UNIT 2 Functions of Sellural Variables

1. Total differentiation, Homogeneous function, Euler's theorem, Implicit function
2. Taylor's theorem → 2 variables
3. Maxima and Minima of a functions
4. Lagrange's multipliers (a) method → 2 Variables
5. Jacobians

Introduction

Function $f(x), f(z)$
$f(y)$, $f(x) = x^2+4x-4$

$f(x,y), f(x,y,z)$ $\quad \frac{df}{dx} = 2x+4$

$f(\pi)$

$f(x,y) = x^4+y^4-4x^2y^2-4$

$\frac{\partial f}{\partial x} = 4x^3 - 8xy^2$ $\quad\Big| -4y^2(2x)$

$\frac{\partial f}{\partial y} = 4y^3 - 8x^2y$ $\quad\Big| -4x^2(2y)$

1. $z = x^2+y^2+3xy$

Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \to 2y+3x$

$2x+3y$

2. $u = e^x \sin y \cos z$

Find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$

$\frac{\partial u}{\partial x} = e^x \sin y \cos z$

$\frac{\partial u}{\partial y} = e^x \cos y \cos z$

$\frac{\partial u}{\partial z} = -e^x \sin y \sin z$

Total differential :- $z = f(x,y)$

$\hookrightarrow dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

Total differentiation

$z = f(x,y)$

$x = g(t)$, $y = h(t)$

$$\boxed{\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}}$$

1. Find $\frac{dz}{dt}$ if $z = xy^2+x^2y$ and $x = at^2$
$y = 2at$

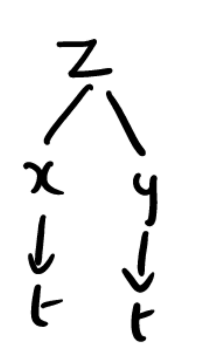
2. Find $\frac{dz}{dt}$ if $z = \sin^{-1}(x-y)$, $x = 3t$
$y = 4t^3$

Ans $\boxed{\frac{dz}{dt} = \frac{3}{\sqrt{1-t^2}}}$

3. Find $\frac{du}{dt}$ if $u = x^3+y^3$, $x = a\cos t$
$y = b\sin t$

4 marks

1. Solution



(1m) $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

(1m) $= (y^2+2xy)(2at) + (2xy+x^2)(2a)$

(2m) $\boxed{\frac{dz}{dt} = 16a^3t^3 + 10a^3t^4}$

3. $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$

$= 3x^2(-a\sin t) + 3y^2(b\cos t)$

$\frac{du}{dt} = 3b^3\sin^2 t\cos t - 3a^3\cos^2 t\sin t$

Formula $\sin^{-1}x \Rightarrow \frac{1}{\sqrt{1-x^2}}$