LSSP Assignment

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1) $(j\Omega)^2 y(j\Omega) + 6(j\Omega) y(j\Omega) + 8 y(j\Omega) = 2X(j\Omega)$

$$y(j\Omega)\left[(j\Omega)^2 + 6(j\Omega) + 8\right] = 2X(j\Omega)$$

$$\frac{Y(j\Omega)}{X(j\Omega)} = \frac{2}{[(j\Omega)^2 + 6(j\Omega) + 8]}$$

$$H(j\Omega) = \frac{2}{(j\Omega)^2 + 6(j\Omega + 8)}$$ → frequency response

Taking inverse of $H(j\Omega)$ [fourier transform]

$$F.T^{-1}[H(j\Omega)] = F.T^{-1}\left[\frac{2}{(j\Omega)^2 + 6(j\Omega) + 8}\right]$$

$$= F.T^{-1}\left[\frac{2}{(j\Omega+4)(j\Omega+2)}\right]$$

using partial fraction method,

$$\frac{2}{(j\Omega+4)(j\Omega+2)} = \frac{A}{j\Omega+4} + \frac{B}{j\Omega+2}$$

$$A(j\Omega+2) + B(j\Omega+4) = 2$$

$$A = -1, \quad B = 1$$

$$\therefore \quad H(j\Omega) = \frac{-1}{j\Omega+4} + \frac{1}{j\Omega+2}$$

$$h(t) = -e^{-4t}u(t) + e^{-2t}u(t)$$ [impulse response]

2) $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$

$H(j\Omega) = \frac{1}{j\Omega+3}$

$Y(j\Omega) = \frac{1}{j\Omega+3} - \frac{1}{j\Omega+4}$

$= \frac{j\Omega+4-j\Omega-3}{(j\Omega+3)(j\Omega+4)} = \frac{1}{(j\Omega+3)(j\Omega+4)}$

$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)}$

$\Rightarrow \frac{1}{(j\Omega+3)} = \frac{\frac{1}{(j\Omega+3)(j\Omega+4)}}{X(j\Omega)}$

$x(j\Omega) = \frac{1}{j\Omega+4}$

$\therefore \quad x(t) = e^{-4t}u(t)$ //

3) $\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 4[y(t)] = \frac{d\,x(t)}{dt} + x(t)$

$\Rightarrow c\lambda^2 e^{\lambda t} + 4c\lambda e^{\lambda t} + 4ce^{\lambda t} = 0$

$ce^{\lambda t}(\lambda^2+4\lambda+4) = 0$

$\lambda^2+4\lambda+4 = 0$

$(\lambda+2)^2 = 0$

$\lambda_1 = 2, \quad \lambda_2 = -2$

$y_h(t) = (C_1+C_2t)e^{\lambda t}$

$y_h(t) = (C_1+C_2t)e^{-2t}$ are the equations.

Particular solution for input $e^{-3t} u(t)$

$\Rightarrow \quad y_p(t) = K e^{at}$

$= K e^{-3t}$

$$\frac{d^2}{dt^2} y(t) + 4 \frac{dy_p(t)}{dt} + 4 y_p(t) = -3e^{-3t} + e^{-3t}$$

$\Rightarrow \quad K(-3)(-3)e^{-3t} + 4K(-3)e^{-3t} + 4Ke^{-3t} = +e^{-3t}(-2)$

$\Rightarrow \quad 9Ke^{-3t} - 12Ke^{-3t} + 4Ke^{-3t} = -2e^{-3t}$

$K(\cancel{e^{-3t}}) = -2(\cancel{e^{-3t}})$

$K = -2 \Rightarrow y_p(t) = -2e^{-3t}$

$\therefore$ Total response $y(t) = y_h(t) + y_p(t)$

$= (C_1 + C_2 t)e^{-2t} - 2e^{-3t}$

$y(0^+) = \frac{9}{4}$ and $\frac{dy}{dt}(0^+) = 5$

$y(0) = \frac{9}{4} = C_1 - 2 \Rightarrow C_1 = \frac{9}{4} + 2 \Rightarrow C_1 = \frac{17}{4}$

$\frac{d}{dt} y(t) = C_1(-2)e^{-3t} + C_2 + (-2)e^{-2t} + C_2 e^{-2t} + 6e^{-3t}$

$\frac{d[y(0)]}{dt} = -2C_1 + C_2 + 6 = 5$

$\Rightarrow -2\left(\frac{17}{4}\right) + C_2 = -1 \Rightarrow -\frac{17}{2} + C_2 = -1$

$$\boxed{C_2 = \frac{15}{2}}$$

$\therefore \quad y(t) = \left(\frac{17}{4} + \frac{15}{2} t\right) e^{-2t} - 2e^{-3t}$ //