

PART-A

1. Diagonalization can be useful in:
 - a) To find a non recursively enumerable language
 - b) To prove undecidability of halting problem
 - c) Both (a) and (b)**
 - d) None of the mentioned

2. If a problem has an algorithm to answer it, we call it _____
 - a) decidable**
 - b) solved
 - c) recognizable
 - d) none of the mentioned

3. Which of the following is true for The Halting problem?
 - a) It is recursively enumerable
 - b) It is undecidable
 - c) Both (a) and (b)**
 - d) None of the mentioned

4. A language L is said to be _____ if there is a turing machine M such that $L(M)=L$ and M halts at every point
 - a) Turing acceptable
 - b) decidable**
 - c) undecidable
 - d) none of the mentioned

5. The language accepted by a turing machine is called _____
 - a) Recursive Enumerable
 - b) Recursive
 - c) Both (a) and (b)**
 - d) None of the mentioned

6. The problems which have no algorithm, regardless of whether or not they are accepted by a turing machine that fails to halts on some input are referred as:
 - a) Decidable

b) Undecidable

c) Computable

d) None of the mentioned

7. An algorithm is called efficient if it runs in _____ time on a serial computer

a) polynomial

b) non polynomial

c) logarithmic

d) none of the mentioned

8. A problem is called _____ if its has an efficient algorithm for itself

a) tractable

b) intractable

c) computational

d) none of the mentioned

9. According to the rice's theorem, If P is a non trivial property, Lp is :

a) infinite

b) decidable

c) undecidable

d) none of the mentioned

10. Which of the following statements are undecidable?

For a given Turing Machine M,

a) does M halt on an empty input tape

b) does M halt for any inputs at all?

c) is L(M) regular? Context free? Turing decidable?

d) all of the mentioned

11. Post Correspondence problem is

a) decidable decision problem

b) undecidable decision problem

c) not a decision problem

d) none of the mentioned

12. PCP stands for?

- a) **Post Correspondence Problem**
- b) Post Corresponding Problem
- c) Pre Correspondence problem
- d) None of the mentioned

13. Can a Modified PCP problem be reduced to PCP?

- a) **yes**
- b) no

14. The complexity class P consist of all the decision problems that can be solved by _____ using polynomial amount of computation time

- a) Push Down automata
- b) DFA
- c) NDFA
- d) **Deterministic Turing machine**

15 What does NP stands for in complexity classes theory?

- a) Non polynomial
- b) **Non-deterministic polynomial**
- c) Both (a) and (b)
- d) None of the mentioned

16. The hardest of NP problems can be:

- a) **NP-complete**
- b) NP-hard
- c) P
- d) None of the mentioned

17. Travelling sales man problem belongs to which of the class?

- a) P
- b) **NP**
- c) Linear
- d) None of the mentioned

18. A problem which is both _____ and _____ is said to be NP complete

- a) **NP, P**
- b) NP, NP hard

- c) P, P complete
- d) None of the mentioned

19. Which of the following does not belong to the closure properties of NP class?

- a) Union
- b) Concatenation
- c) Reversal**
- d) Complement

20. A generalization of P class can be:

- a) PTIME
- b) DTIME
- c) NP**
- d) None of the mentioned

21. Given a context free language if the turing machine does not halt in a given amount of time or check whether it is ambiguous or not makes it

- a) a deducible problem
- b) an undecidable problem**
- c) a decidable problem
- d) a completeness problem

22. Which of the following is decidable (i) Does a given program ever produce an output (ii) if L is a context free language than is also context free language? (iii) if L is a regular language than is also regular ? (iv) if L is recursive than is also recursive ?

- a) (i), (ii), (iii) & (iv)
- b) (iii),(iv)**
- c) (i), (ii)
- d) (ii),(iii),(iv)

23. Which of the following technique is used to find whether a natural language isnt recursive enumerable?

- a) Diagonalization**
- b) Recursive Induction
- c) Both (a) and (b)
- d) None of the mentioned

24. Which of the following are decidable problems?

- a) Can a particular line of code in a program ever be executed?
- b) Do two given CFG's generate the same language
- c) Is a given CFG ambiguous?
- d) None of the mentioned**

25. Which among the following are undecidable theories?
- The first order theory of boolean algebra
 - The first order theory of Euclidean geometry
 - The first order theory of hyperbolic geometry
 - The first order theory of the natural number with addition, multiplication, and equality**
26. $\text{Rec-DFA} = \{ \mid M \text{ is a DFA and } M \text{ recognizes input } w \}$.
Rec-DFA is _____
- Undecidable
 - Decidable**
 - Non finite
 - None of the mentioned
27. Consider three decision problem A, B, C. A is decidable and B is not. Which of the following is a correct option?
- C is undecidable if C is reducible to B
 - C is undecidable if B is reducible to C**
 - C is decidable if A is reducible to C
 - C is decidable if C is reducible to B's complement.
28. Which of the following is incorrect according to rice theorem? Let S be a set of language hat is non trivial:
- there exists a TM that recognizes the language in S
 - there exists a TM that recognizes the language not in S
 - both (a) and (b)**
 - none of the mentioned
29. If the number of steps required to solve a problem is $O(n^k)$, then the problem is said to be solved in:
- non-polynomial time
 - polynomial time**
 - infinite time
 - none of the mentioned
30. The complexity class P consist of all the decision problems that can be solved by _____ using polynomial amount of computation time.
- Push Down automata
 - DFA
 - NDFA
 - Deterministic Turing machine**
31. Which of the following decision problems are undecidable?
- Given NFAs N_1 and N_2 , is $L(N_1) \cap L(N_2) = \Phi$?

II. Given a CFG $G=(N, \Sigma, P, S)$ and string $x \in \Sigma^*$, does

$x \in L(G)$?

III. Given CFGs G_1 and G_2 , is $L(G_1) = L(G_2)$?

IV. Given a TM M , is $L(M) = \Phi$?

a) I and IV only

b) II and III only

c) **III and IV only**

d) II and IV only

32. Which of the following problems is undecidable?

a) **Deciding if a given context-free grammar is ambiguous.**

b) Deciding if a given string is generated by a given context-free grammar.

c) Deciding if the language generated by a given context-free grammar is empty.

d) Deciding if the language generated by a given context-free grammar is finite.

33. Given $\Sigma = \{a, b\}$, which of the following sets is not countable ?

a) Set of all strings over Σ

b) **Set of all languages over Σ**

c) Set of all regular languages over Σ

d) Set of all languages over Σ accepted by Turing machines

34. Which one of the following is not decidable?

a) Given a Turing machine M , a string s and an integer k , M accepts s within k steps

b) **Equivalence of two given Turing machines**

c) Language accepted by a given finite state machine is not empty

d) Language generated by a context free grammar is non-empty

35. Which of the following statements is false?

- a) The Halting Problem of Turing machines is undecidable
- b) Determining whether a context-free grammar is ambiguous is undecidable
- c) Given two arbitrary context-free grammars G_1 and G_2 it is undecidable whether $L(G_1)=L(G_2)$
- d) Given two regular grammars G_1 and G_2 it is undecidable whether $L(G_1)=L(G_2)$**

36. Which of the following statement is false?

- a) Checking the ambiguity of CFL is decidable.
- b) Checking whether a given context free language is regular is decidable.
- c) Checking whether a given context free language is empty is decidable.
- d) Both A and B**

37. Which of the following problems is undecidable?

- a) To determine if two finite automata are equivalent
- b) Membership problem for context free grammar
- c) Finiteness problem for finite automata
- d) Ambiguity problem for context free grammar**

38. A PCP problem is solvable if

- a) $|\Sigma| = 1$**
- b) $|\Sigma| = 4$
- c) $|\Sigma| = 2$
- d) None of these

39. A recursive language is also called

- a) Decidable**
- b) Undecidable
- c) Both (a) and (b)
- d) None of these

40. The complement of recursive language is

- a) Recursive**

- b) Regular
- c) Both (a) and (b)
- d) None of these

PART-B

1 Let $\langle M \rangle$ be the encoding of a Turing machine as a string over $\Sigma = \{0, 1\}$. Let $L = \{ \langle M \rangle \mid M \text{ is a Turing machine that accepts a string of length } 2014 \}$. Then, L is

- (a) Decidable and recursively enumerable
- (b) undecidable but recursively enumerable**
- (c) undecidable and not recursively enumerable
- (d) decidable but not recursively enumerable

Explanation:

There are finite number of strings of length '2014'. So, a Turing machine will take the input string of length '2014' and test it.

If, input string is present in the language then Turing machine will halt in final state.

But, if Turing machine is unable to accept the input string then it will halt in non-final state or go in an infinite loop and never halt.

Thus, ' L ' is undecidable and recursively enumerable.

2. Consider the following statements:

1. The complement of every Turing decidable language is Turing decidable.
2. There exists some language which is in NP but is not Turing decidable.
3. If L is a language in NP, L is Turing decidable.

Which of the above statements is/are True?

- (a) Only 2
- (b) Only 3
- (c) Only 1 and 2
- (d) Only 1 and 3**

Explanation:

1 is true: Complement of Turing decidable is Turing Decidable.

3 is true: All NP problems are Turing decidable.

2 is false: The definition of NP itself says solvable in Polynomial time using non-deterministic Turing machine

3. Which of the following statements are TRUE?

- (1) The problem of determining whether there exists a cycle in an undirected graph is in P
- (2) The problem of determining whether there exists a cycle in an undirected graph is in NP
- (3) If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A

(a) 1, 2 and 3

(b) 1 and 3

(c) 2 and 3

(d) 1 and 2

Explanation:

1 is true because cycle detection can be done in polynomial time using DFS

2 is true because P is a subset of NP

3 is true because NP complete is also a subset of NP and NP means Non-Deterministic Polynomial time solution exists

4. Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?

(a) R is NP-complete

(b) R is NP-hard

(c) Q is NP-complete

(d) Q is NP-hard

Explanation:

(a) Incorrect because R is not in NP. A NP Complete problem has to be in both NP and NP-hard

(b) correct because a NP Complete problem S is polynomial time educable to R

(c) Incorrect because Q is not in NP

(d) Incorrect because there is no NP-complete problem that is polynomial time Turing-reducible to Q

5. Assuming $P \neq NP$, which of the following is true?

- (a) $NP\text{-complete} = NP$
- (b) $NP\text{-complete} \cap P = \phi$
- (c) $NP\text{-hard} = NP$
- (d) $P = NP\text{-complete}$

- (a) A
- (b) B**
- (c) C
- (d) D

Explanation:

The answer is B (no NP-Complete problem can be solved in polynomial time) Because, if one NP-Complete problem can be solved in polynomial time, then all NP problems can be solved in polynomial time. If that is the case, then NP and P set become same which contradicts the given condition.

6. A problem X belongs to P complexity class if there exist _____ algorithm to solve that problem, such that the number of steps of the algorithm is bounded by a polynomial in n, where n is the length of the input.

- (a) 1
- (b) 2
- (c) 3
- (d) all of the mentioned**

Explanation:

A problem X belongs to P complexity class if there exist at least 1 algorithm to solve that problem, such that the number of steps of the algorithm is bounded by a polynomial in n, where n is the length of the input. Thus, all the options are correct.

7. Fill in the blank with reference to Rice's theorem

for any non-trivial property of _____ no general or effective method can decide whether an algorithm computes it with that property.

- (a) Partial functions**
- (b) piecewise functions
- (c) both (a) and (b)
- (d) none of the mentioned

Explanation:

A property of partial functions is called trivial if it holds for all partial computable functions or for none, and an effective decision method is called general if it decides correctly for every algorithm

8. Which of the following set of computable functions are decidable?

- (a) The class of computable functions that are constant, and its complement
- (b) The class of indices for computable functions that are total
- (c) The class of indices for recursively enumerable sets that are co finite
- (d) All of the mentioned**

Explanation:

According to Rice's theorem, if there exists at least one computable function in a particular class C of computable functions and another computable function not in C then the problem deciding whether a particular program computes a function in C is undecidable

9 Which of the following statements is/are FALSE?

- 1. For every non-deterministic TM, there exists an equivalent deterministic TM
- 2. Turing recognizable languages are closed under union and complementation
- 3. Turing decidable languages are closed under intersection and complementation
- 4. Turing recognizable languages are closed under union and intersection

- (a) 1 and 4
- (b) 1 and 3
- (c) 2**
- (d) 3

Explanation:

Statement 1 is true as we can convert every non-deterministic TM to deterministic TM

Statement 2 is false as Turing recognizable languages (RE languages) are not closed under complementation

Statement 3 is true as Turing decidable languages (REC languages) are closed under intersection and complementation

Statement 4 is true as Turing recognizable languages (RE languages) are closed under union and intersection

10. State true or false?

Statement: If a problem X is in NP and a polynomial time algorithm for X could also be used to solve problem Y in polynomial time, then Y is also in NP

(a) **True**

(b) False

Explanation:

This is just a commutative property of NP complexity class where a problem is said to be in NP if it can be solved using an algorithm which was used to solve another NP problem in Polynomial amount of time

PART-C

1. If the number of steps required to solve a problem is $O(n^k)$, then the problem is said to be solved in:

(a) Non-polynomial time

(b) **polynomial time**

(c) infinite time

(d) none of the mentioned

Explanation:

Most of the operations like addition, subtraction, etc as well as computing functions including powers, square roots and logarithms can be performed in polynomial time. In the given question, n is the complexity of the input and k is some non negative integer

2. Consider three decision problems P1, P2 and P3. It is known that P1 is decidable and P2 is undecidable. Which one of the following is TRUE?

(a) **P3 is undecidable if P2 is reducible to P3**

(b) P3 is decidable if P3 is reducible to P2's complement

- (c) P3 is undecidable if P3 is reducible to P2
 (d) P3 is decidable if P1 is reducible to P3

Explanation:

- Option A says $P2 \leq P3$ According to theorem 2 discussed, if P2 is undecidable then P3 is undecidable. It is given that P2 is undecidable, so P3 will also be undecidable. So option (A) is correct.
- Option C says $P3 \leq P2$ According to theorem 2 discussed, if P3 is undecidable then P2 is undecidable. But it is not given in question about undecidability of P3. So option (C) is not correct.
- Option D says $P1 \leq P3$ According to theorem 1 discussed, if P3 is decidable then P1 is also decidable. But it is not given in question about decidability of P3. So option (D) is not correct.
- Option (B) says $P3 \leq P2'$ According to theorem 2 discussed, if P3 is undecidable then P2' is undecidable. But it is not given in question about undecidability of P3. So option (B) is not correct.

3. Consider two languages L1 and L2 each on the alphabet Σ . Let $f : \Sigma \rightarrow \Sigma$ be a polynomial time computable bijection such that $(\forall x) [x \in L1 \text{ iff } f(x) \in L2]$. Further, let f^{-1} be also polynomial time computable. Which of the following CANNOT be true?

- (a) $L1 \in P$ and L2 is finite
 (b) $L1 \in NP$ and $L2 \in P$
 (c) **L1 is undecidable and L2 is decidable**
 (d) L1 is recursively enumerable and L2 is recursive

Explanation:

We have one to one mapping for all instances of L1 to L2.

L1 is given to be undecidable. Further L1 is polynomial time reducible to L2 (By given mapping).

Now if L2 is decidable then there is algorithm to solve L2 in polytime. But then we can solve every instance of L1 in polytime, making L1 also decidable. Contradiction.

4. Let L1 be a recursive language, and let L2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE?

- (a) **L1' is recursive and L2' is recursively enumerable**
- (b) L1' is recursive and L2' is not recursively enumerable
- (c) L1' and L2' are recursively enumerable
- (d) L1' is recursively enumerable and L2' is recursive

Explanation:

Option A is False as L2' can't be recursive enumerable (L2 is RE and RE are not closed under complementation)

Option B is correct as L1' is REC (REC languages are closed under complementation) and L2' is not recursive enumerable (RE languages are not closed under complementation)

Option C is False as L2' can't be recursive enumerable (L2 is RE and RE are not closed under complementation)

Option D is False as L2' can't be recursive enumerable (L2 is RE and RE languages are not closed under complementation) As REC languages are subset of RE, L2' can't be REC as well

5. The problem 3-SAT and 2-SAT are

- (a) both in P
- (b) both NP complete
- (c) **NP-complete and in P respectively**
- (d) undecidable and NP-complete respectively

Explanation:

The Boolean satisfiability problem (SAT) is a decision problem, whose instance is a Boolean expression written using only AND, OR, NOT, variables, and parentheses. The problem is: given the expression, is there some assignment of TRUE and FALSE values to the variables that will make the entire expression true? A formula of propositional logic is said to be satisfiable if logical values can be assigned to its variables in a way that makes the formula true.

3-SAT and 2-SAT are special cases of k-satisfiability (k-SAT) or simply satisfiability (SAT), when each clause contains exactly $k = 3$ and $k = 2$ literals respectively. 2-SAT is P while 3-SAT is NP Complete.

6. Which of the following is/are undecidable?

- 1. G is a CFG. Is $L(G) = \phi$?
- 2. G is a CFG. Is $L(G) = \epsilon^*$?
- 3. M is a Turing Machine. Is $L(M)$ is regular?
- 4. A is a DFA and N is an NFA. Is $L(A) = L(N)$?

- a) 3 only
- b) 3 and 4 only
- c) 1,2, and 3 only
- d) 2 and 3 only**

Explanation:

First is Emptiness for CFG; whether a CFG is empty or not, this problem is decidable

Second is everything for CFC; whether a CFG will generate all possible strings(completeness of CFG), this problem is undecidable

Third is Regularity for REC; whether language generated by TM is regular is undecidable

Fourth is equivalence for regular; whether language generated by DFA and NFA are same is decidable. Second and third will be undecidable. Hence, option(D) is correct.