29. a. Express $f(x)=(\pi-x)^2$ as a Fourier series of periodicity 2π in $0<x<2\pi$ and hence deduce the sum $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

(OR)

b. Compute the first three harmonics of the Fourier series of $f(x)$ given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

30. a. If a string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0}=v_0\sin^3\frac{\pi x}{l},\ 0<x<l,$ determine the transverse displacement $y(x,t)$.

(OR)

b. Solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ subject to (i) $u(0,t)=0$ for $t\geq 0$ (ii) $u(l,t)=0$ for $t\geq 0$ (iii) $u(x,0)=\begin{cases} x & \text{for } 0\leq x\leq l/2 \\ l-x & \text{for } l/2\leq x\leq l \end{cases}$

31. a. Find the Fourier transform of $f(x)=\begin{cases} 1-x^2 & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$ and hence evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$.

(OR)

b. Find Fourier cosine and sine transforms of $e^{-ax}, a>0$ and evaluate $\int_0^{\infty}\frac{dx}{\left(a^2+x^2\right)^2}$ and $\int_0^{\infty}\frac{x^2}{\left(a^2+x^2\right)^2}dx$ if $a>0$.

32.a.i. Find the Z-transform of $\left\{\frac{1}{n(n+1)}\right\}, n\geq 1$.

ii. Find the inverse Z-transform of $x(z)=\frac{z^2}{(z-1/2)(z-1/4)}$ using Convolution theorem.

(OR)

b. Solve $y_{n+2}-7y_{n+1}+12y_n=2^n$ given that $y_0=0,\ y_1=0$ using Z-transforms.

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Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third to Seventh Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The complete integral of $p=2qx$
   (A) $z=ax^2+ay+c$
   (B) $z=ax+ay^2+c$
   (C) $z=ax^2-ay+c$
   (D) $z=ax+by+c$

2. The partial differential equation formed by eliminating arbitrary function in $z=f\left(x^2+y^2\right)$ is
   (A) $xp=yq$
   (B) $xy=pq$
   (C) $xq=yp$
   (D) $x+p=y+q$

3. Solve $\left(D^3-7DD'^2-6D'^2\right)z=0$
   (A) $z=\phi_1(y-x)+\phi_2(y-2x)+\phi_3(y+3x)$
   (B) $z=\phi_1(y-x)+\phi_2(y+2x)+\phi_3(y-3x)$
   (C) $z=\phi_1(y+x)+\phi_2(y-2x)+\phi_3(y+3x)$
   (D) $z=\phi_1(y+x)+\phi_2(y+2x)+\phi_3(y+3x)$

4. The general integral of $z=xp+yq$ is
   (A) $\phi\left(\frac{x}{y},\frac{y}{z}\right)=0$
   (B) $\phi(x+y,\ y+z)=0$
   (C) $\phi\left(x-y,\frac{x}{2}\right)=0$
   (D) $\phi\left(\frac{x}{y},y+z\right)=0$

5. The constant a_0 of the Fourier series for the function $f(x)=k,\ 0\leq x\leq 2\pi$
   (A) k
   (B) 2k
   (C) 0
   (D) k/2

6. The RMS value of $f(x)=x$ in $-1\leq x\leq 1$ is
   (A) 1
   (B) 0
   (C) $1/\sqrt{3}$
   (D) -1

7. Find half-range cosine series of $f(x)=\cos x$ in $(0,\pi)$ the value of a_0 is
   (A) 4
   (B) $2/\pi$
   (C) $4/\pi$
   (D) 0

8. $\int_{-\pi}^{\pi} |x| dx$ is equal to

(A) $2\int_0^{\pi} x dx$ (B) 0

(C) $2\int_0^{\pi} (-x) dx$ (D) $4\int_0^{\pi} x dx$

9. In wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) T/m (B) k/c

(C) m/T (D) k/m

10. One dimensional heat equation is used to find

(A) Temperature (B) Displacement

(C) Time (D) Mass

11. How many initial and boundary conditions are required to solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(A) Two (B) Three

(C) Four (D) Five

12. A rod of length l has its ends A and B are kept at 0°C and 100°C respectively, until steady state conditions prevail. Then the initial condition is given by

(A) $u(x,0) = ax + b + 100l$ (B) $u(x,0) = \frac{100x}{l}$

(C) $u(x,0) = 100lx$ (D) $u(x,0) = (x+l)100$

13. If $F\{f(x)\} = F(s)$, then $F\{f(x-a)\}$

(A) $e^{ias}F(s)$ (B) $e^{ias}F(a)$

(C) $e^{iax}F(a)$ (D) $e^{ias}F(x)$

14. The Fourier transform of $f(x) = e^{-x^2/2}$ is

(A) e^{-s^2} (B) $\frac{1}{e^{s^2/2}}$

(C) $\frac{1}{e^{-s^2/2}}$ (D) $e^{-s^2/2}$

15. $F\{f(x) * g(x)\} =$

(A) $F(s) + G(s)$ (B) $F(s) - G(s)$

(C) $F(s).G(s)$ (D) $F(s)/G(s)$

16. Under Fourier cosine transform of $f(x) = 1/\sqrt{x}$ is

(A) Self-reciprocal function (B) Cosine function

(C) Inverse function (D) Complex function

17. $Z\left[(-1)^n\right]$

(A) $\frac{z+1}{z}$ (B) $\frac{z}{z-1}$

(C) $\frac{z}{z+1}$ (D) $\frac{-z}{z+1}$

18. If $Z[f(t)] = F(z)$, then $\lim_{z\to\infty} F(z)$

(A) $f(0)$ (B) $f(1)$

(C) $\lim_{t\to\infty} f(t)$ (D) $f(\infty)$

19. Find $Z^{-1}\left(\frac{z}{(z-1)^2}\right)$ is

(A) $n+1$ (B) n

(C) $n-1$ (D) $1/n$

20. The poles of $\phi(z) = \frac{z^n}{(z-1)(z-2)}$ are

(A) $z=1, z=2$ (B) $z=-1, z=-2$

(C) $z=1, z=-2$ (D) $z=0, z=2$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2+a)(y^2+b)$.

22. Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y}$.

23. Express $f(x) = x$ in half range sine series of periodicity $2l$ in the range $0 < x < l$

24. Write the possible solutions and correct solution of one dimensional heat equation.

25. Classify the equation $(1+x^2)f_{xx} + (5+2x^2)f_{xy} + (4+x^2)f_{yy} = 2\sin(x+y)$.

26. If $F\{f(x)\} = F(s)$ then $F\{f(x)coax\} = \frac{1}{2}[F(s-a)+(s+a)]$.

27. Find $Z\{\sin n\theta\}$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Solve (i) $9(p^2z+q^2) = 4$ (ii) $x(y-z)p + y(z-x)q = z(x-y)$.

(OR)

b. Solve $(D^3 - 2D^2D')z = \sin(x+2y) + 3x^2y$.