Density of State of Photons

DOS of photon $D(E) = \frac{dn(E)}{dE} \rightarrow (1)$

$n(E)$ is the no. of photons in phase space

$dn(E)$ is the no. of photons in infinitesimal phase space

$$dn(E) = \frac{d^3r\, d^3p}{h^3} \rightarrow (2)$$

$$\therefore n(E) = \int \frac{d^3r\, d^3p}{h^3}$$

$$= \frac{1}{h^3} \int d^3r \int d^3p$$

→ Volume

→ Energy

$$= \frac{1}{h^3} V \int p^2 dp \sin\theta\, d\theta\, d\phi$$

→ solid angle

spherical polar co-ordinates

$$= \frac{V}{h^3} \int p^2 dp\, (4\pi)$$

$$= \frac{4\pi V}{h^3} \int p^2 dp$$

$$\boxed{E = pc}$$

$$\frac{E}{c} = p$$

$$\frac{dE}{c} = dp$$

$$= \frac{4\pi V}{h^3} \int \frac{E^2}{c^2} \cdot \frac{dE}{c}$$

$$= \frac{2\,4\pi V}{h^3 c^3}\int E^2 \cdot dE$$

$$= \frac{4\pi V}{h^3 c^3}\, E^3/3$$

$$n(E) = \frac{4\pi V E^3}{3h^3 c^3} \rightarrow (3)$$

absorption & emission

Photons have two polarization state

$$\therefore n(E) = 2 \cdot \frac{4\pi V E^3}{3h^3 c^3} = \frac{8\pi V E^3}{3h^3 c^3} \rightarrow (4)$$

$$D(E) = \frac{d\,n(E)}{dE} = \frac{8\pi V}{\cancel{3}h^3 c^3} \cdot \cancel{3}E^2$$

$$D(E) = \frac{8\pi V E^2}{h^3 c^3} \rightarrow (5)$$

In terms of frequency $\boxed{E = h\nu}$

$$D(\nu) = \frac{8\pi V \nu^2}{hc^3} \rightarrow (6)$$

Density of states of photons.