MATHS ASSIGNMENT-2

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SECTION: T2
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Q1. Consider two groups:

1^{st} group → 7 boys

2^{nd} group → 5 girls

Ways to arrange 7 boys = $7!$

Ways to arrange 5 girls = $5!$

Ways to arrange 2 groups = $2!$

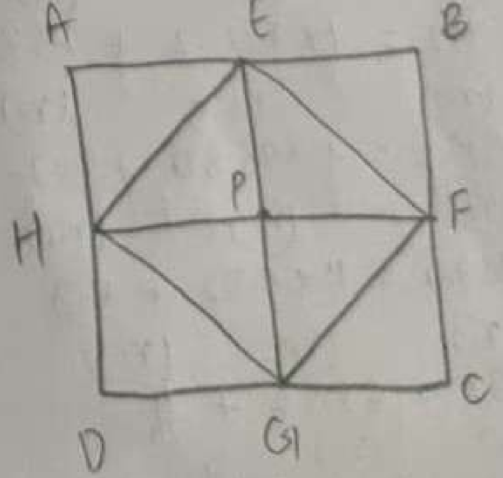
Total no. of ways = $2! \times 5! \times 7!$

⇒ $2 \times 120 \times 5040$

⇒ $\boxed{100800 \text{ ways}}$ //

Q2. Let ABCD → square

E, F, G, H → mid-points of AB, BC, CD, DA respectively.



Let P be a point on the ~~respectively~~ interior of the square.

no. of square = 4

no. of points = 5 (By pigeonhole principle)

Generalisation of pigeonhole principle:

$\left\lfloor \frac{n-1}{m} \right\rfloor + 1 \rightarrow$ pigeons in one pigeonhole

substituting values,

$\left\lceil \frac{5-1}{4} \right\rceil + 1 = 1+1 = \boxed{2}$

$\therefore$ We have atleast 2 points whose distance is exactly $\sqrt{2}$ units

We can divide square ABCD in 4 1x1 squares each of side 1 unit, atleast one such square must contain 2 points exactly $\sqrt{2}$ distance of units apart.

Hence Proved.

Q3.

$a = 28844$, $b = 15712$

Using Euclidean Algorithm,

$$\underset{(a)}{} \quad \underset{(b)}{} \quad \underset{(r_1)}{}$$

(a/b) $28844 = 1 \times 15712 + 13132$

(b/r_1) $\underset{(b)}{15712} = 1 \times \underset{(r_1)}{13132} + \underset{(r_2)}{2580}$

(r_1/r_2) $\underset{(r_1)}{13132} = 5 \times \underset{(r_2)}{2580} + \underset{(r_3)}{232}$

(r_2/r_3) $\underset{(r_2)}{2580} = 11 \times \underset{(r_3)}{232} + \underset{(r_4)}{28}$

(r_3/r_4) $\underset{(r_3)}{232} = 8 \times \underset{(r_4)}{28} + \underset{(r_5)}{8}$

(r_4/r_5) $\underset{(r_4)}{28} = 3 \times \underset{(r_5)}{8} + \underset{(r_6)}{4}$

(r_5/r_6) $\underset{(r_5)}{8} = 2 \times \underset{(r_6)}{4} + \underset{(r_7) \text{ or } (r_{k+1})}{0}$

$\therefore (r_{k+1}) = 0$

r_k is gcd $= 4$

$\gcd(a,b) = 4$

$\gcd(28844, 15712) = 4$ //

Q4.

Given $\gcd(\underset{(a)}{3587}, \underset{(b)}{1819}) = 17$

$\left(\frac{a}{b}\right) \Rightarrow \underset{(a)}{3587} = 1 \times \underset{(b)}{1819} + \underset{(r_1)}{1768}$

$\left(\frac{b}{r_1}\right) \Rightarrow \underset{(b)}{1819} = 1 \times \underset{(r_1)}{1768} + \underset{(r_2)}{51}$

$\left(\frac{r_1}{r_2}\right) \Rightarrow \underset{(r_1)}{1768} = 34 \times \underset{(r_2)}{51} + \underset{(r_3)}{34}$

$\left(\frac{r_2}{r_3}\right) \Rightarrow \underset{(r_2)}{51} = 1 \times \underset{(r_3)}{34} + \underset{(r_4)}{17}$

$\left(\frac{r_3}{r_4}\right) \Rightarrow \underset{(r_3)}{34} = 2 \times \underset{(r_4)}{17} + \underset{(r_{k+1})}{\overset{(r_5)}{0}}$

$\therefore (r_{k+1}) = 0$

$\gcd(a, b) = 17$

$\gcd(3587, 1819) = 17$ //

$17 = 51 - 1 \times 34$

$17 = 51 - 1 \times (1768 - 34 \times 51)$

$17 = 35 \times 51 - 1 \times 1768$

$17 = 35 \times (1819 - 1 \times 1768) - 1 \times 1768$

$17 = 35 \times 1819 - 36 \times 1768$

$17 = 35 \times 1819 - 36 \times (3587 - 1 \times 1819)$

$17 = 71 \times 1819 - 36 \times 3587$

$17 = \underset{(m)}{-36} \times 3587 + \underset{(n)}{71} \times 1819$

$\therefore \boxed{m = -36, \ n = 71}$

Hence, eqn. is $17 = -36 \times 3587 + 71 \times 1819$ //

Q5. $337500 = 2^2 \cdot 3^3 \cdot 5^5$

$21600 = 2^5 \cdot 3^3 \cdot 5^2$

$\gcd(337500, 21600) = 2^{\min(2,5)} \cdot 3^{\min(3,3)} \cdot 5^{\min(5,2)}$

$\Rightarrow 2^2 \cdot 3^3 \cdot 5^2$

$\Rightarrow \boxed{2700}$

$\text{lcm}(337500, 21600) = 2^{\max(2,5)} \cdot 3^{\max(3,3)} \cdot 5^{\max(5,2)}$

$\Rightarrow 2^5 \cdot 3^3 \cdot 5^5$

$\Rightarrow \boxed{27000000}$

$\therefore a \cdot b = \gcd \cdot \text{lcm}$

$\Rightarrow 337500 \cdot 21600 = 2700 \cdot 2700000$

$\Rightarrow 7290000000 = 7290000000$

Hence Proved.

Q6. Given: LHS $= (p \to q) \wedge (p \to r)$

RHS $= p \to (q \wedge r)$

LHS:

$(p \to q) \wedge (p \to r)$

$\Rightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ — ① $\left[[(p \to q) = (\neg p \vee q)] \text{ equivalence relation} \right]$

RHS

$p \to (q \wedge r)$

$\Rightarrow \neg p \vee (q \wedge r)$ — ②

In ②,

$(\neg p \vee q) \wedge (\neg p \vee r)$ — ③ [distributive property of connectors]

Since ① & ③ are equal,

LHS = RHS.

we can say that,

$$\boxed{(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)}$$ //

Q7. The statement $p \to q$ means 'if p then q'.

Converse: The converse of 'if p then q' is 'if q then p'

$\therefore$ converse of $p \to q$: $\boxed{q \to p}$

Inverse: The inverse of 'if p then q' is 'if not p then not q'

$\therefore$ Inverse of $p \to q$: $\boxed{\neg p \to \neg q}$

Thus,

Contrapositive of $p \to q$: $\boxed{\neg q \to \neg p}$

Q8. $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

Applying distributive law,

$((q \vee p) \wedge (q \vee \neg q)) \vee (\neg p \wedge \neg q)$ [$p \vee \neg q \equiv T$]

$\Rightarrow ((q \vee p) \wedge T) \vee (\neg p \wedge \neg q)$

$\Rightarrow (q \vee p) \vee (\neg p \wedge \neg q)$ [Idempotent law ($p \wedge T = p$)]

Applying distributive law again,

$((q \vee p) \vee \neg p) \wedge ((q \vee p) \vee \neg q)$

Using law of associativity,

$(q \vee (p \vee \neg p)) \wedge ((q \vee \neg q) \vee p)$

Now,

$(p \vee \neg p \equiv T)$

Hence,

$(q \vee T) \wedge (T \vee p)$

Using law of domination, $(p \vee T \equiv T)$

$\Rightarrow T \wedge T$

$\Rightarrow \boxed{T}$ //

$\therefore$ The eqtn./argument holds true for every truth values.
Hence, $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$ is a tautology.

<u>Q9.</u> Using mathematical induction

<u>Base step</u>: $n=1$

Keeping $n=1$ in the eqtn. gives:

$1! \geq 2^{1-1}$

$\Rightarrow 1 \geq 2^0$

$\Rightarrow 1 \geq 1 \rightarrow$ true

$\therefore$ The inequality holds true for $n=1$.

let us assume the inequality is true for $n=k$.

for $n=k$,

$k! \geq 2^{k-1}$ — (1)

<u>Inductive Step</u> : let $n = k+1$

inequality begins to be,

$(k+1)! \geq 2^{(k+1)-1}$

$\Rightarrow (k+1)! \geq 2^{k}$

$\Rightarrow (k+1).k! \geq 2^{k}$

Now from eqn.(1), we know $k! \geq 2^{k-1}$

let A be a number bigger than 2^{k-1} but $k! > A$

Now, $2^{k} = 2.2^{k-1}$

we know, $k > 2$ for any natural number k, satisfying eqn.(1),

$\therefore (k+1)(C+D) \rightarrow (k+1) > 3$ & $(C+D) > 0$

$\therefore (k+1).(C+D) > 3.(C+D)$

Since $B = C+D$ & $B > 1$

$\therefore C+D > 1$

$\therefore 3.1 = 3 > 2$, so we know that

$(k+1)! > (k+1).(C+D).2^{(k-1)} > 2.2^{(k-1)} = 2^{k}$

$\therefore \boxed{(k+1)! \geq 2^{k}}$

Hence Proved, for any natural number n, $\boxed{n! \geq 2^{n-1}}$

Q10.

Step	Statement	Reason
(1)	$p \to \neg q$	Rule P
(2)	$q \vee r$	Rule P
(3)	$\neg q \to r$	Rule T, 2, equivalence
(4)	$p \to r$	Rule T, 1, 3, hypothetical syllogism
(5)	$\neg r$	Rule P
(6)	$\neg p$	Rule T, 4, 5, Modus Tollens
(7)	$\neg p \to$	Rule T, 7, Contrapositive
(8)	$\neg s \to p$	Rule P
(9)	s	Rule T, 6, 8, Modus Ponens

∴ The conclusion s was achieved from the given premises using theory of inference.

∴ s is a valid conclusion for the given premises

$p \to \neg q,\ q \vee r,\ \neg s \to p,\ \neg r.$