$x(n) = \{1, 4, 3, 2\}$, $h(n) = \{1, 3, 2, 1\}$   Determine the

Convolution Sum of two sequences.

$x(-1) = 1$     $h(0) = 1$

$x(0) = 4$     $h(1) = 3$

$x(1) = 3$     $h(2) = 2$

$x(2) = 2$     $h(3) = 1$

Lower limit of $x(n)$ is     $-1$

upper limit of $x(n)$ is     $2$

Lower limit of $h(n)$ is     $0$

upper     ,,     "     "     is     $3$

Convolution formula is

$$Y(k) = \sum_{k=-\infty}^{\infty} x(k) \, h(n-k)$$

Need to find the limits for $k$ and $n$.

$$(-1+0) \leq n \leq (2+3)$$

$$\boxed{-1 \leq n \leq 5}$$

$$\boxed{-1 \leq K \leq 2}$$

$$y(n) = \sum_{k=-1}^{2} x(k) h(n-k)$$

$$= x(-1) h(n+1) + x(0) h(n) + x(1) h(n-k) + x(2) h(n-2).$$

· Now put n value from -1 to 5

when n = -1

$$y(-1) = x(-1) h(0) + x(0) h(-1) + x(1) h(-2) + x(2) h(-3)$$

$$= 1 \times 1 + 4 \times 0 + 3 \times 0 + 2 \times 0$$

$$= 1$$

when n = 0

$$y(0) = x(-1) h(1) + x(0) h(0) + x(1) h(-1) + x(2) h(-2).$$

$$= 1 \times 3 + 4 \times 1 + 3 \times 0 + 2 \times 0$$

$$= 3 + 4 = 7$$

when n = 1

$$y(1) = x(-1) h(2) + x(0) h(1) + x(1) h(0) + x(2) h(-1)$$

$$= 1 \times 2 + 4 \times 3 + 3 \times 1 + 2 \times 0$$

$$= 2 + 12 + 3$$

$$= 17$$

When n = 2

$y(2) = x(-1) h(3) + x(0) h(2) + x(1) h(1) + x(2) h(0)$

$\quad = 1 \times 1 + 4 \times 2 + 3 \times 3 + 2 \times 1$

$\quad = 1 + 8 + 9 + 2$

$\quad = 20$

when n = 3

$y(3) = x(-1) h(4) + x(0) h(3) + x(1) h(2) + x(2) h(1)$

$\quad = 1 \times 0 + 4 \times 1 + 3 \times 2 + 2 \times 3$

$\quad = 4 + 6 + 6$

$\quad = 16$

when n = 4

$y(4) = x(-1) h(5) + x(0) h(4) + x(1) h(3) + x(2) h(2)$

$\quad = 1 \times 0 + 4 \times 0 + 3 \times 1 + 2 \times 2$

$\quad = 3 + 4 = 7$

when n = 5

$y(5) = x(-1) h(6) + x(0) h(5) + x(1) h(4) + x(2) h(3)$

$\quad = 1 \times 0 + 4 \times 0 + 3 \times 0 + 2 \times 1$
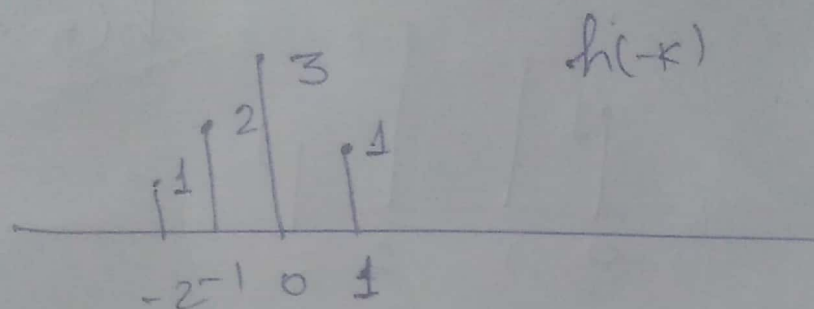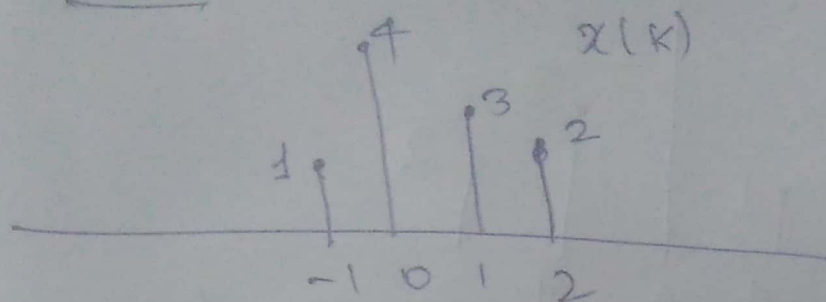
$\quad = 2$

$y(n) = \{ 1, 7, 17, 20, 16, 7, 2 \}$
$\qquad\qquad \uparrow$

# Graphical Method :.
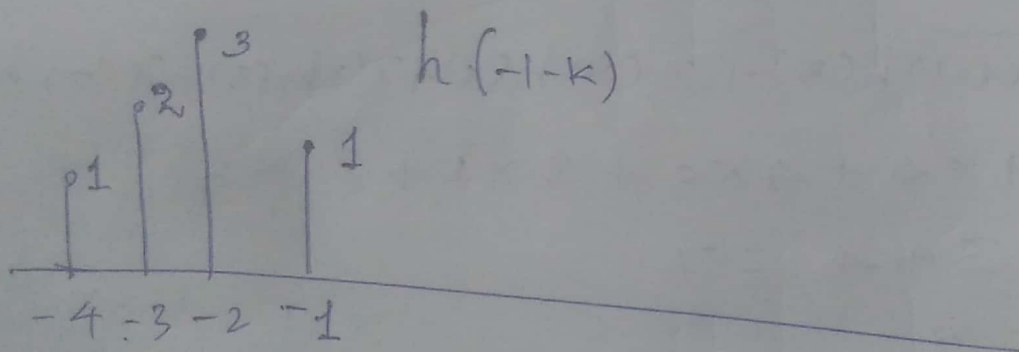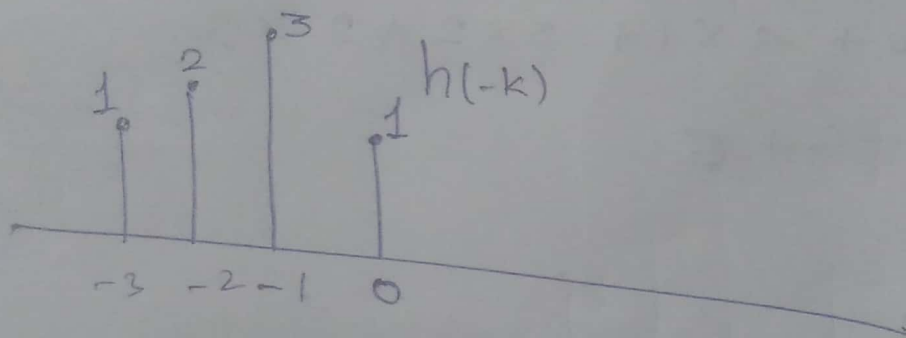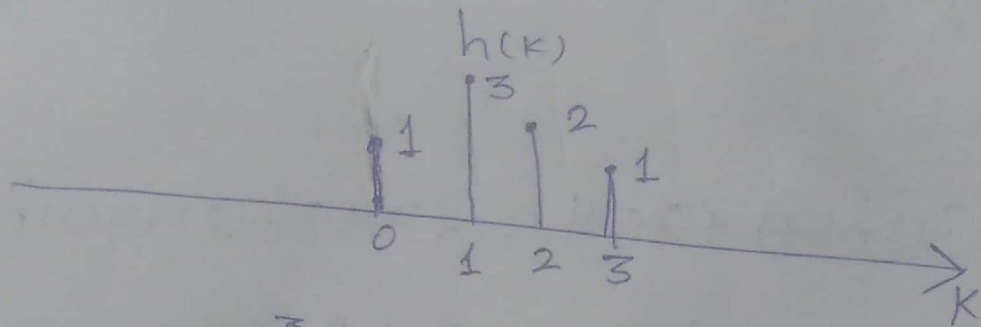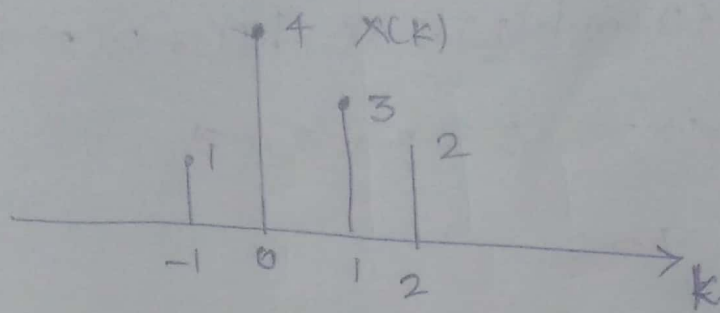


$x(k)$

plotted at $k = -1$ (value 1), $k = 0$ (value 4), $k = 1$ (value 3), $k = 2$ (value 2)

$h(k)$

plotted at $k = 0$ (value 1), $k = 1$ (value 3), $k = 2$ (value 2), $k = 3$ (value 1)

$h(-k)$

plotted at $k = -3$ (value 1), $k = -2$ (value 2), $k = -1$ (value 3), $k = 0$ (value 1)

$h(-1-k)$

plotted at $k = -4$ (value 1), $k = -3$ (value 2), $k = -2$ (value 3), $k = -1$ (value 1)

$$\boxed{y(-1) = 1}$$

When n = 0

$\underline{\underline{y(0)}}$


x(k)

-1 0 1 2


h(-k)

-2 -1 0 1

$$y(0) = 0 + 0 + 3 + 4 = 7$$

When n = 1

$\underline{\underline{y(1)}}$


h(1-k)

-1 0 1 2

$$y(1) = 0 + 2 + 4 + 3 + 2$$
$$= 17.$$

when n = 2

$\underline{y(2)}$


h(2-k)

0 1 2 3

$$y(2) = 1 + 8 + 9 + 2 = 20.$$

when n=3

y(3)

x(k)

3
2
1

-1  0   1   2
          3
       2
      1           1

0   1   2  3

y(3) = 0+4+6+6+0 = 16

when n=4

y(4)

              3    h(4-k)
           2
        1        1

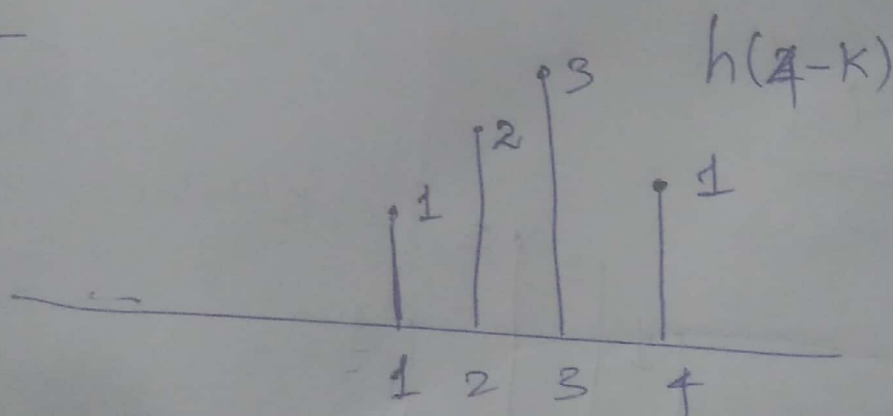1   2   3   4

y(4) = 3+4 = 7
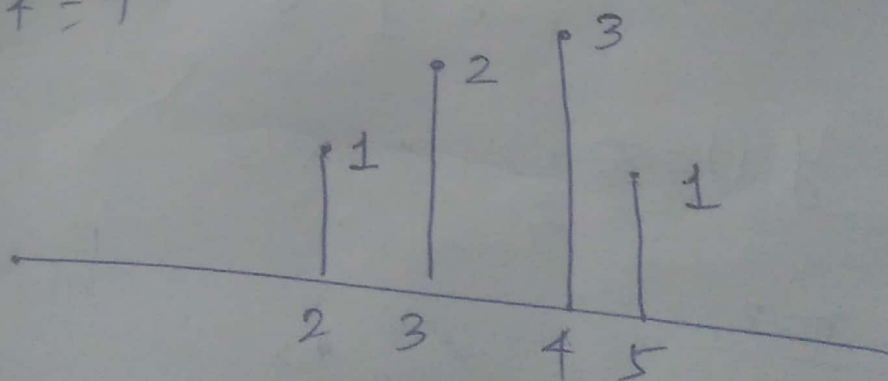
When n=5

y(5)

                 3
              2
           1        1
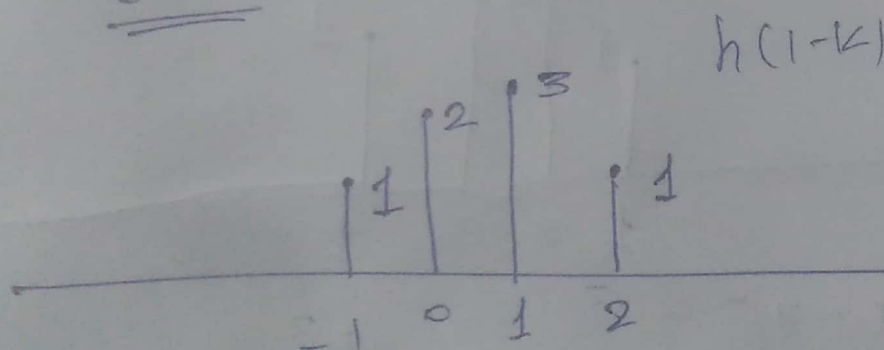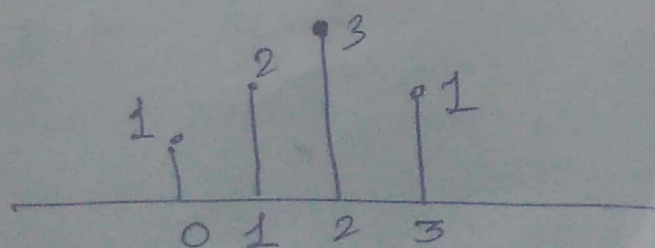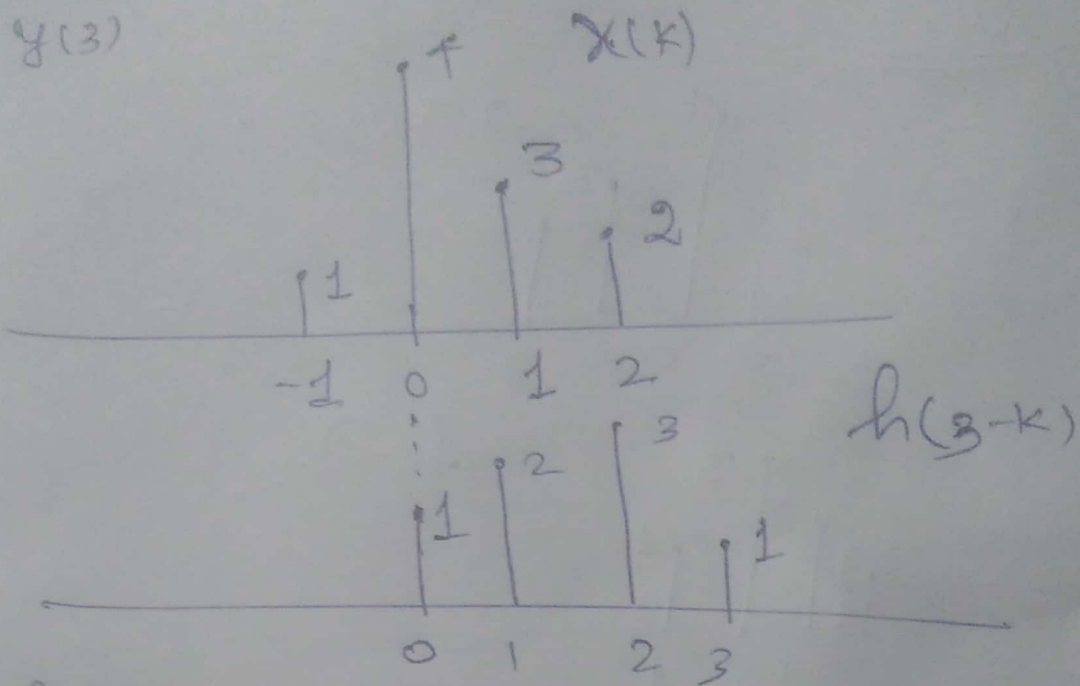
2    3    4  5

y(5) = 2

Cross verification:-

|     | 1 | 4  | 3 | 2 |
|-----|---|----|---|---|
| 1   | 1 | 4  | 3 | 2 |
| 3   | 3 | 12 | 9 | 6 |
| 2   | 2 | 8  | 6 | 4 |
| 1   | 1 | 4  | 3 | 2 |

$$y(n) = \{1, 7, 17, 20, 16, 7, 2\}$$