# Unit 3

## Push Down Automata

* Content free language defined by Special type of automata is push down automata

* Extension of NFA with $\epsilon$ - transition with addition of Stack.

* Stack (LIFO) is used to Store String of Stack Symbols

* Stack is used to read the Symbols, push and pop only at top of Stack

* Can remember an infinite amount of String information.

* PDA can access the infn. on its stack in LIFO way.

* PDA can recognize only CFL

* PDA has finite memory.

PDA involves Seven tuples

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Q – Finite non-empty set of States

$\Sigma$ – finite Set of i/p Symbols

$\Gamma$ – finite Stack alphabet (Set of Symbols pushed to the Stack)

$\delta$ – Transition function.

$$\delta : Q \times (\Sigma \cup \epsilon) \times T \rightarrow Q \times T^*$$

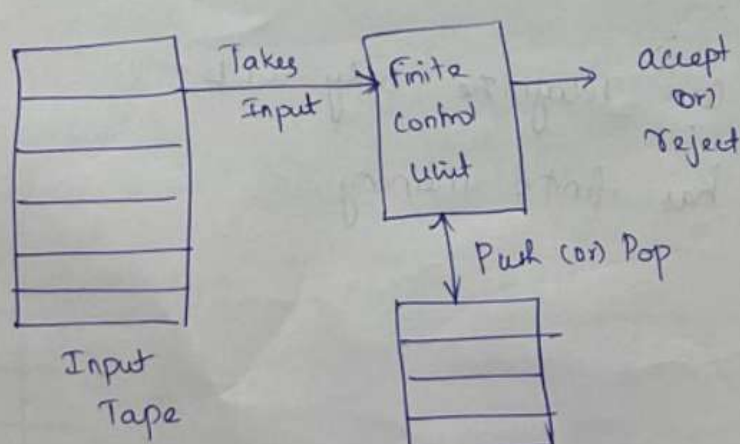
$q_0$ – initial / start Symbol. $(q_0 \in Q)$

$Z_0$ – initial / Start / Bottom of Stack $(Z_0 \in T)$

F – Final Set of accepting States / final States $(F \subseteq Q)$

push Down Automata 3 Components

1. An input tape
2. A finite Control unit
3. A Stack with Infinite Size



$\delta$ takes as argument $\delta(q, a, x)$

i) $q$ is a state in $Q$

ii) $a$ is either i/p symbol in $\Sigma$ (or) $a = \epsilon$

iii) $x$ is stack symbol, that is a member of $\Gamma$

The o/p of $\delta$ is finite set of pairs $(P, \gamma)$

$P$ – New State

$\gamma$ – String of Stack symbols that replaces $x$ at top of Stack.

eg) If $\gamma = \epsilon$, Stack is popped

If $\gamma = x$, stack is unchanged

If $\gamma = YZ$, then $x$ is replaced by $Z$ & $Y$ is pushed onto stack.

Finite state machine.

$$A \xrightarrow{a} B$$

PDA

$$A \xrightarrow{a,\ b \to c} B$$

$a$ – i/p symbol → may be $\epsilon$

$b$ – symbol on top of stack. This symbol is popped.

↓

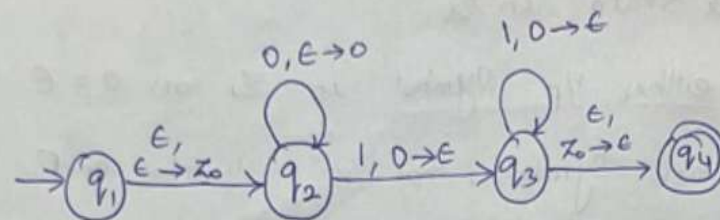
$\epsilon$ means the stack is neither read nor popped.

$c$ – This symbol is pushed onto the stack.

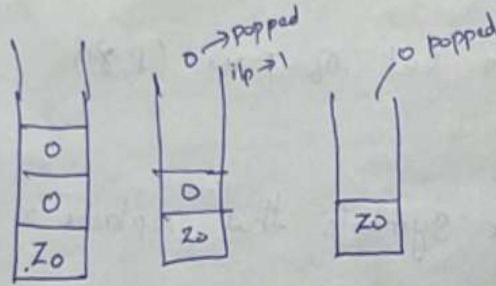
↓

$\epsilon$ means nothing is pushed.

① Construct a PDA that accepts $L = \{0^n 1^n \mid n \geq 0\}$




0011




Now $Z_0$ is top most

reached final st.

## Types of PDA

→ Non-Deterministics PDA (NPDA)

→ Deterministics PDA (DPDA)

## Moves / Transition of PDA

$$Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^* \text{ (DPDA)}$$

$$Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow 2^{Q \times \Gamma^*} \text{ (NPDA)}$$

$Q \times (\Sigma \cup \epsilon) \times \Gamma$ → Implies that a transition is based on

* Current State $q \in Q$
* Next Input $\Sigma \cup \epsilon$
* Stack Symbol (top element of Stack)

$Q \times \Gamma^*$ → Implies that next State reached after transition

## Operation on Stack :

1. **Push**

$$\delta(q_1, \overset{i/p}{a}, \overset{top}{b}) = (q_2, ab)$$

- $q_2$ → next State
- $b$ → Top of Stack
- $a$ → Input Symbol
- $q_1$ → Current Symbol.

ab ⇒ top most symbol on stack is 'b' is replaced by 'a' followed by 'b'. Since 'a' is pushed to stack

2. **POP**

$$\delta(q_1, \overset{i/p}{a}, \overset{top}{b}) = (q_2, \epsilon)$$

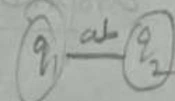
- $q_2$ → next State.
- $b$ → top of Stakk
- $a$ → Input Symbol.
- $q_1$ → Current State

$\epsilon$ → Here 'a' cancels 'b' from stack.

3. Read i/p with no operation on stack.

$$\delta(q_1, a, b) = (q_2, b)$$

No operation performed on stack




## Language of PDA

1. Acceptance by final State
2. Acceptance by Empty stack

Instantaneous description of PDA (ID)

ID is defined as 3 tuple $(q, a, \gamma)$

$q$ - State of PDA

$a$ - remaining i/p

$\gamma$ - Stack contents

$$(q_1, aw, x\beta) \vdash^{*}_{P} (q_2, w, \alpha\beta)$$

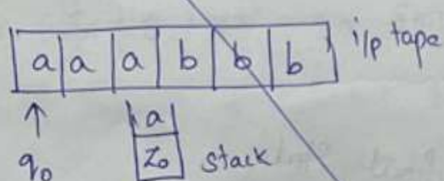
Representation of PDA

1. Transition diagram
2. Transition fn. / moves.

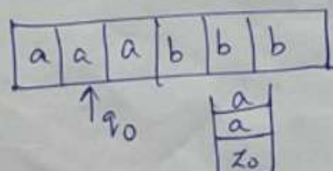
① Design the PDA to accept the language $L = \{a^n b^n \mid n \geq 1\}$ accepting final st. / empty st.

Soln $L = \{ab, aabb, aaabbb \dots\}$.

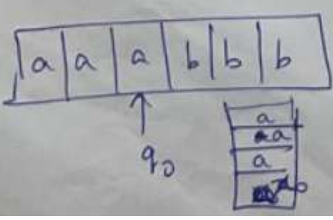
Graphical rep: aaabbb



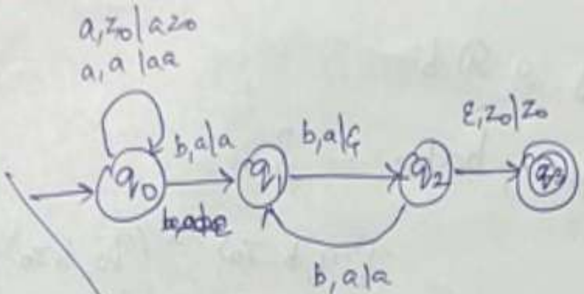

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$




$\delta(q_0, a, a) = (q_0, aa)$




$\delta(q_0, a, a) = (q_0, aa)$




$P_f = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, \{q_3\})$

(or)

Tr. fn:

$\delta(q_0, a, Z_0) = (q_0, aaZ_0)$

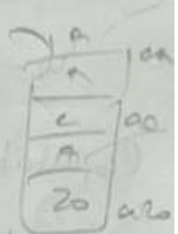
$\delta(q_0, a, a) = (q_0, aaa)$

$\delta(q_0, b, a) = (q_1, \varepsilon)$

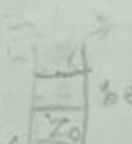
$\delta(q_1, b, a) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$

$\delta(q_0, \varepsilon, Z_0) = (q_2, Z_0)$

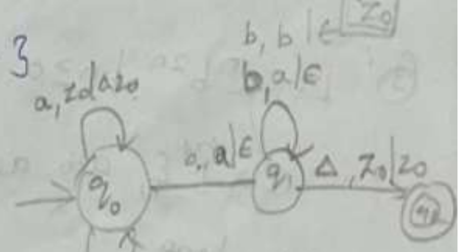


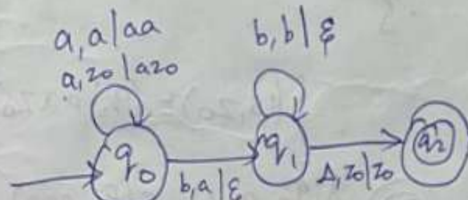
ab, aabb, aaabbb



1. $L = \{a^n b^n \mid n > 0\}$

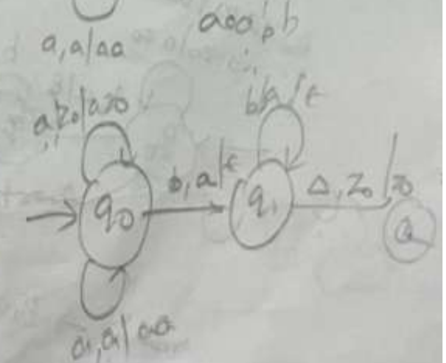
$L = \{ab, aabb, aaabb \cdots\}$




aabb

aaa, b




aabb

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

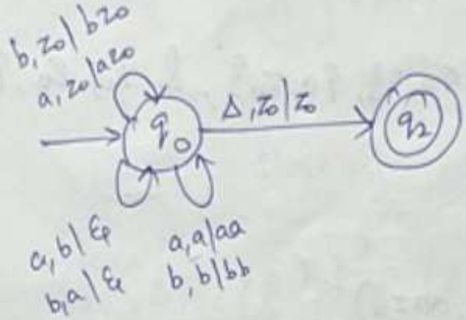
$\delta(q_0, b, a) = (q_1, \varepsilon)$

$\delta(q_1, b, a) = (q_1, \varepsilon)$

$\delta(q_1, \Delta, Z_0) = (q_2, Z_0)$

2. L = equal no. of a & b.

L = { ε, ab, aabb, baba ... }




$\delta(q, b, z_0) = (q_0, bz_0)$

$\delta(q, a, b) = (q_0, \epsilon)$

$\delta(q, b, z_0) = (q_0, bz_0)$

$\delta(q, a, b) = (q_0, \epsilon)$

$\delta(q, \Delta z_0) = (q_2, z_0)$

$w_1 = abab$

$(q_0, abab, z_0) \vdash_P (q_0, bab, az_0)$

$(q_0, w, z_0)$

$\vdash_P (q_0, ab, z_0)$

$\vdash_P (q_0, b, az_0)$

$\vdash_P (q_0, \Delta, z_0)$

$\vdash_P (q_1, z_0)$

accepted

$w = abb$

$(q_0, abb, z_0) \vdash (q_0, bb, az_0)$

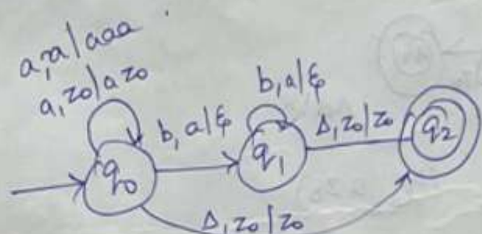
$\vdash_P (q_0, b, z_0)$

$\vdash_P (q_0, \Delta, bz_0)$

Not accepted.

③ $L = a^n b^{2n} \mid n \geq 0$ with a as strategy

L = { ε, abb, aabbbb .... }




$\delta(q, a, z_0) \vdash (q_0, aaz_0)$

$\delta(q_0, a, a) = (q_0, aaa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

$\delta(q_1, \Delta, z_0) = (q_2, z_0)$


read 'a' → Push 2 'a's

$L = \{abb, aabbbb, \cdots\}$

aabbbb

No pop No pop

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

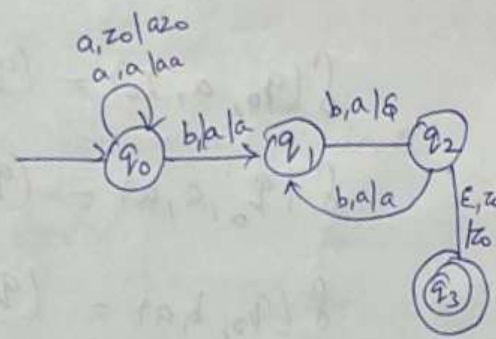
$\delta(q_0, b, a) = (q_1, a)$ odd

$\delta(q_1, b, a) = (q_2, \epsilon)$ even

$\delta(q_2, b, a) = (q_1, a)$ odd

$\delta(q_1, b, a) = (q_2, \epsilon)$ even

$\delta(q_2, \epsilon, Z_0) = (q_3, Z_0)$




$P = (\{q_0, q_1, q_2, q_3\} \{a, b\} \{Z_0, a\}, \{\delta, q_0, Z_0, q_3)$

$\omega =$ aabbbb

$\vdash_P (q_0, abbbb, aZ_0)$

$\vdash_P (q_0, bbbb, aaZ_0)$

$\vdash_P (q_1, bbb, aaZ_0)$

$\vdash_P (q_2, bb, aZ_0)$

$\vdash_P (q_2, b, aZ_0)$

$\vdash_P (q_3, \Delta, Z_0)$

accepted

$\omega =$ aab

$\vdash_P (q_0, aab, aZ_0)$

$\vdash_P (q_0, b, aaZ_0)$

$\vdash_P (q_0, \Delta, aaZ_0)$

Not accepted

④ $L = \{a^n b^m \mid n > m\}$

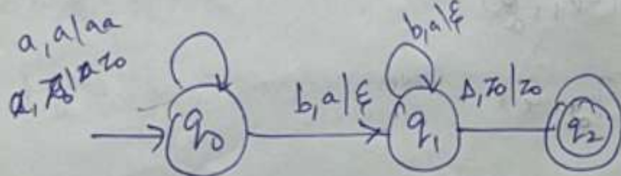
$L = \{aab, aaab, aaabb \cdots\}$

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \varepsilon)$

$\delta(q_1, \Delta, Z_0) = (q_2, Z_0)$




$P_D = \{\{q_0, q_1, q_2\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, q_2\}$

$w_1 = aab$

$\vdash_P (q_0, ab, aZ_0)$

$\vdash_P (q_0, b, aaZ_0)$

$\vdash_P (q_0, \Delta, aZ_0)$

# Deterministic Push Down Automata (DPDA)

Conditions:

1. $\delta(q, a, x)$ has one member for any $q$ in $Q$, $a$ in $\Sigma$ (or) $a = \epsilon$ & $x$ in $\Gamma$
2. $\delta(q, a, x)$ is non-empty for some $a$ in $\Sigma$ then $\delta(q, \epsilon, x)$ must be empty.

① $L = \{wcw^R \mid w \text{ is in } (0+1)^*\}$

$L = \{c, 0c0, 1c1, 01c10, 11c11, 00c00 \ldots\}$

011c110, 001c100 ... 011c110


* after c change state
* perform match operation & pop

011c110

i/p top

$0, Z_0 \mid 0Z_0$

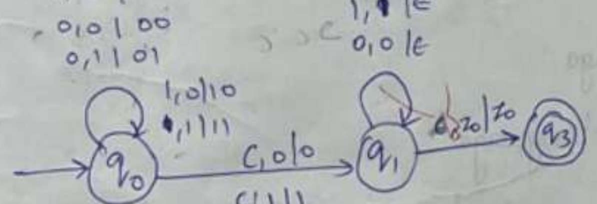
$1, Z_0 \mid 1Z_0$

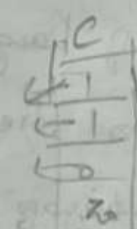
$0, 0 \mid 00$

$0, 1 \mid 01$

$1, 1 \mid \epsilon$

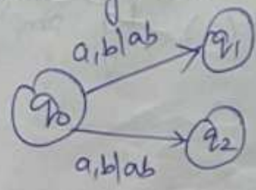
$0, 0 \mid \epsilon$




## Non deterministic PDA (NPDA)

① If two (or) more edges labelled with same i/p and stack symbol




$0, Z_0 \mid 0Z_0$

$1, Z_0 \mid 1Z_0$

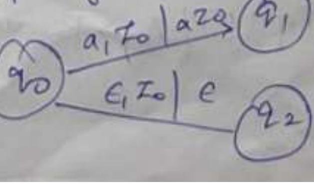
$0, 0 \mid 00$

$1, 1 \mid 11$

② when a state have two edges with same stack symbol & one i/p symbol $\epsilon$

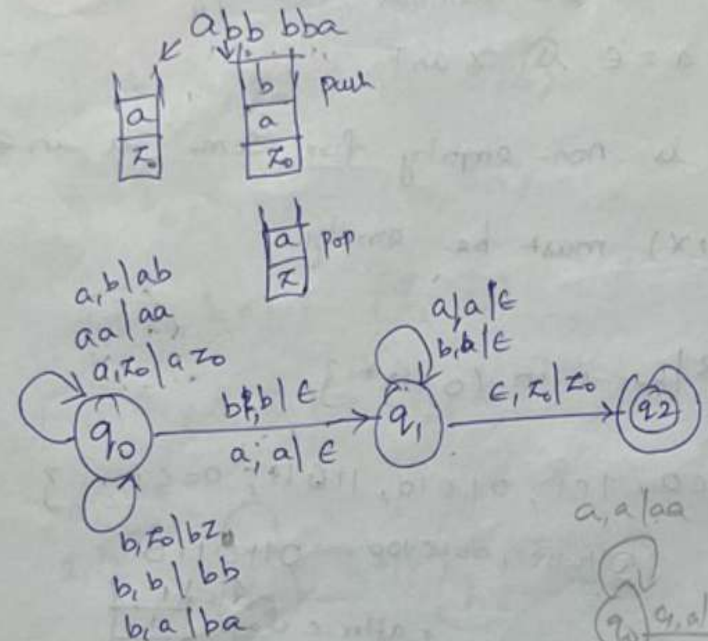
$0, 1 \mid 01$

$1, 0 \mid 10$



① $L = ww^R \quad w \in (a,b)^*$

$L = \{abba, abbbba \ldots\}$




## Equivalence of PDA's And CFL

3 classes of Language:

1. Context free language
2. Language accepted by final state of PDA
3. Lang. accepted by empty stack of PDA.

### CFG to PDA

$G = (V, T, P, S)$

construct PDA that acept $L(G)$ by empty stack.

$P = (\{q\}, T, V \cup T, \delta, q, S)$

$\delta$ is defined by.

(i) For each variable A in CFG.

$\delta(q, \epsilon, A) = \{(q, \beta) \mid A \to \beta$ is prod. of $G\}$

ii) For each terminal 'a' in CFG.

$$\delta(q, a, a) = \{(q, \epsilon)\}$$

① Construct PDA

$$E \rightarrow E+E \mid E*E \mid a$$

Soln

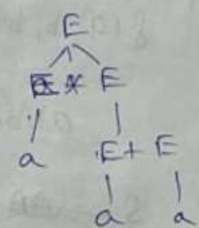
$$P = (\{q\}, \{+, *, a\}, \{E, *, +, a\}, \delta, q, E)$$

Tr. fn. of PDA.

for non terminal.

$$\delta(q, \epsilon, E) = \{(q, E+E)\ (q, E*E)\ (q, a)\}$$

For terminal.




$$\delta(q, +, +) = \{q, \epsilon)\}$$

$$\delta(q, *, *) = \{q, \epsilon\}$$

$$\delta(q, a, a) = \{q, \epsilon\}$$

Inst. description.

$$\omega = a * a + a$$

$$(q, a*a+a, E) \vdash_P (q, a*a+a, E*E)$$

$$\vdash (q, a*a+a, a*E)$$

$$\vdash (q, *a+a, *E)$$

$$\vdash (q, a+a, E)$$

$$\vdash (q, a+a, E+E)$$

$$\vdash (q, a+a, a+E)$$

$$\vdash (q, +a, +E)$$

$$\vdash (q, a, E)$$

$$\vdash (q, a, a)$$

$$\vdash (q, \epsilon, \epsilon)$$

(2) Const. PDA for CFG & test "abbabb" is N(P)

$G = (\{S, A\}, \{a, b\}, R, S)$

$R = \{ S \rightarrow AA \mid a$

$A \rightarrow SA \mid b \}$

Soln 6 tuple

$P = (\{q\}, \overset{T}{\{a, b\}}, \{S, A, a, b\}, \delta, q, S)$

Non termind

$\delta(q, \epsilon, S) = \{(q, AA)\ (q, a)\}$

$\delta(q, \epsilon, A) = \{(q, SA)\ (q, b)\}$

Termind

$\delta(q, a, a) = \{(q, \epsilon)\}$

$\delta(q, b, b) = \{(q, \epsilon)\}$

abbabb

Inst description

yes

$(q, abbabb, S) \vdash (q, abbabb, AA)$

$\vdash_P (q, abbabb, SAA)$

$\vdash_P (q, abbabb, aAA)$

$\vdash_P (q, bbabb, AA)$ a is poped $\because a \rightarrow \epsilon$

$\vdash_P (q, bbabb, bA)$

$\vdash_P (q, babb, A)$ pop 'b'

$\vdash_P (q, babb, SA)$

$\vdash_P (q, babb, AAA)$

$\vdash_P (q, babb, bAA)$

$\vdash_P (q, abb, AA)$ pop b

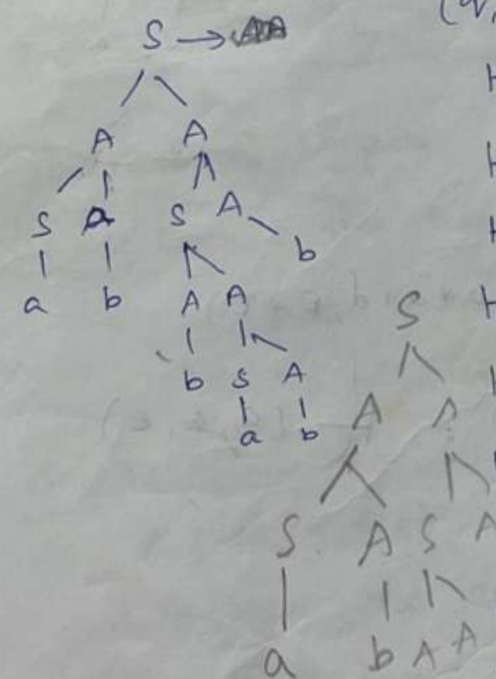
$\vdash_P (q, abb, SAA)$

$\vdash_P (q, abb, aAA)$

$\vdash_P (q, bb, AA)$

$\vdash_P (q, bb, bA)$

$\vdash_P (q, b, A)$ b poped



$\vdash (q, b, b)$

$\vdash (q, \epsilon, \epsilon)$ b poped

3. $S \to 0S1 \mid A$  $V = \{S, A\}$  $w = 0101$

$A \to 1A0 \mid S \mid \epsilon$  $T = \{1, 0\}$

Soln

$P = (\{q\}, \{1,0\}, \{S, A, 1, 0\}, \delta, q, S)$

Non terminal:

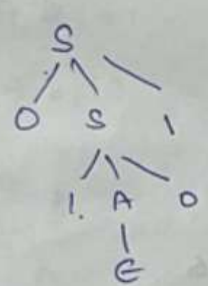
$\delta(q, \epsilon, S) = \{(q, 0S1)\ (q, A)\}$

$\delta(q, \epsilon, A) = \{(q, A0), (q, S), (q, \epsilon)\}$

Terminal:

$\delta(q, 1, 1) = \{(q, \epsilon)\}$

$\delta(q, 0, 0) = \{(q, \epsilon)\}$




$(q, 0101, S) \vdash_P (q, 0101, 0S1)$

$\vdash_P (q, 101, S1)$

$\vdash_P (q, 101, 1A01)$

$\vdash_P (q, 01, A01)$

$\vdash_P (q, 01, \epsilon 01)$

$\vdash_P (q, 1, 1)$

$\vdash_P (q, \epsilon, \epsilon)$

4. $S \to aAA$  $V - \{S, A\}$

$A \to aS \mid bS \mid a$  $T - \{a, b\}$

$P = (\{q\}\ \{a, b\}\ \{S, A, a, b\}, \delta, q, \{S\})$

non terminal:

$\delta(q, \epsilon, S) = (q, aAA)$

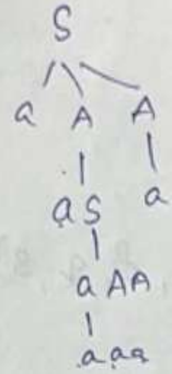
$\delta(q, \epsilon, A) = \{(q, aS)\ (q, bS)\ (q, a)\}$

Terminal:

$\delta(q, a, a) = \{q, \epsilon\}$

$\delta(q, b, b) = (q, \epsilon\}$

aaaaaa




$\delta(q, aaaaaa, S)$

$\vdash_P (q, aaaaaa, aAA)$

$\vdash_P (q, aaaaa, AA)$

$\vdash_P (q, aaaaa, aSA)$

$\vdash_P (q, aaaa, SA)$

$\vdash_P (q, aaaa, aAAA)$

$\vdash_P (q, aaa, AAA)$

$\vdash_P (q, aaa, aAA)$

$\vdash_P (q, aa, AA)$

$\vdash_P (q, aa, aA)$

$\vdash_P (q, a, A)$

$\vdash_P (q, a, a)$

$\vdash_P (q, \epsilon, \epsilon)$

~~PDA CFG~~

~~CFG to PDA~~ CFG to PDA

Let $G = (V, T, R, S)$ be a CFG.

PDA that accepts empty stack.

PDA, $P = (\{q\}, T, V \cup T, \delta, q, S)$

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$

Transition fn $\delta$

1. For each variable A

$\delta(q, \epsilon, A) = \{(q, \beta)\} \mid A \to \beta$ is in $R\}$

2. For each terminal a

$\delta(q, a, a) = (q, \epsilon)$

1. const. a PDA for CFG & test whether "abbabb" is in N(P).

$G = (\{S, A\} \{a, b\}, R, S\}$

$R = \{ S \rightarrow AA \mid a$

$A \rightarrow SA \mid b \}$.

<u>Soln</u>

PDA, $P = (\{q\}, \{a, b\} \{S, A, a, b\}, \delta, q, S)$

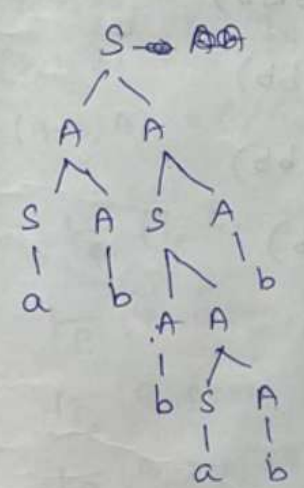
$\delta(q, \epsilon, S) = \{(q, AA)\ (q, a)\}$

$\delta(q, \epsilon, A) = \{(q, SA)\ (q, b)\}$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$q_0$



$(q, abbabb, S) \vdash_P (q, abbabb, AA)$

$\vdash_P (q, abbabb, SAA)$

$\vdash_P (q, abbabb, aAA)$

$\vdash_P (q, bbabb, AA)$

$\vdash_P (q, bbabb, bA)$

$\vdash_P (q, babb, A)$

$\vdash_P (q, babb, SA)$

$\vdash_P (q, babb, AAA)$

$\vdash_P (q, babb, bAA)$

$\vdash_P (q, abb, AA)$

$\vdash_P (q, abb, SAA)$

$\vdash_P (q, abb, aAA)$

$\vdash_P (q, bb, AA)$

$\vdash_P (q, bb, bA)$

$\vdash_P (q, b, A)$

$\vdash_P (q, b, b)$

$\vdash_P (q, \epsilon, \epsilon)$

$\vdash_P (q, \epsilon)$

$\therefore$ "abbabb" is accepted.

(2) $S \to aSb$ Not GNF

$S \to ab$

Soln

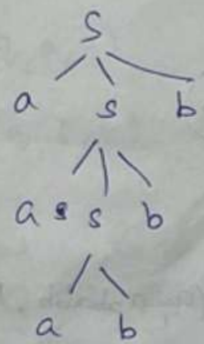
$\delta(q, \epsilon, S) = (q, aSb)$

$\delta(q, \epsilon, S) = \{q, ab)$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

word $\to$ aaabbb




$\delta(q, aaabbb, S) \vdash_P \delta(q, aaabbb, aSb)$

$\vdash_P (q, aabbb, Sb)$

$\vdash_P (q, aabbb, aSbb)$

$\vdash_P (q, abbb, Sbb)$

$\vdash_P (q, abbb, abbb)$

$\vdash_P (q, bbb, bbb)$

$\vdash_P (q, bb, bb)$

$\vdash_P (q, b, b)$

$\vdash_P (q, \epsilon, \epsilon)$

(3) $S \to 0BB$

$B \to 0S \mid 1S \mid 0$

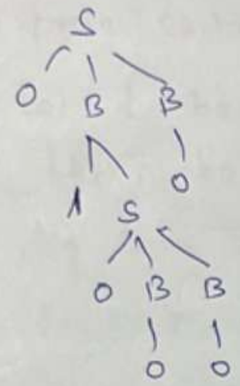
Soln $\delta(q, \epsilon, S) = \{(q, 0BB)\}$

$\delta(q, \epsilon, B) = \{(q, 0S)\ (q, 1S)\ (q, 0)\}$

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

010000



$\delta(q, 010000, S) \vdash_p (q, 010000, 0BB)$

$\vdash_p (q, 10000, BB)$

$\vdash_p (q, 10000, 1SB)$

$\vdash_p (q, 0000, SB)$

$\vdash_p (q, 0000, 0BBB)$

$\vdash_p (q, 000, BBB)$

$\vdash_p (q, 000, 0BB)$

$\vdash_p (q, 00, BB)$

$\vdash_p (q, 00, 0B)$

$\vdash_p (q, 0, B)$

$\vdash_p (q, 0, 0)$

$\vdash_p (q, \epsilon, \epsilon)$

$S \rightarrow 0BB$

$B \rightarrow 0S \mid 1S \mid 0$    It is in GNF

$(q, \epsilon, S, (V \cup T), S, q, \phi)$

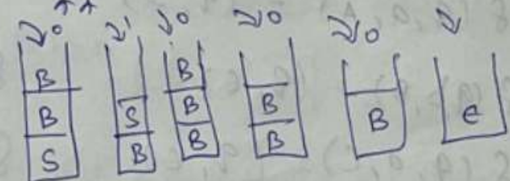
$(q, 0, S) = (q, BB)$

$(q, 0, B) = (q, S)$

$(q, 1, B) = (q, S)$

$(q, 0, B) = (q, \epsilon)$

$W = 010000$



$\vdash_p (q, 010000, S)$

$\vdash_p (q, 10000, BB)$

$\vdash_p (q, 0000, SB)$

$\vdash_p (q, 000, BBB)$

$\vdash_p (q, 00, BB)$

$\vdash_p (q, 0, B)$

$\vdash_p (q, \epsilon, \epsilon)$

$\vdash_p (\epsilon, \epsilon)$

③ $S \to AB$

$B \to b$

$A \to CD$

$C \to a$

$D \to Sa$

Soln

$S \to AB \mid CDB \mid aDB$.

$B \to b$

$A \to CD \mid aD$

$C \to a \quad D \to a$

PDA eq:

$\delta(q, a, S) = (q, DB)$

$\delta(q, a, A) = (q, D)$

$\delta(q, b, B) = (q, \epsilon)$

$\delta(q, a, C) = (q, \epsilon)$

$\delta(q, a, D) = (q, \epsilon)$

$\vdash (q, aab, S)$

$\vdash (q, ab, DB)$

$\vdash (q, b, B)$

$\vdash (q, \epsilon, Z)$

$\vdash (q, \epsilon)$

(or)

$\delta(q, \epsilon, S) = (q, AB)$

$\delta(q, \epsilon, A) = (q, CD)$

$\delta(q, \epsilon, B) = (q, b)$

$\delta(q, \epsilon, C) = (q, a)$

$\delta(q, \epsilon, D) = (q, a)$

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\vdash (q, aab, S)$

$\vdash (q, aab, AB)$

$\vdash (q, aab, CDB)$

$\vdash (q, aab, aDB)$

$\vdash (q, ab, DB)$

$\vdash (q, ab, aB)$

$\vdash q(q, b, B)$

$\vdash q(q, b, b)$

$\vdash q(q, \epsilon, \epsilon)$

# PDA to CFG

Let PDA, $= P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$

CFG, $G = (V, \Sigma, R, S)$

## Variable V

1. Special symbol S
2. $[PXq]$ where P, q are states in Q
   $X$ is in $\Gamma$

## production, R

1. For all states P,

$$S \to [q_0 Z_0 P]$$

2. Let $\delta(q, \overset{i/p}{a}, \overset{top}{X}) = (r, Y_1 Y_2 \ldots Y_k)$ $\quad q, P$

$$[q X r_k] \to a[r Y_1 r_1][r_1 Y_2 r_2] \cdots [r_{k-1} Y_k r_k]$$

For all states $r_1, r_2 \ldots r_k$ $(q, 1, z) = (q, xz)$

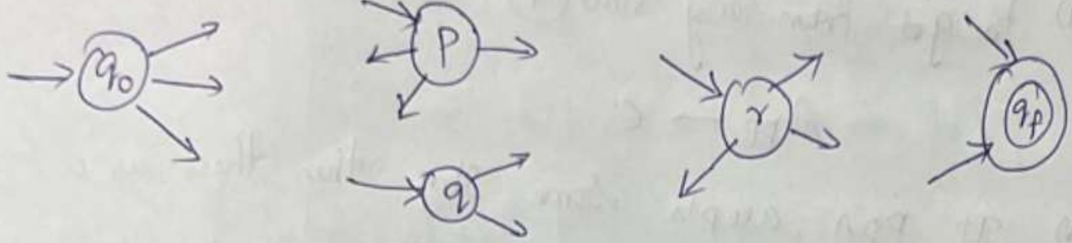
$\delta(q, a, x) = (r, \epsilon)$ $\quad [q, z, q] = [q, x, q][q, z, q]$

$[q X r] \to a$ $\quad (q, z, q) = (q, x, P) \quad (P, z, q)$

$\delta(q, \epsilon, x) = (r, \epsilon)$ $\quad (q, z, P) = (q, x, q) \quad (q, z, P)$

$[q X r] \to \epsilon$

$(q, z, P) = (q, x, P) \quad (P, z, P)$




Step 1 : Simply PDA

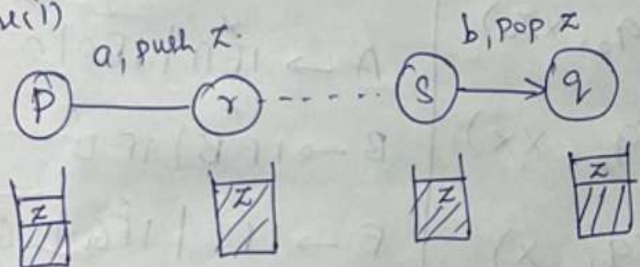
Step 2 : Build CFG.

Non-terminal for every pair of states : $A_{pq}$, $A_{qr}$, $A_{rq_0}$...

Starting non-terminal : $A_{q_0q_f}$

→ consider p & q states in PDA.

case (i)

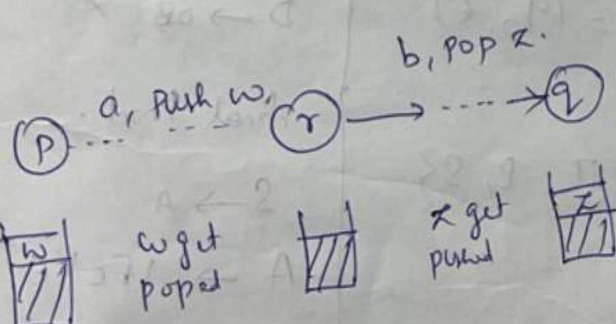



String generated by path

--> "a ..... b"

$A_{pq} \rightarrow a A_{rs} b$

case ii)




$A_{pq} \rightarrow A_{pr} A_{rq}$

iii) to get from any state (P) to itself.

$$A_{PP} \rightarrow \epsilon$$

iv) If PDA accepts some string, then there is a way to go from $q_0$ to $q_f$ that doesn't modify start.

$$A_{q_0 q_f}$$

② Construct CFG.

$$\delta(q_0, 1, z_0) = (q_0, Xz_0)$$

$$\delta(q_0, 1, X) = (q_0, XX)$$

$$\delta(q_0, 0, X) = (q_1, X)$$

$$\delta(q_1, 1, X) = (q_1, \epsilon)$$

$$\delta(q_1, 0, z_0) = (q_0, z_0)$$

$$\delta(q_0, \epsilon, z_0) = (q_0, \epsilon)$$

Soln $CFG = \{V, T, P, S\}$

$T = \{0, 1\}$

P: Start prod.

i) $S \rightarrow \overset{A}{[q_0, z_0, q_0]} - A$

$S \rightarrow \overset{B}{[q_0, z_0, q_1]} - B$

continue

$S \rightarrow A \mid B$

$A \rightarrow 1EA \mid 1FC \mid \epsilon$

$B \rightarrow 1EB \mid 1FD$ ×

$E \rightarrow 1EE \mid 1FG \mid 0G$ ×

$F \rightarrow 1EF \mid 1FH \mid 0H$

$H \rightarrow 1$

$C \rightarrow 0A$

$D \rightarrow 0B$ ×

final Prod.

$S \rightarrow A$

$A \rightarrow 1FC \mid \epsilon$

$F \rightarrow 1FH \mid 0H$

$H \rightarrow 1$

$C \rightarrow 0A$

i) $\delta(q_0, 1, Z_0) = (q_0, XZ_0)$ $\quad 2^2 = 4$

$[q_0, Z_0, q_0] \rightarrow 1[q_0, X, q_0][q_0, Z_0, q_0]$
A $\quad$ E $\quad$ A

$[q_0, Z_0, q_0] \rightarrow 1[q_0, X, q_1][q_1, Z_0, q_0]$
A $\quad$ F $\quad$ C

$[q_0, Z_0, q_1] \rightarrow 1[q_0, X, q_0][q_0, Z_0, q_1]$
B $\quad$ E $\quad$ B

$[q_0, Z_0, q_1] \rightarrow 1[q_0, X, q_1][q_1, Z_0, q_1]$
B $\quad$ F $\quad$ D

ii) $\delta(q_0, 1, X) = (q_0, XX)$

$[q_0, X, q_0] \rightarrow 1[q_0, X, q_0][q_0, X, q_0]$

$[q_0, X, q_0] \rightarrow 1[q_0, X, q_1][q_1, X, q_0]$

$[q_0, X, q_1] \rightarrow 1[q_0, X, q_0][q_0, X, q_1]$

$[q_0, X, q_1] \rightarrow 1[q_0, X, q_1][q_1, X, q_1]$

iii) $\delta(q_0, 0, X) = (q_1, X)$

$[q_0, X, q_0] = 0[q_1, X, q_0]$

$[q_0, X, q_1] = 0[q_1, X, q_1]$

iv) $\delta(q_1, 1, X) = (q_1, \epsilon)$

$[q_1, \cancel{X}, q_1] = 1\cancel{[q_1, \epsilon, q_0]}$ 1

v) $\delta(q_1, 0, Z_0) = (q_0, Z_0)$

$[q_1, Z_0, q_0] = 0[q_0, Z_0, q_0]$

$[q_1, Z_0, q_1] = 0[q_0, Z_0, q_1]$

$\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$[q_0, Z_0, q_0] = \epsilon$

$[q_0, z_0, q_0] - A$ $\quad$ $[q_0, z_0, q_1] = B$

Conformation $[q_1, z_0, q_0] - C$ $\quad$ $[q_1, z_0, q_1] - D$

$[q_0, x, q_0] - E$ $\quad$ $[q_0, x, q_1] - F$

$[q_1, x, q_0] - G$ $\quad$ $[q_1, x, q_1] - H$

(2) $M = \{q_0, q_1\} \{a, b\} \{Z_0\} \delta, q_0, \phi\}$

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, a, a) = (q_0, a a)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

Soln $\quad S \to [q_0\ z_0\ q_0]$ A

$S \to [q_0\ z_0\ q_1]$ B

(i) $\delta(q_0, a, z_0) = (q_0, a z_0)$

$\underset{A}{[q_0, z_0, q_0]} = a \underset{F}{[q_0, a, q_0]} \underset{A}{[q_0, z_0, q_0]}$

$\underset{A}{[q_0, z_0, q_0]} = a \underset{E}{[q_0, a, q_1]} \underset{G}{[q_1, z_0, q_0]}$

$\underset{B}{[q_0, z_0, q_1]} = a \underset{F}{[q_0, a, q_0]} \underset{A}{[q_0, z_0, q_0]}$

$\underset{B}{[q_0, z_0, q_1]} = a \underset{E}{[q_0, a, q_1]} \underset{H}{[q_1, z_0, q_1]}$

ii) $\delta(q_0, a, a) = (q_0, aa)$

$[q_0, \underset{F}{a}, q_0] = a\,[q_0, \underset{F}{a}, q_0]\,[q_0, \underset{F}{a}, q_0]$ +

$[q_0, \underset{F}{a}, q_0] = a\,[q_0, \underset{E}{a}, q_1]\,[q_1, \underset{I}{a}, q_0]$ +

$[q_0, \underset{E}{a}, q_1] = a\,[q_0, \underset{F}{a}, q_0]\,[q_0, \underset{E}{a}, q_1]$ +

$[q_0, \underset{E}{a}, q_1] = a\,[q_0, \underset{E}{a}, q_1]\,[q_1, \underset{D}{a}, q_1]$ ✓

iii) $\delta(q_0, b, a) = (q_1, \epsilon)$

$[q_0, \underset{E}{a}, q_1] = b$.

iv) $\delta(q_1, b, a) = (q_1, \epsilon)$

$[q_1, \underset{D}{a}, q_1] = b$

v) $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$[q_1, \underset{H}{Z_0}, q_1] = \epsilon$

$S \to A \mid B$

$A \to aFA \mid aEG$

$B \to aFA \mid aEH$

$F \to aFF \mid aEI$

$E \to aFE \mid aED \mid b$

$D \to b$

$H \to \epsilon$

After useless prod.

$S \rightarrow B$

~~Aaaa~~ $B \rightarrow aEH$

$E \rightarrow aED \mid b$

$D \rightarrow b$

$H \rightarrow \epsilon$

Pumping lemma CFL

Let L be any CFL. Then there exists a constant 'n', depending only on 'L', such that if $z$ is in L and $|z| \geq n$, then we can write $z = uvwxy$ such that

(i) $|vx| \geq 1$ (or) $|vx| \neq \epsilon$

(ii) $|vwx| \leq n$

(iii) for all $i \geq 0$

$uv^iwx^iy \in L$

1. $L = \{a^n b^n c^n \mid n \geq 0\}$ is not CFL.

soln: $L = \{\epsilon, abc, aabbcc \ldots\}$

$z = aaabbbccc$

$|z| \geq n \qquad 9 \geq 3$

$z = \underbrace{aab}_{u}\underbrace{b}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{ccc}_{y}$

$|vx| \geq 1$

$|vwx| \leq n$

$uv^i wx^i y$

Let $i=0$ $\quad uwy \Rightarrow aaabccc \notin L$

Let $i=1$ $\quad uvwxy \Rightarrow aaabbbccc \in L$.

2. $L = \{0^p \mid p \text{ is prime no}\}$.

$L = \{0^2, 0^3, 0^5, 0^7 \dots\}$

$z = \underset{u\,v\,w\,x\,y}{00000} = 0^5$

$|z| \geq n$ $\quad 5 \geq 5$

$|vwx| \leq 1$ $\quad 3 \leq 5$

$uv^i wx^i y$

$i=0 \Rightarrow 000 \in L$

$i=1 \Rightarrow 00000 \in L$

$i=2 \Rightarrow 0000000 \in L$

$i=3 \Rightarrow 000000000 \notin L$

Not CFL

3

③ Show $L = x^n y^n z^n \mid n \geq 1$ is Context-free or not

Soln: L is context free

$$S = x^n y^n z^n$$

$u v^i w x^i y$

case (i) $n=4$ $S = x^4 y^4 z^4$

v and x each contain only one type of symbol

$$S = \underbrace{x}_{u}\,\underbrace{xx}_{v}\,\underbrace{xyyyy}_{w}\,\underbrace{z}_{x}\,\underbrace{zz}_{y}$$

Let $i=2$ $uv^2wx^2y$

$S = xxxxx\ xyyyyzz\ zz \notin L$

case (ii) Either v or x has more than type of symbol

$n=4$ $$S = \underbrace{xx}_{u}\,\underbrace{xxyy}_{v}\,\underbrace{y}_{w}\,\underbrace{y}_{x}\,\underbrace{zzzz}_{y}$$

$i=2$ $S = xx\ xxyyxxyy\ yyy\ zzzz$

$= x^4 y^2 x^2 y^5 z^4 \notin L$

(4) $L = a^n b^n \mid n \geq 1$

$uv^iwx^iy$

$z = aaabbb$ $n = 3$

$|vx| \geq 1$

(i) $\underbrace{a}_{u}\underbrace{a}_{v}\underbrace{ab}_{w}\underbrace{b}_{x}\underbrace{b}_{y}$

$|vwx| \leq n$

$i = 2$

$v$ & $x$ only one type

aaaabbbb

$a^4 b^4 \in L$

(ii) $\underbrace{aa\ a}_{u}\underbrace{ab}_{v}\ b\ bb$
$w\ x\ y$

$v$ & $x$ have more than one type.

$i = 2$.

aaaababbbbb

$a^5 b^5 \in L$

$\therefore a^n b^n$ is context free