SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB302T- DISCRETE MATHEMATICS

Year/Sem: III/V

Branch: CSE,ECE,EEE

UNIT-I -SET THRORY

1. Let f, g : R →R be defined, respectively by $f(x) = x+1, g(x) = 2x-3$. Find $f+g, f-g$

(a) 3x-2, -x+4 (b) 3x-2, -x-4 (c) 3x-2, x+4 (d) 3x+2, -x+4 **Ans: a**

Solution:

Since $f(x)=x+1, g(x)=2x-3$, f+g = x+1+2x-3 = 3x-2

f-g = x+1 -2x+3 = -x+4

2. Given functions $f: Z \to Z$, $f(n)$=2n+3, $g: Z \to Z$, $g(n)$=3n+2. What is $g \circ f$?

$(a)\ n+11$ $(b)\ 6n+11$ $(c) 11n+6$ $(d)\ 6n+1$ **Ans:b**

Solution:

$(g \circ f)(n) = g(f(n)) = g(2n + 3)$

$= 3(2n+3) + 2 = 6n + 11$

3. Given functions $f: Z \to Z$, $f(n)$=2n+3, $g: Z \to Z$, $g(n)$=3n+2. What is $f \circ g$?

$(a)\ 5n+1$ $(b)\ 6n+1$ $(c) 5n+7$ $(d)\ 6n+7$ **Ans:d**

Solution:

$(f \circ g)(n) = f(g(n))$

$= f(3n + 2) = 2(3n+2) + 3$

$= 6n + 7$

4. If $A=\{1,2,3,4\}, B=\{x,y,z\}$ and $f=\{(1,x),(2,y),(3,z),(4,x)\}$, then the function f is

(a) both 1 – 1 and onto (b) 1 – 1 but not onto

(c) onto but not 1 – 1 (d) neither 1 – 1 nor onto **Ans:c**

Solution:

All the element of B have a pre-image and not 1-1, Also x has two pre-image.

Hence the function f is onto but not 1 – 1.

5. Consider f: Z → Z, f(n) = n + 1 and g: Z → Z, g(n) = n^2

What is g ∘ f and f ∘ g?

(a) $\left\lfloor \frac{n+1}{n} \right\rfloor + 1, n^2$ (b) $n^2+1, (n+1)^2$ (c) n , $(n+1)^2$ (d) $2n+1, n^2+1$ **Ans:b**

Solution:

$(g \circ f)(n) = g(f(n)) = g(n+1) = (n^2+1)$

$(f \circ g)(n) = f(g(n)) = f(n^2) = (n+1)^2$

6. Let a; b; c, and d be distinct symbols and let R = {1; a); (1; b); (2; c)}. Then,find $R^{-1}(\{a\})$ and $R^{-1}(\{d\})$.

(a) {0},{1} (b){φ},{2} (c) {1},{2} (d) {1},{φ} **Ans :d**

Solution:

Domain={1,2} and Range={a,b,c,d} then

$R^{-1}(\{a\}) = 1$, $R^{-1}(\{d\}) = \varphi$

7. Let A={1,2,4,5,6} and B= $\{x: 1 < x \leq 8, x\ is\ an\ \ integer\}$ then ,find $A \cap B$.

(a) {1, 2, 4, 5, 6, } (b) { 2, 3, 4, 5, 6} (c) { 2, 4, 5,6, } (d) { 0,3, 5, 8} **Ans: c**

Solution:

A={1,2,4,5,6}

B = $\{x: 1 < x \leq 8, x \text{ is an integer}\}$ ={2,3,4,5,6,7,8},

$A \cap B = \{2,4,5,6\}$

8.The set $(X/Y) \cup (Y/X)$ denoted by $X \triangle Y$ is called the……….

(a) difference of X and Y (b) union of X and Y

(c) transitive of X and (d) symmetric difference of X and Y **Ans:d**

Solution:

The set $(X/Y) \cup (Y/X)$ denoted by $X \triangle Y$ is called the symmetric difference of X and Y .Ex: $(A/B) = \{18\}, (B/A) = \{3,5\}$ and $A \triangle B = \{3,5,18\}$

9. If the relation R is reflexive, antisymmetric and transitive, then the relation R is called……..

(a) equivalence relation (b) equivalence class (c) partial order relation

(d) partially ordered set **Ans :c**

Solution:

If the relation R is reflexive, antisymmetric and transitive, then the relation R is called partial order relation.

10. Write the solution set of the equation $x^2 + x - 2 = 0$ in roster form.

(a) $\{1, 2\}$ (b) $\{1,1\}$ (c) $\{1, -2,\}$ (d) $\{0,2\}$ **Ans: c**

Solution:

The given equation can be written as $(x - 1)(x + 2) = 0$, i. e., $x = 1, - 2$

Therefore, the solution set of the given equation can be written In roster form as $\{1, - 2\}$.

11. Write the set $\{x : x$ is a positive integer and $x^2 < 40\}$ in the roster form

(a) $\{1, 2,3,4,5,6\}$ (b) $\{1,2,5,6,7\}$ (c) $\{2,4,6,8,10\}$ (d) $\{1,3,5,7,9\}$ **Ans: a**

Solution:

The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is {1, 2, 3, 4, 5, 6}**.**

12. Let A = { a, e, i, o, u } and B = { a, i, u }. Find A ∪ B and A-B

(a) B , A (b) { a, e, i, o, u} {e,o} (c) $\{u, i, e\}$ (d) { a, c, d, e} **Ans: b**

Solution:

We have, A ∪ B = { a, e, i, o, u } = A**.**

A-B = { e,o}

13. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A′.

(a) {1, 2,3,4,5,6} (b) { 1,2,5,6,7} (c) { 2,4,6,8,10 } (d) { 1,3,5,7,9} **Ans: c**

Solution :

We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to A.

$A' = U – A$ ={2,4,6,8,10}

14 . There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C1, 50 to chemical C2, and 30 to both the chemicals C1 and C2. Find the number of individuals exposed to Chemical C1 but not chemical C2 .

(a) 90 (b) 120 (c) 30 (d) 80 **Ans: a**

Solution:

Let U denote the universal set consisting of individuals suffering from the skin disorder, A denote the set of individuals exposed to the chemical C1 and B denote the set of individuals exposed to the chemical C2.

Here n (U) = 200, n (A) = 120, n (B) = 50 and n (A ∩ B) = 30

n (A –B) = n (A) – n (A ∩ B) = 120 –30 = 90

Hence, the number of individuals exposed to chemical C1 but not to chemical C2 is 90.

15. A relation is a……… provided there is exactly one output for each input.

(a) equivalence relation (b) reflexive (c) function (d) partially ordered **Ans :c**

Solution:

A relation is a function provided there is exactly one output for each input. It is NOT a function if one input has more than one output.

16. The relation R is given by {(2,5) , (3,8) , (4,6) , (7, 20)} Is this a function?

(a) function (b) reflexive (c) not function (d) partially ordered **Ans :a**

Solution:

Domain = {2,3,4,7} Range = {5,8,6,20}

Yes: each input is mapped onto exactly one output

17. The relation R is given by {(-3,3) , (1,-2) , (4,4) , (1, 1)}. Is this a function?

(a) symmetric (b) reflexive (c) not function (d) partially ordered **Ans :c**

Solution:

Domain = {-3, 1,4} Range = {3,-2,1,4}

No: input 1 is mapped onto Both -2 & 1,So {(-3,3) , (1,-2) , (4,4) , (1, 1)} is not a function.

18. Which one of the following relations on the set {1, 2, 3, 4} is an equivalent relation

(a) {(2,4), (4,2)} (b) {(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)}

(c) {(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)} (d) {(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)} **Ans:d**

Solution:

Refexive if for each $1 \in A$, (1; 1) ∈ R.

symmetric if for each pair of elements $1,2 \in A$, $(1,2) \in R\ implise\ (2,1) \in R$

transitive if for each triple of elements $1,2,3 \in A$, $(1,1), (2,1) \in R\ implise\ (1,1) \in R$

R ={(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)}

19. If $f(x) = ax + b, g(x) = 1 - x + x^2$ for $x \in R$, and $(g \circ f)(x) = 9x^2 - 9x + 3$. Find the values of a and b.

(a) a = 3, b = -1 (or) a = -3, b = 2 (b) a = 1, b = 3 (or) a = 1, b = 2

(c) a = -3, b = -1 (or) a = -3, b = 2 (d) a = 3, b = 2 (or) a = -3, b = -1 **Ans:a**

Solution:

$(g \circ f)(x) = 9x^2 - 9x + 3$

$(g \circ f)(x) = 1-(ax+b) + (ax+b)^2 = 1-ax-ab+(ax+b)^2 = a^2x^2 + 2axb-ax + b^2+1$

Equating coefficients of x^2 and x,

we get a = 3, b = -1 or a = -3, b = 2

20. If $A = \{1, 2, 3\}$ and f, g are functions from A to A given by f = {(1,2), (2,3), (3,1)}, g = {(1,2), (2,1), (3,3)} then {(1,3), (2,2), (3,1)} is the composition relation of one of the following:

$(a)\ f \circ g$ $(b)\ g \circ f$ $(c)\ f \circ (f \circ g)$ $(d)\ f \circ (g \circ f)$ **Ans:a**

Solution:

f = {(1,2), (2,3), (3,1)}, g = {(1,2), (2,1), (3,3)},

$(f \circ g)(n)$= {(1,3), (2,2), (3,1)}

21. Consider the list of digits { 1, 2,1,4, 2} Is it a set?

(a) not set (b) group (c) set (d) not group **Ans: a**

Solution:

A set is typically expressed by curly braces,{ } enclosing its elements. Observe that the set $\{1, 2, 3, 4\}$, $\{\text{apple}, \text{tomato}, \text{orangeg}\}$, $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

22. The set S that contains no element is called the …………

(a) empty set (b) not null set (c) set (d) singleton set **Ans: a**

Solution:

The set S that contains no element is called the empty set or the null set and is denoted by {φ} or φ .

23. For the relation $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$ determine its domain and range.

(a) $\{1, 2, 3, 4, 5\}$ $\{2\}$ (b) $\{0, 1, 2, 3, 4\}$ $\{3\}$

(c) $\{2, 5, 8, 11, 14, 17\}\ \{1\}$ (d) $\{0, 3, 5, 7, 18\}\ \{1\}$ **Ans: c**

Solution:

Given $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Since 2, 5, 8, 11, 14 and 17 are the elements of the domain of the given relation having their unique images, this relation is a function. Here, domain $=\{2, 5, 8, 11, 14, 17\}$ and range $=\{1\}$

24.The set X = {2; 4; 6; 8; 10} in the predicate notation can be written as………..

(a) $\{x: 0 < x \le 10, x\ is\ an\ even\ integer\}$ (b) $\{x: 1 < x < 11, x\ is\ an\ even\ integer\}$ **Ans :c**

(c) $\{x: 2 \le x \le 10, x\ is\ an\ even\ integer\}$ (d) $\{x: 1 < x < 9, x\ is\ an\ even\ integer\}$

Solution:

$2 \le x \le 10 \Longrightarrow$ Even Integers from 2 to 10 are 2,4,6,8,10

25. Let A={1,2,4,18,} and B= $\{x: 0 < x \le 5, x\ is\ an\ \ integer\}$ then ,find $A \cup B$.

(a) $\{1, 2, 3, 4, 5,\ 18\}$ (b) $\{0,1, 2, 3, 4, 5, 6, 18\}$ (c) $\{0,2, 4, 6, 18\}$ (c) $\{0,3, 5, 7, 18\}$ **Ans: a**

Solution:

A={1,2,4,18,} B= $\{x: 0 \le x \le 5, x\ is\ an\ \ integer\}$ ={1,2,3,4,5},

$A \cup B = \{1,2,3,4,5,18\}$

22. Given functions $f: Z \to Z$, $f(n)=2n+3$, $g: Z \to Z$, $g(n)=3n+2$. What is $f \circ g$?

$(a)\ 5n+1$ $(b)\ 6n+1$ $(c) 5n+7$ $(d)\ 6n+7$ **Ans:d**

Solution:

$(f \circ g)(n) = f(g(n))$

$= f(3n + 2) = 2(3n+2) + 3$

$= 6n + 7$

23. If $A = \{1, 2, 3, 4\}, B = \{x, y, z\}$ and $f = \{(1, x), (2, y), (3, z), (4, x)\}$, then the function f is

(a) both 1 – 1 and onto (b) 1 – 1 but not onto

(c) onto but not 1 – 1 (d) neither 1 – 1 nor onto **Ans:c**

Solution:

All the element of B have a pre-image and not 1-1, Also x has two pre-image.

Hence the function f is onto but not 1 – 1.

24. Consider f: Z → Z, f(n) = n + 1 and g: Z → Z, g(n) = n^2

What is g ∘ f and f ∘ g?

(a) $\left\lfloor \frac{n+1}{n} \right\rfloor + 1, n^2$ (b) $n^2 + 1, (n+1)^2$ (c) n , $(n+1)^2$ (d) $2n+1, n^2+1$ **Ans:b**

Solution:

$$(g \circ f)(n) = g(f(n)) = g(n + 1) = (n^2+1)$$

$$(f \circ g)(n) = f(g(n)) = f(n^2) = (n+1)^2$$

25.. Let a; b; c, and d be distinct symbols and let R = {1; a); (1; b); (2; c)}. Then,find $R^{-1}(\{a\})$ and $R^{-1}(\{d\})$.

(a) {0},{1} (b){φ},{2} (c) {1},{2} (d) {1},{φ} **Ans :d**

Solution:

Domain={1,2} and Range={a,b,c,d} then

$R^{-1}(\{a\}) = 1$, $R^{-1}(\{d\}) = φ$