.1173$

Q8.) $n = 1000$

$P(\text{failure}) = 0.001\ (p)$

$P(\text{not a failure}) = 1 - 0.001$

$= 0.999\ (q)$

i) $P(\text{none are defective}) = \left(\frac{999}{1000}\right)^{1000} = (0.999)^{1000} = 0.3676$

$P(X=0)$

ii) $P(\text{one is defective}) = {}^{1000}C_1\left(\frac{1}{1000}\right)^1\left(\frac{999}{1000}\right)^{999}$

$P(X=1)$

$= 0.3680$

ast 2 are defective) $= P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [0.3676 + 0.3680]$

$= 0.2644$

tmost 3 are defective) $= P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) +$
$P(X=3)$

$= 0.3676 + 0.3680 + {}^{1000}C_2 \left(\frac{1}{1000}\right)^2 \left(\frac{999}{1000}\right)^{998} + {}^{1000}C_3 \left(\frac{1}{1000}\right)^3 \left(\frac{999}{1000}\right)^{997}$

$= 0.922$

Tutorial Sheet-5

PART-A

Q1.) P(defective joint) = 0.001667 $\left(\frac{1}{600}\right)$

P(not a defective joint) $= \frac{e^{-\lambda}\lambda^{x}}{x!}$ Mean, $\lambda = \frac{1}{600} \times 25$

P(X=0) $= 0.041667$

$$= \frac{e^{-\lambda}\lambda^{0}}{0!}$$

$$= e^{-25/600}\left(\frac{25}{600}\right)^{0}$$

$$= e^{-25/600} = 0.95918$$

Sets free from defective joints = 10000 × 0.95918
≃ 9,592

Q2.) P(getting a 6 on a die) = $\frac{1}{6}$ (p)

P(not getting a 6 on die) = $1 - \frac{1}{6} = \frac{5}{6}$ (q)

$$P(X>5) = 1 - P(X \le 5)$$

$$= 1 - [P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)]$$

$$= 1 - p[q^0 + q^1 + q^2 + q^3 + q^4]$$

$$= 1 - \frac{1}{6}\left[\left(\frac{5}{6}\right)^0 + \left(\frac{5}{6}\right)^1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^4\right]$$

$$= 0.4019$$

showing excessive drift) = 0.10 (p) ⇒ q = 0.9

(fifth measuring device will show excessive drift) = ?

$$P(X=5) = p\,q^{x-1}$$

$$= (0.10)(0.9)^4$$

$$= 0.0656$$

Q4.) Mean $= \frac{a+b}{2} \Rightarrow a+b=2$ —— ①

$$a+b=2$$
$$-a+b=4$$
$$2b=6$$
$$\boxed{b=3}$$
$$\boxed{a=2-b=-1}$$
[-1,3]

Var $= \frac{(b-a)^2}{12} \Rightarrow b-a=4$ —— ②

$$f(x) = \frac{1}{4} \; ; \; -1<x<3$$

$$P(X<0) = \int_{-1}^{0} f(x)\,dx = \int_{-1}^{0} \frac{1}{4}\,dx = \frac{1}{4}$$

PART-B

Q5.) $\lambda = \frac{10}{500} = 0.002$

i) P(no defect) $= P(X=0) = \frac{e^{-\lambda}\lambda^x}{x!} = e^{-0.002} = 0.9801$

No. of packets $= 0.9801 \times 20000 = 19{,}604$ packets

ii) P(one defect) $= P(X=1) = \frac{e^{-\lambda}\lambda^x}{x!} = e^{-0.002}(0.002)^1 = 0.01960$

No. of packets $= 0.01960 \times 20000 = 392$ packets

iii) P(two defect) $= P(X=2) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.002}(0.002)^2 \times 20000}{2!}$

≈ 4 packets for 20,000 packets

Q6.)

(xi) No.of accidents	(fi) No.of day's	xi fi	$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$	N×PC
0	19	0	$e^{-1}(1)^0/0! = e^{-1} = 0.3678$	18.39 ~18
1	18	18	$e^{-1}(1)^1/1! = 0.3678$	18.39 ~18
2	8	16	$e^{-2}(1)^2/2! = 0.1839$	9.19 ~9
3	4	12	$e^{-3}(1)^3/3! = 0.0613$	3.065 ~3
4	1	4	$e^{-4}(1)^4/4! = 0.0153$	0.765 ~1
	50	50		verified 50

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{50}{50} = 1 = \lambda$$

Q7.) P(target shot at anyone shot) = 0.8

$p = 0.8$

$p+q=1 \Rightarrow q = 1-p = 1-0.8 = 0.2$

i) $P(X=6) = q^5 p = (0.2)^5(0.8) = 0.00016$

ii) $P(X<5) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= p[q^0+q^1+q^2+q^3]$

$= 0.8[1+0.2+(0.2)^2+(0.2)^3]$

$= 0.9984$

$$= \frac{1}{b-a}$$

$$f_X = \frac{1}{a-(-a)} \text{ in } (-a, a)$$

$$= \frac{1}{2a}$$

$$P(X>1) = \frac{1}{3}$$

$$P(X>1) = \int_1^a f(x)\,dx$$

$$\frac{1}{3} = \frac{1}{2a}\int_1^a 1\,dx$$

$$\frac{1}{3} = \frac{1}{2a}[a-1]$$

$$\Rightarrow \boxed{a=3}$$

$$P(|X|<1) = \int_{-1}^{1} f(x)\,dx$$

$$= \frac{1}{2a}\int_{-1}^{1} 1\,dx$$

$$= \frac{1}{2a}[x]_{-1}^{1}$$

$$= \frac{1}{2a} \times 2$$

$$= \frac{1}{a}$$

$$P(|X|>1) = 1 - P(|X|<1)$$

$$= 1 - \frac{1}{a}$$

As, $P(|X|<1) = P(|X|>1)$

$$\Rightarrow \frac{1}{a} = 1 - \frac{1}{a}$$

$$\Rightarrow \frac{2}{a} = 1$$

$$\Rightarrow \boxed{a=2}$$

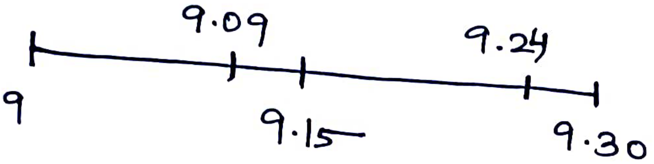
Tutorial sheet – 6

PART–A

Q1) The passenger arrives b/w 9 a.m and 9.30 a.m i.e. interval would $[0, 30]$.

$$f(x) = \frac{1}{b-a} = \frac{1}{30}$$

i) less than 6 minutes

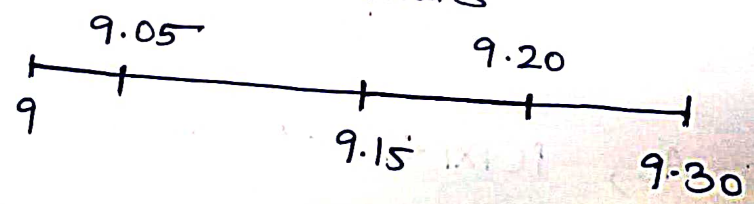


$$P(<6\ \text{min}) = P(9<X<15) + P(24<X<30)$$

$$= \int_{9}^{15} \frac{1}{30}\,dx + \int_{24}^{30} \frac{1}{30}\,dx$$

$$= \frac{1}{30}[6+6] = \frac{12}{30} = 0.4$$

ii) more than 10 minutes



$$P(<10\ \text{min}) = P(0<X<5) + P(15<X<20)$$

$$= \int_{0}^{5} \frac{1}{30}\,dx + \int_{15}^{20} \frac{1}{30}\,dx$$

$$= \frac{1}{30}[5+5] = \frac{1}{3}$$

$(X > 8000+t / X > t) = P(X > 8000)$

$$= \int_{8000}^{\infty} \frac{1}{10000} e^{-\frac{1}{10000}x} dx$$

$$= \cancel{\frac{1}{10000}} \left[\frac{e^{-\frac{1}{10000}x}}{\left(-\cancel{\frac{1}{10000}}\right)} \right]_{8000}^{\infty}$$

$$= \left[0 + e^{-\frac{8000}{10000}} \right]$$

$$= e^{-0.8} = 0.449$$

Mean = 300

$\Rightarrow \lambda = \frac{1}{300}$

) P(More than mean life) $= P(X > 300) = \int_{300}^{\infty} f(x) dx = \int_{300}^{\infty} \frac{1}{300} e^{-\frac{x}{300}} dx$

$$= \frac{1}{300} \left[\frac{e^{-x/300}}{(-x/300)} \right]_{300}^{\infty} = e^{-1} = 0.3678$$

ii) $P(X > 250+100 / X > 250) = P(X > 100)$

$$= \int_{100}^{\infty} f(x) dx = \int_{100}^{\infty} \frac{e^{-x/300}}{300} dx$$

$$= \frac{1}{300} \left[\frac{e^{-x/300}}{(-x/300} \right]_{100}^{\infty} = e^{-1/3}$$

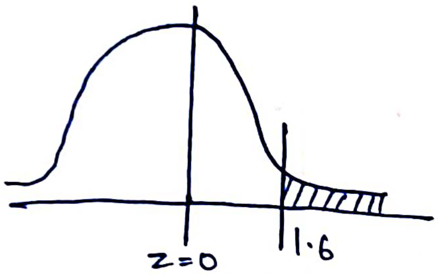
$$= 0.7165$$

Q4.) Mean = 172 cm

SD = 5 cm

$z = \frac{x-172}{5}$

$P(X > 180) = P\left(z \geq \frac{180-172}{5}\right) = P\left(z \geq \frac{8}{5}\right) = P(z > 1.6)$



$= 0.5 - P(0 < x < 1.6)$

$= 0.5 - 0.4452$

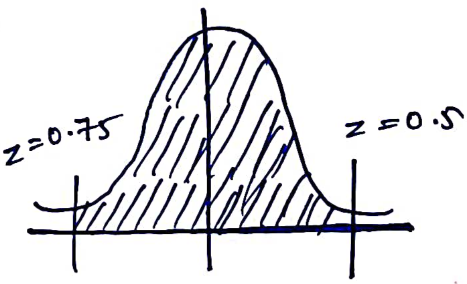
$= 0.0548$

For 1000 soldiers = 54.8 ~ 55

PART-B

Q5.) Mean = 8

SD = 4



i) $P(5 \leq X \leq 10) = ?$

$z = \frac{x-8}{4}$

$P(5 \leq x \leq 10) = P\left(\frac{5-8}{4} \leq x \leq \frac{10-8}{4}\right) = P(-0.75 \leq x \leq 0.5)$

$= P(0 \leq x \leq 0.75) + P(0 \leq x \leq 0.5)$

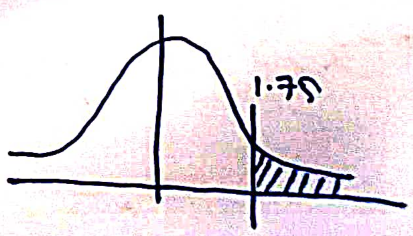
$= 0.2764 + 0.1915 = 0.4619$

ii) $P(X \geq 15) = ?$

$P\left(X \geq \frac{15-8}{4}\right) = P(z \geq 1.75)$

$= 0.5 - P(0 < x < 1.75)$

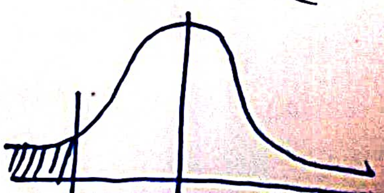
$= 0.5 - 0.4599 = 0.0401$



iii) $P(X \leq 5) = P\left(x \leq \frac{5-8}{4}\right) = P(z \leq -0.75) = 0.5 - P(x < 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

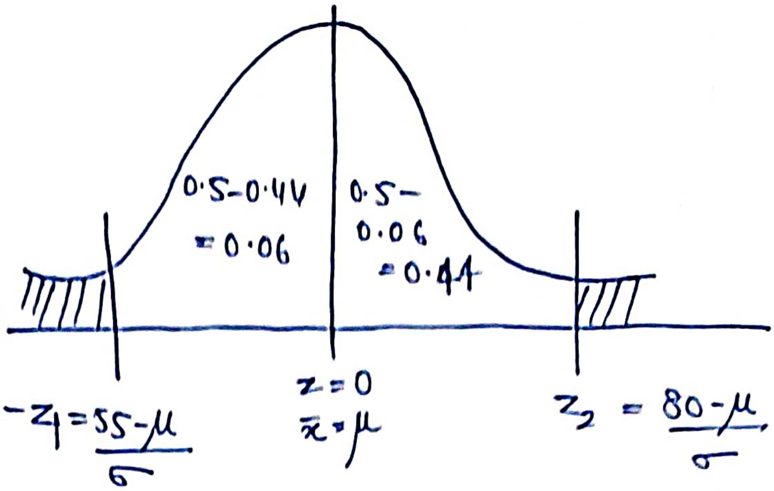


$x < 55$

$(x > 80) = 6\% = 0.06$

$(-z_1 \le z \le 0) = 0.06$

$P(0 \le z < z_2) = 0.44$



$-\frac{(55-\mu)}{\sigma} = 0.16$

$\Rightarrow \mu - 55 = 0.16\sigma$

$\Rightarrow \mu - 0.16\sigma = 55$ —— (1)

S^{lly}, $\frac{80-\mu}{\sigma} = 1.56 \Rightarrow \mu + 1.56\sigma = 80$ —— (2)

$$\begin{array}{r} \cancel{\mu} - 0.16\sigma = 55 \\ \cancel{\mu} + 1.56\sigma = 80 \\ - \quad\quad - \quad\quad - \\ \hline +1.72\sigma = +25 \end{array} \Rightarrow \boxed{\sigma = 14.53}$$

$\mu - 0.16(14.53) = 55$

$\Rightarrow \mu = 55 + 2.3248$

$\Rightarrow \boxed{\mu = 57.32}$