ii. An irregular 6 faced die is such that the probability that it gives 3 odd numbers in 7 throws is twice the probability that it gives 4 odd numbers in 7 throws. How many sets of exactly 7 trials one can expect to give no odd number out of 5000 sets.

30. a. Two independent groups of 10 children were tested to find how many digits they could repeat from memory after hearing them. The results are as follows.

Group A	8	6	5	7	6	8	7	4	5	6
Group B	10	6	7	8	6	9	7	6	7	7

Is the difference between mean scores of the two groups significant?

(OR)

b. Fit a Poisson distribution for the following distribution and also test the goodness of fit.

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

31. a. A one person barber shop can accommodate a maximum of 7 people at a time (6 waiting and 1 getting hair cut). Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the average rate of 3/hr. The barber cuts hair at an average rate of 4/hr.

(i) What is the probability that a customer can get directly into the barber's chair upon arrival?
(ii) What is the expected number of customers waiting for a hair cut?
(iii) How much time can a customer expect to spend in the barber shop?
(iv) What fraction of potential customers are turned away?

(OR)

b. In a railway reservation counter customers arrive at a rate of 32 persons per hour and the clerk can handle and provide assistance using a computer terminal to 40 persons per hour. If the arrival and service follow a Poisson process calculate

(i) The average number of customers waiting for service
(ii) Average waiting time for a customer before getting the service.
(iii) The probability that there will be eleven or more customers in the system
(iv) Average length of the queue that forms from time to time
(v) Average number of customers in the system.

32. a. A salesman's territory consists of 3 cities. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However if he sells either in B or C, then the next day he is twice as likely to sell in A as in the other city. Find the transition probability matrix and classify the states.

(OR)

b. The transition probability matrix of a Markov chain $\{Xn\}, n = 1,2...$ having 3 states 1, 2, and 3 is $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 3/6 & 2/6 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ and the initial distribution is $p^{(0)} = (0 \quad 1 \quad 0)$.

Find (i) $P(X_2 = 3)$ (ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

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B.Tech. DEGREE EXAMINATION, MAY 2019

1st to 7th Semester

15MA207 – PROBABILITY AND QUEUING THEORY

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

(Statistical tables to be provided)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. A random variable X takes two values 0 and 1 with equal probability. The mean of X is
(A) 1/4 (B) 3/4
(C) 1/2 (D) 1

2. $F(-\infty) =$ ____ where F(x) is the CDF of X
(A) 1 (B) 0
(C) 1/2 (D) 1/4

3. $Var(aX+b)$ is equal to
(A) $a^2Var(X)$ (B) $a\,Var(X)$
(C) $a^2Var(X)+b$ (D) $aVar(X)+b$

4. If the first 4 moments of a distribution about $x = 4$ are 1, 4, 10, 45 then the value of mean is
(A) 3 (B) 4
(C) 6 (D) 5

5. If the MGF of a discrete RV X is $e^{4\left(e^{t-1}\right)}$, the variance is
(A) λ (B) λ^2
(C) $\lambda^2+\lambda$ D) 2λ

6. When n is very large and p is nearly _______, the normal distribution is the limiting form of the binomial distribution.
(A) 1 (B) 1/2
(C) 1/4 (D) 3/4

7. The mean of geometric distribution with probability mass function $P(X = r) = pq^r$, $r = 0, 1, 2, ...$ is
(A) $1/q$ (B) $1/p$
(C) q^2/p (D) q/p

8. The mean and variance of a binomial distribution are 4 and 4/3 respectively. The value of n is
(A) 4 (B) 7
(C) 6 (D) 5

9. Rejection of null hypothesis when it is true is a
(A) Type II error (B) Statistics
(C) Type I error (D) Parameter

10. F test is used to test the equality of
(A) Variances (B) Mean deviation
(C) Standard deviation (D) Goodness of fit

11. The formula used to test the difference between sample proportion and population proportion is
(A) $\dfrac{p-P}{\sqrt{PQ/n}}$ (B) $\dfrac{q-P}{\sqrt{Pq/n}}$
(C) $\dfrac{p-P}{\sqrt{q/n}}$ (D) $\dfrac{p-P}{\sqrt{P/n}}$

12. The value of the test statistic z for which the critical region and acceptance region are separated is called the ________.
(A) Unbiased estimate (B) Parameter
(C) Critical value (D) Standard error

13. Which of the following best describe queuing theory?
(A) The study of arrival rate (B) The study of service rates
(C) The study of waiting lines (D) The evaluation of service time costs

14. For a Poisson random variable, λ represents the ______ number of arrivals per time period.
(A) Maximum (B) Minimum
(C) Average (D) Standard deviation of

15. In Kendal's notation $(a/b/c):(d/e)$ symbol 'c' represents
(A) Arrival rate (B) Service rate
(C) Queue discipline (D) Number of servers

16. The traffic intensity of a queuing system is
(A) λ (B) μ
(C) λ/μ (D) μ/λ

17. Identify the probability vector in a Markov chain
(A) $[0.5 \quad 0.5 \quad 0.5]$ (B) $[0 \quad 0.5 \quad 0.4]$
(C) $[0 \quad 0 \quad 0]$ (D) $[1/6 \quad 1/2 \quad 1/3]$

18. ________ is a stochastic matrix.
(A) $\begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}$ (B) $\begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}$
(C) $\begin{bmatrix} 0.4 & 0.7 \\ 0.3 & 0.6 \end{bmatrix}$ (D) $\begin{bmatrix} 1/3 & 1/2 \\ 1/6 & 1/3 \end{bmatrix}$

19. If the tpm of a Markov chain is $\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$ with state space 1, 2, then
(A) States 1 and 2 are persistent (B) State 1 is persistent
(C) State 2 is persistent (D) State 2 is transient

20. If a Markov chain is finite and irreducible all its states are
(A) Reducible (B) Ergodic
(C) Non null persistent (D) Null persistent

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If the CDF of the random variable X is $F(x)=\begin{cases} 0, & x\le 0 \\ 2x^2-x^3, & 0<x<1 \\ 1, & x\ge 1 \end{cases}$

Find (i) the pdf of X and (ii) $P\left(\frac{1}{2}<X<1\right)$.

22. The probability of an individual suffering a bad reaction from an injection of a certain antibiotic is 0.001. Out of 2000 individuals find the probability that exactly 3 sufferers.

23. A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the standard deviation is 10 cm?

24. Write down the formulas for P_0 and P_n in a Poisson queue system in the steady state.

25. Two boys B_1, B_2 and two girls G_1, G_2 are throwing a ball from one to another. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand, each girl throws the ball to each boy with probability 1/2 and never to the other girl. Write the transition probability matrix.

26. The pdf of a random variable X is given by $f(x)=\begin{cases} k & in\ -2<x<2 \\ 0, & elsewhere \end{cases}$. Find (i) k (ii) variance of X.

27. The time (in hrs) required to repair a machine is exponentially distributed with parameter $\lambda=1/2$. What is the conditional probability that a repair take atleast 10 hrs given that its duration exceeds 9 hrs?

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. The probability distribution of a random variable X is given in the following table.

x	0	1	2	3	4	5	6
$p(x)$	k	3k	5k	7k	9k	11k	13k

Find (i) K (ii) $P(X<4)$ (iii) $P(3<X\le 6)$ (iv) find the cdf of X (v) Calculate the minimum value of λ such that $P(X\le\lambda)>0.3$.

(OR)

b.i. If X is uniformly distributed in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ find the pdf of $Y=\tan X$. (4 Marks)

ii. A fair die is tossed 600 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 80 to 120 sixes. (8 Marks)

29. a. Find the mean and standard deviation of marks in an exam where 44% of the candidates obtained marks below 55 and 6% got above 80 marks.

(OR)

b.i. Find the MFG of Poisson distribution and hence find its mean.