Inverse Z-Transform

1. Long division method
2. partial fraction method.
3. Residue method.
4. Convolution method.

Long Division method

Find Inverse Z-transform using long division method

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$ if (a) $x(n)$ is causal (b) $x(n)$ is anticausal

(a)

$$
\begin{array}{r|l}
 & 1+4z^{-1}+7z^{-2}+10z^{-3}+13z^{-4}+16z^{-5} \\
\hline
1-2z^{-1}+z^{-2} & 1+2z^{-1} \\
 & 1-2z^{-1}+z^{-2} \\
\hline
 & 4z^{-1}-z^{-2} \\
 & 4z^{-1}-8z^{-2}+4z^{-3} \\
\hline
 & 7z^{-2}-4z^{-3} \\
 & 7z^{-2}-14z^{-3}+7z^{-4} \\
\hline
 & 10z^{-3}-7z^{-4} \\
 & 10z^{-3}-20z^{-4}+10z^{-5} \\
\hline
 & 13z^{-4}-10z^{-5} \\
 & 13z^{-4}-26z^{-5}+13z^{-6} \\
\hline
 & 16z^{-5}-13z^{-6} \\
 & 16z^{-5}-32z^{-6}+16z^{-7} \\
\hline
 & 19z^{-6}-16z^{-7}
\end{array}
$$

$$\boxed{\begin{array}{l} X(z) = 1+4z^{-1}+7z^{-2}+10z^{-3}+13z^{-4}+16z^{-5}+\dots \\ x(n) = \{\underset{\uparrow}{1}, 4, 7, 10, 13, 16, 19 \dots\} \end{array}}$$

(b) Anticausal

$$2z + 5z^2 + 8z^3 + 11z^4 + 14z^5$$

$$z^{-2} - 2z^{-1} + 1 \;\Big)\; 2z^{-1} + 1$$

$$2z^{-1} - 4 + 2z$$

$$5 - 2z$$

$$5 - 10z + 5z^2$$

$$8z - 5z^2$$

$$8z - 16z^2 + 8z^3$$

$$11z^2 - 8z^3$$

$$11z^2 - 22z^3 + 11z^4$$

$$14z^3 - 11z^4$$

$$14z^3 - 28z^4 + 14z^5$$

$$17z^4 - 14z^5$$

$$\boxed{X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + 14z^5 + \cdots}$$

$$x(n) = \{\ldots\ 14, 11, 8, 5, 2, \underset{\uparrow}{0}\}$$

Find the inverse z-transform using partial fraction method.

$$X(z) = \frac{\frac{1}{4} z^{-1}}{\left(1-\frac{1}{2} z^{-1}\right)\left(1-\frac{1}{4} z^{-1}\right)} \quad ; \text{ ROC } |z| > \frac{1}{2}$$

Solution

$$X(z) = \frac{\frac{1}{4} z^{-1}}{\left(1-\frac{1}{2} z^{-1}\right)\left(1-\frac{1}{4} z^{-1}\right)}$$

$$\frac{X(z)}{z} = \frac{\frac{1}{4}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$\frac{\frac{1}{4}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{2}\right)} = \frac{A}{\left(z-\frac{1}{2}\right)} + \frac{B}{\left(z-\frac{1}{4}\right)}$$

$$\frac{1}{4} = A\left(z-\frac{1}{4}\right) + B\left(z-\frac{1}{2}\right)$$

put $z = \frac{1}{4}$. | put $z = \frac{1}{2}$

$\boxed{B = -1}$ | $\boxed{A = 1}$

$$\frac{X(z)}{z} = \frac{1}{\left(z-\frac{1}{2}\right)} - \frac{1}{\left(z-\frac{1}{4}\right)}$$

$$X(z) = \frac{z}{\left(z-\frac{1}{2}\right)} - \frac{z}{\left(z-\frac{1}{4}\right)}$$; Take Inv. z Transform

$$\boxed{x(n) = \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)}$$