MATHEMATICS (FORMULAE)

UNIT-I

1) [Characteristic Equation]$_{2\times2}$:—

$$\lambda^2 - S_1\lambda + S_2 = 0$$

2) [Characteristic Equation]$_{3\times3}$:—

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

3) For Symmetric Matrix, Eigen Vectors are Orthogonal ($\perp$r).

$$X_1 \cdot X_2 T = 0$$
$$X_1 \cdot X_3 T = 0$$
$$X_2 \cdot X_3 T = 0$$

4) Pdt of Eigen values is equal to $|A|$.

$$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |A|$$

5) Sum of Eigen Values is equal to sum of diagonal elements of [A].

$$(\lambda_1 + \lambda_2 + \lambda_3) = (d_1 + \partial_2 + \partial_3)$$

6) If $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$

then, $P^{-1} = \dfrac{1}{(ad-bc)} \begin{bmatrix} d & (-b) \\ (-c) & a \end{bmatrix}$ cannot use in Cayley-Hamilton Theorem.

7) Diagonalisation :-

$$D = N^T A N$$

8) Quadratic form :-

$$A = \begin{bmatrix} \text{coeff of } x_1^2 & \frac{1}{2}(\text{coeff of } x_1x_2) & \frac{1}{2}(\text{coeff of } x_1x_3) \\ \frac{1}{2}(\text{coeff of } x_1x_2) & \text{Coeff of } x_2^2 & \frac{1}{2}(\text{Coeff of } x_2x_3) \\ \frac{1}{2}(\text{coeff of } x_1x_3) & \frac{1}{2}(\text{coeff. of } x_2x_3) & \text{coeff of } x_3^2 \end{bmatrix}$$

$$\Rightarrow A = \begin{array}{cc} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3} \end{array}$$

9) Index (p) :-

The number of +ve square terms in the canonical form.

10) Signature (s) :-

Diff. b/w no. of +ve terms and -ve terms in canonical form.

11) Rank (r) :-

Nos. of non-zero elements in the eigen values.

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Nature of Canonical form :- (D)

1. If $n = r = p$, then D is positive definite matrix.
2. If $r = p$ but $r < n$, then D is +ve semi-definite matrix.
3. If $n = r$ but $p = 0$, then D is -ve semi-definite matrix.
4. If $r < n$ but $p = 0$, the D is -ve semi definite Matrix.

Sl. no.	Nature	Condition
1)	Positive Definite	$D_n > 0$ (+ve) or All the eigen values are (+)ve.
2)	Negative Definite	$D_n < 0$ (-ve) or All the eigen values are (-)ve.
3)	Positive semi-definite	$D_n > 0$ and atleast one value is zero. (or) All the eigen values > 0 & atleast one value is zero.
4)	Negative Semi-Definite	$D_n < 0$ and atleast one value is zero. (or). All the eigen values < 0 & atleast one value is zero.
5)	Indefinite	All other cases.

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Q) P/T quad. form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_1x_3$ is indefinite.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$D_1 = a_{11} = 1$

$$D_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1(2) - 1(1) = 2 - 1 = 1$$

$$D_3 = |A|$$
$$= |1(6-1)| - |1(3+1)| + |-1(1+2)|$$
$$= 5 - 4 - 3 = (-2)$$

$\therefore$ The nature is Indefinite. (Proved)

Q) Discuss the nature of the quad. form $2x_1x_2 + 2x_2x_3 - 2x_1x_3$ w/o reducing into canonical form.

$$A = \begin{array}{c c} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & +1 \\ -1 & +1 & 0 \end{bmatrix} \end{array}$$

$$D_1 = 0 \; ; \; D_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = (-1)$$

$$D_3 = |A| = 0 - 1(0+1) - 1(1-0)$$
$$= (-1-1) = -2$$

$\therefore$ The nature is Negative Semidefinite.

{UNIT-II}

1) Total differential Equations :-

$z = f(x,y)$

$$\boxed{\partial z = \frac{\partial f}{\partial x}\cdot dx + \frac{\partial f}{\partial y}\cdot dy}$$

$$\boxed{dU = \frac{\partial f}{\partial x_1}\cdot dx_1 + \frac{\partial f}{\partial x_2}\cdot dx_2 + \frac{\partial f}{dx_3}\cdot dx_3 + \cdots}$$

→ where, $U = f(x_1, x_2, \ldots, x_n)$

2) Differentiation of Implicit functions :-

$f(x,y) = 0$

$$\left(\frac{dy}{dx}\right) = \left(\frac{-f_x}{f_y}\right) = \left[\frac{(\partial f/\partial x)}{(\partial f/\partial y)}\right]$$

3) Taylor's Series Expansion :-

$$f(x) = f(x+h)$$
$$= \left\{f(x) + \frac{h}{1!} f(x) + \frac{h^2}{2!} f''(x) + \ldots\right\}$$

4) If u and v are functions of two independent variables x & y, then

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$ is called the Jacobian of u, v w.r.t y.

$$J\left[\frac{u, v}{x, y}\right] \text{ or } \left[\frac{\partial(u,v)}{\partial(x,y)}\right]$$

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5) If (u,v) are functions of (r,s), where (r,s) are functions of (x,y); then

$$\boxed{\frac{\partial(u,v)}{\partial(x,y)} = \left[\frac{\partial(u,v)}{\partial(r,s)} \cdot \frac{\partial(r,s)}{\partial(x,y)}\right]}$$

6) If the Jacobian Value is Zero (0), then u and v are functionally dependent.

So,
$y = e^{-6t}(c\cos t + D\sin t)$ Ans

UNIT-III *) Summarization (Unit 3)

ODE (with constant coefficients) :—

1) $CF = Ae^{m_1 x} + Be^{m_2 x}$; $m_1 \neq m_2$

$CF = (A + Bx)e^{mx}$; $m_1 = m_2 = m$

$CF = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$; $m = \alpha \pm i\beta$.

2) $PI = \left(\frac{1}{f(D)} F(x)\right)$

Type 1 :- $F(x) = 0 \Rightarrow PI = 0$

Type 2 :- $F(x) = e^{ax}$, $PI = \frac{1}{f(a)} F(x)$, $D = a$

Type 3 :- $F(x) = \sin ax$ or $\cos ax$; $PI = \frac{1}{f(D^2 = -a^2)} F(x)$

Type 4 :- $F(x) = x^n$; $PI = [1 \pm Q(D)]^{-1} x^n$

Type 5 :- $F(x) = e^{ax}\cos bx$
(or) $= e^{ax} g(x)$ $e^{ax}\sin bx$
or
$(e^{ax} x^n)$

Type 6 :- $F(x) = x\cos x$
or
$x\sin x$

$$PI = x\left[\frac{1}{f(D)} g(x)\right] - f'(D)\left[\frac{1}{f(D)^2} g(x)\right]$$

Method of variation of parameters :-

$CF = Af_1 + Bf_2$

$PI = Pf_1 + Qf_2$

$$P = -\int \frac{f_2 F(x)}{f_1 f_2' - f_2 f_1'} dx$$

$$Q = \int \frac{f_1 \cdot F(x)}{f_1 f_2' - f_2 f_1'} dx$$

$\boxed{y = CF + PI}$ → for all the models ...

ODE (with Variable Coefficients) :-

A) Euler's :- $D = xD$

Let $x = e^z$

① $z = \log x$

② $xD = D'$

③ $x^2 D^2 = D'(D'-1)$

$D = \frac{d}{dx}$

$D' = \frac{d}{dz}$

A) Legender's :- $D \to (ax+b)D$

Let $ax+b = e^z \Rightarrow$ ① $z = \log(ax+b)$

② $(ax+b)D = aD'$

③ $(ax+b)^2 D^2 = a^2 D'(D'-1)$

UNIT - IV

1) Radius of Curvature :— (ρ)

a) Cartesian form

$$\rho = \left[\frac{\left\{ 1 + \left(\frac{dy}{dx}\right)^2 \right\}^{3/2}}{(d^2y/dx^2)} \right]$$

$$\Rightarrow \rho = \left[\frac{(1 + y_1{}^2)^{3/2}}{y_2} \right]$$

; where $y_1 = \left(\frac{dy}{dx}\right)$

$y_2 = \left(\frac{d^2y}{dx^2}\right)$

b) Parametric form

$x = f(\epsilon)$; $y = g(\epsilon)$

$$\rho = \left[\frac{(x'^2 + y'^2)^{3/2}}{(x'y'' - y'x'')} \right]$$

where x' and y' → first / single derivatives

x'' and y'' → double derivatives.

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c) Polar form

$(r, \theta) \Rightarrow r = f(\theta)$

where, $r_1 = (dr/d\theta)$

$$\rho = \left[\frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - r r_2} \right]$$

$r_2 = (d^2r/d\theta^2)$

2) Centre of curvature

$$\bar{x} = \left[x - \frac{y_1(1 + y_1^2)}{y_2} \right]$$

$$\bar{y} = \left[y + \frac{(1 + y_1^2)}{y_2} \right] ; \; y_1 = (dy/dx)$$

$$y_2 = \left(\frac{d^2y}{dx^2}\right)$$

3) Circle of Curvature

$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2 \rightarrow$ Radius.

4) Evolutes

- First write parametric form.
- Find out centre of curvature.

5) Envelopes

i) $A^2\alpha + B\alpha + C = 0$; $\alpha \rightarrow$ Parameter

ii) Reqd. envelope of curve,

$B^2 - 4AC = 0$

$f(x \cdot y \cdot \alpha) = 0$

$\partial f/\partial x\,(x, y, \alpha) = 0$

solve the eqn & eliminate the parameter.

UNIT-V

$\left\{\frac{1}{n^2}\right\} \rightarrow$ Convergent Series.

$\{n\} \rightarrow$ Divergent Series.

$\{(-1)^n\} \rightarrow$ Oscillates finitely

$\{(-1)^n\}_{n=0}^{\infty} \rightarrow 1, -1, 1, -1, \ldots$
(finite)

$\{(-1)^n n^2\} \rightarrow$ Oscillates Infinitely.

Necessary condition for Convergence & Divergence :-

→ If a +ve term series $\sum_{n=1}^{\infty} u_n$ is Convergent, then

$\lim_{n\to\infty} u_n = 0$; non-zero finite.

→ If $\lim_{n\to\infty} u_n \neq 0$, the series $\sum_{n=1}^{\infty} u_n$ is divergent.

Geometric Series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n$$

i) If $|x| < 1$, then $\sum_{n=0}^{\infty} x^n$ converges

ii) If $|x| \geq 1$, then $\sum_{n=0}^{\infty} x^n$ diverges.

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*) Auxillary Series

$\sum_{n=1}^{\infty} 1/n^p \rightarrow$ converges if $p > 1$

$\hookrightarrow$ diverges if $p \leq 1$

*) Limit Comparison Test

$\lim_{n\to\infty} \frac{u_n}{v_n} =$ finite value (non-zero) [Convergent]

$$v_n = \left(\frac{\text{highest degree of } u_n \text{ in numerator}}{\text{highest degree of } u_n \text{ in denominator}} \right)$$

*) D' Alembert's Ratio Test

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lambda$$

$\lambda < 1 \rightarrow$ Converges

$\lambda > 1 \rightarrow$ diverges

$\lambda = 1 \rightarrow$ Test fails

$\downarrow$ Fails

Raabe's Test

$\downarrow$ Fails

Logarithmic Test

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*) Raabe's Test (apply if D'Alembert's Test fails)

$$\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = l$$

if $l<1 \rightarrow$ Convergent

$l>1 \rightarrow$ divergent

*) Logarithmic Test (apply if Raabe's Test fails)

$$\boxed{\lim_{n\to\infty} n \log \frac{u_n}{u_{n+1}} = l}$$

if $l<1$, Σu_n converges.

if $l>1$, Σu_n diverges.

*) Cauchy Root Test

$$\lim_{n\to\infty} u_n^{1/n} = l$$

$l<1 \rightarrow \Sigma u_n$ is convergent

$l>1 \rightarrow \Sigma u_n$ is divergent.

$l=1 \rightarrow$ test fails.

Alternating Series

An alternating series is given by $\boxed{\Sigma_{n=1}^{\infty} (-1)^n u_n}$

$$\therefore \sum_{n=1}^{\infty} (-1)^n u_n = -u_1 + u_2 - u_3 + u_4 - \ldots$$

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*) Leibnitz Test

① $u_{n+1} < u_n$ for all 'n'

② $\lim_{n \to \infty} u_n = 0$

then, $\sum_{n=1}^{\infty} (-1)^n u_n$ is convergent.

Absolute Convergence → { Alternating series is convergent

∴ $\boxed{\text{if } \sum |(-1)^n u_n| \text{ converges}}$.

Conditional Convergence

∴

→ Convergent series but not absolutely.

ALL THE BEST!!!

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