ii. Solve $9pqz^4 = 4(1+z^3)$.

(OR)

b. Solve the equation $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$.

29. a. Obtain the Fourier series of period $2l$ for the function

$$f(x) = l - x, \text{ in } 0 < x \le l$$
$$= 0, \quad \text{in } l \le x < 2l$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots + \infty = \frac{\pi^2}{8}$.

(OR)

b. Find the Fourier series of $y = f(x)$ in $(0, 2\pi)$ upto the third harmonic using the definition of y given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

30. a. A tightly stretched string of length π is fastened at both ends. The midpoint of the string is displaced by a distance d transversely and the string is released from rest in this position. Find the displacement of any point of the string at any subsequent time.

(OR)

b. A uniform bar of length l through which heat flows is insulted at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given by $k(lx - x^2)$ for $0 < x < l$, find the temperature distribution in the bar after time t.

31. a. Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & for\ |x| \le 1 \\ 0 & for\ |x| > 1 \end{cases}$ hence deduce $\int_0^\infty \left(\frac{\sin x}{x}\right)^4 dx = \frac{\pi}{3}$.

(OR)

b. Find Fourier sine and cosine transforms of e^{-x}. Hence evaluate $\int_0^\infty \frac{x^2}{(x^2+1)^2} dx$.

32. a.i. Find the Z transform of $(n+1)^2$ and $\sin(3n+5)$.

ii. Find the inverse Z-transform of $\frac{z^2}{(z-4)(z-3)}$.

(OR)

b. Solve the equation $y(k+2) + y(k) = 1$, $y(0) = y(1) = 0$, using Z-transform.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2019
3rd to 8th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The complete integral of $pq = 1$ is
   (A) $az = a^2x + y + ac$ (B) $z = ax + ay + c$
   (C) $az = x + y + c$ (D) $z = x + y + c$

2. The partial differential equation formed by eliminating the arbitrary function from $z = f(x^2 + y^2)$ is
   (A) $xp = yq$ (B) $xy = pq$
   (C) $py = qx$ (D) $x + p = y + q$

3. solve $(D^3 - 7DD'^2 - 6D'^3)z = 0$
   (A) $z = f_1(y-x) + f_2(y-2x) + f_3(y+3x)$ (B) $z = f_1(y-x) + f_2(y+2x) + f_3(y-3x)$
   (C) $z = f_1(y+x) + f_2(y-2x) + f_3(y+3x)$ (D) $z = f_1(y-x) + f_2(y-2x) + f_3(y-3x)$

4. The particular integral of $(D^3 - 2D^2D')z = e^{x+2y}$ is
   (A) $\frac{e^{x+2y}}{3}$ (B) $\frac{e^x}{3}$
   (C) e^{x+2y} (D) $\frac{-e^{x+2y}}{3}$

5. The constant a_0 of the Fourier series for the function $f(x) = x^2$ in $(0, 2l)$
   (A) $\frac{l^2}{3}$ (B) $\frac{4l^2}{3}$
   (C) $\frac{8l^2}{3}$ (D) l^2

6. The sum of the Fourier series of $f(x) = x + x^2$, in $-\pi < x < \pi$ at $x = \pi$ is
   (A) π (B) π^2
   (C) $\pi/2$ (D) $\pi^2/2$

7. If $f(x) = x$ in $-l \le x \le l$, then a_n
   (A) $\frac{-2l(-1)^n}{n\pi}$ (B) 0
   (C) l (D) $2l^2/3$

8. The RMS value of $f(x)=x$, in $-1\le x\le 1$ is
(A) 1
(B) 0
(C) $\frac{1}{\sqrt{3}}$
(D) -1

9. Classify the partial differential equation $4u_{xx}+4u_{xy}+u_{yy}=0$
(A) Elliptic
(B) Parabolic
(C) Hyperbolic
(D) Circular

10. The string is stretched between two fixed points $x=0$ and $x=l$, the boundary conditions are (t being positive)
(A) $y(0,t)=0,\ y(x,t)=0$
(B) $y(x,0)=0,\ \left(\frac{\partial y}{\partial t}\right)(x,0)=0$
(C) $y(0,t)=0,\ y(l,t)=0$
(D) $\left(\frac{\partial y}{\partial t}\right)(0,t)=0,\ \left(\frac{\partial y}{\partial t}\right)(l,t)=0$

11. The steady state temperature of a rod of length l whose ends are kept at 30°C and 40°C is
(A) $u=\frac{10x}{l}+30$
(B) $u=\frac{20x}{l}+30$
(C) $u=\frac{10x}{l}+20$
(D) $u=\frac{10x}{l}$

12. One dimensional wave equation is used to find
(A) Temperature
(B) Displacement
(C) Time
(D) Mass

13. If $F\{f(x)\}=F(s)$, then $F\left(e^{-iax}f(x)\right)$ is
(A) $F(s+a)$
(B) $F(s-a)$
(C) $F(as)$
(D) $F(a/s)$

14. $F_c(x.f(x))$ is
(A) $i\frac{dFs(s)}{ds}$
(B) $\frac{-dF_s(s)}{ds}$
(C) $i\frac{dFs(s)}{ds}$
(D) $\frac{dFs(s)}{ds}$

15. The Fourier cosine transform of e^{-ax} is
(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$
(B) $\sqrt{\frac{1}{\pi}}\frac{s}{s^2+a^2}$
(C) $\sqrt{\frac{1}{\pi}}\frac{a}{s^2+a^2}$
(D) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

16. $F(f(ax))$ is
(A) $\frac{1}{s}F(s/a)$
(B) $\frac{1}{a}F(a/s)$
(C) $\frac{1}{s}F(as/a+1)$
(D) $\frac{1}{|a|}F(s/a)$

17. Z-transform of $\frac{1}{n!}$
(A) $\frac{1}{e^z}$
(B) e^{z^2}
(C) $e^{2/z}$
(D) e^{z^3}

18. $z\left(n^2\right)$ is
(A) $\frac{z}{(z-1)^3}$
(B) $\frac{z(z+1)}{(z)^3}$
(C) $\frac{z(z+1)}{(z-1)^3}$
(D) $\frac{z+1}{(z-1)^3}$

19. $z\left(\sin\frac{n\pi}{2}\right)$ is
(A) $\frac{z^2}{z-1}$
(B) $\frac{z}{z^2+4}$
(C) $\frac{z}{z^2+1}$
(D) $\frac{z^2}{z^2+1}$

20. Poles of $\phi(z)=\frac{z^n}{(z-1)(z-2)}$ are
(A) $z=1,\ z=0$
(B) $z=1, z=2$
(C) $z=0, z=2$
(D) $z=0$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Solve $p-q=\log(x+y)$

22. Find the Fourier series of $f(x)=x^2$ in $-\pi\le x\le\pi$.

23. Classify the PDE $(x+1)f_{xx}+2(x+2)f_{xy}+(x+3)f_{yy}=0$.

24. Find the Fourier transform of $f(x)$ given by $f(x)=\begin{cases}x & for\ |x|\le a\\ 0 & for\ |x|>a\end{cases}$.

25. Find $z\left(na^n\right)$.

26. Solve $\left(D^2-DD'\right)z=\cos x\cos 2y$.

27. Find $Z(f(n))$ where $f(n)=an^2+bn+c$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a.i. Find the partial differential equation of all planes which are at a constant distance k from the origin.

8. The RMS value of $f(x)=x$ in $-1\le x\le 1$ is

(A) 1 (B) 0

(C) $\frac{1}{\sqrt{3}}$ (D) -1

9. Classify the partial differential equation $4u_{xx}+4u_{xy}+u_{yy}=0$

(A) Elliptic (B) Parabolic

(C) Hyperbolic (D) Circular

10. The string is stretched between two fixed points $x=0$ and $x=l$, the boundary conditions are (t being positive)

(A) $y(0,t)=0,\ y(x,t)=0$ (B) $y(x,0)=0,\ \left(\frac{\partial y}{\partial t}\right)(x,0)=0$

(C) $y(0,t)=0,\ y(l,t)=0$ (D) $\left(\frac{\partial y}{\partial t}\right)(0,t)=0,\ \left(\frac{\partial y}{\partial t}\right)(l,t)=0$

11. The steady state temperature of a rod of length l whose ends are kept at 30°C and 40°C is

(A) $u=\frac{10x}{l}+30$ (B) $u=\frac{20x}{l}+30$

(C) $u=\frac{10x}{l}+20$ (D) $u=\frac{10x}{l}$

12. One dimensional wave equation is used to find

(A) Temperature (B) Displacement

(C) Time (D) Mass

13. If $F\{f(x)\}=F(s)$, then $F\left(e^{-iax}f(x)\right)$ is

(A) $F(s+a)$ (B) $F(s-a)$

(C) $F(as)$ (D) $F(a/s)$

14. $F_c(x.f(x))$ is

(A) $i\frac{dFs(s)}{ds}$ (B) $\frac{-dF_s(s)}{ds}$

(C) $i\frac{dFs(s)}{ds}$ (D) $\frac{dFs(s)}{ds}$

15. The Fourier cosine transform of e^{-ax} is

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$ (B) $\sqrt{\frac{1}{\pi}}\frac{s}{s^2+a^2}$

(C) $\sqrt{\frac{1}{\pi}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

16. $F(f(ax))$ is

(A) $\frac{1}{s}F(s/a)$ (B) $\frac{1}{a}F(a/s)$

(C) $\frac{1}{s}F(as/a+1)$ (D) $\frac{1}{|a|}F(s/a)$

17. Z-transform of $\frac{1}{n!}$

(A) $\frac{1}{e^z}$

(C) $e^{2/z}$

18. $Z(n^2)$ is

(A) $\frac{z}{(z-1)^3}$

(C) $\frac{z(z+1)}{(z-1)^3}$

19. $z\left(\sin\frac{n\pi}{2}\right)$ is

(A) $\frac{z^2}{z-1}$

(C) $\frac{z}{z^2+1}$

20. Poles of $\phi(z)=\frac{z^n}{(z-1)(z-2)}$ are

(A) $z=1, z=0$

(C) $z=0, z=2$

PART – I

Answer A

21. Solve $p-q=\log(x+y)$

22. Find the Fourier series of $f(x)=$

23. Classify the PDE $(x+1)f_{xx}+2($

24. Find the Fourier transform of f

25. Find $z(na^n)$.

26. Solve $(D^2-DD')z=\cos x$

27. Find $Z(f(n))$ where $f(n)$

P

28. a.i. Find the partial differentia origin.

B.Tech Degree Examinations, May 2019

Code / Title : 15MA201 - Transforms and Boundary Value Problems

Date of Exam : 20-05-2019 (FN)

Max. Marks : 100

Part - A

$20 \times 1 = 20$

1. (A.) $az = a^2x + y + ac$.
2. (C.) $py = qx$
3. (A.) $f_1(y-x) + f_2(y-2x) + f_3(y+3x)$
4. (D.) $\frac{-e^{x+2y}}{3}$
5. (C.) $\frac{8l^2}{3}$
6. (B.) π^2
7. (B.) 0
8. (C.) $\frac{1}{\sqrt{3}}$
9. (B) Parabolic
10. (C.) $y(0,t) = 0$; $y(l,t) = 0$.
11. (A.) $u = \frac{10x}{l} + 30$.
12. (B.) displacement
13. (B.) $F(s-a)$
14. (D.) $\frac{d}{ds}[F_s(s)]$
15. (D.) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2+s^2}$
16. (D.) $\frac{1}{|a|} F\left(\frac{s}{a}\right)$
17. (A) $1/e^z$
18. (C.) $\frac{z(z+1)}{(z-1)^3}$
19. (C.) $\frac{z}{z^2+1}$
20. (B.) $z = 1$ and $z = 2$

PART- B

21. $p-q = \log(x+y)$

Aux. eq'ns are $\frac{dx}{1} = \frac{dy}{-1} = \frac{dz}{\log(x+y)}$. (1m)

$\frac{dx}{1} = \frac{dy}{-1} \Rightarrow x = -y+a \Rightarrow \boxed{x+y=a}$ (1m)

$\frac{dx}{1} = \frac{dz}{\log(x+y)} = \frac{dz}{\log(a)} \Rightarrow x = \frac{1}{\log(a)} z + b \Rightarrow \boxed{x - \frac{z}{\log(x+y)} = b}$ (1m)

$\Rightarrow \boxed{\phi\left(x+y,\ x - \frac{z}{\log(x+y)}\right) = 0}$ (1m)

22. $f(x) = x^2$ in $-\pi < x < \pi$. $\Rightarrow$ Even function $\Rightarrow \boxed{b_n = 0}$ (1m)

$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2\pi^2}{3}$ | $a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\, dx = \frac{4(-1)^n}{n^2}$ (2m)

$\boxed{f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx}$ (1m)

23. $(x+1) f_{xx} + 2(x+2) f_{xy} + (x+3) f_{yy} = 0$

$A = x+1$ | $B = 2(x+2)$ | $C = x+3$. $\longrightarrow$ (1m)

$B^2 - 4AC = 4(x+2)^2 - 4(x+1)(x+3)$

$= 4[x^2+4x+4 - x^2 - 4x - 3]$

$= 4 > 0. \Rightarrow$ Hyperbolic at all regions $\longrightarrow$ (3m)

24. $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x) e^{isx} dx = \frac{1}{\sqrt{2\pi}}\int_{-a}^{a} x e^{isx} dx$ $\longrightarrow$ (1m)

$= \frac{1}{\sqrt{2\pi}}\left[\frac{x e^{isx}}{is} - \frac{e^{isx}}{(is)^2}\right]_{-a}^{a}$ $\longrightarrow$ (3m)

$= \frac{2i}{s^2}\frac{1}{\sqrt{2\pi}}\cdot[\sin sa - as\cos sa]$

25. $Z[na^n] = -z \cdot \frac{d}{dz}\left[\frac{z}{z-a}\right] = \frac{az}{(z-a)^2}$ (2m) (2m)

26. $(D^2 - DD')z = \cos x \cos 2y$.

MARKS CAN BE GIVEN TO ANY OTHER VALID METHOD ALSO.

Aux. Eq'n $\Rightarrow m^2 - m = 0 \Rightarrow m = 0, 1$. $CF = \phi_1(y) + \phi_2(y+x)$ $\longrightarrow$ (1m)

$\longrightarrow$ (1m)

$$PI = \frac{1}{D(D-D')} \cdot \frac{D(D+D')}{D(D+D')} \cos x \cdot \cos 2y$$

$$= \frac{D(D+D')}{D^2(D^2-D'^2)} \cos x \cos 2y = \frac{D(D+D')}{(-3)} (\cos x \cos 2y)$$

$$= \frac{-1}{3}\left[-\cos x \cos 2y + 2\sin x \sin 2y\right]$$ $\longrightarrow$ (1m)

$$\therefore z = \phi_1(y) + \phi_2(y+x) - \frac{1}{3}\left[2\sin x \sin 2y - \cos x \cos 2y\right]$$ $\longrightarrow$ (1m)

27. $Z[an^2 + bn + c] = a \cdot Z[n^2] + b[Z(n)] + c\, Z(1)$. $\longrightarrow$ (1m)

$$Z(n^2) = Z(n \cdot n) = -z \cdot \frac{d}{dz}\left(\frac{z}{(z-1)^2}\right) = \frac{z(z+1)}{(z-1)^3}$$ $\longrightarrow$ (1m)

$$\therefore Z[an^2 + bn + c] = \frac{az(z+1)}{(z-1)^3} + \frac{bz}{(z-1)^2} + \frac{cz}{(z-1)}$$ $\longrightarrow$ (2m)

PART – C

$5 \times 12 = 60$

28(a) (i) Eq'n of plane: $x\cos\alpha + y\, ~~\sin\alpha~~ \cos\beta + z\cos\gamma = k$

$\cos\alpha = a$; $\cos\beta = b$; $\cos\gamma = c$ are direction cosines of normal to plane.

$\Rightarrow a^2 + b^2 + c^2 = 1$.

$\therefore$ Eq'n of plane is $ax + by + \sqrt{1-a^2-b^2}\, z = k$. $\longrightarrow$ (2m)

Partially diff., $\Rightarrow$ $\left.\begin{array}{l} a + \sqrt{1-a^2-b^2}\, p = 0 \\ b + \sqrt{1-a^2-b^2}\, q = 0 \end{array}\right\}$

$$\Rightarrow \frac{a}{p} = \frac{b}{q} = -\sqrt{1-a^2-b^2} = \lambda \text{ (say)}.$$ $\longrightarrow$ (1m)

$\Rightarrow a = \lambda p$ and $b = \lambda q$ and $\sqrt{1-\lambda^2(p^2+q^2)} = -\lambda$

$\Rightarrow 1-\lambda^2(p^2+q^2) = \lambda^2$

$\Rightarrow \lambda = -\dfrac{1}{\sqrt{1+p^2+q^2}}$ $\longrightarrow$ 2m

$\therefore$ Eq'n of plane becomes (∵ λ is −ve)

$\boxed{z = px+qy+k\sqrt{1+p^2+q^2}}$ $\longrightarrow$ 1m

28(a)(ii) $9pqz^4 = 4(1+z^3)$.

Solution is of the form $z = z(u) = z(x+ay)$ where $\longrightarrow$ 1m

a is arbitrary constant.

$\Rightarrow p = \dfrac{dz}{du}$ and $q = a\cdot\dfrac{dz}{du}$. $\longrightarrow$ 1m

$\therefore \Rightarrow 9a\left(\dfrac{dz}{du}\right)^2 z^4 = 4(1+z^3) \Rightarrow \dfrac{\sqrt{a}}{2}\,\dfrac{3z^2}{\sqrt{1+z^3}}\,dz = du$ $\longrightarrow$ 2m

Integrating $\Rightarrow \sqrt{a}\cdot\sqrt{1+z^3} = u+b$

Substituting $u = x+ay$ and squaring $\Rightarrow$ $\longrightarrow$ 2m

$\boxed{a(1+z^3) = (x+ay+b)^2}$ is complete solution.

(OR)

28(b) $(D^2+2DD'+D'^2)z = x^2y+e^{x-y}$.

Aux. Eqn. : $m^2+2m+1=0 \Rightarrow m=-1,-1$ $\longrightarrow$ 3m

$\boxed{CF: f_1(y-x)+x f_2(y-x)}$

$PI_1 = \dfrac{1}{(D+D')^2}x^2y = \dfrac{1}{D^2}\left(1+\dfrac{D'}{D}\right)^{-2}(x^2y)$ $\longrightarrow$ 4m

$= \dfrac{1}{D^2}\left[x^2y - \dfrac{2}{D}x^2\right] = \dfrac{x^4y}{12} - \dfrac{x^5}{30}$.

$$PI_2 = \frac{1}{(D+D')^2} e^{x-y} = \frac{x^2}{2!} e^{x-y} \longrightarrow 4m$$

$$\therefore \boxed{z = f_1(y-x) + x \cdot f_2(y-x) + \frac{x^4 y}{12} - \frac{x^5}{30} + \frac{x^2}{2} e^{x-y}} \longrightarrow 1m$$

29(a) $f(x) = \begin{cases} l-x ; & 0 < x \le l \\ 0 ; & l \le x < 2l. \end{cases}$; period $= 2l$.

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx = \frac{l}{2} \longrightarrow (3m)$$

$$a_n = \frac{1}{l}\int_0^{l} (l-x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{l}\left[(l-x)\left(\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (-1)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^l$$

$$= \frac{l}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$\boxed{a_n = \begin{cases} \frac{2l}{n^2\pi^2} ; & n \text{ is odd} \\ 0 ; & n \text{ is even} \end{cases}} \longrightarrow (3m)$$

$$b_n = \frac{1}{l}\int_0^{2l} (l-x)\sin\frac{n\pi x}{l}\,dx = \frac{1}{l}\left[(l-x)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (-1)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^l$$

$$\boxed{b_n = \frac{l}{n\pi}} \longrightarrow (3m)$$

$$\boxed{\therefore f(x) = \frac{l}{4} + \frac{2l}{\pi^2}\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2}\cos\frac{n\pi x}{l} + \frac{l}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{l}} \longrightarrow (1m)$$

At $x=0$, $f(0) = \frac{f(0-)+f(0+)}{2} = \frac{l}{2}$.

At $x=0$, $\frac{l}{2} = \frac{l}{4} + \frac{2l}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right] \Rightarrow \boxed{\left[\frac{1}{1^2}+\frac{1}{3^2}+\cdots\right] = \frac{\pi^2}{8}} \longrightarrow (2m)$

(OR)

29(b) Exclude the point $x = 2\pi$. Here $\underline{n=6}$.

x	y	$\cos x$	$y\cos x$	$\sin x$	$y\sin x$	$\cos 2x$	$y\cos 2x$	$\sin 2x$	$y\sin 2x$	$\cos 3x$	$y\cos 3x$	$\sin 3x$
0	1.98	1	1.98	0	0	1	1.98	0	0	1	1.98	0
$\pi/3$	1.3	0.5	0.65	0.866	1.1258	-0.5	-0.65	0.866	1.1258	-1	-1.3	0
$2\pi/3$	1.05	-0.5	-0.525	0.866	0.9093	-0.5	-0.525	-0.866	-0.9093	1	1.05	0
π	1.3	-1	-1.30	0	0	1	1.30	0	0	-1	-1.3	0
$4\pi/3$	-0.88	-0.5	0.44	-0.866	0.7620	-0.5	0.44	0.866	-0.762	1	-0.88	0
$5\pi/3$	-0.25	0.5	-0.125	-0.866	0.2165	-0.5	0.125	-0.866	0.2165	-1	0.25	0
Σ	4.5		1.12		3.013		2.67		-0.329		-0.2	

(5m)

$\Sigma y \sin 3x = 0$

$a_0 = \frac{2}{6}\Sigma f(x) = 1.5.$

$a_1 = \frac{2}{6}(1.12) = 0.37$

$b_1 = \frac{2}{6}(3.013) = 1.004$

$a_2 = \frac{2}{6}(2.67) = 0.89$

$b_2 = \frac{2}{6}(-0.329) = -0.110$

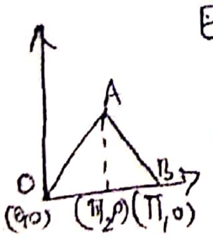
$a_3 = \frac{2}{6}(-0.2) = -0.07$

$b_3 = 0.$ (6m)

$$\therefore y = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \cdots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \cdots$$

$$\boxed{y = 0.75 + 0.37\cos x + 0.89\cos 2x - 0.07\cos 3x + 1.004\sin x - 0.110\sin 2x} \rightarrow (1m)$$

30(a)



Eq'n of OA : $\frac{y-0}{d-0} = \frac{x-0}{\pi/2 - 0} \Rightarrow y = \frac{2xd}{\pi}$; $0 < x < \pi/2$

Eq'n of AB: $\frac{y-d}{0-d} = \frac{x-\pi/2}{\pi-\pi/2} \Rightarrow y = \frac{2d(\pi - x)}{\pi}$; $\pi/2 < x < \pi$.

$$\therefore y(x,0) = \begin{cases} \frac{2xd}{\pi} & ; 0 < x < \pi/2, \\ \frac{2d(\pi - x)}{\pi} & ; \pi/2 < x < \pi. \end{cases} \rightarrow (2m)$$

The other boundary conditions are

$$\left.\begin{array}{l} y(0,t)=0 \;;\; t\geq 0 \\ y(\pi,t)=0 \;;\; t\geq 0 \\ \left(\dfrac{\partial y}{\partial t}\right)_{t=0} = 0 \;;\; 0<x<\pi. \end{array}\right\} \rightarrow (2m)$$

The required PDE is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \rightarrow (1m)$$

Solution is of the form $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$.

At $x=0 \Rightarrow \boxed{A=0.}$

At $x=\pi \Rightarrow \sin\lambda\pi = 0 \quad (\because B\neq 0)$

$\Rightarrow \lambda\pi = n\pi \Rightarrow \boxed{\lambda = n}$

At $\left(\dfrac{\partial y}{\partial t}\right)_{t=0} \Rightarrow \boxed{D=0.}$

$$\therefore\; y(x,t) = \sum_{n=1}^{\infty} B_n \sin nx \cos nat. \longrightarrow (2m)$$

At $t=0 \Rightarrow$

$$\sum_{n=1}^{\infty} B_n \sin nx = \begin{cases} \dfrac{2xd}{\pi} & ; 0<x<\pi/2 \\ \dfrac{2d(\pi-x)}{\pi} & ; \pi/2<x<\pi. \end{cases} \quad (3m)$$

This is half-range Fourier Sine Series.

$$\therefore\; B_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx = \frac{2}{n^2}\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 \;;\; \text{if } n \text{ is even} \\ \dfrac{2}{n^2}\sin\dfrac{n\pi}{2} \;;\; \text{if } n \text{ is odd.} \end{cases}$$

$$\therefore\; y(x,t) = \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2}\sin(2n-1)\left(\frac{\pi}{2}\right)\sin(2n-1)x\cos(2n-1)at$$

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} \frac{2(-1)^{n-1}}{(2n-1)^2}\sin(2n-1)x\cos(2n-1)at} \rightarrow (2m)$$

(OR)

30.(b) Required PDE is $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{\alpha^2}\dfrac{\partial u}{\partial t}. \longrightarrow (1m)$

Boundary conditions are

$$\left.\begin{array}{l} u(0,t)=0 \;;\; t\geq 0 \\ u(l,t)=0 \;;\; t\geq 0 \\ u(x,0)=k(lx-x^2) \;;\; 0<x<l \end{array}\right\} (2m)$$

Solution is of the form $u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2\lambda^2 t}$

At $x=0 \Rightarrow \boxed{A=0}$ | At $x=l \Rightarrow \sin\lambda l = 0$ ($\because B \neq 0$)

$\Rightarrow \boxed{\lambda = \frac{n\pi}{l}}$

$$\Rightarrow u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\, e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}} \longrightarrow (3m)$$

At $t=0 \Rightarrow k(lx - x^2) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$; $0 < x < l$.

This is a half range fourier sine series.

$$\Rightarrow B_n = \frac{2}{l}\int_0^l k(lx - x^2)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{4kl^2}{n^3\pi^3}\left[1-(-1)^n\right] \longrightarrow (5m)$$

$$\therefore \boxed{u(x,t) = \sum_{n=1}^{\infty} \frac{4kl^2}{n^3\pi^3}(1-(-1)^n)\sin\frac{n\pi x}{l}\, e^{-\frac{\alpha^2 n^2\pi^2 t}{l^2}}} \rightarrow (1m)$$

31(a) $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx = \frac{1}{\sqrt{2\pi}}\int_{-1}^{+1}\left[1-|x|\right]\left[\cos sx + i\sin sx\right]dx$.

$$= \frac{2}{\sqrt{2\pi}}\int_0^1 (1-x)\cos sx\,dx \quad \{\because 1-|x| \text{ is even}\}.$$

$$= \sqrt{\frac{2}{\pi}}\left(\frac{1-\cos s}{s^2}\right) \longrightarrow (6m)$$

Using Parseval's identity, $\frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos s}{s^2}\right)^2 ds = \int_{-1}^{1}(1-|x|)^2 dx$. $\rightarrow (1m)$

$$\Rightarrow \frac{4}{\pi}\int_0^{\infty}\frac{(1-\cos s)^2}{s^4}ds = 2\int_0^1 (1-x)^2 dx = \frac{2}{3}.$$

$$\Rightarrow \frac{16}{\pi}\int_0^{\infty}\frac{\sin^4(s/2)}{s^4}ds = \frac{2}{3}$$

Take $\frac{s}{2} = x \Rightarrow \boxed{\int_0^{\infty}\frac{\sin^4 x}{x^4}dx = \frac{\pi}{3}} \longrightarrow (5m)$

1.(b) $f(x) = e^{-x}$.

$$F_s(e^{-x}) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-x} \sin sx\, dx = \sqrt{\frac{2}{\pi}} \frac{s}{s^2+1} \longrightarrow (3m)$$

$$F_c(e^{-x}) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-x} \cos sx\, dx = \sqrt{\frac{2}{\pi}} \frac{1}{s^2+1} \longrightarrow (3m)$$

Using Parseval's identity in $F_s(e^{-x})$,

$$\int_0^{2\pi} \frac{2}{\pi} \frac{s^2}{(s^2+1)^2} ds = \int_0^\infty e^{-2x} dx = \left(\frac{e^{-2x}}{-2}\right)_0^\infty = \frac{1}{2}.$$

$$\Rightarrow \int_0^{2\pi} \frac{s^2}{(s^2+1)^2} ds = \frac{\pi}{4}.$$

Let $s = x$ $\Rightarrow$ $\boxed{\int_0^{2\pi} \frac{x^2}{(x^2+1)^2} dx = \frac{\pi}{4}}$. $\longrightarrow (6m)$

3.2(a) (i) $Z\left((n+1)^2\right) = Z(n^2 + 2n + 1) = Z(n^2) + 2Z(n) + Z(1)$

Now $Z(n^2) = Z(n \cdot n) = -z \frac{d}{dz}\left(\frac{z}{(z-1)^2}\right) = \frac{z(z+1)}{(z-1)^3}$.

$$\therefore \boxed{Z\left((n+1)^2\right) = \frac{z(z+1)}{(z-1)^3} + \frac{2z}{(z-1)^2} + \frac{z}{(z-1)}}. \longrightarrow (3m)$$

$$Z(\sin(3n+5)) = Z\left[(\sin 3n)(\cos 5) + (\cos 3n)(\sin 5)\right]$$

$$= (\cos 5) Z(\sin 3n) + (\sin 5) Z(\cos 3n).$$

$$= (\cos 5)\left(\frac{z \sin 3}{z^2 - 2z\cos 3 + 1}\right) + (\sin 5)\left(\frac{z(z-\cos 3)}{z^2 - 2z\cos 3 + 1}\right)$$

$$= \frac{z \cos 5 \sin 3 + z^2 \sin 5 - z \sin 5 \cos 3}{z^2 - 2z\cos 3 + 1}$$

$$\boxed{Z[\sin(3n+5)] = \frac{z^2 \sin 5 - z \sin 2}{z^2 - 2z\cos 3 + 1}}. \longrightarrow (3m)$$

32(a)(ii) $Z^{-1}\left[\frac{z^2}{(z-4)(z-3)}\right] = Z^{-1}\left[\frac{z}{z-4}\right] * Z^{-1}\left[\frac{z}{z-3}\right]$ {By Convolution} $\longrightarrow$ (1m)

We know $Z^{-1}\left[\frac{z}{z-a}\right] = a^n.$

$$\therefore Z^{-1}\left[\frac{z^2}{(z-4)(z-3)}\right] = (4)^n * (3)^n \longrightarrow (1m)$$

$$= \sum_{k=0}^{n} (4)^{n-k}(3)^k \longrightarrow (1m)$$

$$= (4)^n \sum_{k=0}^{n} \left(\frac{3}{4}\right)^k$$

$$= 4^n\left[\frac{1-(3/4)^{n+1}}{1-3/4}\right]$$

$$= 4^{n+1}\left[1-\left(\frac{3}{4}\right)^{n+1}\right].$$

$$= 4^{n+1} - 3^{n+1} \longrightarrow (3m)$$

32.(b.) $Y(k+2)+Y(k)=1$; $y(0)=y(1)=0.$

$$\Rightarrow z^2 Y(z) + Y(z) = Z(1) = \frac{z}{z-1}$$

$$\Rightarrow \boxed{Y(z) = \frac{z}{(z-1)(z^2+1)}} \longrightarrow (3m)$$

Consider $\frac{Y(z)}{z} = \frac{1}{(z-1)(z^2+1)} = \frac{A}{(z-1)} + \frac{Bz+C}{(z^2+1)}$

Solving we get, $\boxed{A = 1/2 ;\ B = C = \frac{-1}{2}.} \longrightarrow (3m)$

$$\Rightarrow \frac{Y(z)}{z} = \frac{1}{2}\left[\frac{1}{z-1} - \frac{z+1}{z^2+1}\right]$$

$$\Rightarrow Y(z) = \frac{1}{2}\left[\frac{z}{z-1} - \frac{z^2}{z^2+1} - \frac{z}{z^2+1}\right] \longrightarrow (4m)$$

Taking inverse Z-transform

$$\Rightarrow \boxed{y(k) = \frac{1}{2}\left[1 - \cos\frac{n\pi}{2} - \sin\frac{n\pi}{2}\right].} \longrightarrow (2m)$$

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