Wednesday, November 24, 2021   1:35 PM

Method of Variation of Parameters

(2) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\tan x$

by method of Variation of parameters.

(3) **Home work**

$\frac{d^2y}{dx^2} + y = \text{cosec}\, x$ by method of Variation of Parameters

(2) Sol $(D^2+2D+5)y = \underbrace{e^{-x}\tan x}_{F(x)}$

A.E $\quad m^2+2m+5=0$

$m = \frac{-2 \pm \sqrt{4-20}}{2}$

$m = -1 \pm 2i$

$CF = e^{-x}\left[A\cos 2x + B\sin 2x\right]$

$CF = A\left(e^{-x}\cos 2x\right) + B\left(e^{-x}\sin 2x\right)$

$CF = A f_1 + B f_2$

$f_1 = \underbrace{e^{-x}\cos 2x}_{(uv)' = u'v + uv'}$ | $f_2 = e^{-x}\sin 2x$

$f_1' = -2e^{-x}\sin 2x - e^{-x}\cos 2x$ | $f_2' = 2e^{-x}\cos 2x - e^{-x}\sin 2x$

$Dem = f_1 f_2' - f_2 f_1'$

$Dem = 2e^{-2x}$

$PI = P f_1 + Q f_2$

$P = -\int \frac{f_2}{Dem} F(x)\,dx$

$= -\int \frac{\cancel{e^{-x}}\sin 2x}{2\cancel{e^{-2x}}} \cancel{e^{-x}}\tan x\,dx$

$= -\frac{1}{2}\int 2\sin x\cancel{\cos x}\left(\frac{\sin x}{\cancel{\cos x}}\right)dx$

$= -\frac{1}{2}\cdot 2\int \sin^2 x\,dx$

$= -\int\left(\frac{1-\cos 2x}{2}\right)dx$

$= -\left(\frac{x}{2} - \frac{\sin 2x}{4}\right)$

$\boxed{P = -\frac{x}{2} + \frac{\sin 2x}{4}}$

$\sin 2x = 2\sin x\cos x$

$\tan x = \frac{\sin x}{\cos x}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$Q = \int \frac{f_1}{Dem} F(x)\,dx$

$= \int \frac{\cancel{e^{-x}}\cos 2x}{2\cancel{e^{-2x}}} \cancel{e^{-x}}\tan x\,dx$

$= \frac{1}{2}\int \cos 2x\tan x\,dx$

$\left(\cos 2x = 2\cos^2 x - 1 \quad \tan x = \frac{\sin x}{\cos x}\right)$

$= \frac{1}{2}\int\left(2\cos^2 x - 1\right)\frac{\sin x}{\cos x}\,dx$

$= \frac{1}{2}\int\left(\sin 2x - \tan x\right)dx$

$= \frac{1}{2}\left[-\frac{\cos 2x}{2} - \left(-\log(\cos x)\right)\right]$

$\boxed{Q = -\frac{\cos 2x}{4} + \frac{1}{2}\log(\cos x)}$

$PI = P f_1 + Q f_2$

$\boxed{y = CF + PI}$

(3) Homework

Answer $\quad CF = A\cos x + B\sin x$

$\boxed{P = -x}$

$\boxed{Q = \log(\sin x)}$

$PI = -x\cos x + \log(\sin x)\sin x$

$\boxed{y = A\cos x + B\sin x + \quad "}$