

1. $a^n c b^n, n \geq 1$ $w = acb, aacbb, aaacbbb$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$ ----- by final state

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$ -----by null stack

2. $a^n b^n c^m / n, m \geq 1$ $w = abc, abcc, aabbc, bc(\text{invalid})$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, c, z_0) = (q_2, z_0)$

$\delta(q_2, c, z_0) = (q_2, z_0)$ -----by final state

3. $a^n b^m c^n / n, m \geq 1$

CFG to PDA

Construct a PDA 'P' that accepts $L(G)$ by empty stack as follows

$$P = \{q\}, T, VUT, \delta, q, S$$

δ is defined by

- for each variable 'A'
 $\delta(q, \epsilon, A) = \{(q, \beta) \mid A \rightarrow \beta \text{ is a production of } P\}$
- for each terminal 'a', $\delta(q, a, a) = \{(q, \epsilon)\}$

$$S \rightarrow aSa \mid bSb \mid c$$

$$\delta(q, \epsilon, S) = \{(q, aSa), (q, bSb), (q, c)\}$$

$$\delta(q, a, a) = \{(q, \epsilon)\}$$

$$\delta(q, b, b) = \{(q, \epsilon)\}$$

$$\delta(q, c, c) = \{(q, \epsilon)\}$$

$$w = abcba$$

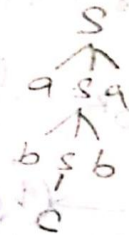
$$(q, abcba, S) \vdash (q, abcba, aSa)$$

$$\vdash (q, bcba, Sa)$$

$$\vdash (q, bcba, bsba)$$

$$\vdash (q, cba, sbqa) \vdash (q, cba, cba)$$

$$\vdash (q, ba, ba) \vdash (q, a, a) \vdash (q, \epsilon)$$



$$3. S \rightarrow aAA, A \rightarrow aS \mid bS \mid a$$

$$W = abaaaa$$

$$4. S \rightarrow AB$$

$$B \rightarrow b$$

$$A \rightarrow CD$$

$$C \rightarrow a$$

$$D \rightarrow a$$

$$W = aab$$

$$\begin{aligned}
 2. \quad E &\rightarrow I \mid E+E \mid E \times E \mid (E) \\
 I &\rightarrow a \mid Ia \mid 0 \mid Io \\
 W &= aa^*a0 \\
 \delta(q, \epsilon, E) &= \{ (q, I), (q, E+E), (q, E \times E), (q, (E)) \} \\
 \delta(q, \epsilon, I) &= \{ (q, a), (q, 0), (q, Ia), (q, Io) \}
 \end{aligned}$$

$$\Delta(q, +, +) = (q, \epsilon)$$

$$\Delta(q, *, *) = (q, \epsilon)$$

$$\Delta(q, (, ()) = (q, \epsilon)$$

$$\Delta(q,),) = (q, \epsilon)$$

$$\Delta(q, a, a) = (q, \epsilon)$$

$$\Delta(q, 0, 0) = (q, \epsilon)$$

(E^*E)

(I^*E)

(Ia^*E)

$(aa^*E) (a^*E) (^*E) (E) (I) (I0) (a0) (0) (\epsilon)$

PDA to CFG

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \emptyset)$ be a PDA accepting L by empty store

Construct $G = (V, T, P, S)$ such that $L(G) = N(M)$

$P: S \rightarrow [q_0, z_0, p]$ for each P in Q

Rules for mapping

1. If $\delta(q_1, a, A)$ contains $(P, B_1, B_2, \dots, B_m)$ $m \neq 0$
 P contains
 $[q_1, A, q_{m+1}] \rightarrow a[P, B_1, q_2][q_2, B_2, q_3][q_3, B_3, q_4] \dots [q_m, B_m, q_{m+1}]$
 $x \in \Sigma \cup \{\epsilon\}$
2. If $\delta(q, a, A)$ contains (P, ϵ) $m = 0$
 $[q, A, P] \rightarrow a \in P$

EX:1 $M = (\{q_0, q_1\}, \{0, 1\}, \{z_0, x\}, \delta, q_0, z_0, \emptyset)$

(i) $\delta(q_0, 0, z_0) = \{(q_0, xz_0)\}$

(ii) $\delta(q_0, 0, x) = \{(q_0, xx)\}$

(iii) $\delta(q_0, 1, x) = \{(q_1, \epsilon)\}$

(iv) $\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$

(v) $\delta(q_1, \epsilon, x) = \{(q_1, \epsilon)\}$

(vi) $\delta(q_1, \epsilon, z_0) = \{(q_1, \epsilon)\}$

What is $N(M)$?

SOL: step 1: $S \rightarrow [q_0, z_0, q_0] / S \rightarrow [q_0, z_0, q_1]$

Step 2: $[q_0, X, q_1] \rightarrow 1$

$[q_1, X, q_1] \rightarrow 1$

$[q_1, X, q_1] \rightarrow \epsilon$

$[q_1, Z_0, q_1] \rightarrow \epsilon$

$[q_0, z_0, q_1]$

Step 3: (i) $\delta(q_0, 0, z_0) = \{(q_0, XZ_0)\}$

$[q_0, z_0, q_0] = 0[q_0, X, q_0][q_0, Z_0, q_0]$

$[q_0, z_0, q_0] = 0[q_0, X, q_1][q_1, Z_0, q_0]$

$[q_0, z_0, q_1] = 0[q_0, X, q_0][q_0, Z_0, q_1]$

$[q_0, z_0, q_1] = 0[q_0, X, q_1][q_1, Z_0, q_1] \text{-----} \rightarrow$

(ii) $\delta(q_0, 0, X) = \{(q_0, XX)\}$

$[q_0, X, q_0] = 0[q_0, X, q_0][q_0, X, q_0]$

$[q_0, X, q_0] = 0[q_0, X, q_1][q_1, X, q_0]$

$[q_0, X, q_1] = 0[q_0, X, q_0][q_0, X, q_1]$

$[q_0, X, q_1] = 0[q_0, X, q_1][q_1, X, q_1] \text{-----} \rightarrow$

Final Productions

A

$S \rightarrow [q_0, z_0, q_1]$

B

$[q_0, X, q_1] \rightarrow 1$

D

$[q_1, X, q_1] \rightarrow 1$

D

$[q_1, X, q_1] \rightarrow \epsilon$

C

$[q_1, Z_0, q_1] \rightarrow \epsilon$

A \rightarrow OBC

$[q_0, z_0, q_1] = 0[q_0, x, q_1][q_1, z_0, q_1]$

B \rightarrow OBD | 1

$[q_0, X, q_1] = 0[q_0, x, q_1][q_1, X, q_1]$

S \rightarrow A

A \rightarrow OBC

B \rightarrow OBD | 1

C $\rightarrow \epsilon$

D $\rightarrow 1 | \epsilon$ $0^m 1^n / m, n \geq 1$ 01, 0011, 00001

$$2. \delta(q_0, 1, z_0) = (q_0, xz_0)$$

$$\delta(q_0, 1, x) = (q_0, xx) \checkmark$$

$$\delta(q_0, 0, x) = (q_1, x) \checkmark$$

$$\delta(q_1, 0, z_0) = (q_0, z_0)$$

$$\delta(q_0, \epsilon, z_0) = (q_0, \epsilon) \checkmark$$

$$\delta(q_1, 1, x) = (q_1, \epsilon) \checkmark$$

$$\delta(q_0, 0, x) = (q_1, x)$$

$$[q_0, x, q_0] \rightarrow 0 [q_1, x, q_0]$$

$$[q_0, x, q_1] \rightarrow 0 [q_1, x, q_1]$$

Final Productions

$$S \rightarrow [q_0, z_0, q_0]$$

$$[q_0, z_0, q_0] \rightarrow \epsilon$$

$$[q_1, x, q_1] \rightarrow 1$$

$$[q_0, z_0, q_0] \rightarrow 1 [q_0, x, q_1] [q_1, z_0, q_0]$$

$$[q_0, x, q_1] \rightarrow 1 [q_0, x, q_1] [q_1, x, q_1]$$

$$[q_0, x, q_1] \rightarrow 0 [q_1, x, q_1]$$

$$[q_1, z_0, q_0] \rightarrow p [q_0, z_0, q_0]$$

$$(i) \quad \delta(q, 1, z) = \{(q, xz)\}$$

$$(ii) \quad \delta(q, 1, x) = \{(q, xx)\}$$

$$(iii) \quad \delta(q, \epsilon, x) = \{(q, \epsilon)\}$$

$$(iv) \quad \delta(q, 0, z) = \{(p, x)\}$$

$$(v) \quad \delta(p, 1, x) = \{(p, \epsilon)\}$$

$$(vi) \quad \delta(p, 0, z) = \{(q, z)\}$$

$$S \rightarrow [q, z, q]$$

$$S \rightarrow [q, z, p]$$

