

Pumping Lemma

1. PL is used to prove that a language is not regular
2. It cannot be used to prove that a language is regular

If A is a RL, then A has a pumping length ' P ' such that any string ' S ' where $|S| \geq P$ may be divided into 3 parts $S=XYZ$ such that the following conditions must be true

- (1) $xy^iz \in A$ for every $i \geq 0$
- (2) $|Y| > 0$
- (3) $|XY| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

- > Assume that A is Regular
- > It has to have a Pumping Length (say P)
- > All strings longer than P can be pumped $|S| \geq P$
- > Now find a string ' S ' in A such that $|S| \geq P$
- > Divide S into $x y z$
- > Show that $x y^i z \notin A$ for some i
- > Then consider all ways that S can be divided into $x y z$
- > Show that none of these can satisfy all the 3 pumping conditions at the same time
- > S cannot be Pumped == CONTRADICTION

Using PL prove that the language $A = \{a^n b^n \mid n \geq 0\}$ is not regular

Assume A is regular

Pumping length = P

$S = a^P b^P$

Divide S into $X Y Z$

$P=7$

$S = aaaaaaabbabbbb$

Case 1

$XY^iZ \rightarrow XY^2Z$ / example $i=2$

Y is in ' a ' part

$aa aaaaaaa abbbbbb$

aa aaaa abbbbbb

$x \quad y \quad z$

$11 \neq 7$

case 2

Y is in 'b' part

aaaaaaabb bbbbbbbb b

$7 \neq 11$

aaaaaaabb bbbb b

x y z

case 3

aaaaa aabbaabb bbbbb

Y is in 'a' and 'b' part

not follow pattern

aaaaa aabb bbbbbb

x y z

$|XY| \leq P$

$P=7$

CASE 1

$6 \leq 7$

CASE 2

$13 \leq 7$

CASE 3

$9 \leq 7$

ALL THREE CASES CDTN NOT SATISFIED

2. $A = \{YY \mid Y \in (0,1)^*\}$

Assume A is regular

It must have a pumping length = P

$S = 0^p 1 0^p 1$

$S \rightarrow X Y Z$

$P=7$

0000000100000001

00 0000 0100000001

X Y Z

$XY^iZ \rightarrow XY^2Z$ / example $i=2$

00 00000000 0100000001

String doesnot lie in the language

$|y| > 0$

$|XY| \leq P$

$6 \leq 7$