TESTING OF HYPOTHESIS

Sampling:

A statistical population is defined as the totality of all objects under investigation. In most situations, the population may be considered infinitely large.

A finite subset of a population is called a <u>Sample</u> and the process of selection of such samples is called Sampling.

Why sampling?

When the entire population is analysed and studied, it is known as complete Enumeration. Sampling is considering a part of the population & analysing its properties. Sampling is prefered for the foll. reasons.

(1) When the information is urgently required.
(2) To save time (3) Sometimes the population may be incomplete (destroyed or missing) (4) Analysing entire population is more expensive (5) For some investigations sampling is the only method (eg) Blood testi

Parameters and statistics.

Statistical measures calculated on the basis of population values are called <u>Parameters</u>.

Eg Mean of the Population (μ)
S.D. of the Population (σ).

And the Statistical measures computed on the basis of sample observations are called __Statistics__

Eg Mean of the Sample ($\bar{x}$)
S.D. of the sample (s)

In practice, Parameter values are not known and their __estimates__ based on sample values are generally used.

__Sampling distribution.__

__Defn__

The Probability distribution of the Statistic that would be obtained if the number of samples, each of same size, were infinitely large is called the __Sampling distribution__

__Eg__: In a Population select several random samples of size 'n' and if its mean is given by $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n$

then

Sample No:	1	2	3	...	n
Mean	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$		$\bar{x}_n$

is known as the __Sampling distribution of Mean.__

|||ly Sampling distri. of S.D can also be found.

__Standard Error:__

The standard deviation of any sampling distribution is known as the __Standard error__

Eg. The S.D. of the sampling dis. of mean is defined as the standard error of the mean.

Testing of Hypothesis

A Statistical hypothesis is an assumption about the parameters of the population. The testing of hypothesis is a decision rule to accept or to reject the hypothesis on the basis of an observed sample.

Testing of hyp. is a two fold problem of either accepting or rejecting the hyp.

Null Hypothesis & Alternate Hypothesis

When an hypothesis is set up, we assume that there is no significant difference between the sample statistic & population statistic (or) between two sample statistic. Such a hyp. of no difference is called a Null hypothesis denoted by H_0

A hypothesis that is complementary to the null hypo. is called an Alternate hypothesis denoted by H_1.

Eg $H_0 : \mu = \mu_0$, $H_1 : \mu \neq \mu_0$ (or) $\mu < \mu_0$ (or) $\mu > \mu_0$.

One tailed and Two tailed tests

If μ is the population parameter and if μ_0 is the corresponding sample statistic and if as

Set the null hypo. $H_0: \mu = \mu_0$ then the alter. crit

hypo. H_1 which is complementary to H_0 can

any one of the following.

(i) $H_1: \mu \neq \mu_0$. (ii) $H_1: \mu > \mu_0$ (or) $\mu < \mu_0$

when H_1 given by $\underline{\mu_1 \neq \mu_0}$ is called two tailed alter. hyp, then the test is known as <u>two tailed test</u> and

if H_1 given by $\underline{\mu > \mu_0 \text{ (or) } \mu < \mu_0}$ is called one tailed alter. hyp, then the test is known as <u>one tailed test</u>

Errors in Testing of Hypothesis

The decision to accept or reject a null hypothesis H_0 is made on the basis of Information supplied from the observed sample and the conclusion drawn may not always be true in respect of the population.

	H_0: True	H_0: False.
Reject H_0	Type I error 'α'	Correct
Accept H_0	Correct	Type II error 'β'

Similar to good product being rejected by the consumer.

The error committed in rejecting H_0, when it is really true is called <u>Type I error</u> (Producer's risk)

Similar to accept a product of inferior quali

The error committed in accepting H_0, when it is false is called <u>Type II error</u> (consumer's risk)

Critical Region & Level of Significance (LOS)

To accept that the difference between a sample statistic and the corresponding parameter is significant, when the sample statistic lies in a certain region or interval. Then that region is called the critical region (or) Region of rejection

The region complementary to the critical region is called the region of acceptance.

LOS

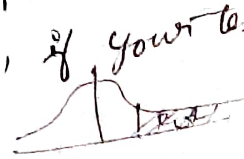
The Probability that a random value (α) of the statistic lies in the critical region is called the level of significance and is usually expressed as a percentage

In otherwords, the total area of the critical region expressed as α% is the LOS.

Critical values (or) Significant values

The value of the test statistic 'z' for which the critical region and acceptance region are separated is called the critical value or the significant value of z & is denoted by z_α where α is the LOS. It is clear that the values of z_α depends not only on α also on the nature of alternate hypothesis

[A critical value is a line on a graph that splits the graph into sections. One or two of the sections is the rejection region, if your test value falls into that region, then you reject H_0]



The critical values for some standard LOS both for 2 tailed & one tailed tests

Nature of test	1% (0.01)	2% (0.02)	5% (0.05)	10% (0.1
Two-tailed	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 2.33$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
one. tailed	$\lvert Z_\alpha \rvert = 2.33$	$\lvert Z_\alpha \rvert = 2.055$	$\lvert Z_\alpha \rvert = 1.645$	$\lvert Z_\alpha \rvert = 1.28$

Procedure to test hypothesis

(1) Null hyp. H_0 is defined

(2) Alternate hyp. H_1 is also defined after a careful study of the problem & also the nature of the test.

(3) LOS 'α' is fixed or taken from the problem if specified & Z_α is noted.

(4) The test statistic $z = \dfrac{t - E(t)}{SE(t)}$ is computed

where t is the statistic in large samples which follows a normal distribution with mean $E(t)$ & S.D equal to $SE(t)$

(5) comparison is made between $|Z|$ & Z_α

If $|Z| < Z_\alpha$, H_0 is accepted (or) H_1 is rejected

(ie) It is concluded that the diff. between t & $E(t)$ is not significant at α% LOS

On the other hand if $|z| > z_\alpha$,

Ho is rejected (or) H_1 is accepted

(ii) It is concluded that the diff. between t & E(t) is significant at α% Los.

Confidence Limits (or) fiducial limits

The interval within which the Population Parameter is expected to lie is called the confidence interval for that parameter. The end Points of the confidence Interval are called confidence limits or fiducial limits

The 95% confidence limits for E(t) is $\{t - 1.96\,SE(t),\ t + 1.96\,SE(t)\}$

III^{ly} The 99% Confidence limits for E(t) is $\{t - 2.58\,SE(t),\ t + 2.58\,SE(t)\}$

Large samples

If the Size of the sample n > 30, then the sample is said to be a large sample.

Type I and II errors are denoted by α and β.
The probability α of committing Type I error is the L.O.S

UNIT-3

TESTING HYPOTHESIS

LARGE SAMPLE TEST
TEST 1
TEST OF SIGNIFICANCE OF THE DIFFERENCE BETWEEN SAMPLE PROPORTION AND POPULATION PROPORTION

1. Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20% was wrong. Based on the particular day's production, find also the 95% confidence limits for the % of top quality product.
2. A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one.
3. The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in metropolitan hospital and only 63 patients died. Can you consider the hospital is efficient?
4. A salesman in a departmental store claims that at most 60% of the shoppers showed that 35 of them left without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making purchase. Are these sample results consistent with the claim of the sales man? Use a Level of significance of 5%.
5. In a sample of 1000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in the state at 1% level of significance?

TEST 2
TEST OF SIGNIFICANCE OF THE DIFFERENCE BETWEEN TWO SAMPLE PROPORTIONS

1. A random sample of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favor of the proposal. Test the hypothesis that proportions of men and women in favor of the proposal are same against that they are not at 5% LOS?
2. A sample of 300 spare parts produced by a machine contained 48 defectives. Another sample of 100 spare parts produced by another machine contained 24 defectives. Can you conclude that the first machine is better than the second?
3. In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5 % of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?
4. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is a significant decrease in the consumption of tea after the increase in duty.
5. 15.5% of a random sample of 1600 undergraduates was smokers, whereas 20% of a random sample of 900 post graduates was smokers in a state. Can we conclude that less number of undergraduate is smokers than the post graduates?

TEST 3
TEST OF SIGNIFICANCE OF THE DIFFERENCE BETWEEN SAMPLE MEAN AND POPULATION MEAN

1. A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165cm and the SD is 10cm?
2. A random sample of 100 students gave a mean weight of 58 kg and an SD of 4 kg. Find the 95% and 99% confidence limits of mean of the population.

3. The mean breaking strength of the cables supplied by a manufactured is 1800, with an SD of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS?
4. An IQ test was given to a large group of boys in the age group of 18 to 20 years, who scored an average of 62.5 marks. The same test was given to a fresh group of 100 boys of the same age group. They scored an average of 64.5 marks with an SD 12.5 marks. Can we conclude that the fresh groups of boys have better IQ?

TEST 4
TEST OF SIGNIFICANCE OF THE DIFFERENCE BETWEEN TWO SAMPLE PROPORTIONS

1. In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the sample have been drawn from the same population with SD 4?
2. Test significance of the difference between the means of the samples drawn from two normal populations with the same SD using the following data.

	SAMPLE SIZE	MEAN	SD
I	100	61	4
II	200	63	6

3. A simple sample of heights of 6400 English men has a mean of 170 cm and an SD of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172cm and an SD of 6.3cm. Do the data indicate that Americans are on the average taller than the Englishmen?
4. The average marks scored by 32 boys are 72 with an SD of 8, while that for 36 girls is 70 with an SD of 6. Test at 1% LOS whether the boys performs better than girls.

TEST 5
TEST OF SIGNIFICANCE OF THE DIFFERENCE BETWEEN SAMPLE SD AND POPULATION SD

1. In a certain random sample of 72 items, the SD is found to be 8. Is it reasonable to suppose that it has been drawn from a population with SD 7?
2. In a random sample of 200 items, drawn from a population with SD 0.8, the sample SD is 0.7. Can we conclude that the sample SD is less than the population SD at 1% LOS?
3. A manufacturer of electric bulbs, according to a certain process, finds the SD of the life of lamps to be 100hrs. He wants to change the process, if the new process results in a smaller variation in the life of lamps. In adopting a new process, a sample of 150 bulbs gave an SD of 95hrs. Is the manufacturer justified in changing the process?

TEST 6
TEST OF SIGNIFICANCE OF THE DIFFERENCE BETWEEN TWO SAMPLE SDs

1. The SD of a random sample of 1000 is found to be 2.6 and SD of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same SD.
2. The SD of a random sample of 900 members is 4.6 and that of another independent sample of 600 members is 4.8. Examine if the two samples could have been drawn from a population with SD 4.

TEST 7
CHI-SQUARE TEST

1. The following table gives the number of air-craft accidents that occurred during the various days of a week. Test whether the accidents are uniformly distributed over the week.

DAY	MON	TUE	WED	THURS	FRI	SAT
NO. OF ACCIDENTS	15	19	13	12	16	15

2. The following table shows the distribution of digits in the numbers chosen at random from a telephone directory:

Digits	0	1	2	3	4	5	6	7	8	9	Total
Frequency	1026	1107	997	966	1075	933	1107	972	964	853	10,000

3. The following data show defective articles produced by 4 machines

MACHINE	A	B	C	D
Production time	1	1	2	3
Number of defective	12	30	63	98

   Do the data indicate a significant difference in the performance of the machine?
4. The theory predicts the proportion of beans in the four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory?
5. A survey of 320 families with 5 children each revealed the following distributions:

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

   Is this result consistent with the hypothesis that males and female births are equally probable?
6. Fit a Poisson distribution to the following data and test the goodness of fit

X	0	1	2	3	4	5	6
f	275	72	30	7	5	2	1

TEST 8
CHI-SQUARE TEST-INDEPENDENCE OF ATTRIBUTES

1. The following data are collected on two characters

	SMOKERS	NON-SMOKERS
LITERATES	83	57
ILLITERATES	45	68

   Based on this, can you say that there is no relation between smoking and literacy?
2. A total number of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them 1872 were men and rest were women. 2257 individuals were in favor of the proposal and 917 were opposed to it. 243 men were undecided and 443 women are opposed to the proposal. Do you justify or contradict the hypothesis that there is no association between sex and attitude.
3. The following table gives for a sample of married women, the level of education and the marriage adjustment score.

Level of Education	Marriage adjustment				
	VERY LOW	LOW	HIGH	VERY HIGH	
COLLEGE	24	97	62	58	241
HIGH SCHOOL	22	28	30	41	121
MIDDLE SCHOOL	32	10	11	20	73
	78	135	103	119	435

   Can you conclude from the above data that the higher the level of education, the greater is the degree of adjustment in marriage?