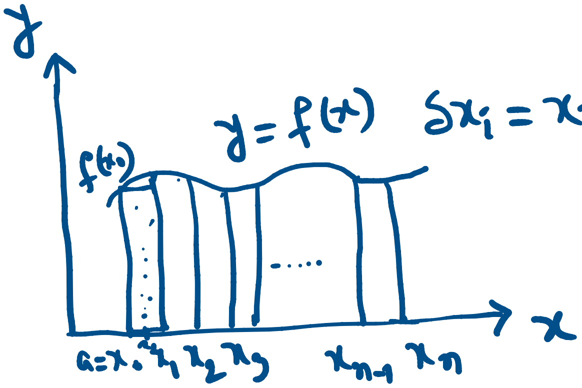
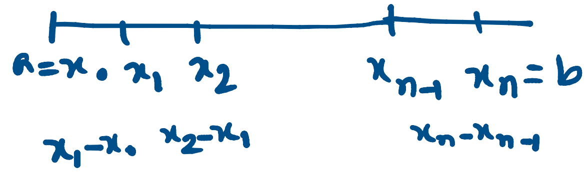
$\boxed{\text{Example}}$ $f(x) = x^2, \quad 1 \le x \le 2$

$$\int_1^2 x^2\,dx = \left.\frac{x^3}{3}\right|_1^2 = \frac{1}{3}\left(2^3 - 1^3\right) = \frac{1}{3}(7) = 7/3$$



$\delta x_i = x_i - x_{i-1}$



$f(x_1^*)\,\delta x_1 + f(x_2^*)\,\delta x_2 + \cdots\cdots$

$x_i^* \in [x_{i-1}, x_i]$

Riemann's Sum:

$$\sum_{i=1}^{n} f(x_i^*)\,\delta x_i$$

$$\Rightarrow \int_a^b f(x)\,dx = \underset{\max(\delta x_i)\to 0}{\mathrm{Lt}} \sum_{i=1}^{n} f(x_i^*)\,\delta x_i$$

$\delta x = x_1 - x_0 = x_2 - x_1 = \cdots = x_n - x_{n-1}$

$$= \underset{n\to\infty}{\mathrm{Lt}} \sum_{i=1}^{n} f(x_i^*)\,\delta x$$

When a function $f(x)$ is integrated with respect to x between the limits a and b, we get the single integral which is defined by

$$\int_a^b f(x)\,dx \longrightarrow \text{Area under the curve.}$$

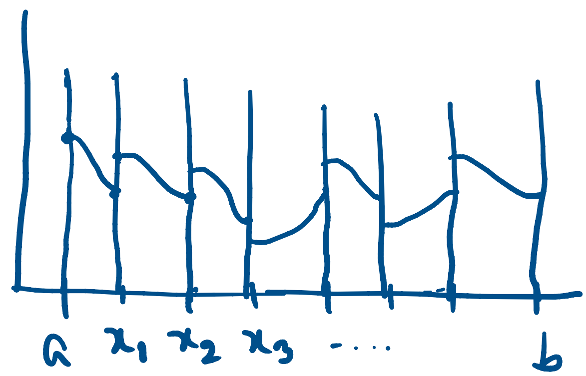
a curve.

$\boxed{\text{Note}}$ If $f(x)=1$, then the integral represent the length of the interval.

$\boxed{\text{Note}}$ If f is piecewise continuous function $a \le x \le b$, then $\int_a^b f(x)\,dx$ exists.

piecewise continuous:

$$f(x_i^-) \neq f(x_i^+)$$



If the function has finite number of discontinuity at $x_1, x_2, \ldots, x_n$ and the limits of the function in each subinterval is finite and left hand limit and right hand limits are not equal i.e,

$$f(x_i^-) = \text{finite} \qquad x_i \in [a,b]$$

$$f(x_i^+) = \text{finite} \qquad x_i \in [a,b].$$

$$f(x_i^-) \neq f(x_i^+)$$

Double integral:

If the integrand is a function $f(x,y)$ and if it is integrated with respect to x and y repeatedly the limits x_0 and x_1 (for x) and between the limits y_0 and y_1 (for y) we get a double integral that is denoted by

$$\int_{y_0}^{y_1}\int_{x_0}^{x_1} f(x,y)\,dx\,dy$$

$\boxed{\text{Ex}}$ Evaluate $\displaystyle\int_0^2\int_0^1 4xy\,dx\,dy$

$$= \int_0^2 4y\cdot\left[\frac{x^2}{2}\right]_0^1 dy$$

$$= \int_0^2 4y\cdot\frac{1}{2}\,dy$$

$$= 2\left[\frac{y^2}{2}\right]_0^2 = 4$$