Tuesday, October 12, 2021 10:43 AM

Revision

① Write the characteristic equation
$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **Ans** $\lambda^2 - 2\lambda - 3 = 0$

② Find the eigen value of $\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$ **Ans** $\lambda = 2, -2$

③ Find the eigen value of A^2

$\begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$ **Ans** 9, 16, 25

④ The sum of the eigen values of a matrix is equal to <u>sum of a diagonal element</u>
trace of A (or)

⑤ The eigen values of a unit matrix are all equal and equal to <u>1</u>

⑥ If 1, 5 are the two eigen values of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ what is the third eigen value. **Ans** $\lambda = 1$

⑦ Find the sum and product of the eigen values of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$
Ans 6, 6

⑧ Find the eigen values of $-3A^{-1}$ for matrix $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$
Ans 3, −1

⑨ Find the sum of the squares of the eigen values
of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$
Ans: 53

⑩ If the sum of two eigen values and trace of 3x3 matrix A are equal
Find the value of $|A|$ **Ans** 0

⑪ Find the eigen value of $A - 3I$ for matrix $\begin{pmatrix} 3 & 2 \\ 3 & 3 \end{pmatrix}$ **Ans** $\sqrt{6}, -\sqrt{6}$

⑫ Eigen values of a 3rd order matrix A are 1, 2, −1
Find the value of trace of A. **Ans** 2

⑬ What are the eigen values of a null matrix of 3rd order?
Ans $\lambda = 0, 0, 0$

⑭ Write matrix for the Q.F **Ans** $\begin{bmatrix} 3 & -2 & 4 \\ -2 & -2 & 6 \\ 4 & 6 & -1 \end{bmatrix}$
$3x^2 - 2y^2 - z^2 + 12yz + 8zx - 4xy$

⑮ One of the eigen values $\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ 4 & -1 & -8 \end{bmatrix}$ is −9
Ans 7, −7
Find other two eigen values

⑯ For what values of k are the vectors $(2, 3, 0)^T$ $(1, 2, 0)^T$ and $(8, 13, k)^T$ linearly dependent?
Ans $k = 0$ $|A| = 0$

⑰ All eigen values of a real skew symmetric matrix are ___
Ans Purely imaginary

⑱ All ei " " symmetric matrix are <u>real</u>

⑲ If one of the eigen values of a matrix is zero, then the matrix is <u>singular</u>

⑳ Find the nature of the Q.F
$2y_1^2 + 3y_2^2 + 6y_3^2$ **Ans** positive definite

⑮
✓ $\lambda_1 + \lambda_2 + \lambda_3 = 7 - 8 - 8 = -9$
✓ $\lambda_1 \lambda_2 \lambda_3 = |A| = 441$

$\boxed{\lambda_3 = -9}$ $\lambda_1 + \lambda_2 = 0$
$\boxed{\lambda_1 = -\lambda_2}$

$\lambda_1 \lambda_2 = \frac{441}{-9} = -49$
$\cancel{-}\lambda_2 \lambda_2 = \cancel{-}49$
$\lambda_2^2 = 49$
$\lambda_2 = \pm 7$
$\boxed{\lambda = 7, -7}$

⑯ linearly depen $|A| = 0$

$$\begin{vmatrix} 2 & 1 & 8 \\ 3 & 2 & 13 \\ 0 & 0 & k \end{vmatrix} = 0$$

$\Rightarrow \boxed{k = 0}$

⑩ $\cancel{\lambda_1} + \cancel{\lambda_2} = \cancel{\lambda_1} + \cancel{\lambda_2} + \lambda_3$
$|A|$ $\boxed{\lambda_3 = 0}$
Product $\lambda_1 \lambda_2 \lambda_3 = |A|$
$\boxed{0 = |A|}$

⑪ $\begin{pmatrix} 3 & 2 \\ 3 & 3 \end{pmatrix}$
$\begin{vmatrix} 3-\lambda & 2 \\ 3 & 3-\lambda \end{vmatrix} = 0$
$\lambda^2 - 6\lambda + 3 = 0$
$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm 2\sqrt{6}}{2}$
$= 3 \pm \sqrt{6}$
$\lambda = 3 + \sqrt{6}$ $\lambda = 3 - \sqrt{6}$

$A - 3I \Rightarrow \lambda = \cancel{3} + \sqrt{6} - \cancel{3}$, $\lambda = \cancel{3} - \sqrt{6} - \cancel{3}$
$\boxed{\lambda = \pm\sqrt{6}}$