

## Unit V Work sheet

1. Two friends Ramu and Somu playing head or tail game. The game has four rounds. While playing each round before flipping the coin, they are asked to take a number plate from the bowl which contains number plates of 1, 2 and 3. If they got number 1 only one chance is given the flip the coin. If they got number 2, two chances are given to flip the coin. If they got number 3, three chances are given to flip the coin. Following are the observations of each round.

### Round 1

Ramu – Number Plate1: Tail

Somu – Number Plate2: Tail Head

### Round 2

Ramu – Number Plate1: Head

Somu – Number Plate2: Tail Head

### Round 3

Ramu – Number Plate3: Head Tail Head

Somu – Number Plate2: Head Tail

### Round 4

Ramu – Number Plate2: Tail Tail

Somu – Number Plate1: Tail

Create two list from the observations named R for Ramu's outcome and S for Somu's outcome. Check whether the PCP has a solution?

### Solution:

Assume H = Head

T = Tail

List R = (T, H, HTH, TT)

List S = (TH, TH, HT, T)

Now we have to find out a sequence that strings formed by R and S are identical.

Such a sequence is 1, 2, 1, 3, 3, 4.

Hence from the R and S list.

1	2	1	3	3	4	1	2	1	3	3	4
T	H	T	HTH	HTH	TT	TH	TH	TH	HT	HT	T

(or)

	List A	List S
i	$w_i$	$x_i$
1	T	TH
2	H	TH
3	HTH	HT
4	TT	T

Take  $N=5$

Take the combination 1 2 1 3 3 4

By concatenating strings in this series

~~$w_1 w_2 w_1 w_3 w_3 w_4$~~

$w_1 w_2 w_1 w_3 w_3 w_4 = x_1 x_2 x_1 x_3 x_3 x_4$

$THTHTHHTHTT = THTHTHHTHTT$

Instance of PCP = 1 2 1 3 3 4

2. Consider the following solitaire game. You are given an  $m \times m$  board where each one of the  $m^2$  positions may be empty or occupied by either a red stone or a blue stone. Initially, some configuration of stones is placed on the board. Then, for each column you must remove either all of the red stones in that column or all of the blue stones in that column. (If a column already has only red stones or only blue stones in it then you do not have to remove any further stones from that column.) The objective is to leave at least one stone in each row. Finding a solution that achieves this objective may or may not be possible depending upon the initial configuration. Let  $SOLITAIRE = \{G \in \mathcal{G} \mid G \text{ is a game configuration with a solution}\}$ .
  - i. Prove that  $SOLITAIRE$  is in NP (8 Marks)
  - ii. Show that 3SAT is reducible to  $SOLITAIRE$  (10 marks)
  - iii. A problem which is both \_\_\_\_\_ and \_\_\_\_\_ is said to be NP complete. (1 mark)
    - a) P and NP
    - b) NP and NP hard
    - c) P and NP hard
    - d) P and P hard
  - iv. Consider three decision problem A, B, C. A is decidable and B is not. Which of the following is a correct option?
    - a) C is undecidable if C is reducible to B
    - b) C is undecidable if B is reducible to C

- c) C is decidable if A is reducible to C  
 d) C is decidable if C is reducible to B's complement.
3. To organize graduation day ceremony in the University, volunteers are invited from student's community. Equal number of boys and girls are selected to organize the event.
- Construct Turing machine with transition table or transition function that accepts only if the number of boys and girls in the organizing team is equal. (5 Marks)
  - Write the codes for the above Turing machine by encoding all the transition functions. (5 Marks)
  - Construct the Modified PCP (MPCP) instance for the above Turing machine. (8 Marks)
  - Check whether the string 01 has solution or accepted using MPCP instance. (5 Marks)

4.

In Erode, there was a big turmeric market, where daily lots of import and export will be processed. Market manager feels difficult in counting the turmeric import & export bags. For counting first he needs to calculate the complexity in that process. Help him to evaluate the time complexity to count the bags of import and export. (5 marks)

Counting sort takes  $O(n+k)$  time and  $O(n+k)$  space, where  $n$  is the number of items we're sorting and  $k$  is the number of possible values.

We iterate through the input items twice - once to populate counts and once to fill in the output array. Both iterations are  $O(n)$  time. Additionally, we iterate through counts once to fill in next index, which is  $O(k)$  time.

The algorithm allocates three additional arrays: one for counts, one for next index, and one for the output. The first two are  $O(k)$  space and the final one is  $O(n)$  space.