

## SRM Institute of Science and Technology Department of Mathematics 18MAB204T- Probability and Queueing Theory Module – IV Tutorial Sheet – I1

## **Questions** If a customer has to wait in a (M/M/1): $(\infty/FIFO)$ queue system, what is his average waiting time in the queue, if $\square = 8$ per hour and $\mu = 12$ per hour? 2 What is the probability that a customer has to wait more than 15min to get his service completed in a in a (M/M/1): ( $\infty$ /FIFO) queue system, if $\square = 6$ per hour and $\mu = 10$ per hour 3 In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the interarrival time follows an exponential distribution and the service time is also exponential with an average 36 minutes. Calculate the following: (i) the mean queue size (line length) (ii) the average number of trains in the queue. (iii) the probability that the queue size exceeds 10. (iv) If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii) If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and if it takes exactly 1.5 min to reach the correct seat after purchasing the ticket, (a) Can he expect to be seated for the start of the picture? (b) What is the probability that he will be seated for the start of the picture? (c) How early must he arrive in order to be 99% sure of being seated for the start of the picture? Arrivals at a telephone booth are considered to be Poisson with an average time of 10min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 min. (a) Find the average number of persons waiting in the system. (b) What is the probability that a person arriving at the booth will have to wait in the queue? (c) What is the probability that it will take him more than 10 min altogether to wait for phone and complete his call? (d) Estimate the fraction of the day when the phone will be in use. (e) The telephone department will install a second booth when convinced that an arrival has to wait on the average for at least 3 min for phone. By how much the flow of arrivals should increase in order to justify a second booth? Customers arrive at the first-class ticket counter of a theatre at a rate of 12 per hour. There is one clerk servicing the customer at the rate of 30 per hr. (i) What is the probability that there is no customer at the counter? (ii) What is the probability that there are more than 2 customers at the counter? (iii) What is the probability that there is no customer waiting to be served? (iv) What is the probability that a customer is being served and nobody is waiting?

(v) Probability that a customer has to wait for at most 4 minutes in the queue?

- Customers arriving at a watch repair shop according to Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes. (i) Find the average number of customers in the shop.
  - (ii) Find the average time a customer spends in the shop.
  - (iii) Find the average number of customers in the queue.
  - (iv) What is the probability that the server is idle?
- 8 Customers arrive at a one-man barber shop according to a Poisson process with a mean interarrival time of 20 min. Customers spend an average of 15 min in the barber's chair.
  - (a) What is the expected number of customers in the barber shop and also in the queue?
  - (b) What is the probability that a customer will not have to wait for a hair-cut?
  - (c) How much can a customer expect to spend in the barber shop?
  - (d) Management will put another chair and hire another barber when a customer's average waiting time in the shop exceeds 1.25h. How much must the average rate of arrivals increase to warrant a second barber?
  - (e) What is the average time customers spend in the queue?
  - (f) What is the probability that the waiting time in the system is greater than 30 min?
  - (g) What is the probability that there are more than 3 customers in the system?