Saturday, November 20, 2021 4:24 PM

2) Linear Differential equations with Variable coefficients

a) Cauchy's Homogeneous linear equation
(or)
Linear differential equations with Euler's type

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_n y = F(x)$$

$a_1, a_2 . \, a_n$ are Constant

Put $x = e^z$

$\Rightarrow \log x = z$

$d/dx = D$

$d^2/dx^2 = D^2$

$d/dz = D'$

✓ $xD = D'$

✓ $x^2D^2 = D'(D'-1)$

$x^3D^3 = D'(D'-1)(D'-2)$

$x^4D^4 = D'(D'-1)(D'-2)(D'-3)$

① Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 4\sin(\log x)$

Sol

$$[x^2D^2 + xD + 1]y = 4\sin(\log x)$$

$\frac{d}{dx} = D$

$\frac{d^2}{dx^2} = D^2$

Put $\boxed{x = e^z \quad z = \log x}$ $\boxed{xD = D' \quad x^2D^2 = D'(D'-1)}$

$$[D'(D'-1) + D' + 1]y = 4\sin z$$

$$[D'^2 - \cancel{D'} + \cancel{D'} + 1]y = 4\sin z$$

$$(D'^2 + 1)y = 4\sin z$$

A.E D' by m

$m^2 + 1 = 0$

$m = \pm i$

$CF = A\cos z + B\sin z$

$$PI = 4 \cdot \frac{1}{D'^2+1} \sin z$$

D'^2 by $-a^2 = -1$

$$= 4 \cdot \frac{1}{0}$$

$$= 4z \cdot \frac{1}{2D'} \sin z$$

$D' = \frac{d}{dz}$

$\int \sin z \, dz$

$$PI = 2z(-\cos z)$$

$$y = A\cos z + B\sin z - 2z\cos z$$

$$\boxed{y = A\cos(\log x) + B\sin(\log x) - 2\log x \cos(\log x)}$$

② Solve $(x^2D^2 + 4xD + 2)y = x\log x$