b. A and B shoot independently until each has hit his own target. The probabilities of their hitting the target at each shot are 3/5 and 5/7 respectively. Find the probability that B will require more shots than A.

30. a. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumer's of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is a significant decrease in the consumption of tea after the increase in duty.

(OR)

b. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In a experiment among 1600 beans, the numbers in the four groups were 882, 313,287 and 118. Does the experiment support the theory?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min (i) Find the average number of persons waiting in the system (ii) what is the probability that a person arriving at the booth will have to wait in the queue? (iii) Estimate the fraction of the day when the phone will be in use (iv) what is the average length of the queue that forms from time to time.

(OR)

b. The local on-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customer arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time) (i) what percentage of time is the barber idle? (ii) what fraction of the potential customers are turned away? (iii) what is the expected number of customers waiting for a haircut (iv) how much time can a customer expected to spend in the barber shop?

32. a. The transition probability matrix of a Markov chain $\{X_n\}$, n = 1, 2, ,3 .....having 3 states 1, 2, and 3 is

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (i) $P\{X_2 = 3\}$ *and* (*ii*) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. show that the process is Markovian. Find the transition matrix and classify the states.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018

3rd to 7th Semester

15MA207 – PROBABILITY AND QUEUING THEORY

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. If c is a constant (non random variable) then E(c) is
   (A) 0 (B) 1
   (C) cf(c) (D) c

2. $Var(4X+8)$ is
   (A) $12Var(X)$ (B) $4Var(X)+8$
   (C) $16Var(X)$ (D) $16Var(X)+8$

3. The expectation of the number on a die when thrown
   (A) 1 (B) 7/2
   (C) 3 (D) 2

4. the $E[X^2]=8$ and $E(X)=2$, then Var(X) is
   (A) 3 (B) 2
   (C) 1 (D) 4

5. The MGF of binomial distribution is
   (A) $(p+q^{et})^n$ (B) $(pe^t+q)^n$
   (C) $(pe^t+qe^{-t})^{-n}$ (D) $(pe^{-t}+q)^n$

6. Mean of the Poisson distribution is
   (A) λ (B) $\lambda+1$
   (C) λ^2 (D) $\lambda-1$

7. If the probability of success on each trial is 1/3. What is the expected no of trials required for the first success
   (A) 2 (B) 3
   (C) 4 (D) 5

8. If X is uniform distributed in (0, 10) then P(X>8) is
   (A) 1/5 (B) 1/10
   (C) 3/5 (D) 1/3

9. The form of the alternative hypothesis can be
   (A) One-tailed (B) Two-tailed
   (C) Neither one nor two-tailed (D) One or two tailed

10. What is the standard deviation of a sampling distribution called?
(A) Sampling error (B) Sample error
(C) Standard error (D) Simple error

11. A failing student is passcd by an cxaminer, it is an example of
(A) Type I error (B) Type II error
(C) Unbiased decision (D) Difficult to tel

12. The degree of freedom for t-test based on n observations is
(A) 2n–1 (B) n–2
(C) 2(n–1) (D) n–1

13. What stands for 'd' in the queue model $(a/b/c:d/e)$
(A) Queue discipline (B) System capacity
(C) Service time (D) Number of servers

14. The probability of no customer in the system in $(M/M/I):(\infty/FIFO)$ model is
(A) λ/μ (B) $\frac{\lambda}{\mu}-1$
(C) $1-\frac{\lambda}{\mu}$ (D) $\frac{\lambda}{\mu}+1$

15. The probability that the number of customers in the system exceeds K, in$(M/M/I):(\infty/FIFO)$ model
(A) $\left(\frac{\lambda}{\mu}\right)^{K+1}$ (B) $\left(\frac{1}{\mu}\right)^{K-2}$
(C) $\left(\frac{\lambda}{\mu}\right)^{K+2}$ (D) $\left(\frac{\lambda}{\mu}\right)^{K}$

16. The average waiting time of a customer in the system in $(M/M/1:\infty/FIFO)$ model _____.
(A) $\frac{1}{\mu-\lambda}$ (B) $\frac{1}{\lambda-\mu}$
(C) $\frac{1}{\lambda+\mu}$ (D) $\frac{1}{\lambda^2}$

17. If P is a tpm of the regular chain, then
(A) $p\pi = \pi+1$ (B) $\pi p = \pi$
(C) $\pi p^2 = \pi$ (D) $\pi p = \pi-1$

18. Ergodic means
(A) Irreducible and periodic (B) Irreducible and aperiodic
(C) Not irreducible (D) Regular

19. In a transition probability matrix, the sum of all elements of any row is
(A) 0 (B) 1
(C) 2 (D) –1

20. If the one step transition probability does not depend on the step, then the Markov chain is
(A) Reducible (B) Regular
(C) Homogeneous (D) Non homogeneous

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A RV X has mean $\mu = 12$ and variance $\sigma^2 = 9$ and an unknown probability distribution. Find $P(6<X<18)$.

22. If a boy is throwing stones at a target, what is the probability that his 10^{th} throw is his 5^{th} hit, if the probability of hitting the target at any trial is 1/2?

23. A salesman in a departmental store claims that atmost 60% of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of salesman? Use 5% LOS.

24. In the usual notation of a $(M/M/I):(\infty/FIFO)$ queue system is $\lambda = 12$ per hour and $\mu = 24$ per hour, find the average number of customers in the system and in the queue.

25. A gambler has ₹2. He bets ₹1 at a time and wins ₹1 with probability 1/2. He stops playing if he loses ₹2 or wins ₹4. What is the tpm of the related Markov chain?

26. Given the random variable X with density function
$f(x)=\begin{cases}2x, & 0<x<1\\ 0, & elsewhere\end{cases}$ find the pdf of $Y = 8X^3$.

27. X is normally distributed and the mean of X is 12 and standard deviation is 4. Find
(i) $P(X \geq 20)$ (ii) $P(X \leq 20)$

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution.

x	–2	–1	0	1	2	3
p(x)	0.1	K	0.2	2K	0.3	3K

(i) Find K (ii) Evaluate P(X < 2) and P(–2 <X < 2) (iii) Find cdf of X (iv) evaluate the mean of X.

(OR)

b. The cdf of a continuous random variable X is given by
$F(x) = 0, x<0$
$= x^2, 0 \leq x < 1/2$
$= 1-\frac{3}{25}(3-x)^2, \frac{1}{2} \leq x < 3$
$= 1, x \geq 3$
Find the pdf of X and evaluate $P(|X| \leq 1)$ *and* $P\left(\frac{1}{3} \leq X < 4\right)$ using both the pdf and cdf.

29. a. Fit a Poisson distribution for the following distribution.

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

(OR)