Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours   Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. If $E\left[X^2\right] = 8$ and $E[X] = 2$ then $Var(X)$ is
   (A) 3
   (B) 2
   (C) 1
   (D) 4

2. The expectation of the number on a die when thrown is
   (A) 1
   (B) 7/2
   (C) 3
   (D) 2

3. If C is a constant (non-random variable) then E(C)
   (A) 0
   (B) 1
   (C) Cf(c)
   (D) –1

4. The expected value of a random variable is equal to its
   (A) Zero
   (B) Variance
   (C) Mean
   (D) Standard deviation

5. The mean of binomial distribution is 20 and S.D is 4. Find 'n'
   (A) 160
   (B) 180
   (C) 100
   (D) 150

6. The MGF of a geometric distribution is
   (A) $\dfrac{1}{1-qe^t}$
   (B) $\dfrac{1}{1-pe^t}$
   (C) $\dfrac{q}{1-pe^t}$
   (D) $\dfrac{pe^t}{1-qe^t}$

7. If X is uniformly distributed in (0, 10), then P(X > 8) is
   (A) 1/5
   (B) 1/10
   (C) 3/5
   (D) 1/3

8. The random variable X is exponentially distributed with parameter 2. Find mean
   (A) 1/2
   (B) 4/9
   (C) 2/3
   (D) 5/7

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30. a.i. A simple sample of heights of 6400 Englishmen has a mean of 170 cm and SD of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D of 6.3 cm. Do the data indicate that the Americans are on the average taller than the Englishmen?

ii. Table below gives the number of air craft accidents that occurred during various days of a week. Test whether the accidents are uniformly distributed over the week.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No.of accidents	15	19	13	12	16	15

(OR)

b. The nicotine contents in two random samples of tobacco are given below:

Sample 1	21	24	25	26	27	-
Sample 2	22	27	28	30	31	36

Can you say that the two samples came from the same population?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 mins.
   (i) Find the average number of persons waiting in the system.
   (ii) What is the probability that a person arriving at the booth will have to wait in the queue?
   (iii) Estimate the fraction of the day when the phone will in use.
   (iv) What is the average length of the queue that forms from time to time?

(OR)

b. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is expontial with mean rate of 20 per hour.
   (i) Find the effective arrival rate at the clinic.
   (ii) What is the probability that an arriving patient will not wait?
   (iii) What is the expected waiting time until a patient is discharged from the clinic?

32. a. The transition probability matrix of a Markov Chain $\{X_n\}$, $n = 1, 2, 3 \ldots$ having 3 states 1, 2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)} = (0.7 \quad 0.2 \quad 0.1)$. Find
   (i) $P\{X_2 = 3\}$ (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. show that the process is Markovian. Find the transition matrix and classify the states.

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9. If a teacher is trying to prove that the new method of teaching mathematics is more effective than traditional one, he/she will conduct a
   - (A) Two tailed test
   - (B) One tailed test
   - (C) Neither one tailed nor two tailed test
   - (D) Both one tailed and two tailed test

10. Type I error
    - (A) Reject H_0 when it is true
    - (B) Accept H_0 when it is false
    - (C) Accept H_0 when it is true
    - (D) Reject H_0 when it is false

11. The standard deviation of any sampling distribution is called
    - (A) Sampling error
    - (B) Sample error
    - (C) Standard error
    - (D) Simple deviation

12. For F-test the value of F should be
    - (A) Less than 1
    - (B) Greater than 1
    - (C) Equal to 1
    - (D) Equal to 0

13. What stand for 'd' in the queue model $(a/b/c : d/e)$
    - (A) Queue discipline
    - (B) System capacity
    - (C) Service time
    - (D) Number of servers

14. The probability of number of customers in the system in $(M/M/1):(\infty/FIFO)$ model is
    - (A) $\dfrac{\lambda}{\mu}$
    - (B) $\dfrac{\lambda}{\mu}-1$
    - (C) $1-\dfrac{\lambda}{\mu}$
    - (D) $\dfrac{\lambda}{\mu}+1$

15. The average waiting time of a customer in the system in $(M/M/1):(\infty/FIFO)$ model is
    - (A) $\dfrac{1}{\mu-\lambda}$
    - (B) $\dfrac{1}{\lambda-\mu}$
    - (C) $\dfrac{1}{\lambda+\mu}$
    - (D) $\dfrac{1}{\lambda^2}$

16. Consider an $(M/M/1):(\infty/FIFO)$ queue. The probability that the waiting time of a customer in the system exceeds t, $\left[P(W_s>t)\right]$ is
    - (A) $e^{-(\lambda-\mu)t}$
    - (B) $e^{-\lambda t}$
    - (C) $e^{-(-\mu-\lambda)t}$
    - (D) $e^{-\mu t}$

17. The steady state probability vector π of a discrete Markov chain with tpm P satisfies the matrix equation
    - (A) $P\pi=0$
    - (B) $\pi P=\pi$
    - (C) $P^t\pi=0$
    - (D) $\pi(1-P)=0$

18. In a tpm, the sum of all elements of any row is
    - (A) 1
    - (B) 0
    - (C) 2
    - (D) 1

19. Ergodic means
    - (A) Irreducible and periodic
    - (B) Irreducible and aperiodic
    - (C) Not irreducible
    - (D) Regular

20. If the one step transition probability does not depend on the step, then the Markov chain is
    - (A) Reducible
    - (B) Regular
    - (C) Homogeneous
    - (D) Non Homogeneous

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Let x be a continuous random variable with pdf $f(x)=\begin{cases} x/2\;;\;1<x<5 \\ 0\quad;\;otherwise \end{cases}$

    Find the pdf of $y=2x-3$.

22. If the probability that an applicant for a driver's licence will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the fourth trial and (ii) in fewer than 4 trials?

23. The fatality rate of typhoid patients is believed to be 17.26 percent. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital as efficient?

24. If $\lambda=3$ per hour, $\mu = 4$ per hour and maximum capacity K = 7 in a $(M/M/1):(K/FIFO)$ system, find the average number of customers in the system.

25. If the tpm of a Markov chain is $\begin{pmatrix} 1 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the steady state distribution of the chain.

26. A random variable X has mean $\mu = 12$ and variance $\sigma^2 = 9$ and an unknown probability distribution. Find $P(6 < X <18)$.

27. State and prove memoryless property of the exponential distribution.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. A random variable X has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) 'a' (ii) $P(X<3)$ (iii) $P(0<X<5)$ and (iv) Cumulative Distribution Function.

(OR)

b. A random variable x has the pdf $f(x)=Kx^2e^{-x}$; $x \ge 0$. Find K, mean, variance and $E\left[3x^2-2x\right]$.

29. a. Fit a binomial distribution for the following data and also calculate the expected frequencies.

x	0	1	2	3	4	5	6	Total
f	5	18	28	12	7	6	4	80

(OR)

b. As a result of a test on 20,000 electric fans manufactured by a company, it was found that the life time of the fans is normally distributed with average life of 2040 hours and the standard deviation of 60 hours. On the basis of the information, estimate that the number of fans expected to run for (i) more than 2150 hours (ii) less than 1960 hours (iii) more than 1960 hrs and less than 2160 hours.