SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB302T- DISCRETE MATHEMATICS

Year/Sem: III/V

Branch: CSE,ECE,EEE

Unit 3- LOGICS

1. The proposition $(P \vee Q) \leftrightarrow (Q \vee P)$ is

(a) Contradiction (b) tautology (c) contra positive (d) Converse **Ans : b**

Solution:

P	Q	$P \vee Q$	$Q \vee P$	$(P \vee Q) \leftrightarrow (Q \vee P)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

It is a Tautology.

2.The proposition $(P \wedge Q) \wedge \neg(P \vee Q)$ is **Ans :a**

(a) Contradiction (b) tautology (c) contra positive (d) Converse

Solution:

P	Q	$P \wedge Q$	$P \vee Q$	$\neg(P \vee Q)$	$(P \wedge Q) \wedge \neg(P \vee Q)$
T	T	T	T	F	F

T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

3. Symbolic form of "If either ram takes calculus or krish takes sociology ,then arun will take English".

   (a)$(a \vee b) \rightarrow c$ (b) $(a \wedge b) \rightarrow c$ (c) $(\neg a \vee b) \rightarrow c$ (d) $(a \vee \neg b) \rightarrow c$ **Ans : a**

   Solution:

   Let a: Ram takes calculus

   b: Krish takes sociology

   c: Arun will take English. Symbolically form is $(a \vee b) \rightarrow c$

4. Symbolic form of " If tigers have wings then the earth travels round the sun". **Ans : a**

   (a) $P \rightarrow Q$ (b) $P \rightarrow \neg Q$ (c) $\neg P \rightarrow Q$ (d) $P \wedge Q$

   Solution:

   Let P: Tiger have wings, Q: Earth travels round the symbolic form is $P \rightarrow Q$

   The given proposition is a contradiction

5. Give the converse of the implication " If it is raining , then I get wet". **Ans :d**

   (a) If I do not get wet ,then it is raining (b) If I do not get wet ,then it is not raining

   (c) If I get wet ,then it is not raining (d) If I get wet ,then it is raining

   Solution:

   If $P \rightarrow Q$, then the converse of the implication is given by $Q \rightarrow P$.

   Hence, If I get wet ,then it is raining.

6. Give the contra positive of the implication " If it is raining , then I get wet". **Ans :b**

   (a) If I do not get wet ,then it is raining (b) If I do not get wet ,then it is not raining

(c) If I get wet ,then it is not raining (d) If I get wet ,then it is raining

Solution:

If $P \rightarrow Q$, then the contrapositive of the implication is given by $\neg Q \rightarrow \neg P$

Let P: It is raining , Q: I get wet, $\neg P$: It is not raining, $\neg$Q: I do not get wet

If I do not get wet ,then it is not raining.

7. Give the contra positive of the implication “ Only if Raju studies well he pass the test”. **Ans :c**

(a) If Raju does not study, then he will pass the test (b) If Raju study, then he will pass the test

(c)If Raju does not study, then he will not pass the test

(d) If Raju does not study, then he will pass the test

Solution:

Let P: Raju studies well Q: He will pass the test.

Contrapositive for $P \rightarrow Q$ is $\neg Q \rightarrow \neg P$

8. The proposition $p \wedge \sim p$ is a

(a) Contradiction (b) tautology (c) contra positive (d) Converse **Ans : a**

Solution :

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

From the above truth table, The proposition $p \wedge \sim p$ is a Contradiction.

9. p, $p \rightarrow q$, $q \rightarrow r \Rightarrow$ **Ans :c**

(a) p (b) q (c) r (d) $\neg$ p

Using modus ponens p, $p \rightarrow q$, we get q.

Again, Using modus ponens q, $q \rightarrow r$, we get r

10. Logical equivalence of $p \rightarrow (q \rightarrow r)$ is

(a) $(p \wedge q) \rightarrow r$ (b) $(p \wedge q) \rightarrow q \vee r$ (c) (d) $\neg r$ **Ans: a**

Solution:

$p \rightarrow (q \rightarrow r)$	$\equiv \neg p \vee (\neg q \vee r)$	as $p \rightarrow p \equiv (\neg p) \vee q$
	$\equiv (\neg p \vee \neg q) \vee r$	Associativity
	$\equiv \neg(p \wedge q) \vee r$	De Morgan
	$\equiv (p \wedge q) \rightarrow r$	as $p \rightarrow p \equiv (\neg p) \vee q$

11. Logical equivalence of $\neg(p \leftrightarrow q) \equiv$ -----------

(a) $\neg p \leftrightarrow q$ (b) $(p \wedge q) \rightarrow q$ (c) q (d) $\neg r$ **Ans: a**

Solution:

$\neg(p \leftrightarrow q) \equiv \neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q)$	De Morgan
$\equiv (\neg p \vee \neg q) \wedge (p \vee q)$	De Morgan, Double negation
$\equiv (\neg p \wedge p) \vee (\neg p \wedge q) \vee (\neg q \wedge p) \vee (\neg q \wedge q)$	Distributivity
$\equiv (\neg p \wedge q) \vee (\neg q \wedge p)$	Constants
$\equiv (\neg p \wedge q) \vee (\neg\neg p \wedge \neg q)$	Double negation
$\equiv \neg p \leftrightarrow q$	

12. The dual of $\neg(p \vee q) \wedge r$ and T is are **Ans :a**

(a) $\neg(p \wedge q) \vee r$, F (b) $(p \wedge q)$, T (c) $\neg(p \wedge q)$, F (d) $(p \wedge q) \vee r$, T

Solution:

The dual P $*$ of a formula P involving the connectives $\vee$, $\wedge$, $\neg$ is obtained by interchanging $\vee$ with $\wedge$. Therefore, the dual of $\neg(p \vee q) \wedge r$, T are $\neg(p \wedge q) \vee r$ and F.

13. Logical equivalence of $p \rightarrow q, p \rightarrow r$, $p \rightarrow q \wedge r$ *is* **Ans :b**

a) F (b) (T (c) $\neg(p \wedge q)$ (d) $(p \wedge q) \vee r$

Solution:

Suppose p is T . Since $p \rightarrow q$ is T , q is T . Since $p \rightarrow r$ is T , r is T . Then $q \wedge r$ is T . Hence $p \rightarrow q \wedge r$ is T .

14. $\neg (p \vee (\neg p \wedge q)) \equiv$

Ans :d

(a) $\neg p \rightarrow (p \rightarrow q)$ (b) $p \rightarrow (p \rightarrow q)$ (c) $\neg p \rightarrow (\neg p \rightarrow q)$ (d) $\neg p \wedge \neg q$.

Solution:

Using equivalence laws properties

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ DeMorgan

$\equiv \neg p \wedge (p \vee \neg q)$ DeMorgan

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ Distributivity

$\equiv F \vee (\neg p \wedge \neg q)$ Because $\neg p \wedge p \equiv F$

$\equiv \neg p \wedge \neg q$ Because $F \vee r \equiv r$ for any r

15. What is the valid inference from the premises $P \rightarrow Q, Q \rightarrow R$ *and* P. **Ans :b**

(a) P (b) R (c) Q (d) $P \rightarrow R$

Solution:

(1) $P \rightarrow Q$ Rule P

(2) $Q \rightarrow R$ Rule P

(3) P Rule P

(4) $P \rightarrow R$ Rule T, Hypothetical Syllogism(1), (2)

(5) R Rule T, Modus ponens, (3), (4)

16. $(p \wedge q) \rightarrow (p \vee q) \equiv$ **Ans :a**

(a)T (b) F (c) p v q (d) p-q

Solution:

$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ DeMorgan

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ Commutativity and Associativity

$\equiv T \vee T \equiv T$ Because $\neg p \vee p \equiv T$

17. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ **Ans :a**

(a) $p \rightarrow (q \wedge r)$ (b) $p - p \rightarrow (q \wedge r)$

(c) $p \rightarrow (q \wedge r)$ vp (d) $p \rightarrow p$ v $(q \wedge r)$

Solution:

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv (\neg p \vee q) \wedge (\neg p \vee r)$

$\equiv \neg p \vee (q \wedge r)$

$\equiv p \rightarrow (q \wedge r)$

18. The proposition $p \rightarrow q \equiv$ -------------

(a) $p \rightarrow (q \wedge p)$ (b) $\neg q \rightarrow p$ (c) $p \rightarrow q$ v p (d) $p \rightarrow p$ v q **Ans :b**

Solution:

$p \rightarrow q$ is false if and only if p is true and q is false
if and only if $\neg p$ is false and $\neg q$ is true
if and only if $\neg q \rightarrow \neg p$ is
false. Hence $p \rightarrow q \equiv \neg q \rightarrow p$.

19. $p, p \rightarrow q \Rightarrow$

(a) p (b) q (c) $p \rightarrow q$ v p (d) $p \rightarrow p$ v q **Ans :b**

Solution:

Suppose p and $p \rightarrow q$ are T (under an assignment). Suppose q is F (under the same assignment). As $p \rightarrow q$ is T , p must be F . This is a contradiction

20. $\neg q, p \rightarrow q \Rightarrow$

(a) $\neg p$ (b) q (c) $p \rightarrow q$ v p (d) $p \rightarrow p$ v q **Ans :a**

Solution :

Suppose $\neg q$ and $p \to q$ are T . If $\neg p$ is F , then p is T . Now that $p \to q$ is T , we see that q is T . This is a contradiction.

21.The proposition p $\wedge \neg p$ is a

(a) Tautology (b) Contradiction (c) Implication (d) Quantifier **Ans :a**

Solution :

P	$\neg p$	p $\wedge \neg p$
T	F	F
F	T	F

It is a contradiction.

22. The proposition p $\vee \neg p$ is a

(a) Tautology (b) Contradiction (c) Implication (d) Quantifier **Ans :a**

Solution :

P	$\neg p$	p $\vee \neg p$
T	F	T
F	T	t

It is a Tautology

23. The proposition $p \to q,\ q \to r \Rightarrow p \to r\ is$

(a) F (b) T (c) contingency (d) p $\vee$ q **Ans : b**

Solution:

Suppose $p \to r$ is F . Then p is T and r is F . As r is F and $q \to r$ is T , q must be F . As q is F and $p \to q$ is T , p is F , a contradiction.

24. The proposition is $p \to q, p \to r \Rightarrow p \to q \wedge r$

(a) p $\wedge$q (b) $p \to r.$ (c) T (d) q **Ans : c**

Solution :

Suppose p is T . Since $p \to q$ is T , q is T . Since $p \to r$ is T , r is T . Then $q \wedge r$ is T . Hence

$p \to q \wedge r$ is T .

25. The proposition is $p \to r,\ q \to r \Rightarrow p \vee q \to r$

(a) F (b) $p \to r$. (c) q (d) T **Ans : d**

Solution :

Suppose $p \vee q \to r$ is F . Then $p \vee q$ is T and r is F . Since r is F and the premise $p \to r$ is T , we have p is F . Similarly, the premise $q \to r$ gives q is F . Now, the three statements p is F , q is F and $p \vee q$ is T lead to a contradiction**.**