Tuesday, November 2, 2021 12:36 PM

Taylor's theorem - Problems

④ Expand e^{xy} at $(1,1)$ as the Taylor's series.

Sol

$$f(x,y) = f(a,b) + \left[(x-1)f_x(1,1) + (y-1)f_y(1,1)\right]$$
$$+ \frac{1}{2!}\left[(x-1)^2 f_{xx}(1,1) + (y-1)^2 f_{yy}(1,1) + 2(x-1)(y-1)f_{xy}(1,1)\right]$$
$$+ \frac{1}{3!}\left[(x-1)^3 f_{xxx}(1,1) + (y-1)^3 f_{yyy}(1,1) + 3(x-1)^2(y-1)f_{xxy}(1,1) + 3(x-1)(y-1)^2 f_{xyy}(1,1)\right] + \cdots$$

$$f(x,y) = e^{xy}$$
$$f(1,1) = e^{1\cdot 1} = e$$

$$f_x = \frac{\partial f}{\partial x} = e^{xy}\cdot y = y e^{xy} \qquad f_x = e$$
$$f_y = \frac{\partial f}{\partial y} = e^{xy} x \qquad f_y = e$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = y e^{xy} y = y^2 e^{xy}$$
$$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\underset{u}{x}\, \underset{v}{e^{xy}}\right)$$
$$= x e^{xy} y \cdot 1 + e^{xy}\cdot 1$$
$$= (xy+1)e^{xy}$$
$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = x^2 e^{xy}$$

At point $(1,1)$

$$f_{xx} = e \qquad f_{xy} = 2e$$
$$f_{yy} = e$$

$$f_{xxx} = \frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right) = y^3 e^{xy}$$
$$f_{yyy} = \frac{\partial^3 f}{\partial y^3} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right) = x^3 e^{xy}$$
$$f_{xxy} = \frac{\partial^3 f}{\partial x^2 \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x \partial y}\right) = \frac{\partial}{\partial x}\left(xy e^{xy} + e^{xy}\right) = y(x e^{xy} y + e^{xy}) + e^{xy} y$$
$$= (xy^2 + 2y)e^{xy}$$
$$f_{xyy} = \frac{\partial^3 f}{\partial x \partial y^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right) = 2x e^{xy} + x^2 y e^{xy}$$

$$f_{xxx} = e \qquad f_{xxy} = 3e$$
$$f_{yyy} = e \qquad f_{xyy} = 3e$$

$$f(x,y) = e\left[1 + (x-1) + (y-1) + \frac{1}{2}\left((x-1)^2 + (y-1)^2 + 4(x-1)(y-1)\right) + \frac{1}{6}\left((x-1)^3 + (y-1)^3 + 9(x-1)^2(y-1) + 9(x-1)(y-1)^2\right) + \cdots\right]$$

MCQ

① Find $f_{xy}(0,0)$ for $e^x \sin y$ $\quad f_{xy} = e^x \cos y = 1$

② Find $f(0,0)$ for $e^x \log(1+y)$ $\quad f(0,0) = 0$

③ Find $f_{xy}(0, \pi/2)$ for $e^{2x}\cos 2y$ $\quad f_{xy} = -4e^{2x}\sin 2y = 0$ $\quad \to -4\sin 2\pi/2$

④ Find $f_{xy}(0,0)$ for $e^x \log(1+y)$ $\quad f_{xy} = \frac{e^x}{1+y} = 1$

⑤ Find f_{xx} for (y^2/x^3)

$$f_{xx} = 12x^{-5}y^2$$

(a) $\frac{12y^2}{x^5}$