SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB302T- DISCRETE MATHEMATICS

Year/Sem: III/V

Branch: CSE,ECE,EEE

UNIT-2 -COMBINATORICS

(1) If $np_4 = 360$, then the value of n.

(a) 5 (b) 6 (c) 7 (d) 8 **Ans: b**

Solution: Given

$np_4 = 360$

n(n-1)(n-2)(n-3)=(n^2-n) (n^2-5n+6)

(n^2-n) (n^2-5n+6) = 360

Therefore n = 6

(2) If $10p_r = 360$, then the value of r.

(a) 3 (b) 6 (c) 7 (d) 8 **Ans: a**

Solution: Given

$10p_r = 360$

10 x 9 x 8 x….. to r factors =720 =10 x 9 x 8

Therefore r = 3

(3) Which one of the following is correct?

$np_r = nc_r$ (b) $np_r = rnc_r$ (c) $np_r = r! \times nc_r$ (d) $r! \times np_r = nc_r$

Solution:

$np_r = \frac{n!}{(n-r)!}$ $nC_r = \frac{n!}{r!(n-r)!}$

Ans: d

$$r! \times np_r = nc_r$$

(4) How many numbers of four digits can be formed with the digits 1,2,3,4 and 5 (no repetition)?

(a) 120 (b) 150 (c) 100 (d) 110

Solution:

n = 5 and r = 4

required number = $5p_4$ =120 **Ans: a**

(5) How many numbers between 400 and 1000 can be made with the digits 2,3,4,5,6 and 0?

(a) 50 (b) 60 (c) 65 (d) 70 **Ans: b**

Solution:

$3 \times 5p_2$=60

Hundreds place can be filled up by any one of the digits 4,5,6 and remaining two places can be filled up by remaining five digits in 5p2

(6) How many ways can 6 boys and 4 girls sit in a row if the boys are to sit together and the girls are to sit together ?

(a) 34500 (b) 34200 (c) 34560 (d) 34650

Ans: c

Solution:

No of ways can 6 boys and 4 girls sit in a row if the boys are to sit together and the girls are to sit together is

$2 \times 6! \times 4! = 34560$

In two ways , the boys can be arranged in 6! ways and the girls can be arranged in 4! ways .

(7) If $15C_{3r} = 15C_{r+3}$, then what is the value of r?

(a) 8 (b) 5 (c) 6 (d) 3 **Ans: d**

Solution :

$15C_{3r} = 15C_{r+3,}$

3r = r + 3 gives r = 1.5 that is not possible r is integer

3r + r + 3 = 15 hence r = 3

(8) If $nC_6 = n - 3C_3 = 33{:}\,4$, then the value of n is .

(a) 10 (b) 9 (c) 11 (d) 8 **Ans: c**

Solution:

$\frac{n!}{(n-3)!} \times \frac{3!}{6!} = \frac{33}{4}$

n(n-1) (n-2) =33.6.5

n(n-1)(n-2)=11.10.9 Therefore n=11.

(9) A club consisting of 6 men and 7 women , in how many ways can we select a committee of 3 men and 4 women ?

(a) 500 (b) 700 (c) 600 (d) 800 **Ans: b**

Solution: They can be selected in C(6,3) . C(7,4) = 700 ways

(10) A club consisting of 6 men and 7 women , in how many ways can we select a committee of 4 persons has at most one man ?

(a) 230 (b) 245 (c) 254 (d) 250

Ans: b

Solution :

No of ways we select a committee of 4 persons has at most one man

$C(6,0).C(7,4) + C(6,1).C(7,3) = 6C_0.7C_4 + 6C_1.7C_3 = 245$ ways

(11) A club consisting of 6 men and 7 women , in how many ways can we select a committee of 4 persons that has persons of both sexes ?

(a) 625 (b) 650 (c) 665 (d) 664 **Ans: c**

Solution:

No of ways we select a committee of 4 persons that has persons of both sexes

$C(6,1).C(7,3) + C(6,2).C(7,2) + C(6,3).C(7,1)$

$= 6C_1.7C_4 + 6C_2. 7C_2 = 665$ ways.

(12) There are 3 piles of identical red , blue and green balls where each pile contains at least 10 balls.In how many ways can 10 balls be selected with no restriction?

(a) 64 (b) 65 (c) 66 (d) 67 **Ans: c**

Solution:

n=3 and r=10

$C(n+r-1 , r) = C(12,10) = 12C_{10} = 66$

(13) If we select 10 points in the interior of an equilateral triangle of side 1, then there must be at least two points whose distance apart is

(a) Less than 1/3 (b) greater than 1/3 (c) equal to 1/3 (d) less than 3 **Ans: a**

Solution :

9 sub triangles may be regarded as 9 pigeon holes and 10 interior points may be regarded as 10 pigeons .The distance between any two interior points of any sub triangle is less than 1/3.

(14) If the linear combination of a and b is $gcd\ (a,b)\ = ma + nb$,then

(a) m and n integers (b) m and n are rationals (c) m and n are real

(d) m and n are only positive integers **Ans: d**

Solution:

If gcd (a, b) is defined by the expression, $gcd\ (a,b)\ = ma + nb$ where m,n are positive integers and a,b is both not zero, then the expression is called Bezout's Identity and m,n can be calculated by extended form of Euclidean algorithm.

(15) Which one of the following is hold true?

(a) gcd (ka,kb) = gcd (a,b) (b) gcd(ka,kb) = k gcd(a,b)

(c) gcd (ka,kb) = gcd (a,b) / k (d) gcd(ka,kb) = k2 gcd(a,b) **Ans: b**

Solution :

Let d = gcd (a,b) then ma+nb = d where m and n are integers .

m(ka)+n(kb)

= kd = k.gcd(a,b)

(16) The value of gcd (1819,3587) is

(a) 15 (b) 16 (c) 17 (d) 51 **Ans: c**

Solution:

By division algorithm , 3587=1.1819+1768 ;

1819 =1.1768+51 ;

1768 = 34.51+34 ;

51 = 1.34 +17 ;

34 =2.17+0

(17) The value of lcm (231,1575) is

(a)51775 (b) 51765 (c) 17325 (d) 51985 **Ans: c**

Solution:

lcm (231,1575)= $3^{(0,1)} \times 7^{(1,1)} \times 11^{(0,1)} \times 15^{(0,2)}$

= $3.7.11.15^2$ = 17325

(18) If gcd (a,4) = gcd(b,4) = 2 ,then gcd (a+b,4) is

(a) 4 (b) 5 (c) 6 (d) 8 **Ans: a**

Solution:
let a=2m and b=2n m and n are odd integers a+b = 2(m+n) =2.2r
Therefore gcd(4r,4) = 4

(19) If a and b be two positive integers , then lcm (a, b). gcd (a, b) is
Solution:
(a) 1 (b) ab (c) a/b (d) a+b **Ans: b**
Solution:
By Theorem: lcm(a, b) × gcd(a, b) = ab for any positive integers a, b.

(20) If 25 dictionaries in a library contain a total of 40,325 pages , then one of the dictionaries must have atleast
(a) 1614 (b) 1615 (c) 1610 (d) 1618 **Ans: a**

Solution: $\left[\frac{m-1}{n}\right] + 1 = \left[\frac{40325-1}{25}\right] + 1 = 1614$
no. of pages = m = 40325 ; no. of dictionaries = n = 25

(21) If gcd (a,b) =1, then for any integer c ,
(a) gcd(ac,b) = gcd(c,b) (b) gcd (a,b) = gcd (c,b)
(c) gcd(a,b) = gcd(c,b) (d) gcd (ac,b) = a. gcd(c,b) **Ans: a**

Solution:
By Theorem: If gcd (a,b) =1 then gcd (ac,b) = gcd(c,b)
Hence the solution is gcd (ac,b) = gcd (c,b)

(22) The value of gcd (231,1575) is
(a) 9 (b) 15 (c) 7 (d) 5 **Ans: c**

Solution: gcd (231,1575)
$= 3^{(0,1)} \times 7^{(1,1)} \times 11^{(0,1)} \times 15^{(0,2)}$
$= 3^0.7^1.11^0.15^0 = 7$

(23) Among 100 people, at least …… of them were born in the same month.
(a) 10 (b) 9 (c) 8 (d) 7 **Ans: b**

Solution:
no. of pages = m =100
no. of dictionaries = n = 12
$\left[\frac{m-1}{n}\right] + 1 = \left[\frac{100-1}{12}\right] + 1 = 9$

(24) How many positive integers not exceeding 1000 are divisible by 7 or 11?

(a) 200 (b) 210 (c) 205 (d) 220 **Ans: d**

Solution:

$$|A| = \left[\frac{1000}{7}\right] = 142; |B| = \left[\frac{1000}{11}\right] = 90;$$

$|A \cap B| = \left[\frac{1000}{77}\right] = 12$ by principle of inclusion and exclusion

$|A \cup B| = |A| + |B| - |A \cap B| = 220$

(25) How many positive integers not exceeding from 1 to 100 are not divisible by 5 or 7?

(a) 65 (b) 68 (c) 63 (d) 64 **Ans: b**

Solution:

$|A| = \left[\frac{100}{5}\right] = 20; |B| = \left[\frac{100}{7}\right] = 14;$

$|A \cap B| = \left[\frac{100}{35}\right] = 2$ by principle of inclusion and exclusion

$|A \cup B| = |A| + |B| - |A \cap B| = 32$ not divisible =100-32=68