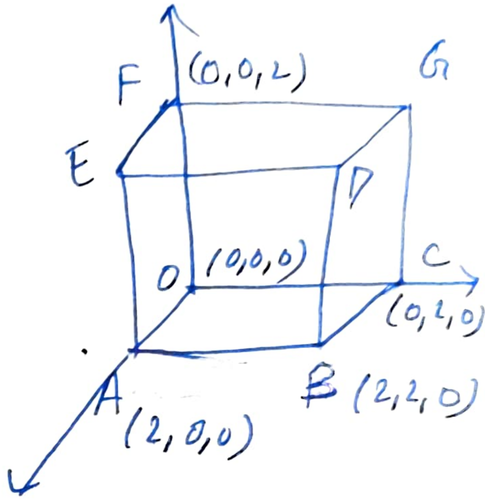
26·5·22

MATHS ASSIGNMENT - 2

KA2111032010006
SHAURYA SRINET

Q1. By Stoke's theorem,

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl } \vec{F}\cdot \hat{n}\, ds$$



$$\vec{F} = (y-z+2)\hat{i} + (yz+4)\hat{j} - xy\,\hat{k}$$

$$\vec{F}\cdot d\vec{r} = (y-z+2)\,dx + (yz+4)\,dy - xy\,dz$$

LHS

$$\int F\cdot dr = \int_{OA} + \int_{AB} + \int_{BC} + \int_{CO}$$

Along OA,

$y=0$ $\quad z=0$ $\quad$ dx exists $x=0$ to 2.

$dy=0$ $\quad dz=0$

$\therefore \vec{F}\cdot d\vec{r} = 2dx$

So, $\int_{OA} \vec{F}\cdot d\vec{r} = \int_0^2 2\,dx = 2[x]_0^2 = 4$ //

Along AB,

$x=2$ $\quad z=0$ $\quad$ dy exists $x=0$ to 2

$dx=0$ $\quad dz=0$

$\therefore \vec{F}\cdot d\vec{r} = 4\cdot dy$

So, $\int_{AB} \vec{F}\cdot d\vec{r} = \int_0^2 4\,dy = 4[y]_0^2 = 8$ //

Along BC,

$y=2$ $\quad z=0$ $\quad$ dx exists from 2 to 0.

$dy=0$ $\quad dz=0$

$\therefore \vec{F}\cdot d\vec{r} = \int_2^0 4\cdot dx = -\int_0^2 4\,dx = -8$ //

Along CO,

$x=0 \quad z=0$ dy exists from 2 to 0.

$dx=0 \quad dz=0$

$\therefore \vec{F}\cdot d\vec{r} = \int_2^0 4\cdot dy = -8$

Thus, $\int_C \vec{F}\cdot d\vec{r} = 4+8-8-8$

$\Rightarrow \boxed{-4}$

$$\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ y-z+2 & yz+4 & -xz \end{vmatrix}$$

$\Rightarrow \hat{i}(-x-y) - \hat{j}(-y+1) + \hat{k}(0-1)$

$\Rightarrow (-x-z)\hat{i} + \hat{j}(y-1) - \hat{k}$.

RHS

$$\iint_S \text{curl } \vec{F}\cdot\hat{n}\, ds = \iint_{ABDE} + \iint_{OCGF} + \iint_{BCGD} + \iint_{OAEF} + \iint_{DEFG}$$

	Surface	$\hat{n}$	ds	values of x, y, z	curl $\vec{F}\cdot\hat{n}$	
S_1	ABDE	$\hat{i}$	dydz	$x=2$	$-x-y$	$\Rightarrow -z-y$
S_2	OCGF	$-\hat{j}$	dydz	$x=0$	$x+y$	$\Rightarrow y$
S_3	BCGD	$\hat{j}$	dzdx	$y=2$	$y-1$	$\Rightarrow 1$
S_4	OAEF	$-\hat{j}$	dzdx	$y=0$	$-(y-1)$	$\Rightarrow 1$
S_5	DEFG	$\hat{k}$	dxdy	$z=2$	-1	$\Rightarrow -1$

S_1; $\iint_{ABDE} \text{curl } \vec{F}\cdot\hat{n}\, ds = \int_0^2\int_0^2 (-2-y)\, dy\, dz$

$$\Rightarrow -\int_0^2\int_0^2 (z+y)\,dy\,dz \Rightarrow -\int_0^2 \left[zy + \frac{y^2}{2}\right]_0^2 dz$$

$$\Rightarrow -\int_0^2 (4+2)\,dz \Rightarrow -6\,[z]_0^2 \Rightarrow -12 //$$

S_2; $\iint_{OCGF} \text{curl}\,\vec{F}\cdot\hat{n}\,ds = \int_0^2\int_0^2 y\,dy\,dz$

$$\Rightarrow \int_0^2 \left[\frac{y^2}{2}\right]_0^2 dz \Rightarrow \int_0^2 2\,dz \Rightarrow 4 //$$

S_3; $\iint_{BCGD} \text{curl}\,\vec{F}\cdot\hat{n}\,ds = \int_0^2\int_0^2 dz\,dx$

$$\Rightarrow \int_0^2 [z]_0^2\,dx \Rightarrow 4 //$$

S_4; $\iint_{OAEF} \text{curl}\,\vec{F}\cdot\hat{n}\,ds = \int_0^2\int_0^2 1.\,dz\,dy$

$$\Rightarrow 4 //$$

S_5; $\iint_{OEFG} \text{curl}\,\vec{F}\cdot\hat{n}\,ds = \int_0^2\int_0^2 -1.\,dx\,dy.$

$$\Rightarrow \int_0^2 [-x]_0^2\,dy \Rightarrow -4 //$$

$\therefore$ RHS $= -12+4+4+4-4$

$$\Rightarrow \boxed{-4}.$$

$\therefore$ Hence, RHS = LHS

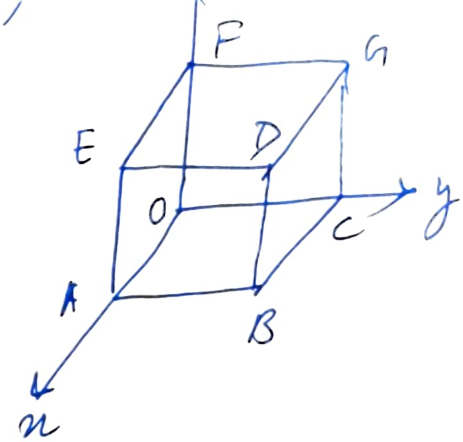
& Stokes Theorem is Verified.

Al

PTO

Q2. By Gauss Divergence Theorem,

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \operatorname{div}\vec{F}\,.\,dv$$



RHS $\iiint_V (\operatorname{div}\vec{F})\,dv$

$\therefore\ V = \int_0^1\int_0^1\int_0^1 (4z - 2y + y)\,dz\,dy\,dx$

$\Rightarrow V = \int_0^1\int_0^1 \left[\frac{4z^2}{2} - yz\right]_0^1 dy\,dx$

$\Rightarrow \int_0^1\int_0^1 [2 - y]\,dy\,dx$

$\Rightarrow \int_0^1 \left[2y - \frac{y^2}{2}\right]_0^1 dx$

$\Rightarrow \int_0^1 (2 - 1/2)\,dx \Rightarrow \int_0^1 3/2\,dx \Rightarrow 3/2\,[x]_0^1$

$\Rightarrow \boxed{3/2}$

LHS

	Surfaces	$\hat{n}$	ds	Values of x, y, z	$\vec{F}\cdot\hat{n}$
S_1	ABDE	$\hat{i}$	$dy\,dz$	$x = 1$	$4z$
S_2	OCGF	$-\hat{i}$	$dy\,dz$	$x = 0$	0
S_3	BCGD	$\hat{j}$	$dz\,dx$	$y = 1$	-1
S_4	OAEF	$-\hat{j}$	$dz\,dx$	$y = 0$	0
S_5	DEFG	$\hat{k}$	$dx\,dz$	$z = 1$	y
S_6	OABC	$-\hat{k}$	$dx\,dz$	$z = 0$	0

$\iint_{S_1} = \int_0^1\int_0^1 4z\,dy\,dz \Rightarrow \int_0^1 [4yz]_0^1\,dz \Rightarrow \int_0^1 4z\,dz$

$\Rightarrow \frac{4z^2}{2} \Rightarrow 2$ //

$$\therefore \iint_{S_2} = \int_0^1\int_0^1 0\, dy\, dz \Rightarrow 0$$

$$\therefore \iint_{S_3} = \int_0^1\int_0^1 -dz\,dx \Rightarrow \int_0^1 -1\,dx \Rightarrow -1$$

$$\therefore \iint_{S_4} = \int_0^1\int_0^1 0 \cdot dz\,dx \Rightarrow 0$$

$$\therefore \iint_{S_5} = \int_0^1\int_0^1 y\,dx\,dy \Rightarrow \int_0^1 y[x]_0^1\, dz \Rightarrow \int_0^1 y\,dz \Rightarrow 1/2$$

$$\therefore \iint_{S_6} = \int_0^1\int_0^1 0 \cdot dx\,dy \Rightarrow 0$$

$$LHS = 2 + 1/2 - 1 + 0 + 0 + 0 \Rightarrow \boxed{3/2}$$

Hence, LHS = RHS

Gauss Divergence Theorem is Verified.

Q3. $f(t) = \begin{cases} t & , 0 < t < 1 \\ 2-t & , 1 < t < 2 \end{cases}$

$$\therefore L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$$

$$\Rightarrow \frac{1}{1-e^{-2s}}\left[\int_0^1 e^{-st} f(t)\,dt + \int_1^2 e^{-st} f(t)\,dt\right]$$

$$\Rightarrow \frac{1}{1-e^{-2s}}\left[\int_0^1 e^{-st}\, t\,dt + \int_1^2 e^{-st}(2-t)\,dt\right]$$

$$\Rightarrow \frac{1}{1-e^{-2s}}\left[\left[t\left(\frac{e^{-st}}{-s}\right) - 1\left(\frac{e^{-st}}{s^2}\right)\right]_0^1 + \left[(2-t)\left(\frac{e^{-st}}{-s}\right) + (-1)(-1)\left(\frac{e^{-st}}{s^2}\right)\right]_1^2\right]$$

$$\Rightarrow \frac{1}{1-e^{-2s}}\left[\cancel{\left(\frac{e^{-s}}{-s}\right)}-\left(\frac{e^{-s}}{s^2}\right)+\frac{e^{0}}{s^2}+\frac{e^{-2s}}{s^2}-\frac{e^{-s}}{s^2}+\cancel{\frac{e^{-s}}{s}}\right]$$

$$\Rightarrow \frac{1}{1-e^{-2s}}\left[\frac{e^{-2s}+1-2e^{-s}}{s^2}\right]$$

$$\Rightarrow \frac{1}{1-e^{-2s}}\left[\frac{1-2e^{-s}+(e^{-s})^2}{s^2}\right]$$

$$\Rightarrow \frac{1}{1-(e^{-s})^2}\left[\frac{(1-e^{-s})^2}{s^2}\right]$$

$$\Rightarrow \frac{1}{(1+e^{-s})(1-e^{-s})} \times \frac{(1-e^{-s})^2}{s^2}$$

$$\Rightarrow \frac{(1-e^{-s})}{1+e^{-s}}\left(\frac{1}{s^2}\right)$$

$$\Rightarrow \left(\frac{1-e^{-s/2}\cdot e^{-s/2}}{1+e^{-s/2}\cdot e^{-s/2}}\right)\left(\frac{1}{s^2}\right)$$

$$\Rightarrow \frac{1}{s^2}\left(\frac{1-\frac{e^{-s/2}}{e^{s/2}}}{1+\frac{e^{-s/2}}{e^{s/2}}}\right) \Rightarrow \frac{1}{s^2}\left(\frac{e^{s/2}-e^{-s/2}}{e^{s/2}+e^{-s/2}}\right)$$

$$\Rightarrow \boxed{\frac{1}{s}\tanh\left(\frac{s}{2}\right)} //$$

PTO $\longrightarrow$

Q4: $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$

$\therefore L^{-1}[F(s)\cdot G(s)] = f(t) * g(t)$

$\therefore f(t) * g(t) = \int_0^t f(u) g(t-u)\, du$

$\Rightarrow L^{-1}\left[\dfrac{s}{s^2+a^2}\cdot\dfrac{s}{s^2+b^2}\right] = L^{-1}[F(s)\cdot G(s)]$

$\therefore F(s) = \dfrac{s}{s^2+a^2}$, $G(s) = \dfrac{s}{s^2+b^2}$

$f(t) = \cos at$, $g(t) = \cos bt$

$\therefore f(t) * g(t) = \int_0^t \cos au \cos b(t-u)\, du$

$\Rightarrow \int_0^t \dfrac{\cos(au+bt-bu)+\cos(au-bt+bu)}{2} \cdot du$

$\Rightarrow \dfrac{1}{2}\int_0^t \cos[(a-b)u+bt] + \cos[(a+b)u-bt]\, du$

$\Rightarrow \dfrac{1}{2}\left[\sin\left(\dfrac{(a-b)u+bt}{a-b}\right)\right]_0^t + \sin\left(\dfrac{(a+b)u-bt}{a+b}\right)\Bigg]_0^t$

$\Rightarrow \dfrac{1}{2}\ \dfrac{\sin at - \sin bt}{(a-b)(a-b)} + \dfrac{\sin at + \sin bt}{(a+b)(a+b)}$

$\Rightarrow \dfrac{(a+b)(\sin at - \sin bt) + (a-b)(\sin at + \sin bt)}{2(a^2-b^2)}$

$\Rightarrow \dfrac{a \sin at - b \sin bt}{a^2-b^2}$ //

Surfaces,

$$x(t) = L^{-1}\left[\frac{2s^2 - s - 2}{(s+1)(s^2-2s+1)}\right]$$

$$\Rightarrow x(t) = L^{-1}\left[\frac{2s^2 - s - 2}{(s+1)(s-1)^2}\right]$$

$$\Rightarrow \frac{2s^2-s-2}{(s+1)(s-1)^2} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\therefore 2s^2 - s - 2 = \frac{A}{s+1} + \frac{B(s-1)+C}{(s-1)^2}$$

So,

$$\Rightarrow \frac{2s^2-s-2}{\cancel{(s+1)(s-1)^2}} = \frac{A(s-1)^2 + B(s-1)(s+1) + C(s+1)}{\cancel{(s+1)(s-1)^2}}$$

$$\Rightarrow 2s^2 - s - 2 = A(s^2+1-2s) + B(s^2-1) + C(s+1)$$

$$\Rightarrow 2s^2 - s - 2 = (A+B)s^2 + (-2A+C)s + A - B + C$$

$\therefore A + B = 2 \qquad A - B + C = -2$

$-2A + C = -1$

$B = 2 - A \qquad \Rightarrow A - (2 - A) + C = -2$

$\Rightarrow A - 2 + A + C = -2$

$\Rightarrow 2A + C = 0$

$-2A + C = -1$

$(+) \quad (+) \quad (+)$

$2C = -1$

$\Rightarrow C = -1/2$

$-2A - \frac{1}{2} = -1$

$\Rightarrow -2A = -\frac{1}{2}$

$\Rightarrow A = 1/4$

$\frac{1}{4} - B - \frac{1}{2} = -2$

$\Rightarrow B = \frac{1}{4} - \frac{1}{2} + 2$

$\Rightarrow B = \frac{1-2+8}{4}$

$\Rightarrow B = 7/4$

So, $x(t) = L^{-1}\left[\frac{1/4}{s+1} + \frac{7/4}{s-1} + \frac{-1/4}{(s-1)^2}\right]$

$\Rightarrow \frac{1}{4}e^{-t} + \frac{7}{4}e^{t} + \frac{1}{2}L^{-1}\left[\frac{1}{s^2}\right]_{s\to s-1}$

$\Rightarrow \boxed{\frac{1}{4}e^{-t} + \frac{7}{4}e^{t} + \frac{1}{2}(t)e^{-t}}$ //

Q5. $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^{-t}$, $x(0)=2$, $x'(0)=1$

let $x'' = \frac{d^2x}{dt^2}$

$x' = \frac{dx}{dt}$

$\Rightarrow x'' - 2x' + x = e^{-t}$

Applying LT on both sides, we get,

$L\{x''\} - 2L\{x'\} + L\{x\} = L\{e^{-t}\}$

$\Rightarrow [s^2 L\{x(t)\} - sx(0) - x'(0)] - 2[sL\{x(t)\} - x(0)] + L\{x(t)\} = \frac{1}{s+1}$

$\Rightarrow [s^2 L\{x(t)\} - 2s - 1] - 2[sL\{x(t)\} - 2] + L\{x(t)\} = \frac{1}{s+1}$

$\Rightarrow L\{x(t)\}(s^2-2s+1) - 2s - 1 + 4 = \frac{1}{s+1}$

$\Rightarrow L\{x(t)\}(s^2-2s+1) = \frac{1}{s+1} + 2s - 3$

$\Rightarrow L\{x(6)\}(s^2-2s+1) = \frac{(2s-3)(s+1)+1}{(s+1)}$

$\therefore L\{x(t)\} = \frac{(2s-3)(s+1)+1}{(s+1)(s^2-2s+1)} \Rightarrow \boxed{\frac{2s^2-3s+2s-3+1}{(s+1)(s^2-2s+1)}}$ //