C 3152

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fifth Semester

Computer Science and Engineering

CS 1303 — THEORY OF COMPUTATION

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is a finite automation? Give two examples.

2. Enumerate the differences between DFA and NFA.

3. Verify whether $L = \{a^{2n} \mid n \geq 1\}$ is regular.

4. Mention the closure properties of regular languages.

5. Let the productions of a grammar be $S \to 0B \mid 1A$, $A \to 0 \mid 0S \mid 1AA$, $B \to 1 \mid 1S \mid 0BB$. For the string 0110 find a rightmost derivation.

6. Define the languages generated by a PDA using the two methods of accepting a language.

7. State pumping lemma for context - free language.

8. Define a Turing Machine.

9. Differentiate between recursive and recursively enumerable languages.

10. Mention any two undecidability properties for recursively enumerable languages.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove the following by the principle of induction

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)\ (2n+1)}{6}. \qquad (6)$$

(ii) For the finite state machine M given in the following table, test whether the strings 101101,11111 are accepted by M. (4)

State	Input 0	Input 1
$\rightarrow (q_0)$	q_0	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

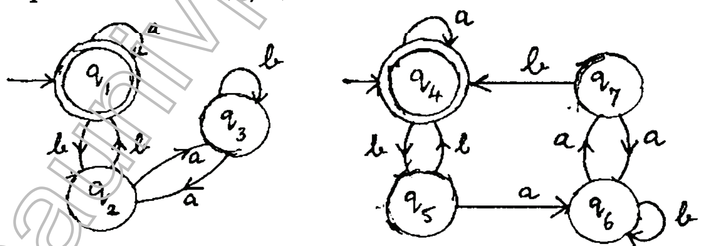
(iii) Construct a DFA that accepts all the strings on $\{0,1\}$ except those containing the substring 101. (6)

Or

(b) (i) Prove that there is no string x in (a, b)* such that $ax = xb$. (6)

(ii) Construct a non-deterministic finite automation accepting the same set of strings over {a, b} ending in aba. Use it to construct a DFA accepting the same set of strings. (10)

12. (a) (i) Verify whether the finite automata M_1 and M_2 given below are equivalent over {a, b}.



Finite Automaton M_1 Finite Automaton M_2

(8)

(ii) Construct transition diagram of a finite automaton corresponding to the regular expression (a b + c*)* b. (8)

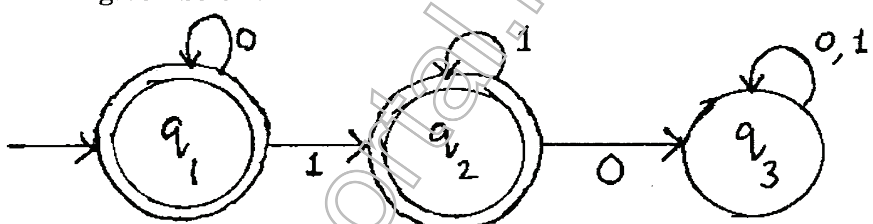
Or

(b) (i) Construct a minimum state automaton equivalent to a given automaton M whose transition table is given below.

State	Input a	Input b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
q_6	q_1	q_3

(8)

(ii) Find the regular expression corresponding to the finite automaton given below. (8)



13. (a) (i) Find a derivation tree of $a*b+a*b$ given that $a*b+a*b$ is in L(G) where G is given by $S \rightarrow S+S \mid S*S, S \rightarrow a \mid b$. (6)

(ii) Show that the grammar $S \rightarrow a \mid abSb \mid aAb, A \rightarrow bS \mid aAAb$ is ambiguous. (6)

(iii) Consider the following productions:

$S \rightarrow aB \mid bA$

$A \rightarrow aS \mid bAA \mid a$

$B \rightarrow bS \mid aBB \mid b.$

For the string aaabbababba, find a leftmost derivation. (4)

Or

(b) (i) Construct a PDA accepting by empty stack the language $\{a^m b^m c^n \mid m, n \geq 1\}$. (8)

(ii) Show that set of all strings over {a, b} consisting of equal number of a's and b's is accepted by a deterministic PDA. (8)

14. (a) (i) Find a grammar in Chomsky normal form equivalent to $S \rightarrow aAbB,\ \ A \rightarrow aA \mid a,\ \ B \rightarrow bB \mid b$. (8)

(ii) Convert the grammar $S \rightarrow AB,\ \ A \rightarrow BS \mid b,\ \ B \rightarrow SA \mid a$ into Greibach normal form. (8)

Or

(b) (i) Show that $L = \{a^n b^n c^n \mid n \geq 1\}$ is not a context-free language. (6)

(ii) Design a Turing Machine that computes $x + y$ where x and y are positive integers. (8)

(iii) What are the features of a Universal Turing Machine? (2)

15. (a) (i) Show that "If a language L and its compliment $\overline{L}$ are both recursively enumerable, then both languages are recursive". (6)

(ii) Show that halting problem of Turing Machine is undecidable. (5)

(iii) Does PCP with two lists $x = (b, b\,ab^3, ba)$ and $y = (b^3,\ ba,\ a)$ have a solution? (5)

Or

(b) (i) Show that the characteristic function of the set of all even numbers is recursive. (6)

(ii) Let $\sum = \{0, 1\}$. Let A and B be the lists of three strings each, defined as

	List A	List B
i	w_i	x_i
1	1	111
2	10111	10
3	10	0

Does this PCP have a solution? (5)

(iii) Show that it is undecidable for arbitrary CFG's G_1 and G_2 whether $L(G1) \cap L(G_2)$ is a CFL. (5)