T 8127

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fifth Semester

Computer Science and Engineering

CS 1303 — THEORY OF COMPUTATION

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Construct a finite automata for the language $\{0^n \mid n \bmod 3 = 2, n \geq 0\}$.

2. Let R be any set of regular languages. Is $\cup R_i$ regular? Prove it.

3. Consider the alphabet $\Sigma = \{a, b, (,), +, *, ., \in\}$. Construct a context free grammar that generates all strings in Σ^* that are regular expressions over the alphabet $\{a, b\}$.

4. Write a CFG to generate the set $\{a^m b^n c^p \mid m + n = p \text{ and } p \geq 1\}$.

5. Can you say the language generated by a CFG in CNF is finite or infinite? If so, how? If not, why?

6. Define the languages generated by a PDA using final state of the PDA and empty stack of that PDA.

7. What is the class of language for which the TM has both accepting and rejecting configuration? Can this be called a context free language?

8. The binary equivalent of a positive integer is stored in a tape. Write the necessary transitions to multiply that integer by 2.

9. Show that the following problem is undecidable.

   "Given two CFGs G_1 and G_2, is $L(G_1) \cap L(G_2) = \phi$?".

10. Define L_d.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that a language L is accepted by some $\in$ –NFA if and only if L is accepted by some DFA. (8)

(ii) Consider the following $\in$ –NFA. Compute the $\in$-Closure of each state and find it's equivalent DFA. (8)

	$\in$	A	b	c
p	$\{q\}$	$\{p\}$	ϕ	ϕ
q	$\{r\}$	ϕ	$\{q\}$	ϕ
$*r$	ϕ	ϕ	ϕ	$\{r\}$

Or

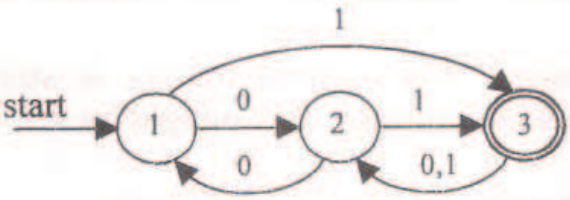
(b) (i) Prove that a language L is accepted by some DFA iff L is accepted by some NFA.

(ii) Convert the following NFA to it's equivalent DFA.

	0	1
p	$\{p, q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	ϕ
$*s$	$\{s\}$	$\{s\}$

12. (a) (i) Explain the construction of NFA with $\in$ transition from any given regular expression. (8)

(ii) Find the regular expression for the set of all strings denoted by R_{13}^2 from the deterministic finite automata given below : (8)



Or

(b) Find whether the following languages are regular or not.

(i) $L = \{w \in \{a,b\} \mid w = w^R\}$. (4)

(ii) $L = \{0^n 1^m 2^{n+m}, n,m \geq 1\}$. (4)

(iii) $L = \{1^k \mid k = n^2, n \geq 1\}$. (4)

(iv) $L_1 / L_2 = \{x \mid \text{for some } y \in L_2,\ xy \in L_1\}$, where L_1 and L_2 are any two languages and L_1 / L_2 is the quotient of L_1 and L_2. (4)

13. (a) (i) Prove that if $L = N(P_N)$ for some PDA $P_N = (Q, \Sigma, \Gamma, \delta_N, q_0, Z_0)$, then there is a PDA P_F such that $L = L(P_F)$. (8)

(ii) Construct a PDA for $\{a^n b^m a^{2(m+n)} \mid n,m \geq 0\}$. (8)

Or

(b) (i) Show that the grammar $S \to a\ S\ b\ S \mid b\ S\ a\ S \mid \in$ is ambiguous and what is the language generated by this grammar? (6)

(ii) Write a grammar to recognize all prefix expressions involving all binary arithmetic operators. Construct parse tree for the sentence "$- * + a\ b\ c / d\ e$" using your grammar. (6)

(iii) Suppose G is a CFG and w, of length l, is in $L(G)$. How long is a derivation of w in G if G is in CNF and if G is in GNF? (4)

14. (a) (i) Show that every CFL without $\in$ can be generated by a CFG in CNF. (4)

(ii) Simplify the following grammar and find it's equivalent in CNF. (8)

$S \to bA \mid aB \quad A \to bAA \mid aS \mid a \qquad B \to aBB \mid bS \mid b$

(iii) Find the GNF equivalent of the grammar $S \to AA \mid 0$, $A \to SS \mid 1$. (4)

Or

(b) (i) Design a Turing Machine M for $f(x,y,z) = 2(x + y) - z$, $z < 2(x + y)$ and x,y,z are stored in the tape in the form $0^x 10^y 10^z 1$. (12)

(ii) Show that if L is accepted by a multi tape Turing machine, it is accepted by single tape Turing machine also. (4)

15. (a) Find whether the following languages are recursive or recursively enumerable.

(i) Union of two recursive languages. (4)

(ii) Union of two recursively enumerable languages. (4)

(iii) L if L and complement of L are recursively enumerable. (4)

(iv) L_u. (4)

Or

(b) (i) Show that "Finding whether the given CFG is ambiguous or not" is undecidable by reduction technique. (4)

(ii) Consider the Turning Machine M and $w = 01$,

where $M = (\{q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_1, B, \{q_3\})$ and

δ is given by

q_i	$\delta(q_i, 0)$	$\delta(q_i, 1)$	$\delta(q_i, B)$
q_1	$(q_2, 1, R)$	$(q_2, 0, L)$	$(q_2, 1, L)$
q_2	$(q_3, 0, L)$	$(q_1, 0, R)$	$(q_2, 0, R)$
q_3	–	–	–

Reduce the above problem to Post's Correspondence Problem and find whether that PCP has a solution or not. (12)