SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB302T- DISCRETE MATHEMATICS

Year/Sem: III/V

Branch: CSE,ECE,EEE

UNIT-4 -GROUP THRORY

1. Let $(G,*)$ be the group then for each $a, b \in G$ the value of $(a * b)^{-1}$ is
   (a) $(ab)^{-1}$ (b) $a^{-1}b^{-1}$ (c) $a^{-1} + b^{-1}$ (d) $b^{-1} * a^{-1}$ **Ans: d**

   Solution: $(a * b) * (b^{-1} * a^{-1}) = a * (b * b^{-1}) * a^{-1}$ (By Associative Law)

   $= a * e * a^{-1}$ (By Inverse Law)

   $= a * \ a^{-1}$ (By Identity Law)

   $= \ e$

   Hence Inverse of $a * b$ is $b^{-1} * a^{-1} \implies (a * b)^{-1} = b^{-1} * a^{-1}$

2. Let $G = \{1, -1\}$ then under usual multiplication $(G, .)$ is

   (a) Group (b) Sub Group (c) Cyclic Group (d) Not a Group **Ans: a**

   Solution: Cayley Table of G is

•	1	-1
1	1	-1
-1	-1	1

   From the above table G satisfies Closure law, since multiplication is associative in any number set, it is true here also. Hence associative axiom is satisfied. 1 is the Identity element. Inverse of 1 is 1 and Inverse of -1 is -1. Hence $(G, .)$ is a group.

3. Let $(G,*)$ be the set of all non-zero real numbers defined by the binary operator $a * b = \frac{ab}{2}, \forall a, b \in G$ & G is Abelian Group. Then Identity element e of G is
   (b) 4 (b) 2 (c) 1 (d) 0 **Ans: b**

Solution: $a * e = a \qquad \forall\, a \in G$

$\frac{ae}{2} = a \;\Rightarrow\; e = 2$

4. The fourth root of unity $\{1, -1, i, -i\}$ where $\sqrt{-1} = i$ forms an Abelian group under multiplication. Then Inverse of -1 and i are
   (a) $-1, i$ (b) 1,1 (c) $i, -i$ (d) $-1, -i$ **Ans: d**

   Solution: Cayley Table

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

From the above table 1 is the Identity element.

$-1 \times -1 = 1 = e$ & $i \times -i = -i^2 = 1 = e$

Inverse of $-1, i$ are $-1, -i$

5. Which of the following statements are true?
   I. Identity element of a group is unique and Inverse of each element is finite.
   II. Identity element of a group is unique and Inverse of each element is same.
   III. Identity element of a group is unique and Inverse of each element is unique.
   IV. Identity element and Inverse element are equal for any group.
   (a) I,II (b) III (c) IV (d) III&IV **Ans: b**

   Solution: From Properties of Group.

6. An algebraic structure __________ is called a semigroup.
   a) $(P,*)$ b) $(Q,+,*)$ c) $(P,+)$ d) $(+,*)$ **Ans: a**

   Solution: An algebraic structure $(P,*)$ is called a semigroup if $a * (b * c) = (a * b) * c$ for all $a, b, c \in S$ or the elements follow associative property under $*$.

7. A cyclic group is always __________
   a) abelian group b) monoid c) semigroup d) subgroup **Ans: a**

Solution: A cyclic group is always an abelian group but every abelian group is not a cyclic group. For instance, the rational numbers under addition is an abelian group but is not a cyclic one.

8. If $x * y = x + y + xy$ then $(G,*)$ is ______________

a) Monoid b) Abelian group c) Commutative semigroup d) Cyclic group

Ans: c

Solution: Let x and y belongs to a group G.Here closure and associativity axiom holds simultaneously. Let e be an element in G such that $x * e = x$ then
$x + e + xe = a \Rightarrow e(1 + x) = 0 \Rightarrow e = 0/(1 + x) = 0.$
So, identity axiom does not exist but commutative property holds. Thus, $(G,*)$ is a commutative semigroup.

9. From the group $G = [\{0,1,2,3,4\}, +_5]$, order of the element 4 is
(a) 0 (b) 1 (c) 3 (d) 5 **Ans: d**

Solution: Identity element of G is e =0

$$O(0) = 1, O(1) = O(2) = O(3) = O(4) = O(5) = 5$$

10. From the Multiplicative group $G = \{1, \omega, \omega^2\}$ & $\omega^3 = 1$ the order of ω^2 is
(b) 1 (b) 2 (c) 3 (d) 0 **Ans: c**

Solution: Identity element of G is e =1

$$O(\omega^2) = (\omega^2)^3 = 1 = e \text{ Hence } O(\omega^2) = 3$$

11. If $(M,*)$ is a cyclic group of order 73, then number of generator of G is equal to ______
a) 89 b) 23 c) 72 d) 17 **Ans: c**

Solution: We need to find the number of co-primes of 73 which are less than 73. As 73 itself is a prime, all the numbers less than that are co-prime to it and it makes a group of order 72 then it can be of {1, 3, 5, 7, 11….}.

12. The dihedral group having order 6 can have degree ______________
a) 3 b) 26 c) 326 d) 208 **Ans: a**

Solution: A symmetric group on a set of three elements is said to be the group of all permutations of a three-element set. It is a dihedral group of order six having degree three.

13. Suppose (2, 5, 8, 4) and (3, 6) are the two permutation groups that form cycles. What type of permutation is this?
a) odd b) even c) acyclic d) prime **Ans: b**

Solution: There are four permutations (2, 5), (2, 8), (2, 4) and (3, 6) and so it is an even permutation.

14. Invariant permutations of two functions can form ___________
a) groups b) lattices c) graphs d) rings **Ans: a**

Solution: Suppose, there are two functions f_1 and f_2 which belong to the same equivalence class since there exists an invariant permutation say, π(a permutation that does not change the object itself, but only its representation), such that: $f2 * \pi \equiv f1$. So, invariant permutations can form a group, as the product (composition) of invariant permutations is again an invariant permutation.

15. The transpositions of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix}$ are
(a) (1 6) (2 5) (2 3) (b) (1 6) (1 5) (1 3)
(c) (2 5) (2 3) (2 6) (d) (1 3) (1 5) (1 6) **Ans: a**

Solution: f = (1 6) (2 5 3) (4) (7)

= (1 6) (2 5 3)

= (1 6) (2 5) (2 3)

16. Non-trivial subgroups of $(Z_6, +_6)$ are
(a) {[0], [3]} , {[2], [4]}
(b) {[0], [3]} , {[0], [2], [4]}
(c) {[0], [3]} , {[2], [4]}, {[1], [4]}
(d) {[1], [0], [3]} , {[2], [4]}, {[1], [4]} **Ans: b**

Solution:

$+_6$	[0]	[3]
[0]	[0]	[3]
[3]	[3]	[0]

$+_6$	[0]	[2]	[4]
[0]	[0]	[2]	[4]
[2]	[2]	[4]	[0]
[4]	[4]	[0]	[2]

Both are closed under $+_6$. Hence they are subgroups.

17. Let G be a finite group with two sub groups M & N such that $|M| = 56$ and $|N| = 123$. Determine the value of $|M \cap N|$.
a) 1 b) 56 c) 14 d) 78

Solution: We know that gcd(56, 123)=1. So, the value of |M⋂N|=1

18. Let K be a group with 8 elements. Let H be a subgroup of K and H<K. It is known that the size of H is at least 3. The size of H is ___________
a) 8 b) 2 c) 3 d) 4 **Ans: d**

Solution: For any finite group G, the order (number of elements) of every subgroup L of G divides the order of G. G has 8 elements. Factors of 8 are 1, 2, 4 and 8. Since given the size of L is at least 3(1 and 2 eliminated) and not equal to G(8 eliminated), the only size left is 4. Size of L is 4.

19. A function is defined by $f(x) = 2x$ and $f(x + y) = f(x) + f(y)$ is called ______________
a) isomorphic b) homomorphic c) cyclic group d) heteromorphic

Ans: a

Let $(G,*)$ and $(G', +)$ are two groups. The mapping f: $G \rightarrow G'$ is said to be isomorphism if two conditions are satisfied 1) f is one-to-one function and onto function and 2) f satisfies homomorphism.

20. How many different non-isomorphic Abelian groups of order 8 are there?

a) 5 b) 4 c) 2 d) 3 **Ans: c**

Solution: The number of Abelian groups of order P^m (let, P is prime) is the number of partitions of m. Here order is 8 i.e. 2^3 and so partition of 3 are {1, 1} and {3, 0}. So number of different abelian groups are 2.

21. Let $(Z,\oplus,\odot)$ be the set of Integers with Binary operators defined by $a \oplus b = a + b - 1$, $a \odot b = a + b - ab, \forall a, b \in Z$. Then Z is

(a) Commutative Ring with Identity

(b) Non Commutative Ring

(c) Commutative Ring without Identity

(d) Not a Ring **Ans: a**

Solution:

$(Z,\oplus)$ is Abelian Group,

$(Z,\oplus,\odot)$ Satisfies the properties of Ring along with Identity and commutative.

22. If $Q(\sqrt{2}) = \{a + b\sqrt{2}) : \quad a, b \ \in Q\}$ is a Field with respect to addition and Multiplication. Then the Inverse of each element of Q with respect to Addition is

(a) $\sqrt{2}$ (b) $a - \sqrt{2}$ (c) $a - b\sqrt{2}$ (d) $-a - b\sqrt{2}$ **Ans:d**

Solution: Since $(Q\sqrt{2}, +)$ is an abelian group , $e = 0 + 0\sqrt{2}$

$$\left(a + b\sqrt{2}\right) + \ \left(a + b\sqrt{2}\right)^{-1} = 0 + 0\sqrt{2}$$

$$\left(a + b\sqrt{2}\right)^{-1} = -a - b\sqrt{2}, \forall a, b \in Q\sqrt{2}$$

23. Let $(Z, +, \ .)$ and $(2Z, +, \ .)$ be two Rings. $f: Z \rightarrow 2Z$ given by $f(x) = 2x, \forall x \epsilon Z$ is

(a) Ring Homomorphism

(b) Group Homomorphism

(c) Not a Ring Homomorphism

(d) Group Isomorphism **Ans: c**

Solution: $f(x). f(y) = 2x. 2y = 4xy \neq f(xy)$

24. The only Idempotent elements of an Integral Domain are

(a) 0 &1 (b) 0&2 (c) 1&2 (d) 1&3 **Ans: a**

Solution: Let $(R, +, .)$ be an Integral domain. Let $a \in R$ be an Idempotent element

Then $a^2 = a \Longrightarrow a^2 - a = 0$

$\Longrightarrow a(a-1) = 0$

Since R has no Zero Divisors $a = 0$ & 1 only.

25. If $x = 11010$ and $y = 10101$ then $H(x, y)$ is
(a) 6 (b) 4 (c) 3 (d) 5 **Ans: b**

Solution: $H(x, y) = |x \oplus y| = |01111| = 4 = \text{No of Positions in the Strings}$

26. If the message $w \in B^2$ and let $G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ then $e(1\ 1)$ is

(a) (0 0 0 0 0) (b) (1 0 1 1 0) (c) (0 1 0 1 1) (d) (1 1 1 0 1) **Ans: d**

Solution : $e(1\ 1) = (1\ 1)\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} = (1\ 1\ 1\ 0\ 1)$

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