# B.Tech Degree Examination, May 2023.

## 18MAB302T – Discrete Mathematics for Engineers

## Answer Key.

### PART–A.

| | | | |
|---|---|---|---|
| 1) c) $\phi$. | 7) B | 13) A | 19) C |
| 2) C | 8) A | 14) D | 20) B |
| 3). any one | 9) D | 15) B | |
| 4) A) | 10) D | 16) D | |
| 5) A | 11) D | 17) A | |
| 6) D | 12) B | 18) B | |

### PART–B

21). $(fo(goh))(x) = \dfrac{1}{(1+x^2)^3} - \dfrac{4}{(1+x^2)^{+1}} \longrightarrow (2m)$

$((fog)oh)(x) = \dfrac{1}{(1+x^2)^3} - \dfrac{4}{(1+x^2)}. \longrightarrow (2m)$

22). when 5 occupies first place $= \dfrac{6!}{2!} = 360$

when 6 or 7 occupies first place $= \dfrac{6!}{2!\,2!} \longrightarrow (2m)$

$\therefore$ No of numbers exceeding 50,00,000 $\Big\} = 360 + 180 + 180$

$= 720. \longrightarrow (2m)$

23.

| $p$ | $q$ | $\neg p$ | $q \to \neg p$ | $p \leftrightarrow q$ | $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | F | T | F | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

$\longrightarrow$ (4m)

24). Let $(G, *)$ be a cyclic group with $a \in G$ as a generator. Let $b, c \in G$. Then $b = a^m$ and $c = a^n$ where $m$ and $n$ are integers. Now $b * c = a^m * a^n = a^n * a^m = c * b$.

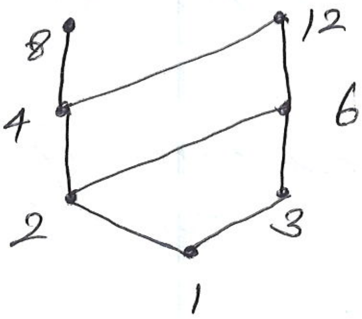
Hence $(G, *)$ is an abelian group. $\longrightarrow$ (4m)

25) Since every edge is incident with exactly two vertices every edge contributes 2 to the sum of the degree of vertices. $\therefore \sum_i \deg(v_i) = 2e$. $\longrightarrow$ (4m)

26). $S = \{1, 2, 3, 4, 6, 8, 12\}$.

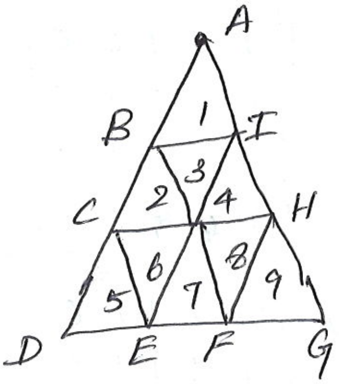
$R = \{x \le y \mid x \text{ divides } y,\ x, y \in S\} = \{(1,1), (1,2), (1,3), (1,4),$
$(1,6), (1,8), (1,12), (2,2), (2,4), (2,6), (2,8), (2,12),$
$(3,3), (3,6), (3,12), (4,4), (4,8), (4,12), (6,6), (6,12),$ $\longrightarrow$ (1m)
$(8,8), (12,12)\}$.

Hasse Diagram:




$\longrightarrow$ (3m)

27)




Let ADG be the given equilateral triangle. The pairs of points B, C; E, F and H, I are the points of trisection of the sides AD, DG and GA respectively and each of side length 1/3.

No. of pigeons = 10 ; No. of pigeonholes = 9 =

Then by pigeon hole principle, atleast one sub triangle must contain 2 interior points. $\longrightarrow$ (4m)

## PART - C

28) a) $A = \{1, 2, 3, 4, 5\}$ ; $R = \{(1,2), (2,1), (2,3), (3,4), (4,2), (4,4), (5,1), (5,5)\}$.

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \longrightarrow (1m)$$

| $W_i$ | 1's entry in column $W_{0-1}$ | 1's entry in row $W_{0-1}$ | Relation | Matrix relation |
|---|---|---|---|---|
| $W_0$ | – | – | – | $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow (1m)$ |
| $W_1$ | 2, 5 | 2 | (2,2), (5,2) | $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ |

$\longrightarrow$ (2m)

| | | | | |
|---|---|---|---|---|
| $W_2$ | 1, 2, 4, 5 | 1, 2, 3 | (1,1), (1,2), (1,3), (2,1), (2,2), (2,3) (4,1), (4,2), (4,3) (5,1), (5,2) (5,3) | $\begin{bmatrix} 1&1&1&0&0\\ 1&1&1&0&0\\ 0&0&0&1&0\\ 1&1&1&1&0\\ 1&1&1&0&1 \end{bmatrix}$ → (2m) |
| $W_3$ | (1, 2, 4, 5) | 4 | (1,4), (2,4), (4,4) (5,4) | $\begin{bmatrix} 1&1&1&1&0\\ 1&1&1&1&0\\ 0&0&0&1&0\\ 1&1&1&1&0\\ 1&0&0&1&1 \end{bmatrix}$ → (2m) |
| $W_4$ | 1, 2, 3, 4, 5. | 1, 2, 3, 4 | (1,1), (1,2) (1,3), (1,4) (2,1), (2,2), (2,3) (2,4) (3,1), (3,2), (3,3) (3,4) (4,1), (4,2), (4,3) (4,4) (5,1), (5,2), (5,3), (5,4) | $\begin{bmatrix} 1&1&1&1&0\\ 1&1&1&1&0\\ 1&1&1&1&0\\ 1&1&1&1&0\\ 1&1&1&1&1 \end{bmatrix}$ → (2m) |
| $W_5$ | 5 | (1, 2, 3, 4, 5) | (5,1), (5,2) (5,3), (5,4), (5,5) | $\begin{bmatrix} 1&1&1&1&0\\ 1&1&1&1&0\\ 1&1&1&1&0\\ 1&1&1&1&0\\ 1&1&1&1&1 \end{bmatrix} = R^{\infty}$ → (2m) |

28) b) **Proof:** $f: A \to B$ ; $g: B \to C$ $g \circ f: A \to C$

Since $f, g$ are invertible, $g \circ f$ is also invertible

so $(g \circ f)^{-1} : C \to A$ exists.

Since $g^{-1}: C \to B$ and $f^{-1}: B \to A$ ; $f^{-1} \circ g^{-1} : C \to A$

can be formed. and $(g \circ f)^{-1}, f^{-1} \circ g^{-1} : C \to A$. → (6m)

Now for any $a \in A$, let $b = f(a)$ and $c = g(b)$.

$\therefore (g \circ f)(a) = g(f(a)) = g(b) = c$

$\Rightarrow (g \circ f)^{-1}(c) = a \to (1)$

By the assumption $a = f^{-1}(b)$ and $b = g^{-1}(c)$

$\therefore (f^{-1} \circ g^{-1})(c) = f^{-1}(g^{-1}(c)) = f^{-1}(b) = a \rightarrow (2)$

from (1) & (2) $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. $\longrightarrow$ (6m)

29) a) GCD(512, 320). By Euclidean Algorithm.

$512 = 1 \times 320 + 192$

$320 = 1 \times 192 + 128$

$192 = 1 \times 128 + 64$

$128 = 2 \times 64 + 0$

$\boxed{GCD(512, 320) = 64}$ $\rightarrow$ (7m)

$64 = 192 - 128$

$\boxed{64 = 2 \times 512 - 3 \times 320}$

$\therefore m = 2;\ n = -3$.

$\longrightarrow$ (5m)

29) b) Let A, B, C be set of integers that lie between 1 and 500 both inclusive and that divisible by 3, 5, 7.

$|A| = \left\lfloor \frac{500}{3} \right\rfloor = 166$; $|B| = \left\lfloor \frac{500}{5} \right\rfloor = 100$; $|C| = \left\lfloor \frac{500}{7} \right\rfloor = 71$.

$n(A \cap B) = \left\lfloor \frac{500}{15} \right\rfloor = 33$ | $n(A \cap C) = \left\lfloor \frac{500}{21} \right\rfloor = 23$ | $n(B \cap C) = \left\lfloor \frac{500}{35} \right\rfloor = 14$.

$n(A \cap B \cap C) = \left\lfloor \frac{500}{105} \right\rfloor = 4$. $\longrightarrow$ (6m)

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(C \cap B) + n(A \cap B \cap C)$ $\longrightarrow$ (4m)

$= 271$.

$\therefore$ Not divisible by 3, 5, 7 $= 500 - 271$

$= 229$ $\longrightarrow$ (2m)

30) a) (i)

$(\neg p \vee q) \wedge (p \wedge (p \wedge q))$

$\equiv (\neg p \vee q) \wedge ((p \wedge p) \wedge q) \equiv (\neg p \vee q) \wedge (p \wedge q)$

$\equiv (\neg p \wedge (p \wedge q)) \vee (q \wedge (p \wedge q))$

$\equiv ((\neg p \wedge p) \wedge q) \vee (q \wedge p)$

$\equiv (F \wedge q) \vee (p \wedge q) \equiv F \vee (p \wedge q) \equiv (p \wedge q)$. $\longrightarrow$ (6m)

(ii)

| S.NO | Statement | Reason. |
|---|---|---|
| 1. | $(p \vee q) \to \neg r$ | P |
| 2. | $\neg r \to (s \wedge \neg t)$ | P |
| 3. | $(p \vee q) \to (s \wedge \neg t)$ | T(1,2) HS. |
| 4. | $p \vee q$ | P |
| 5 | $s \wedge \neg t$ | T(4,3) M.P |
| 6. | $(s \wedge \neg t) \to (a \vee b)$ | P |
| 7. | $a \vee b$ | T(5,6) M.P $\longrightarrow$ (6m) |

30) b)

p : Rama gets his degree

q : He will go for a job.

r : He will get married soon.

s : He goes for higher study. $\longrightarrow$ (2m)

Symbolic Form:-

$p \to q$, $q \to r$, $s \to \neg r$, $p \wedge s$ are inconsistent.

$\longrightarrow$ (2m)

30) b) cont.

| Step No. | statement | Reason. |
|---|---|---|
| 1. | $p \to q$ | P |
| 2. | $q \to r$ | P. |
| 3. | $p \to r$ | T(1, 2) HS |
| 4. | $p \wedge s$ | P |
| 5 | $p$ | T(4) |
| 6. | $s$ | T(4) |
| 7. | $s \to \neg r$ | P |
| 8. | $\neg r$ | T(6, 7) MP |
| 9. | $r$ | T(5, 3) MP. |
| 10 | $r \wedge \neg r$ | T(8, 9) conjunction |
| 11. | F | T(10) negation law. |

→ (4m)

→ (4m)

31) a) i) Let $(G, *)$ be a group. Let $(H_1, *) \subseteq (G, *)$ and $(H_2, *) \subseteq (G, *)$. $\therefore$ $H_1 \cap H_2$ is a nonempty set. since identity elt 'e' is common to both $H_1$ and $H_2$

Let $a \in H_1 \cap H_2$ Then $a \in H_1$ and $a \in H_2$.

Let $b \in H_1 \cap H_2$ then $b \in H_1$ and $b \in H_2$.

$H_1$ is a subgroup of G. $\therefore$ $a * b^{-1} \in H_1$

$H_2$ is a subgroup of G. $\therefore$ $a * b^{-1} \in H_2$

$\Rightarrow a * b^{-1} \in H_1 \cap H_2$.

$\therefore (H_1 \cap H_2, *) \subseteq (G, *)$.

Let $(H_1, *) = (2Z, +)$ and $(H_2, *) = (3Z, +)$ then $(H_1 \cup H_2, *) = (2Z \cup 3Z, +) \Rightarrow 2 \in H_1 \cup H_2$, $3 \in H_1 \cup H_2$ But $5 \notin H_1 \cup H_2$

Hence $(H_1 \cup H_2, *) \not\subseteq (G, *)$. → (4m)

31) a) (ii) Let $a+ib$ and $c+id$ be any two elts of $C$.

then $f((a+ib)+(c+id)) = f((a+c)+i(b+d)) = a+c$.

$= f(a+ib) + f(c+id)$ Hence $f$ is a homomorphism from $C$ to $R$.

The identity of $R$ is the real number $0$.

The images of all complex numbers with real part $0$ are each equal to $0$, the identity of $R$ under $f$.

Hence, the kernel of $f$ is the set of all purely imaginary numbers. $\longrightarrow$ (4m)

31) b) $H = [A^T / I_{n-m}]$ ; $e: B^2 \to B^5$; $m=2$, $n=5$.

$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ Then $H = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

Generator matrix $G = [I_m / A]$

$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \longrightarrow$ (6m)

$w \in B^2 = \{00, 01, 10, 11\}$ ; $e(w) = (wG)$

$e(00) = [00000]$ ; $e(10) = [10011]$

$e(01) = [01011]$ ; $e(11) = [11000]$ $\longrightarrow$ (6m).

Hence generated code words are

0000, 01011, 10011, 11000.

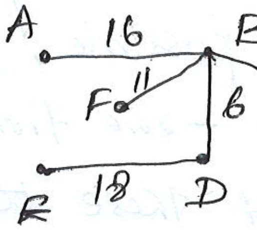
32) a)

| S.No | Edges | Weight | Included in spanning tree or not |
|---|---|---|---|
| 1. | BC | 5 | Yes |
| 2. | BD | 6 | Yes |
| 3. | CD | 10 | No (B–C–D–B) |
| 4. | FB | 11 | Yes. |
| 5. | FD | 14 | No (F–B–D–F) |
| 6. | AB | 16 | Yes |
| 7. | ED | 18 | Yes |
| 8. | AE | 19 | No |
| 9. | EF | 40 | No. |
| 10. | AF | 21 | No. |

→ (7m)

no of vertices = 6.   No. of edges needed to form spanning tree = 5.

Minimum Spanning Tree



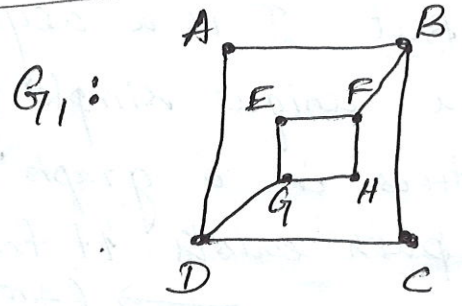

→ (2m)

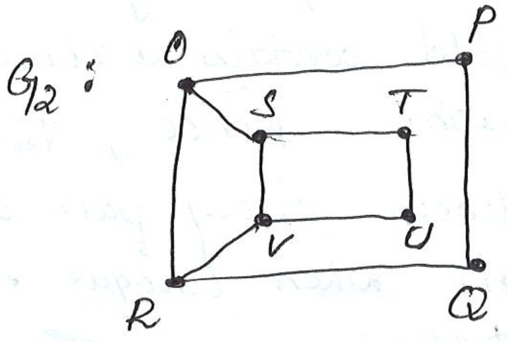
Minimum Distance

= 5+6+11+16+18

= 56.   → (2m)

32) b) (i)



In $G_1$:

$d(A) = 2 = d(E) = d(H) = d(C)$.

$d(B) = 3 = d(F) = d(G) = d(D)$ ⟶ (2m)

In $G_2$:

$d(P) = d(T) = d(U) = d(Q) = 2$

$d(O) = d(S) = d(V) = d(R) = 3$. ⟶ (2m)

In $G_1$ Vertex B having degree 3 is incident with vertices A and C having degree 2 whereas in $G_2$: vertex O having degree 3 is incident with vertices P and R having degree 2 and 3. Hence $G_1 \not\cong G_2$. ⟶ (4m)

They are not isomorphic.

32) b) (ii) Proof: (i) Let the undirected graph T be a tree. Then, by defn. of a tree T is connected. Hence, there is a simple path between any pair of vertices say $v_i$ and $v_j$. If possible, let there be two paths between $v_i$ and $v_j$ — one from $v_i$ to $v_j$ and other from $v_j$ to $v_i$. Union of these two paths would contain a circuit. But T is a acyclic graph. Hence, there is a unique simple path between every pair of vertices in a graph T.

(ii) When unique simple path exists it forms a tree. ⟶ (4m)

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