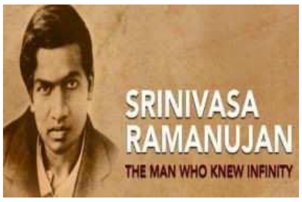
SRM Institute of Science and Technology
Kattankulathur

DEPARTMENT OF MATHEMATICS

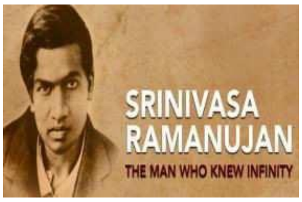
18MAB101T Calculus and Linear Algebra

UNIT –I Matrices



Sl.No.	Tutorial Sheet -1	Answers
	Part – A	
1	**If A=** $\begin{pmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$ **, find the eigenvalues of (i) A (ii) A^{-1} (iii) adj A (iv) A^3**	**(i) 3,4,1 (ii) 1/3, ¼, 1 (iii) 12, 4, 3 (iv) 27, 64, 1**
2	**Two of the eigenvalues of A=** $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ **are 1 and 2. Find the eigenvalues of A^2.**	**1, 4, 9**
3	**Find the sum and product of the eigenvalues of the matrix A=** $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$	**-1, 45**
4	**Find the eigenvalues and eigenvectors of the matrix A=** $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	**1, 5** $\begin{pmatrix} 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
5	**Find the characteristic equation of A=** $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$	$\lambda^3-11\lambda^2+36\lambda-36=0$
	Part – B	
6	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	**2, 3, 5** $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$
7	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}$	**1, 1, 7** $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
8	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{pmatrix}$	**-1, -1, -1** $\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ -7 \end{pmatrix}, \begin{pmatrix} 6 \\ 7 \\ 0 \end{pmatrix}$

9	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	**0, 3, 15** $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$
10	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$	**8, 2, 2** $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

	SRM Institute of Science and Technology Kattankulathur **DEPARTMENT OF MATHEMATICS** **18MAB101T Calculus and Linear Algebra** **UNIT –I Matrices**	
Sl.No.	**Tutorial Sheet -2**	**Answers**
	Part – A	
1	Verify Cayley Hamilton theorem and find A^4 when A=$\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$.	$A^4=\begin{bmatrix}25 & 0\\0 & 25\end{bmatrix}$
2	Two eigen values of the matrix A=$\begin{bmatrix}2 & 2 & 1\\1 & 3 & 1\\1 & 2 & 2\end{bmatrix}$ are equal to 1 each. Find the eigen values of A^{-1}	A= 1 , 1, 5 $A^{-1}=1,1,1/5$
3	The matrix A is $\begin{bmatrix}-1 & 0 & 0\\2 & -3 & 0\\1 & 4 & 2\end{bmatrix}$. Find the eigen values of A^2	A= -1 , 3, 2 A^2 = 1 , 9 , 4
4	Verify Cayley Hamilton theorem and find A^{-1} when A=$\begin{bmatrix}3 & 1 & 1\\-1 & 5 & -1\\1 & -1 & 3\end{bmatrix}$.	$A^{-1}=1/20\begin{bmatrix}7 & -2 & -3\\1 & 4 & 1\\-2 & 2 & 8\end{bmatrix}$
5	Verify Cayley Hamilton theorem and find A^{-1} when A=$\begin{bmatrix}1 & -1 & 1\\0 & 1 & 0\\2 & 0 & 3\end{bmatrix}$	$A^{-1}=\begin{bmatrix}3 & 3 & -1\\0 & 1 & 0\\-2 & -2 & 1\end{bmatrix}$
6	Obtain the matrix A^6-25A^2+122A where A=$\begin{bmatrix}0 & 0 & 2\\2 & 1 & 0\\-1 & -1 & 3\end{bmatrix}$.	**Ans** : $\begin{bmatrix}-34 & 0 & -20\\-20 & -54 & 0\\10 & 10 & -74\end{bmatrix}$
7	If A =$\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$, Prove that $A^3-3A^2-9A-5I=0$. Hence find A^4 and A^{-1}.	$A^4=\begin{bmatrix}209 & 208 & 208\\208 & 209 & 208\\208 & 208 & 209\end{bmatrix}$
8	Diagonalise the matrix A=$\begin{bmatrix}2 & 1 & -1\\1 & 1 & -2\\-1 & -2 & 1\end{bmatrix}$ when	$A^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 4\end{bmatrix}$

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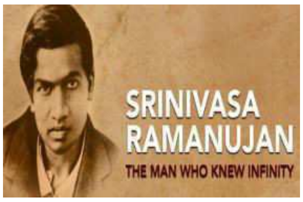
18MAB101T Calculus and Linear Algebra

UNIT –I Matrices



Sl.No.	Tutorial Sheet -3	Answers
	Part – A	
1	Write the Quadratic form $Q=x^2-2y^2+3z^2-4xz+5yz+6xz$ as product of matrices.	$Q=X^TAX$ where $X^T=[x\ y\ z]$ $A=\begin{pmatrix}1 & -2 & 3\\ -2 & -2 & 5/2\\ 3 & 5/2 & 3\end{pmatrix}$
2	Write the Q.F where $A=\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 9\\ 3 & 9 & 3\end{pmatrix}$	$x^2+4y^2+3z^2+4xy+18yz+6xz$
3	Determine the nature of the quadratic form (i) $6x^2+3y^2+14z^2+4yz+18xz+4xy$ (ii) $2xy+2yz-2xz$ without reducing into canonical form.	(i) $D_1=6$, $D_2=14$, $D_3=$ -ve Q.F is indefinite. (ii) $D_1=0$, $D_2=-1$, $D_3=-2$ Q.F is indefinite.
	Part – B	
4	Reduce the quadratic form $Q=3x^2+5y^2+3z^2-2xy-2yz+2xz$ to canonical form and hence find its nature, rank, index and signature.	$A=\begin{pmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{pmatrix}$ $\lambda^3-11\lambda^2+36\lambda-36=0$ $\lambda=2,3,6$ $Q=2y_1^2+3y_2^2+6y_3^2$ nature=positive definite index=3 signature=3 rank=3
5	Reduce the quadratic form $Q=x_1^2+2x_2x_3$ to canonical form and hence find its nature, rank, index and signature.	$\lambda^3-\lambda^2-\lambda+1=0$ $\lambda=1,1,-1$ $Q=y_1^2+y_2^2-y_3^2$ nature=indefinite index=2 signature=1 rank=3
6	Reduce the quadratic form $Q=x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$ to canonical form and hence find its nature, rank, index and signature.	$\lambda^3-4\lambda^2+3\lambda=0$ $\lambda=0,1,3$ $Q=0y_1^2+y_2^2+3y_3^2$ nature=positive semi definite index=2 signature=2 rank=2

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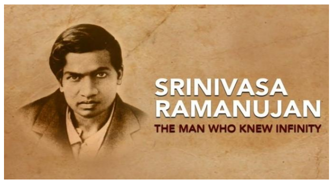
18MAB101T CALCULUS & LINEAR ALGEBRA
UNIT-2 Functions of Several Variables



Sl.No.	Tutorial Sheet-1	Answers
	PART – B	
1	If $u = x^2y^3,\ x = \log t,\ y = e^t$, find $\frac{du}{dt}$	$\frac{e^{3t}\log t(2+3t\log t)}{t}$
2	If $u = \log\left(x^3+y^3+z^3-3xyz\right)$ then prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$	
3	If $z = f(x+ct)+\phi(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$	
4	If $f(x,y) = x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, verify $f_{xy} = f_{yx}$.	$f_{xy} = \frac{x^2-y^2}{x^2+y^2}$
5	Obtain the Maclaurin's series of $e^x\cos y$.	$1+x+\frac{1}{2}(x^2-y^2)+$ $+\frac{1}{6}(x^3-3xy^2)+\ldots$
	PART – C	
6	If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z} = 0$.	
7	Using Taylor's series, verify that, $\log(1+x+y) = (x+y)-\frac{1}{2}(x+y)^2+\frac{1}{3}(x+y)^3\ldots.$	
8	**Let $\phi = \phi(u,v)$ where $u = e^x\cos y$ and $v = e^x\sin y$, show that $v\frac{\partial\phi}{\partial x}+u\frac{\partial\phi}{\partial y} = (u^2+v^2)\frac{\partial\phi}{\partial v}$.**	
9	Find the expansion for $f(x,y) = \tan^{-1}(xy)$ and hence compute the value of $f(0.9,-1.2)$. Hint.: Use the point (1,-1) for the expansion.	**-0.8229**
10	Expand $e^x\sin y$ in power of x and y near the point $\left(-1,\frac{\pi}{4}\right)$ as far as the terms of the third degree. Ans.: $\frac{1}{e\sqrt{2}}\left\{1+(x+1)+\left(y-\frac{\pi}{4}\right)+\frac{1}{2}\left[(x+1)^2+2(x+1)\left(y-\frac{\pi}{4}\right)-\left(y-\frac{\pi}{4}\right)^2\right]+\ldots\right\}$	

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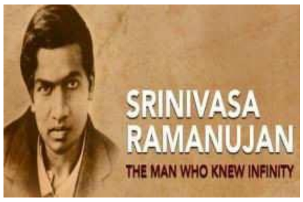
18MAB101T Calculus and Linear Algebra

UNIT –II

Functions of Several Variables



Sl.No.	Tutorial Sheet -2	Answers
1	Find the extreme values of a function $x^2+y^2+6x+12$	(-3, 0) is the stationary point, minimum value = 3
2	Find the maxima and minima of the function $x^3+3xy^2-15x^2-15y^2+72x$	Max. value is 112 ,when x = 1, y = 2
3	Find the dimensions of the rectangular box, open at the top of maximum capacity whose surface is 432sq.cm.	X = 12, y = 12 and z = 6.
4	A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.	Hence s is minimum when $x = y = (2V)^{1/3} = 2z$.
5	Find the minimum value of xy^2z^3 subject to $x + y + z = 24$	The extreme points are (4, 8, 12) and the minimum value is $4\times8^2\times12^3$.
6	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	$V=\frac{8abc}{3\sqrt{3}}$.
7	Find the minimum value of $x^2 + y^2 + z^2$ given that $ax + by + cz = p$.	$f=\frac{p^2}{a^2+b^2+c^2}$
8	Identify the saddle point and extreme points of $f(x, y) = x^4 – y^4 - 2x^2 + 2y^2$	(i) The points (0, 1), (0, -1) are maximum point. (ii) The points $(\pm1, 0)$ are minimum point. (iii) The points $(\pm1,\pm1)$ are saddle points.

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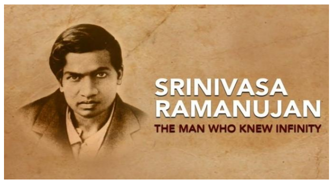
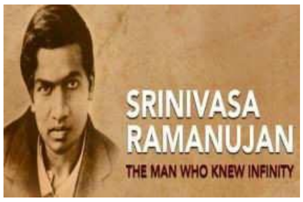
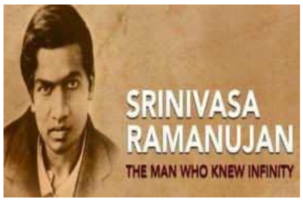
18MAB101T CALCULUS & LINEAR ALGEBRA
UNIT-2 Functions of Several Variables



Sl.No.	Tutorial Sheet-3	Answers
	PART – B	
1	If $x = u(1-v), y = uv$ verify that $\dfrac{\partial(x,y)}{\partial(u,v)} \cdot \dfrac{\partial(u,v)}{\partial(x,y)} = 1$.	
2	If $u = x^2, v = y^2$ find $J = \dfrac{\partial(u,v)}{\partial(x,y)}$.	$J = 4xy$
3	If $x = r\cos\theta, y = r\sin\theta, z = z$ find $\dfrac{\partial(x,y,z)}{\partial(r,\theta,z)}$.	$J = r$
4	If $u = xyz, v = xy + yz + zx, w = x + y + z$ find $J = \left(\dfrac{\partial(u,v,w)}{\partial(x,y,z)}\right)$.	$J = (x-y)(y-z)(z-x)$
5	The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.	$50°C$
	PART – C	
6	If $x = r\cos\theta, y = r\sin\theta$ verify that $\dfrac{\partial(x,y)}{\partial(r,\theta)} \cdot \dfrac{\partial(r,\theta)}{\partial(x,y)} = 1$.	
7	If we transform from 3D-Cartesian co-ordinates (x,y,z) to spherical polar co-ordinates (r,θ,ϕ) show that $\dfrac{\partial(x,y,z)}{\partial(r,\theta,z)} = r^2\sin\theta$.	
8	Find the $J = \left(\dfrac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)}\right)$ if $y_1 = \dfrac{x_2x_3}{x_1}$; $y_2 = \dfrac{x_1x_3}{x_2}$; $y_3 = \dfrac{x_2x_1}{x_3}$.	$J = 4$
9	Examine the functional dependence of the functions $u = y + z$; $v = x + 2z^2$; $w = x - 4yz - 2y^2$. If so find the relationship.	$J = 0,\ v - w = 2u^2$
10	Find the shortest and longest distance from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$, using Lagrange's method of constrained maxima and minima.	$\sqrt{6}$ and $3\sqrt{6}$

	SRM Institute of Science and Technology Kattankulathur	
	DEPARTMENT OF MATHEMATICS	
	18MAB101T Calculus and Linear Algebra	
	UNIT –3 Ordinary Differential Equations	
Sl.No.	**Tutorial Sheet -1**	**Answers**
	Part – A	
1	**Solve** $\left(D^2-7D+12\right)y=0$	$y=Ae^{3x}+Be^{4x}$
2	**Solve** $\left(D^2-2D+4\right)y=0$	$y=\left(Ax+B\right)e^{2x}$
3	**Solve** $\left(3D^2+D-14\right)y=0$	$y=Ae^{-(7/3)x}+Be^{2x}$
4	**Solve** $\left(D^2+2D+5\right)y=0$	$y=e^{-x}(A\cos 2x+B\sin 2x)$
5	**Solve** $\left(D^2+16\right)y=0$	$y=(A\cos 4x+B\sin 4x)$
6	**Solve** $\left(D^2+2D+2\right)y=e^{-2x}+\cos 2x$	$y=e^{-x}(A\cos x+B\sin x)+\frac{1}{2}e^{-2x}+\frac{1}{5}\sin 2x-\frac{1}{10}\cos 2x$
7	**Solve** $\left(D^2-5D+6\right)y=x^2+3x-1$	$y=Ae^{2x}+Be^{3x}+\frac{1}{6}\left[x^2+\frac{14}{3}x+\frac{26}{9}\right]$
8	**Solve** $\left(D^2+D+1\right)y=x^2e^{-x}$	$y=e^{-\frac{1}{2}x}(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x)+e^{-x}\left(x^2+2x\right)$
9	**Solve** $\left(D^2+4\right)y=x\sin x$	$y=(A\cos 2x+B\sin 2x)+\frac{x}{3}\sin x-\frac{2}{9}\cos x$
10	$\left(D^2-2D+1\right)y=e^x\sin x$	$y=\left(Ax+B\right)e^x-e^x\sin x$

	SRM Institute of Science and Technology Kattankulathur	
	DEPARTMENT OF MATHEMATICS	
	18MAB101T Calculus and Linear Algebra	
	UNIT –III – Ordinary Differential Equations	
Sl.No.	**Tutorial Sheet -2**	**Answers**
1	**Solve** $\left(x^2D^2 - xD + 1\right)y = 0$	$y = x(A\log x + B)$
2	**Solve** $\left(x^2D^2 + 4xD + 2\right)y = 0$	$y = \dfrac{A}{x} + \dfrac{B}{x^2}$
3	**Solve** $\left(x^2D^2 + 1\right)y = 0$	$y = \sqrt{x}\left[A\cos\left(\dfrac{\sqrt{3}}{2}\log x\right) + B\sin\left(\dfrac{\sqrt{3}}{2}\log x\right)\right]$
4	**Solve** $\left((x+2)^2D^2 + 4(x+2)D + 1\right)y = 0$	$y = (A\log(x+2) + B)(x+2)$
5	**Solve** $\left((2x+1)^2D^2 - 2(2x+1)D - 12\right)y = 6x + 5$	$y = A(2x+1)^3 + \dfrac{B}{2x+1} - \dfrac{3(2x+1)}{16} - \dfrac{1}{6}$
6	**Solve** $\left(x^2D^2 + xD - 9\right)y = \dfrac{5}{x^2}$	$y = Ax^3 + \dfrac{B}{x^3} - \dfrac{1}{x^2}$
7	**Solve** $\left(x^2D^2 + xD + 1\right)y = 4\sin\left(\log x\right)$	$y = (A\cos\left(\log x\right)x + B\sin\left(\log x\right))$ $-2\log x\left(\cos\left(\log x\right)\right)$
8	**Solve** $\left(x^2D^2 - 4xD + 6\right)y = x^2 + \log x$	$y = (Ax^2 + Bx^3) - x^2\log x$ $+\dfrac{\log x}{6} + \dfrac{5}{36}$
9	**Solve** $\left(x^2D^2 - xD + 1\right)y = \dfrac{\log x}{x}$	$y = x(A\log x + B) +$ $\dfrac{1}{27x^2}\left[3\left(\log x\right)^2 + 4\left(\log x\right) + 2\right]$

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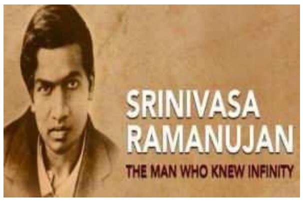
UNIT –III – Ordinary Differential Equations



	Tutorial Sheet -3	Answers
1	**Solve** $\left(D^2+a^2\right)y=\tan ax$ **by the method of variation of parameter**	$y=(c_1\cos ax+c_2\sin ax)$ $-\frac{1}{a^2}\cos ax\log\left[\sec ax+\tan ax\right]$
2	**Solve** $\left(D^2+1\right)y=\sec ax$ **by the method of variation of parameter**	$y=(c_1\cos x+c_2\sin x)$ $-\cos x\log\left(\cos x\right)+x\sin x$
3	**Solve** $\left(D^2+1\right)y=\cos ecx$ **by the method of variation of parameter**	$y=(c_1\cos x+c_2\sin x)$ $+\sin x\log\left(\sin x\right)-x\cos x$
4	**Solve** $\left(D^2+2D+5\right)y=e^{-x}\tan x$ **by the method of variation of parameter**	$y=e^{-x}(c_1\cos 2x+c_2\sin 2x)$ $+\left[-\frac{1}{2}x+\frac{\sin 2x}{4}\right]e^{-x}\cos 2x$ $+\left[-\frac{-\cos 2x}{2}+\frac{1}{2}\log\left(\cos x\right)\right]e^{-x}\sin 2x$
5	**Solve** $\frac{dx}{dt}-y=0;\frac{dy}{dt}+x=0$	$x=A\cos t+B\sin t$ $y=-A\sin t+B\cos t$
6	**Solve** $\frac{dx}{dt}+y=e^t;x-\frac{dy}{dt}=t$	$x=-A\sin t+B\cos t+\frac{1}{2}e^t+t$ $y=A\cos t+B\sin t+\frac{1}{2}e^t-1$
7	$\frac{dx}{dt}+2x-3y=t;\frac{dy}{dt}-3x+2y=e^{2t}$	$x=Ae^t-Be^{-5t}-\frac{2}{5}t+\frac{3}{7}e^{2t}-\frac{13}{25}$ $x=Ae^t+Be^{-5t}-\frac{3}{5}t+\frac{4}{7}e^{2t}-\frac{12}{25}$

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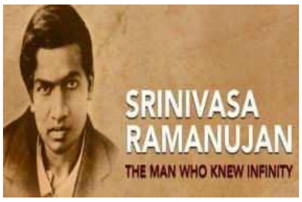
UNIT - IV



	Tutorial Sheet -1	Answers
1.	Find the radius of the curve $y=e^x$ at (0, 1)	$\rho = 2\sqrt{2}$
2.	Find the radius of curvature at the point $\left(\frac{1}{4},\frac{1}{4}\right)$ on the curve $\sqrt{x}+\sqrt{y}=1$.	$\rho = 1/\sqrt{2}$
3.	Show that the radius of curvature at any point of the catenary $y=c\cosh(x/c)$ is y^2/c. Also find ρ at (0, c).	$\rho = C$
4.	Find the radius of curvature at the point (c, c) on the curve $xy=c^2$	$\rho = c\sqrt{2}$
5.	Find ρ at any point $P(at^2, 2at)$ on the parabola $y^2 = 4ax$.	$\rho = 2a(1+t^2)^{3/2}$
6.	Find the radius of curvature at any point $x=a\cos^3\theta, y=a\sin^3\theta$ of the curve $x^{2/3}+y^{2/3}=a^{2/3}$. Also show that $\rho^3 = 27axy$.	$\rho = 3a\sin 2\theta/2$
7.	Show that the radius of curvature at any point of the curve $x=ae^{\theta}(\sin\theta-\cos\theta)$, $y=ae^{\theta}(\sin\theta+\cos\theta)$ is twice the perpendicular distance of the tangent at the point from the origin.	
8.	Prove that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$.	
9.	Show that the line joining any point θ on $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ to its centre of curvature is bisected by the line y = 2a.	
10.	Find the circle of curvature of the curvature $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$.	$(x-\frac{3a}{4})^2+(y-\frac{3a}{4})^2=\frac{a^2}{2}$

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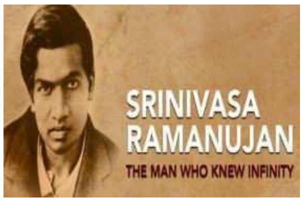
18MAB101T Calculus and Linear Algebra

UNIT - IV



	Tutorial Sheet -2	Answers
1.	State two properties of the evolute of the curve.	
2.	Find the envelope of the family of straight lines $y = mx + am^2$, m being the parameter	Ans: $x^2 + 4ay = 0$
3.	Define envelope of a family of curves.	
4.	Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = a\sec\alpha, \alpha$ being the parameters.	$\boldsymbol{Ans}: \boldsymbol{y^2 - 4a(a-x) = 0}$
5.	Define involutes and evolutes.	
6.	Find the equation of the circle of curvature at (c, c) on $xy = c^2$.	$\boldsymbol{Ans}: (\boldsymbol{x} - \boldsymbol{2c})^2 + (\boldsymbol{y} - \boldsymbol{2c})^2 = (\sqrt{2}\boldsymbol{c})^2$
7.	Find the equation of the evolute of the a) parabola $y^2 = 4ax$; b) ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; c) hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; d) rectangular hyperbola $xy = c^2$ and e) curve $x^{2/3} + y^{2/3} = a^{2/3}$.	Ans: a) $27ay^2 = 4(x-2a)^3$ b) $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$ c) $(ax)^{\frac{2}{3}} - (by)^{\frac{2}{3}} = (a^2 + b^2)^{\frac{2}{3}}$ d) $(x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}} = (4c)^{\frac{2}{3}}$ e) $(x+y)^{\frac{2}{3}} + (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$
8.	Show that the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$ is another equal cycloid.	
9.	Find the evolute of the tractrix $x = a\left(\cos t + \log\tan\left(\frac{t}{2}\right)\right), y = a\sin t$.	Ans: y = a $\boldsymbol{\cosh\frac{x}{a}}$
10.	Show that the evolute of the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ is a circle.	

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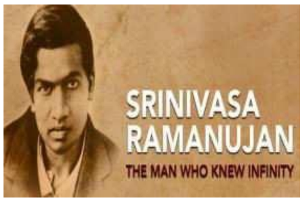

DEPARTMENT OF MATHEMATICS

18MAB101T Calculus and Linear Algebra

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	Tutorial Sheet -3	Answers
1.	Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$	
2.	Evaluate $\int_0^1 x^6(1-x)^9\,dx$	$\frac{6!\ \ 9!}{16!}$
3.	Evaluate $\int_0^{\pi/2}\sin^6\theta\cos^{10}\theta\,d\theta$	$\frac{1}{512}\,\frac{225*63}{8!}\pi$
4	Evaluate $\int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta$	$\frac{\pi}{\sqrt{2}}$
5.	Evaluate $\int_0^{\infty}e^{-x}\sqrt{x}\,dx$	$\frac{\sqrt{\pi}}{2}$
6.	Evaluate $\int_0^{\infty}e^{-4x}x^{16}\,dx$	$\frac{16!}{4^{17}}$
7.	Evaluate $\int_0^1\frac{dx}{\sqrt{1-x^4}}$	$\frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$
8.	Evaluate $\int_0^{\infty}e^{-x^4}x^4\,dx$	$\frac{1}{4}\Gamma\left(\frac{5}{4}\right)$

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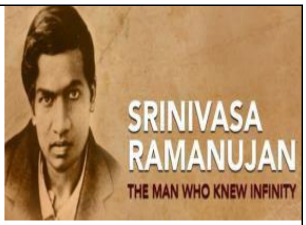
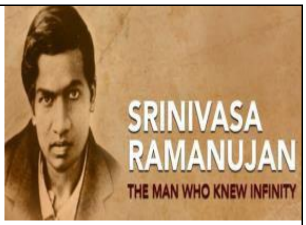
18MAB101T -CALCULUS AND LINEAR ALGEBRA

UNIT V: SEQUENCE & SERIES

Tutorial Sheet -1



Sl.No.	Questions	Answer
	Part – A	
1	Show that the sequence $\left\{\frac{n+1}{2n+7}\right\}$ is convergent.	
2	Examine the nature of the sequence: $\{2^n\}$	Divergent.
3	Examine the nature of the sequence: $\{3+(-1)^n\}$	Oscillatory.
4	Test for convergence of the series: $\frac{1}{4.7.10}+\frac{4}{7.10.13}+\frac{9}{10.13.16}+.......\infty$	Divergent.
5	Test for convergence of the series: $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+.......\infty$	Convergent.
	Part – B	
6	Test for convergence of the series: $\sum_{n=1}^{\infty}\frac{n^3+1}{2^n+1}$	Convergent.
7	Test for convergence of the series: $\frac{x}{1}+\frac{1}{2}.\frac{x^2}{3}+\frac{1.3}{2.4}.\frac{x^3}{5}+\frac{1.3.5}{2.4.6}.\frac{x^5}{7}+.....\infty, x>0$	Convergent for $0<x<1$. Divergent for $x>0$.
8	Test for convergence of the series: $\sum_{n=1}^{\infty}\sqrt{\frac{n}{n+1}}x^n, x>0$.	Convergent for $0<x<1$. Divergent for $x\geq 0$.
9	Test for convergence of the series: $\sum\frac{x^n}{n!}$	Convergent for all x.
10	Test for convergence of the series: $\sum\frac{x^n}{1+x^n}$	Convergent for $0<x<1$. Divergent for $x\geq 0$.

SRM INSTITUTE OF SCIENCE & TECHNOLOGY (Deemed to be University u/s 3 of UGC Act, 1956)	**SRM Institute of Science and Technology** **Kattankulathur** **DEPARTMENT OF MATHEMATICS** **18MAB101T -CALCULUS AND LINEAR ALGEBRA** **UNIT V: SEQUENCE & SERIES** **Tutorial Sheet -2**	
Sl.No.	**Questions**	**Answer**
	Part – A	
1	Test for convergence of the series: $\sum \frac{n^3}{3^n}$.	Convergent.
2	Test for convergence of the series: $\sum (\log n)^{-2n}$.	Convergent.
3	Test for convergence of the series: $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+.....\infty$	Convergent.
4	Test for convergence of the series: $\sum \left(\frac{n+1}{2n+7}\right)^n$	Convergent.
5	Test for convergence of the series: $1+\frac{x}{2}+\frac{x^2}{3^2}+\frac{x^3}{4^3}+.....\infty, x>0$	Convergent.
	Part – B	
6	Test for convergence of the series: $\frac{2}{3.4}+\frac{2.4}{3.5.6}+\frac{2.4.6}{3.5.7.8}+\frac{2.4.6.8}{3.5.7.9.10}+.....\infty$	Convergent.
7	Test for convergence of the series: $\frac{3}{4}.\frac{x}{5}+\frac{3.6}{4.7}.\frac{x^2}{3}+\frac{3.6.9}{4.7.10}.\frac{x^3}{11}+....\infty, x>0$	Convergent for $0<x\le 1$. Divergent for $x>0$.
8	Test for convergence of the series: $\sum \frac{1.3.5...(2n-1)}{2.4.6...2n}x^n$.	Convergent for $0<x<1$. Divergent for $x\ge 0$.
9	Test for convergence of the series: $\sum \frac{(n!)^2}{(2n)!}x^n$.	Convergent for $x^2<4$. Divergent for $x^2\ge 4$.
10	Test for convergence of the series: $\frac{x}{1}+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+.....\infty$	Convergent for $x<\frac{1}{e}$.Divergent for $x\ge\frac{1}{e}$

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Sl.No.	**Questions**	**Answer**
	Part – A	
1	Define absolutely convergent with an example.	
2	Define conditionally convergent with an example.	
3	Test for convergence of the series: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$	Convergent.
4	Test for convergence of the series: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2n+3)}{2n}$	Oscillatory
5	Test whether the series is absolutely convergent or not: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$.	Conditionally convergent
	Part – B	
6	Test for convergence of the series: $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}x^n}{n(n-1)}, 0<x<1.$	Convergent.
7	State the values of x for which the series is convergent. $\frac{x}{1}-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.....\infty.$	$-1<x\le 1$
8	Prove that the exponential series $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\infty$ is absolutely convergent and convergent for all values of x .	
9	Discuss the convergence of the series $\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-.....\infty,$ if $0<x<1.$	Convergent.
10	Prove that the series $\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}-.....\infty$ converges absolutely.	