b.i. Solve $\left(D^3 - 3DD'^2 - 6D'^3\right)z = x^2y + \sin(x+2y)$.

ii. Solve $4\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 3z$ subject to $z = e^{-5y}$ when $x = 0$ by the method of separation of variables.

29. a.i. Find the half range sine series for $f(x) = \begin{cases} x & in\ 0 < x \le \pi/2 \\ \pi - x & in\ \pi/2 < x \le \pi \end{cases}$

ii. Expand $f(x) = x^2$, when $-l < x < l$ in Fourier series of periodicity $2l$ and hence deduce $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots = \dfrac{\pi^2}{6}$.

(OR)

b. Compute the first three harmonics of the Fourier series of f(x) given by the following table.

x	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

30. a. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $kx(l-x)$. Find the displacement of the string.

(OR)

b. A rod of length 30cm, long has its end points A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function u(x, t) taken $x = 0$ at A.

31. a. Find the Fourier transform of $f(x) = \begin{cases} \left(1 - x^2\right) & if\ |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$ and hence evaluate $\displaystyle\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$.

(OR)

b.i. Using Parseval's identity, evaluate $\displaystyle\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.

ii. Find $F_c\left(xe^{-ax}\right)$ and $F_s\left(xe^{-ax}\right)$.

32. a.i. Find the Z^{-1} transform of $\dfrac{10z}{(z-1)(z-2)}$ using Residue method.

ii. Find the Z^{-1} transform of $\dfrac{z}{z^2 + 7z + 10}$ using partial fraction method.

(OR)

b.i. Find the inverse Z-transform of $\dfrac{z(z+2)}{z^2 + 2z + 4}$ using long division method.

ii. Solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 4^n$ given $y(0) = 0, y(1) = 1$.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019

Third Semester

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. The PDE formed by eliminating the arbitrary constants a and b from $z = \left(x^2 + a\right)\left(y^2 + b\right)$ is
(A) $4xyz = pq$ (B) $xyz = pq$
(C) $xy = 4pq$ (D) $p(q+1) = qz$

2. The equation $z = px + qy + f(p, q)$ is known as
(A) Euler equation (B) Lagrange's equation
(C) Bernoulli's equation (D) Clairaut's equation

3. Solution of $pq = y$ is
(A) $z = ax + by + cz^2$ (B) $ax^2 + by^2 + cz^2 = 1$
(C) $z = ax + \dfrac{y^2}{2a} + b$ (D) $(x-a)^2 + (y-b)^2 + (z-c)^2 = c^2$

4. Solution of $\left(D^2 - 4DD' + 4D'^2\right)z = 0$
(A) $z = f(x+2y) + g(y+2x)$ (B) $z = f(y+2x) + g(y+2x)$
(C) $z = f(y-2x) + g(x-2y)$ (D) $z = f(y-2x) + xg(y-2x)$

5. One dimensional wave equation $\alpha^2\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}$ is
(A) Hyperbolic (B) Parabolic
(C) Elliptic (D) Hypersonic

6. The solution of wave equation which is periodic in x and t is
(A) $\left(Ae^{\lambda x} + Be^{-\lambda x}\right)\left(Ce^{\lambda at} + De^{-\lambda at}\right)$ (B) $(A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$
(C) $(Ax + B)(Cx + D)$ (D) $(A\cos\lambda x + B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$

7. The temperature at any particular point does not vary with time is known as
(A) Unsteady state (B) Transient state
(C) Steady state (D) Untransient state

8. One dimensional heat equation is
(A) $\dfrac{\partial^2 u}{\partial t\partial x} = \alpha^2\dfrac{\partial u}{\partial x}$ (B) $\dfrac{\partial u}{\partial t} = \alpha^2\dfrac{\partial u}{\partial x}$
(C) $\dfrac{\partial^2 u}{\partial t^2} = \alpha^2\dfrac{\partial^2 u}{\partial x^2}$ (D) $\dfrac{\partial u}{\partial t} = \alpha^2\dfrac{\partial^2 u}{\partial x^2}$

9. The value of bn in the expansion of Fourier series for $f(x)=x^2$ in $-\pi<x<\pi$ with periodicity 2π is
(A) 0 (B) 1
(C) −1 (D) 2

10. Parseval's identity in Fourier series is
(A) $\bar{y}^2=\frac{a_0^2}{2}+\frac{1}{4}\sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right)$ (B) $\bar{y}^2=\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right)$
(C) $\bar{y}^2=\frac{a_0^2}{2}-\sum_{n=1}^{\infty}\left(a_n^2-b_n^2\right)$ (D) $\bar{y}^2=a_0^2+\sum_{n=1}^{\infty}\left(a_n\cos nx+b_n\sin nx\right)$

11. The value of a_0 for the function $f(x)$ of periodicity 2 for $f(x)=\begin{cases}0 & in\ -1<x<0\\ 1 & in\ 0<x<1\end{cases}$ is
(A) −1 (B) 2
(C) 1 (D) −2

12. RMS value of the function $f(x)=x-x^2$ in $-1<x<1$ is
(A) $\sqrt{\frac{15}{8}}$ (B) $\sqrt{\frac{7}{8}}$
(C) $\sqrt{\frac{8}{7}}$ (D) $\sqrt{\frac{8}{15}}$

13. Fourier sine transform of $\frac{1}{x}$
(A) $\sqrt{\frac{\pi}{2}}$ (B) $\sqrt{\frac{2}{\pi}}$
(C) $\sqrt{\frac{\pi}{4}}$ (D) $\sqrt{\frac{4}{\pi}}$

14. The Fourier transform of $f(x)=\begin{cases}1 & for\ |x|<a\\ 0 & for\ |x|>a>0\end{cases}$ is
(A) $\frac{\cos as}{s}$ (B) $\sqrt{\frac{2}{\pi}}\frac{\sin as}{s}$
(C) $\sqrt{\frac{2}{\pi}}\frac{\cos as}{s^2}$ (D) $\sqrt{\frac{\pi}{2}}\frac{\sin as}{s^2}$

15. The Fourier sine transformation of $f(x)=e^{-ax}$, $a>0$ is
(A) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2-a^2}$ (B) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$
(C) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2-a^2}$

16. Modulation theorem on Fourier transformation is
(A) If $F[f(x)]=F(s)$, then $F[f(x)\cos ax]=\frac{1}{2}[F(x-a)+F(x+a)]$
(B) If $F[f(x)]=F(s)$, then $F[f(x)\cos ax]=\frac{1}{2}[F(x+a)e^{iasx}]$
(C) If $F[f(x)]=F(s)$, then $F[f(x)\cos ax]=\frac{1}{2}e^{-aisx}[F(x-a)]$
(D) If $F[f(x)]=F(s)$, then $F[f(x)\cos ax]=\frac{1}{2}[F(s-a)+F(s+a)]$

17. Z-transform of K is i.e $Z(K)=$ __________.
(A) $\frac{KZ}{z-1}$ (B) $\frac{KZ}{z+1}$
(C) $\frac{KZ^2}{z^2+1}$ (D) $\frac{KZ}{(z+1)^2}$

18. If $F(Z)=\frac{10Z}{(Z-1)(Z-2)}$ then $f(0)=$ _____ using final value theorem.
(A) 1 (B) 0
(C) −1 (D) 2

19. Z transform of $n(n-1)$ is i.e $Z\left[\left(n^2-n\right)\right]$ is
(A) $\frac{2z}{(z+1)^2}$ (B) $\frac{2z}{z+1}$
(C) $\frac{2z}{(z-1)^3}$ (D) $\frac{2z}{z-1}$

20. $Z=\left(na^n\right)=$ _______.
(A) $\frac{az}{(z+a)^2}$ (B) $\frac{az}{(z+a)}$
(C) $\frac{az}{z-a}$ (D) $\frac{az}{(z-a)^2}$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Obtain the partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
22. Find the singular solution of the PDE $z=px+qy+p^2-q^2$.
23. Obtain half range cosine series for $f(x)=x$ in $0<x<\pi$.
24. Find the Fourier series of periodicity 2 for $f(x)$ given by
$f(x)=\begin{cases}0 & in\ -1<x<0\\ 1 & in\ 0<x<1\end{cases}$
25. A *taut* string of length $2l$ is fastened at both ends. The midpoint of the string is taken to a height 'h' and then released from rest in that position. Write the boundary condition for this problem.
26. Find Fourier cosine transform of $f(x)=e^{-ax}$, $a>0$ and hence deduce the inversion formula.
27. Find $Z\left(r^n\cos n\theta\right)$ and $Z\left(r^n\sin n\theta\right)$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Form the partial differential equation by eliminating arbitrary function 'f' from $f\left(x+y+z,\ xy+z^2\right)=0$.
ii. Solve $\left(y^2+z^2-x^2\right)p-2xyq+2xz=0$.

(OR)