ii. Solve $z = px + qy + p^2q^2$.

**(OR)**

b.i. Solve $\left(D^3 - 3D^2D' + 4D'^3\right)z = e^{x+2y}$.

ii. Find the general solution of $p\tan x + q\tan y = \tan z$.

29. a. Expand $f(z) = x^2$ when $-\pi < x < \pi$ in a fourier series of periodicity $2\pi$. Hence deduce that, $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\infty = \frac{\pi^2}{6}$; $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\infty = \frac{\pi^2}{12}$.

**(OR)**

b. Compute the first two harmonics of the fourier series of f(x) given by the following table.

| x | 0 | π/3 | 2π/3 | π | 4π/3 | 5π/3 | 2π |
|---|---|---|---|---|---|---|---|
| f(x) | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

30. a. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y$ at any time and at any distance from the end $x = 0$.

**(OR)**

b. A rod, 30cm, long has its end A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then reduced to 0°C and kept so. Find the resulting temperature function u(x, t) taken $x = 0$ at A.

31. a. Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence find the values of $\int_0^\infty \frac{\sin^4 t}{t^4} dt$.

**(OR)**

b.i. Find the Fourier sine transform of $\frac{x}{x^2 + a^2}$ and fourier cosine transform of $\frac{1}{a^2 + x^2}$..

ii. Using Parseval's identify evaluate $\int_0^\infty \frac{dx}{\left(a^2 + x^2\right)^2}$.

32. a.i. Find $z[\cos\omega t]$.

ii. Find the inverse z-transform of $\frac{z^2}{(z-a)^2}$ using covolution theorem.

**(OR)**

b. Solve $y_{n+2} - 4y_{n+1} + 4y_n = 0$ given $y_0 = 1$ *and* $y_1 = 0$.

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**B.Tech. DEGREE EXAMINATION, MAY 2019**
Third Semester

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
*(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)*

**Note:**
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

**PART – A (20 × 1 = 20 Marks)**
Answer **ALL** Questions

1. The complete integral of p = q is
   (A) $z = ax + by$ (B) $z = a(x+y) + b$
   (C) $z = ax + by + c$ (D) $z = ax - by + a$

2. The complete solution of $z = px + qy + p^2 - q^2$ is
   (A) $ax + by + a^2 - b^2$ (B) $ax - by + a^2b^2$
   (C) $ax + by + a^2b^2$ (D) $ax + by + a^2 + b^2$

3. Solve $\left(D^3 - 2D^2D'\right)z = 0$
   (A) $z = \phi_1(y) + x\,\phi_2(y) + \phi_3(y - 2x)$ (B) $z = \phi_1(y) + \phi_2(y) + x\phi_3(y + 2x)$
   (C) $z = \phi_1(y) + x\,\phi_2(y) + \phi_3(y + 2x)$ (D) $z = \phi_1(y) - \phi_2(y) - \phi_3(y + 2x)$

4. Find particular integral of $\left(D^3 - 2D^2D'\right)z = e^{x+2y}$
   (A) $PI = \frac{-1}{3}e^{x+2y}$ (B) $PI = \frac{1}{3}e^{x+2y}$
   (C) $PI = \frac{1}{3}e^{x-2y}$ (D) $PI = \frac{-1}{3}e^{2x+y}$

5. The partial differential equaltion $xf_{xx} + yf_{yy} = 0,\ x > 0, y > 0$
   (A) Hyperbolic (B) Elliptic
   (C) Parabolic (D) None of these

6. $\cos x$ is a periodic function with period
   (A) π (B) π/2
   (C) 2π (D) 4π

7. Which one of the following function is an even function?
   (A) *sinx* (B) *x*
   (C) $e^x$ (D) $x^2$

8. Half range sine series for f(x) in (0, $\pi$) is
(A) $\sum_{n=1}^{\infty} a_n \cos nx$
(B) $\frac{a_0}{2} + \sum a_n \cos nx$
(C) $\sum_{n=1}^{\infty} b_n \sin nx$
(D) $\frac{a_0}{2} - \sum a_n \cos nx$

9. The proper solution of the problems on vibration of string is
(A) $y(x, t) = \left(Ae^{\lambda x} + Be^{-\lambda x}\right)\left(Ce^{\lambda at} + De^{-\lambda at}\right)$
(B) $y(x, t) = (Ax + B)(Ct + D)$
(C) $y(x, t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$
(D) $y(x,t) = (Ax + B)$

10. One dimensional wave equation used to find
(A) Temperature
(B) Displacement
(C) Time
(D) Mass

11. The proper solution of $u_t = \alpha^2 u_{xx}$ is
(A) $u = (Ax + B)e$
(B) $u = (A\cos\lambda x + B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$
(C) $u = \left(Ae^{\lambda x} + Be^{-\lambda x}\right)e^{\alpha^2\lambda^2 t}$
(D) $u = At + B$

12. Heat flows from ______temperature.
(A) Higher to lower
(B) Uniform
(C) Lower to higher
(D) Stable

13. The Fourier transform of $f(x) = e^{-x^2/2}$ is
(A) $e^{-s^2}$
(B) $e^{-s^2/2}$
(C) $e^{s^2}$
(D) $e^{s^2/3}$

14. If $F(f(x)) = F(s)$, then $F(f(ax))$ is
(A) $\frac{1}{|a|}F\left(\frac{s}{a}\right),\ a \neq 0$
(B) $\frac{1}{|a|}F\left(\frac{a}{s}\right),\ a \neq 0$
(C) $\frac{1}{s}F\left(\frac{as}{x}\right),\ s \neq 0$
(D) $F\left(\frac{a}{s}\right)$

15. $F[f(x) * g(x)] =$
(A) $F(s) + G(s)$
(B) $F(s) - G(s)$
(C) $F(s)G(s)$
(D) $\frac{F(s)}{G(s)}$

16. If $F[f(x)] = F(s)$ then $F[f(x)\cos ax]$
(A) $[F(a) + F(s-a)]/2$
(B) $[F(sa) + F(s+a)]/2$
(C) $\frac{1}{2}[F(s-a) + F(s+a)]$
(D) $F(s-a) - F(s+a)$

17. $z\left[(-1)^n\right]$
(A) $\frac{z+1}{z}$
(B) $\frac{z}{z-1}$
(C) $\frac{z}{z+1}$
(D) $\frac{-z}{z+1}$

18. $z\left[\cos\frac{n\pi}{2}\right]$ is
(A) $\frac{z}{z^2+1}$
(B) $\frac{z^2}{z^2+1}$
(C) $\frac{z}{z^2-1}$
(D) $\frac{z^2}{z^2-4}$

19. $z^{-1}\left[\frac{az}{(z-a)^2}\right]$ is
(A) $a^{n-1}$
(B) $n\,a^{n+1}$
(C) $na^{n-1}$
(D) $na^n$

20. $z[f(n) \times g(n)] =$ is
(A) $F(z)G^{-1}(z)$
(B) $F^{-1}(z)G^{-1}(z)$
(C) $F(z)G(z)$
(D) $F^{-1}(z)G(z)$

**PART – B (5 × 4 = 20 Marks)**
Answer **ANY FIVE** Questions

21. Solve $\sqrt{p} + \sqrt{q} = 1$.

22. Find $a_n$ for the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$.

23. Write down any four assumptions made in deriving one dimensional wave equation.

24. State and prove change of scale property in fourier transform.

25. Find z(t).

26. Find the Fourier sine transform of $f(x) = e^{-x}$.

27. Find $F_c\left(e^{-ax}\right)$.

**PART – C (5 × 12 = 60 Marks)**
Answer **ALL** Questions

28. a.i. Form the partial differential equation by eliminating f from $z = xy + f\left(x^2 + y^2 + z^2\right)$.