SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB302T- DISCRETE MATHEMATICS

Year/Sem: III/V

Branch: CSE,ECE,EEE

UNIT-5 -GRAPH THEORY

1. If a graph contains 21 edges and 3 vertices of degree 4 and all other vertices of degree2, then the total number of vertices is

(a) 18 (b) 14 (c) 81 (d) 41 **Ans: a**

Solution:

Let n be the total number of vertices,

By hand shaking theorem,

$d(v_i) = 2|E|$

3x4+2(n-3)=42

n=18

The total number of vertices is 18

2. The number of edges in the complete graph on 5 vertices is

(a) 10 (b) 11 (c) 12 (d) 13 **Ans: a**

Solution:

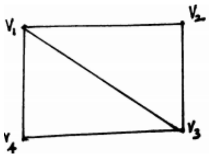
Given: n=5

The number of edges in the complete graph on 'n' vertices is $\frac{n(n-1)}{2}$ …… (1)

Sub n=5 in the equation (1)

The number of edges is 10

3. The name of the below graph is



(a) Multi graph (b) simple graph (c) Pseudo graph (d) null graph **Ans: b**

Solution:

By the definition of simple graph,

Since, there is only one edge between a pair of vertices

The given graph is a simple graph

4. What is the degree of v_5 for the following graph

(a) 2 (b) 3 (c) 4 (d) 1 **Ans:c**

Solution: The degree of a vertex in an undirected graph is the number of edges incident with it, with the exception that a loop at a vertex contributes twice to the degree of that vertex.

In the given graph there are 4 edges incident with the vertex v_5

Therefore $d(v_5) = 4$

5. If G = (V, E) is an undirected graph with e edges then, $\sum_i \deg(v_i)$ is

(a) 2e (b) 3e (c) 4e (d) e **Ans: a**

Solution:

By Handshaking theorem,

The sum of the degrees of all the vertices of an undirected graph is twice the number of edges of the graph and hence even

i.e $\sum_i \deg(v_i) = 2e$

6. The number of edges in a bipartite graph with 2 vertices is

(a) at most 5 (b) at most 4 (c) at most 3 (d) at most 2 **Ans: d**

Solution:

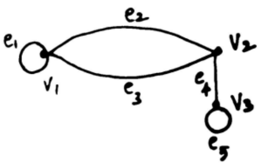
Given n=2

Since the number of edges in a bipartite graph with n vertices is at most $\frac{n^2}{2}$

Using the above formula, we get $\frac{2^2}{2}$ =2

Therefore, the number of edges in a bipartite graph with n vertices is at most 2

7. Which one of the matrix is the incident matrix for the following graph



(a) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **Ans: b**

Solution:

The given G = (V, E) is an undirected graph with 3 vertices and 5 edges then the 3x5 matrix $A = \lfloor a_{ij} \rfloor$, where

a_{ij} = 1, when edge e $_j$ is incident on v $_i$

a_{ij} = 0 otherwise.

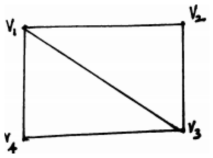
By above condition, satisfied in option ……

8. The diagonal entry of an adjacency matrix is

(a) 0 (b) 1 (c) 2 (d) 3 **Ans: a**

Solution:
By the definition of adjacency matrix,
Since a simple graph has no loops,
Each diagonal entry of matrix is zero

9. Which one of the matrix is the adjacency matrix for the following graph



(a) $\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ **Ans: d**

Solution:
The adjacency matrix $A = \lfloor a_{ij} \rfloor$, where

a_{ij} =1 if $v_i v_j$ is an edge of G

a_{ij} =0 otherwise.

The adjacency matrix of simple graph is symmetric and a simple graph has no loops. All the conditions are satisfied in option d

10. The maximum degree of any vertex in a simple graph with 4 vertices is

(a) 3 (b) 4 (c) 5 (d) 6 **Ans: a**

Solution: Given Number of vertices (n) = 4
The maximum degree of any vertex in a simple graph with n vertices is n-1
Using the above formula,
In a simple graph, the maximum degree with 4 vertices is 4-1=3

11. A tree with 6 vertices has ….edges.

(a) 3 (b) 4 (c) 5 (d) 6 **Ans: c**

Solution:
By the property of tree,
A tree with n vertices has (n-1) edges.
Given n (vertices) = 6
Using the above property, we get (6-1) = 5edges.

12 .The maximum number of edges in a simple disconnected graph G with 4vertices and 2 components is

(a) 3 (b) 4 (c) 5 (d) 6 **Ans: a**

Solution:

By the theorem,

The maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$ ……….(1)

Given: n=4 and k=2 Sub in (1),

We get the maximum number of edges has 3

13. If a graph G (either connected or not) has exactly two vertices of odd degree there is a ……… joining these two vertices.

(a) path (b) circuit (c) loop (d) edges **Ans: a**

Solution:

If v_1 and v_2 be the only vertices of G which are of odd degree

Also, the number of odd vertices is even,

Clearly there is a path connecting v_1 and v_2

14. In a 7-node directed cycle graph, the number of Hamiltonian cycle to be

(a) 180 (b) 720 (c) 360 (d) 540 **Ans: c**

Solution:

A Hamiltonian cycle in a connected graph G is defined as a closed path that traverses every vertex of G exactly once except the starting vertex, at which the path also terminates.

In an n- complete graph, there are $\frac{(n-1)!}{2}$

Using the above formula, $\frac{(7-1)!}{2}$=360

15. Which of the following is FALSE in the case of a spanning tree of a graph G?

(a) It is a tree that spSolution G (b) It is a sub graph of the G
(c) It includes every vertex of the G (d) It can be either cycle or acyclic

Ans: d

Solution:

A graph can have many spanning tress.

Each spanning tree of a graph G is a sub graph of the graph G, and spanning trees are always acyclic.

16. Consider a complete graph G with 4 vertices. The graph G has-----spanning trees.

(a) 15 (b) 8 (c) 16 (d) 13 **Ans: c**

Solution:

A graph can have spanning trees. And a complete graph with n vertices has $n^{(n-2)}$ spanning trees.So the complete graph with 4 vertices has $4^{(4-2)}$=16 spanning trees

17. The travelling salesman problem can be solved using-----

(a) A spanning tree (b) A minimum spanning tree
(c) Bellman-Ford algorithm (d) DFS traversal **Ans: b**

Solution:

In travelling salesman problem we have to find the shortest possible route that visits every city exactly once and returns to the starting point for the given a set of cities. So, travelling salesman problem can be solved by contracting the minimum spanning tree.

18. Which of the following is NOT the algorithm to find the minimum spanning tree of the given graph
(a) Boruvka's algorithm (b) Prim's algorithm
(c) Kruskal's algorithm (d) Bellman-Ford algorithm **Ans: d**

Solution: The Boruvka's algorithm, Prim's algorithm and Kruskal's algorithm are the algorithms that can be used to find the minimum spanning tree of the given graph.
The Bellman-Ford algorithm is used to find the shortest path from the single source to all other vertices.

19. What is the weight of the minimum spanning tree using the Kruskal's algorithm?

(a) 24 (b) 23 (c) 15 (d) 19 **Ans: d**

Solution:

Kruskal's algorithm constructs the minimum 'spanning tree by constructing by adding the edges to spanning tree one-one by one.
So, the weight of the minimum spanning tree is19

20. What will be the chromatic number for an empty graph having n vertices?

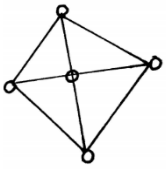
(a) 0 (b) 1 (c) 2 (d) n **Ans: b**

Solution:
An empty graph is a graph without edges.
So the chromatic number for such a graph will be 1

21. What will be the chromatic number for an bipartite graph having n vertices?
(a) 3 (b) 1 (c) 2 (d) n **Ans: c**

Solution:
A bipartite graph is graph such that no two vertices of the same set are adjacent to each other. So the chromatic number for such a graph will be 2

22. What will be the chromatic number of the following graph?



(a) 3 (b) 1 (c) 2 (d) n **Ans: a**

Solution:
The given graph will require 3 unique colors so that no two vertices connected edge will have the same colour.
So its chromatic number will be 3

23. A graph with chromatic number less than or equal to k is called?

(a) k chromatic (b) k colourable
(c) k chromatic colourable (d) k colourable chromatic **Ans: b**

Solution :

Any graph that has a chromatic number less than or equal to k is called colourable. Whereas a graph with chromatic number k is called k chromatic

24. If the chromatic number of a line graph is 4 then the chromatic index of the graph will be
(a) 0 (b) 1 (c) 2 (d) 4 **Ans: d**

Solution:
The chromatic index of a graph is always equal to the chromatic number of its line graph. So the chromatic index of the graph will be 4

25. How many unique colors will be required for proper vertex colouring of an empty graph having n vertices?
(a) 0 (b) 1 (c) 2 (d) n **Ans: b**

Solution:
An empty graph is a graph without any edges. So the number of unique colors required for proper coloring of the graph will be 1.

;