ii. Let X be a random variable which follows an exponential distribution whose pdf is $f(x)=\frac{1}{3}e^{-x/3}$, $x>0$. Find (i) $P(X>3)$ (ii) $P\left[|X|<5\right]$.

30. a. The following data relate to the marks obtained by 11 students in 2 tests, one held at the beginning of a year and the other at the end of the year after intensive coaching.

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

Do the data indicate that the students have benefited by coaching?

(OR)

b. The following figures show the distribution of digits in numbers chosen at random from a telephone directory.

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

Test whether all the digits are equally distributed in the directory.

31. a. Customers arrive at one-man barbar shop according to a Poisson process with a mean inter arrival time of 12 minutes. Customers spend an average of 10 minutes in the barbar chair.
   (i) What is the expected number of customers in the barbar shop and in the queue?
   (ii) Calculate the percentage of time an arrival can walk straight into the barbar's chair without having to wait.
   (iii) Management will provide another chair and hire another barbar, when a customer's waiting time in the shop exceeds 1.25 hours. How much must the average rate of arrivals increase to warrant to a second barbar.
   (iv) What is the average time customers spend in the queue?
   (v) What is the probability that the waiting time in the system is greater than 30 minutes?

(OR)

b. Patients arrive clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Investigation time per patient is exponential with mean rate of 20 per hour.
   (i) Determine the effective arrival at the clinic.
   (ii) What is the probability that an arriving patient will not wait?
   (iii) What is the expected waiting time until a patient is discharged from the clinic?
   (iv) What is the probability that an arriving patient can enter the system without waiting?

32. a. The transition probability matrix of a Markov chain $\{X_n\}=1,2,3....$ having 2 states 1, 2 and 3 and the initial distribution is $P(0)=[0.7\quad 0.2\quad 0.1]$

$$P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$$. Find (i) $P(X_2=3)$ (ii) $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$

(OR)

b. A man drives a car or catches a train to go to office each day. He never goes 2 days in a row by a train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week the man tossed a fair die and drove to work if and only if an even number appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives in the long run.

* * * * *

Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2019
Fourth Semester

MA1014 - PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. A random variable X has the p.d.f given by $f(x)=\begin{cases}2e^{-2x}; & x\ge 0\\ 0 & ; x<0\end{cases}$ then the MGF is
   (A) $\frac{2}{2-t}$ (B) $\frac{3}{3-t}$
   (C) $2(2-t)^{-3}$ (D) $3(3-t)^{-2}$

2. If the random variable X has the pdf $f(x)=\begin{cases}Kx^3, & 0<x<1\\ 0 & , otherwise\end{cases}$ then the value of K is
   (A) 3 (B) 4
   (C) 1/2 (D) 3/4

3. If $E(x^2)=8$ and $E(x)=2$ then $Var(x)$ is
   (A) 3 (B) 2
   (C) 1 (D) 4

4. A continuous random variable X has a pdf $f(x)=3x^2; 0\le x\le 1$, find the value of b such that $P(x>b)=0.05$
   (A) $\left(\frac{16}{20}\right)^{1/3}$ (B) $\left(\frac{19}{20}\right)^{1/3}$
   (C) $\left(\frac{13}{20}\right)^{1/3}$ (D) $\left(\frac{15}{19}\right)^{1/3}$

5. The MGF of binomial distribution is
   (A) $\left(p+qe^{t}\right)^n$ (B) $\left(p+qe^{-t}\right)^n$
   (C) $\left(pe^{t}+q\right)^n$ (D) $\left(pe^{-t}+q\right)^n$

6. Poisson distribution is limiting case of
   (A) Geometric distribution (B) Normal distribution
   (C) Binomial distribution (D) Exponential distribution

7. If X has uniform distribution in (–1, 3) then $p(x > 0)$
(A) 1/2 (B) 3/4
(C) 1/3 (D) 1/4

8. If X is exponentially distributed with mean 10 then the pdf is
(A) $10e^{-10x}, x \geq 0$ (B) $\frac{1}{10}e^{-10x}, x \geq 0$
(C) $\frac{1}{10}e^{x/10}, x \geq 0$ (D) $\frac{1}{10}e^{-x/10}, x \geq 0$

9. Type I error occurs when
(A) The null hypothesis is incorrectly accepted when it is false (B) The null hypothesis is incorrectly rejected when it is true
(C) The sample mean differs from the population mean (D) The test is biased

10. A _______ is numerical characteristic of a sample and a _____is a numerical characteristic of a population.
(A) Sample, population (B) Population, sample
(C) Static, parameter (D) Parameter, static

11. Which of the following value is not typically used for α
(A) 0.01 (B) 0.05
(C) 0.10 (D) 0.25

12. Which hypothesis is always in an equality form
(A) Null hypothesis (B) Alternative hypothesis
(C) Simple hypothesis (D) Composite hypothesis

13. The symbolic notation of queuing model is represented by
(A) Kendall (B) Euler
(C) Fisher (D) Neumann

14. The traffic intensity of a queuing system is
(A) λ (B) μ
(C) λ/μ (D) μ/λ

15. Which term refers to "A customer who leaves the queue is too long;"
(A) Balking (B) Reneging
(C) Jockeying (D) Leaving

16. In which basis the service is provided in queuing theory
(A) LCFO (B) LIFO
(C) FCFS (D) FCLS

17. Markov process is one in which the future value is independent of______ values.
(A) Present (B) Past
(C) Future (D) None

18. Ergodic means
(A) Irreducible and periodic (B) Irreducible and aperiodic
(C) Not irreducible (D) Regular

19. Transition matrix is a ________ with sum of the row as 1.
(A) Zero matrix (B) Square matrix
(C) Rectangular matrix (D) Day order

20. Chapman-Kolomogrov theorem states that
(A) $\left[p_{ij}^{(n)}\right]=\left[p_{ij}\right]^n$ (B) $\left[p(n)\right]=\left[p_{ij}\right]^n$
(C) $\left[n\,p_{ij}\right]=\left[p_{ij}\right]^n$ (D) $p_{ij}[n]=\left[p_{ij}\right]^n$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If the random variable 'X' takes the values 1, 2, 3, 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution.

22. Find the MGF of Poisson distribution and hence find mean.

23. A random variable 'X' has a uniform distribution over (0, 10) find (i) P(X < 2) (ii) P(X > 8) (iii) P(3 < X < 9).

24. The average number of articles produced by two machines per day are 200 and 250 with standard deviations 20 and 25 respectively on the basis of records of 25 days production. Can you consider both the machines equally efficient at 1% level of significance?

25. A telephone exchange receives one call for every 3 minutes. If the rate of arrivals follows Poisson distribution and service time follows exponential distribution. Find out (i) Expected waiting time for a call (ii) expected waiting time in the system (iii) expected number of customers in the system.

26. Find the nature of the states of the Markov chain with the tpm
$$P=\begin{bmatrix}0 & 1 & 0\\ 1/2 & 0 & 1/2\\ 0 & 1 & 0\end{bmatrix}.$$

27. Let X be a continuous RV with pdf $f(x)=\begin{cases}x/12, & 1<x<5\\ 0, & otherwise\end{cases}$. Find the pdf of $Y=2X-3$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28.a.i. If a random variable X has the mgf $M_X(t)=\frac{3}{3-t}$. Obtain mean, variance and SD of X. (5 Marks)

ii. Given the following table

X	–3	–2	–1	0	1	2	3
p(x)	0.05	0.10	0.3	0	0.3	0.15	0.10

Compute (i) E(X) (ii) Var(X) (iii) E(X+3). (7 Marks)

(OR)

b. A fair die is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

29. a. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation.

(OR)

b.i. Fit a binomial distribution to the following data.

x	0	1	2	3	4
f	2	14	20	34	20