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MATHS ASSIGNMENT-1

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SECTION: T2
DEPT: CSE IOT

Q1: $A-(B\cap C) = (A-C) \cup (A-B)$

LHS

$A-(B\cap C) \Rightarrow \{x : x\in A \text{ and } x\notin (B\cap C)\}$

$\Rightarrow \{x : x\in A \text{ and } (x\notin B \text{ or } x\notin C)\}$

$\Rightarrow \{x : (x\in A \text{ and } x\notin B) \text{ or } (x\in A \text{ and } x\notin C)\}$

$\Rightarrow \{x : x\in (A-B) \text{ or } x\in (A-C)\}$

$\Rightarrow \{x : (A-B) \cup (A-C)\}$

$\Rightarrow (A-B)\cup(A-C)$

$\therefore$ LHS = RHS.

Q2: no. of relations from set A to A $= 2^{\text{no. of elements in } A\times A}$
$= 2^{n^2}$

Now, no. of reflexive relations from A to A $= 2^{n^2-n}$

And, no. of symmetric relations from A to A $= 2^{n(n+1)}$

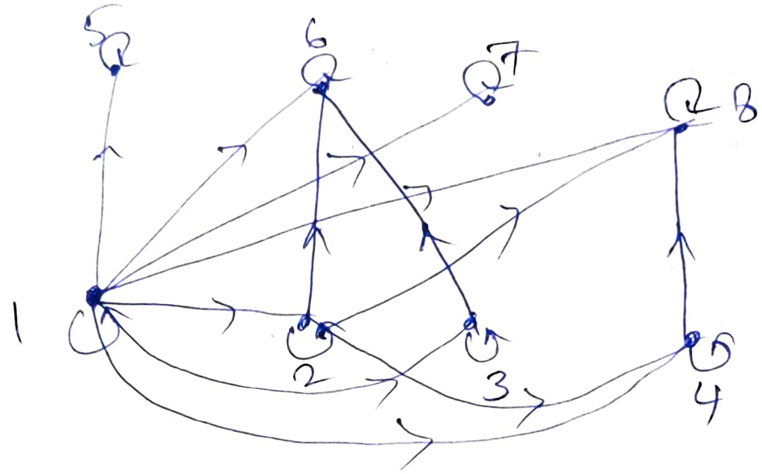
PTO.

Q3. aRb if a divides b,

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)\ (1,7)\ (1,8)$
$(2,2)\ (2,4)\ (2,6)\ (2,8)$
$(3,3)\ (3,6)\ (4,4)\ (4,8)\ (5,5)\ (6,6)\ (7,7)\ (8,8)\}$

Digraph:



Q4. $f(x) = x-2$, $g(x) = x+2$

$f \circ g = f(g(x)) = f(x+2) = (x+2) - 2 \Rightarrow x$

$g \circ f = g(f(x)) = g(x-2) = (x-2)+2 \Rightarrow x$

Thus $f \circ g = g \circ f$.

$\longrightarrow$

PTO.

Q5- $A = \{ 3^n \mid n \in N \}$.

Reflexive

$a \in A$; $\frac{a}{a} = 1$ (i.e. $a = 3^n, n \in N$)
↓
integer

Thus, aRa is reflexive.

Anti Symmetric

aRb & bRa ⇒ $\frac{b}{a}$ & $\frac{a}{b}$ → both are integers.

So, $a = b$

Thus, it is anti-symmetric.

Transitive

aRb & bRc ⇒ $\frac{b}{a}$ & $\frac{c}{b}$ → integers

For ratio $\frac{c}{a}$ to be an integer, c must be a multiple of b & b must be a multiple of a. Since b/a & c/a are integers, c/a is also an integer. Thus aRc holds transitive.

So, R is a partial ordering on A.

Q6.

To prove : $A \cap B = A \cap C$ need not always imply $B = C$

let $A = \{0\}$, $B = \{0, 2, 3\}$, $C = \{0, 4, 5\}$

$A \cap B = \{0\}$, $A \cap C = \{0\}$

Here, $A \cap B = A \cap C = \{0\}$

However, $B \neq C$ & $2 \in B$ & $2 \notin C$

Thus, $A \cap B = A \cap C$ need not always imply $B = C$.

To prove : $A \cup B = A \cup C$ and $A \cap B = A \cap C$ imply $B = C$.

① Suppose $A \cup B = A \cup C \rightarrow$ any element $\in$ (A or in B) is also any element in C B' $\rightarrow B \subseteq C$ & $C \subseteq B \Rightarrow B = C$

② Suppose $A \cap B = A \cap C \rightarrow$ any element thats both in A & B is also both in A and C and vice versa. Thus, any element in $B \in C$ and any element in $C \in B \Rightarrow B \subseteq C$ & $C \subseteq B \rightarrow B = C$

Q7: $R = \{(1,1)(1,2)(2,2)(3,3)(4,3)(4,4)(4,5)(5,5)(6,6)\}$

$A = \{1, 2, 3, 4, 5, 6\}$

Symmetric closure of R is given by

$R = R \cup R^{-1} = R \cup \{(1,1)(2,1)(2,2)(3,3)(3,4)(4,4)(5,4)(5,5)$
$(6,6)\}$

$R = \{(1,1)(1,2)(2,2)(3,3)(4,3)(4,4)(4,5)(5,5)(6,6)(2,1)(3,$
$(5,4)\}$

warshall's algorithm

$$W_0 = M_R = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$$W_1 = \begin{array}{c|cccccc} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

P_i	Q_j
1	1
	2
	(1,1)
	(1,2)

$$W_2 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

P_i	Q_j
1	2
2	
	(1,2)
	(2,2)

$$W_3 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

P_i	Q_j
3	3
4	
	(3,3)
	(4,3)

$$W_4 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

P_i	Q_j
4	3
	4
	5
	(4,3)
	(4,4)
	(4,5)

$$W_5 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

P_i	Q_j
6	6
	(6,6)

Thus, transitive closure of R is

$$M_R^{\infty} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} = W_6$$

$R^{\infty} = \{(1,1)\,(1,2)\,(2,2)\,(3,3)\,(4,3)\,(4,4)\,(4,5)\,(5,5)\,(6,6)\}$

Q8. $A = \{0, 2, 5, 10, 11, 15\}$

$R = \{(0,0)(0,2)(0,5)(0,10)(0,11)(0,15)(2,2)(2,5)$
$(2,10)(2,11)(2,15)(5,5)(5,10)(5,11)(5,15)(10,10)(10,11)$
$(10,15)(11,11)(11,15)(15,15)\}$

Digraph:

Hasse Diagram

Q9.

let $x_1, x_2 \in Z$ & $f(x_1) = f(x_2)$ are both even & odd

If they are both odd,

then $2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$ or

If they are both even,

then $-2x_1 = -2x_2 \Rightarrow x_1 = x_2$

So, if $f(x_1) = f(x_2)$, we get $x_1 = x_2$, So f is one-one let $y \in N$, if y is odd,

then its pre image is $\frac{y+1}{2}$ since

$$f\left(\frac{y+1}{2}\right) = 2\left(\frac{y+1}{2}\right) - 1 = y \quad \left[\text{as } \left(\frac{y+1}{2}\right) > 0\right]$$

If y is even, then its pre image is $-y/2$ since

$$f(-y/2) = -2(-y/2) - 1 = y \quad \left[\text{as } (-y/2) \leq 0\right]$$

Thus, for any $y \in N$, the pre-image is $\frac{y+1}{2} \in Z$ or $-y/2 \in Z$. Hence $f(x)$ is onto.

Q10. $(-1, 1) \in R$, Now $f(x) = x^2$

$f(-1) = (-1)^2 = 1$

$f(1) = 1^2 = 1$

$f(x_1) = f(x_2)$ but $x_1 \neq x_2$ since $1 \neq -1$.

Thus, f is not one-one

let $f(x) = y$ such that $y \in R \Rightarrow x = \pm\sqrt{y}$

$\Rightarrow$ Put $y = 3 \Rightarrow x = \pm\sqrt{3}$ & $x \in R$.

Thus f is onto

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