29.a. Find the half-range cosine series for $f(x)=x, 0\le x\le \pi$. Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots=\frac{\pi^2}{8}$.

(OR)

b. Find the Fourier sine series upto third harmonic for the function $y=f(x)$ in $(0,\pi)$ from the table.

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y	2.34	2.2	1.6	0.83	0.51	0.88	2.34

30.a. A tightly stretched string of length l has its end fastened at $x=0$, $x=l$. At $t=0$, the string is in the form $f(x)=\lambda x(l-x)$ and then released. Find the displacement y at any time and at any distance from the end x=0.

(OR)

b. Find the solution of the equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions.

(i) $u(0,t)=0$ (ii)

(ii) $u(l,t)=0 \; for \; t>0$

(iii) $u(x,0)=\begin{cases} x, 0\le x\le l/2 \\ l-x, l/2\le x\le l \end{cases}$

31.a. Find the Fourier transform of $f(x)$ given by $f(x)=\begin{cases} a^2-x^2; & if\ |x|<a \\ 0 & ; if\ |x|>a>0 \end{cases}$ hence prove that $\int_0^{\alpha}\left(\frac{\sin x - x\cos x}{x^3}\right)dx=\frac{\pi}{4}$.

(OR)

b.i. Find the Fourier transform of $e^{-a|x|}$ and hence evaluate $\int_0^{\infty}\frac{1}{\left(x^2+a^2\right)^2}dx, a>0$. (8 Marks)

ii. If $F[f(x)]=F(s)$, then $F(f(x)\cos ax)=\frac{1}{2}[F(s+a)+F(s-a)]$. (4 Marks)

32.a.i. Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem.

ii. Find $Z^{-1}\left[\frac{z^2}{(z+2)\left(z^2+4\right)}\right]$ by method of partial fraction.

(OR)

b. Using Z-transform solve $u_{n+2}+4u_{n+1}+3u_n=3^n$ with $u_0=0, u_1=1$.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
First to Eighth Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. Find the complete integral of $p^2+q^2=1$
(A) $z=ax+by+c$ (B) $z=ax+by$
(C) $z=a(x+y)+b$ (D) $z=ax-by+a$

2. Solve $pq=xy$
(A) $z=k\frac{x}{2}+\frac{1}{k}y/2+c$ (B) $z=k\frac{x^2}{2}+\frac{1}{k}\frac{y^2}{2}+c$
(C) $z=k(x+y/2)+c$ (D) $z=k(x/2-y)+c$

3. Solve $\left(D^3-3DD'^2+2D'^3\right)z=0$
(A) $z=\phi_1(y+2x)+\phi_2(y-x)+\phi_3(y+x)$ (B) $z=\phi_1(y+6x)+\phi_2(y-x)+x\phi_3(y-2x)$
(C) $z=\phi_1(y-2x)+\phi_2(y+x)+x\phi_3(y+x)$ (D) $z=\phi_1(y+3x)+\phi_2(y-2x)+\phi_3(y+4x)$

4. Find the particular integral of $\left(D^2+3DD'+4D'^2\right)z=e^{x-y}$
(A) $\frac{e^{x+y}}{2}$ (B) $\frac{e^{2x+y}}{2}$
(C) $\frac{e^{x-2y}}{2}$ (D) $\frac{e^{x-y}}{2}$

5. $\int_{-1}^{1}|x|\,dx$ is equal to
(A) $\int_0^1 x\,dx$ (B) $2\int_0^1 x\,dx$
(C) $2\int_0^1(-x)dx$ (D) $4\int_0^{\pi/2}(-x)dx$

6. The constant a_0 of the Fourier series for the function $f(x)=x$ is $0\le x\le 2\pi$
(A) π (B) 3π
(C) 2π (D) 0

7. The RMS value of $f(x)=x$ in $-1 \le x \le 1$ is
(A) 1 (B) 0
(C) -1 (D) $1/\sqrt{3}$

8. For half range cosine series of $f(x)=\cos x$ in $(0,\pi)$ the value of a_0 is
(A) 0 (B) 4
(C) $2/\pi$ (D) $4/\pi$

9. The partial differential equation $u_{xx}+2u_{xy}+u_{yy}=0$ of the form.
(A) Elliptic (B) Parabolic
(C) Hyperbolic (D) None of these

10. The proper solution of $u_t=\alpha^2 u_{xx}$ is
(A) $u=(Ax+B)C$ (B) $u=(A\cos\lambda x+B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$
(C) $u=\left(Ae^{\lambda x}+Be^{-\lambda x}\right)e^{\alpha^2\lambda^2 t}$ (D) $u=At+B$

11. One dimensional wave equation is used to find
(A) Temperature (B) Time
(C) Displacement (D) Mass

12. The amount of heat required to produce a given temperature change in a body is proportional to
(A) Weight of the body (B) Mass of the body
(C) Density of the body (D) Tension of the body

13. The steady state temperature of a rod of length l whose ends are kept at 30 and 40 is
(A) $u=\frac{10x}{l}+30$ (B) $u=\frac{20x}{l}+30$
(C) $u=\frac{10x}{l}+20$ (D) $u=\frac{10x}{l}$

14. The Fourier cosine transform of e^{-ax} is
(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$ (B) $\sqrt{\frac{1}{\pi}}\frac{s}{s^2+a^2}$
(C) $\sqrt{\frac{1}{\pi}}\frac{a}{s^2-a^2}$ (D) $\sqrt{\frac{2}{\pi}}\frac{a}{\left(s^2+a^2\right)}$

15. $F[xf'(x)]=$
(A) $\frac{dF(s)}{ds}$ (B) $i\frac{dF(s)}{ds}$
(C) $-i\frac{dF(s)}{ds}$ (D) $-\frac{dF(s)}{ds}$

16. $F[f(x)*g(x)]=$
(A) F(s) + G(s) (B) F(s) – G(s)
(C) F(s)G(s) (D) F(s) / G(s)

17. What is Z-transform of na^n ?
(A) $\frac{az}{(z-a)^2}$ (B) $\frac{z}{(z-a)^2}$
(C) $\frac{a}{(z-a)^2}$ (D) $\frac{z}{(z-a)^3}$

18. Region of convergence of a $Z\left[a^n\right]$ is
(A) $|z|<a$ (B) $|z|>a$
(C) $|z|>|a|$ (D) $|z|<|a|$

19. Find $Z^{-1}\left[\frac{z}{z-a}\right]$
(A) a^{n+1} (B) a
(C) a^n (D) a^{n-1}

20. The difference equation formed by eliminating 'a' in $u_n=a\,2^{n+1}$ is
(A) $u_{n+1}-2u_n=0$ (B) $u_{n+1}=0$
(C) $u_{n+1}-u_n=0$ (D) $u_n=0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Form a partial differential equation by eliminating arbitrary constants 'a' and 'b' from $z=(x+a)^2\cdot(y+b)^2$.

22. Find the general solution of $\left(5D^2-12DD'-9D'^2\right)z=0$.

23. Find a Fourier sine series for the function $f(x)=1, 0<x<\pi$.

24. Find the RMS value of $f(x)=1-x$ in $0<x<1$.

25. What are all the solutions of one dimensional wave equation?

26. Prove that $F\left(e^{iax}f(x)\right)=F(s+a)$, where $F(f(x)=F(s))$.

27. Find the Z-transform of $(n+1)(n+2)$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28.a.i. Form the partial differential equation by eliminating f from $xyz=f\left(x^2+y^2-z^2\right)$.

ii. Solve $(3z-4y)p+(4x-2z)q=2y-3x$.

(OR)

b. Solve $\left(D^2-6DD'+5D'^2\right)z=e^x\sinh y+xy$.