DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

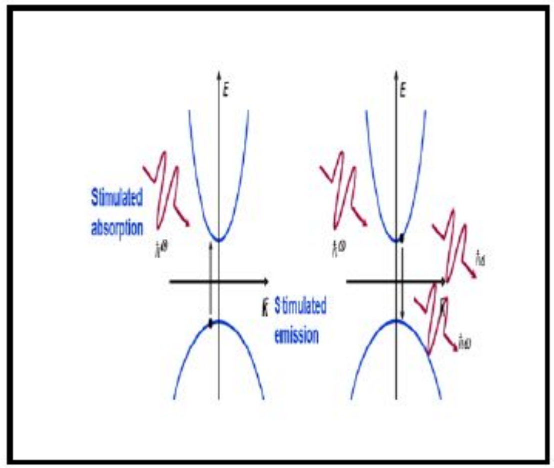
18PYB103J –Semiconduuctor Physics

Joint density of states

(Conservation of energy and momenta of
electron
with photon interacts)

Optical Joint Density of States

How many states are possible for photon interaction of energy hγ in valence and conduction band is given by optical joint density of states. To determine the density of state ργ with which a photon of energy hγ interacts under a condition of energy and momentum conservation in a direct band gap semiconductor.



To approximate this relation for a direct band-gap semiconductor by two parabolas,

$$E_2 = E_c + \frac{\hbar^2 K^2}{2m_c}$$

$$E_1 = E_v - \frac{\hbar^2 K^2}{2m_v}$$

$$h\gamma = E_2 - E_1$$

$$h\gamma = E_g + \frac{\hbar^2 K^2}{2m_v}$$

$$K^2 = \frac{2m_v}{\hbar^2}(h\gamma - E_g)$$

Here, substitute the value of K^2 in eq (1) & eq (2)

$$E_2 = E_c + \frac{m_v}{m_c}(h\gamma - E_g)$$

Similarly,

$$E_1 = E_v - \frac{m_v}{m_c}(h\gamma - E_g)$$

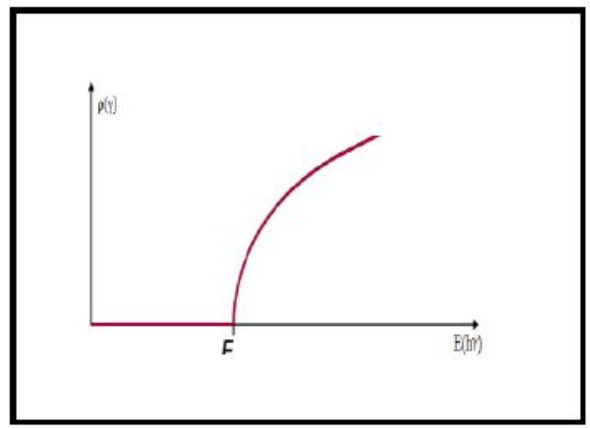
The one-to-one correspondence between E_2 and γ permits us to readily relative $\rho(\gamma)$ to the density of states $\rho_c(E_2)$ in conduction band by use of the incremental relation

$$\rho_c(E_2)dE = \rho(\gamma)\, d\gamma$$

Here $\rho_c(E_2)dE$ is no of states between E_2 *and* dE_2 and $\rho(\gamma)\, d\gamma$ is the number of states per unit volume of energy between $h\gamma$ and $h(\gamma+d\gamma)$ to interact.

Therefore,

$$\rho(\gamma) = \rho_c(E_2)\frac{dE}{d\gamma}$$



$$\rho\,(\gamma) = \frac{(2m_v)^{3/2}}{\pi\hbar^2}(h\gamma - E_g)^{1/2} \text{ for } h\gamma \geq E_g$$

The density of states which a photon of energy $h\gamma$ interact increases with $h\gamma \geq E_g$ in accordance with a square root law. Similarly One-to-One correspondence between E_1 and $\rho(\gamma)$ in equation, together with $\rho(\gamma)$ E_1, results in an expression for $\rho(\gamma)$ identical.

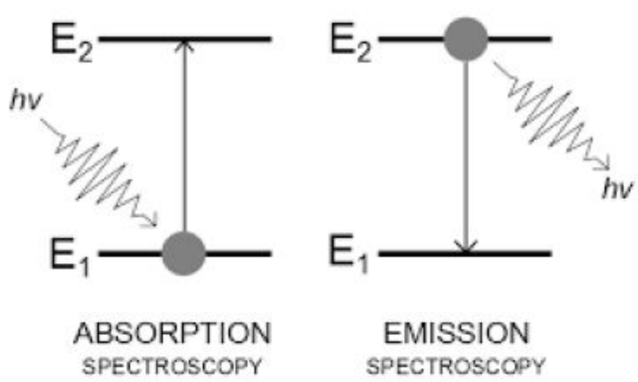
Transition Rate due to electron-photon interaction

Transition Rate due to electron-photon interaction

The interaction rate for t absorption of a photon is shown in figure. Assuming an electrons is initially at the solid state a is given by Fermi's Golden rule (using time-dependent perturbation theory)

$$W_{abs} = \frac{2\pi}{\hbar}\left|< b\left|H'(r)\right|a >\right|^2 \delta\,(E_b - E_a - \hbar\omega)$$

Absorption Emission



Transition Rate due to electron-photon interaction

In general Transition probability for Fermi's golden rule

$$\lambda_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \, Pf$$

Where, $|M_{if}|^2$ - is Matrix element for interaction $|M_{if}|^2 = |< b|H'(r)|a >|^2$ and,

Pf – is the number of continuum state per unit volume or density of final state.

(Pf = $\delta(E_b - E_a - \hbar\omega)$).

Where $E_b > E_a$ has been assumed. The total upward transition rate per unit volume(S^{-1}, cm^{-3}) in the crystal taking into account the probability that state a is occupied and state b is empty is

$$R_{a-b} = \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} |H'_{ba}|^2 \, \delta(E_b - E_a - \hbar\omega) f_a (1 - f_b)$$

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Where we sum over the initial and final states and assume that the Fermi-Dirac distribution f_a is the probability that the state a is occupied. A similar expression holds for f_b with E_a replaced by E_b, and $(1 - f_b)$ is probability that the state b is empty. The prefactor 2 takes into account the sum over spins, and the matrix element H'_{ba} is given by

$$H'_{ba} = |< b|H'(r)|a >|^2 = \int \psi^*(r) H'(r) \psi_a(r) d^3r$$

Similarly, The transition rate for the emission of a photon (fig.2) if an electron is initially at state b is.

$$W_{ems} = \frac{2\pi}{\hbar} |< a|H'^{+}(r)|b >|^2 \, \delta\,(E_a - E_b + \hbar\omega)$$

Transition Rate due to electron-photon interaction

The downward transition rate per unit volume (S^{-1} cm^{-3}) is

$$R_{b-a} = \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} |H'^{+}{}_{ab}|^2 \, \delta\,(E_a - E_b + \hbar\omega) f_b\,(1 - f_a)$$

Using the even property of the delta function, $\delta\,(-x) = \delta\,(x) and\ |H'{}_{ba}| = |H'^{+}{}_{ab}|$.

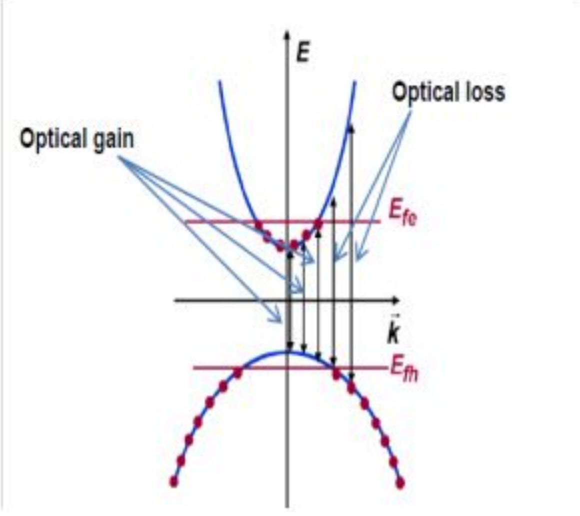
The net upward transition rate per unit volume can be written as,

$$R = R_{a\to b} - R_{b\to a}$$

$$R = \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} |H'{}_{ba}|^2 \, \delta\,(E_b - E_a - \hbar\omega)(f_a - f_b)$$

Optical Gain in Semiconductor

Each downward transition generator a new photon while upward absorbs one. If the number of downward transition for seconds exceeds the number of upward transition there will be a net generation of photons and optical gain can be achieved. The condition for optical gain is net stimulated emission is greater than absorption process.



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Optical Gain in Semiconductor

Optical gain in the material is attained when we injected a carrier density beyond E2 such that the Quasi-Fermi level are separated by an energy greater (E_{fa}-E_{fb}). The process of stimulated downward transition is called optical gain and the process of upward transition is called optical loss. The simple formula for optical gain is

$$g \equiv \frac{1}{\varphi}\frac{d\varphi}{dz}$$

Where φ is photon flux (number of photons per cross section area unit in the unit time) and Z is the direction of electromagnetic field propagation is equal to

$$R = R_{a\to b} - R_{b\to a}$$

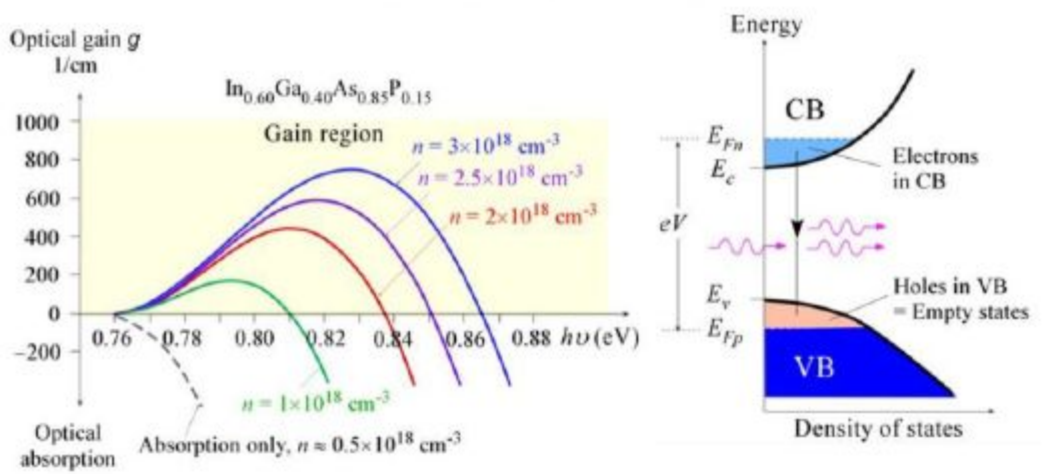
Optical Gain in Semiconductor

So the resultant gain we explained as

$$g = \frac{1}{\varphi} \cdot \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_a - E_{ab} + \hbar\omega)(f_b - f_a)$$

The gain and absorption (Loss) profiles as a function of energy is shown in Fig.

Optical Gain Curve



Density of States for Photons

To define the density of states for photons we assume that the photon is enclosed in a large cube of side length L, such that volume is $V= L^3$. The wave function of photon is a plane wave $e^{ik.\vec{r}}$ .We use the periodic boundary conditions that the wave function should be periodic in the x,y and z directions with a period L.

Because of the wave function has to be zero at boundaries. We have Quantization of wave number

$L.\ K = n2\pi$

$K_x = l\ \frac{2\pi}{L}$; $K_y = l\ \frac{2\pi}{m}$; $K_z = l\ \frac{2\pi}{n}$

The volume of state in K space is $(\frac{2\pi}{L})^3$

Density of States for Photons

Let us look at the integral using the number of states with a differential volume in the K-space.

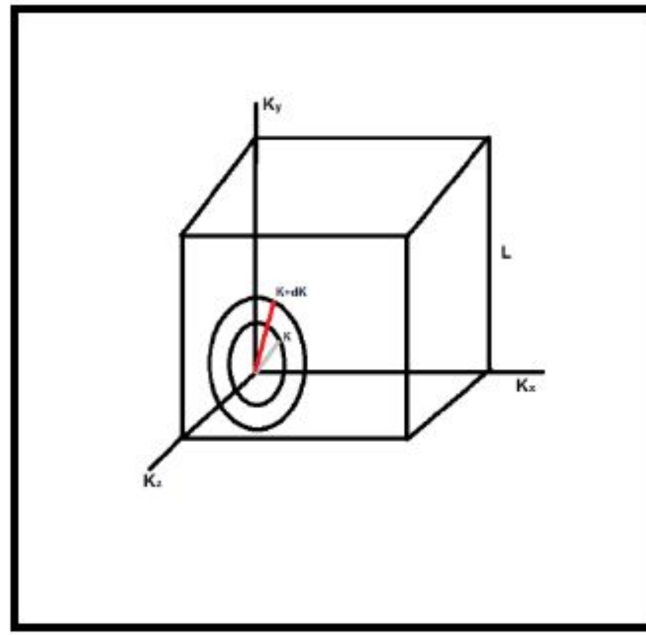
$$\frac{d^3K}{(\frac{2\pi}{L})^3} = \frac{K^2 dk d\Omega}{(\frac{2\pi}{L})^3}$$

Where $d\Omega$ is the differential solid angle.

Therefore

$$N(E_{21}) = \frac{2}{V}\sum_{K}\delta\ (E_2 - E_1 - E_k)$$

$$N(E_{21}) = 2\int \frac{K^2 dk d\boldsymbol{\Omega}}{(2\pi)^3}\delta\ (E_2 - E_1 - E_k)$$



$$E_k = \hbar\omega_k = \frac{\hbar KC}{n_r}$$

Where, C/n_r is the speed of light in medium with refractive index of n_r. Here integration over solid angle is 4π.

Density of States for Photons

$$K = \frac{E_k n_r}{\hbar C}$$

$$dK = \frac{n_r 2\pi}{hC} dE_k$$

$$N(E_{21}) = 2\int \frac{K^2 dk d\Omega}{(2\pi)^3} \delta\ (E_{21} - E_k)$$

$$N(E_{21}) = 2\int \frac{K^2}{(2\pi)^3} \frac{n_r 2\pi}{hC} dE_k\,(4\pi)\delta\ (E_{21} - E_k)$$

$$N(E_{21}) = 2\int \frac{1}{(2\pi)^3} \left(\frac{E_k n_r 2\pi}{hC}\right)^2 \frac{n_r 2\pi}{hC} dE_k (4\pi)\ \delta\ (E_{21} - E_k)$$

$$N(E_{21}) = \frac{2 \times 4\pi \times (2\pi)^3 (n_r)^3}{(2\pi)^3 (hC)^3} \int (E_k)^3\ dE_k\ \delta\ (E_{21} - E_k)$$

$$N(E_{21}) = \frac{8\pi (n_r)^3}{(hC)^3} E_{21}^2 \quad [\hbar = \frac{h}{2\pi}; h = \hbar\, 2\pi]$$

$$N(E_{21}) = \frac{8\pi E_{21}^2 (n_r)^3}{8\pi^3 \hbar^3 C^3} = \frac{E_{21}^2 (n_r)^3}{\pi^2 \hbar^3 C^3}$$

Which is the number of states with photon energy E_{21} per unit volume per energy interval.

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Thank you