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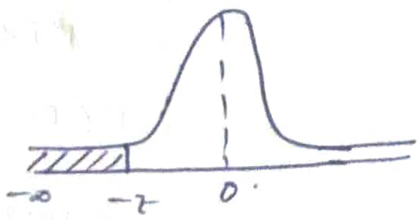
* In an examination, a student is failed, got 2nd class, 1st class and distinction according if he scores 45%, 45% to 60%, 60% to 75% and above 75% resptively. In a particular exam 10% of a student failed and 5% got distinction, find the percentage of students who have got 1st & 2nd class.

Sol :- $X \longrightarrow$ percentage of marks

Given $P(x < 45) = 0.10$ $\quad \left[z = \frac{x-\mu}{\sigma}\right]$

$P(-\infty < x < 45) = 0.10$

$P\left(-\infty < z < \frac{45-\mu}{\sigma}\right) = 0.10$



$0.5 - P\left(0 < z < \frac{-45+\mu}{\sigma}\right) = 0.10$

$P\left(0 < z < \frac{-45+\mu}{\sigma}\right) = 0.4.$

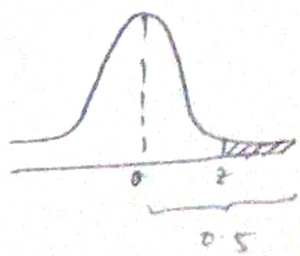
$\Rightarrow \quad \frac{\mu - 45}{\sigma} = 1.28.$

$\mu - 1.28\sigma = 45$ — (1).

Given $P(x > 75) = 0.05$ [change in question)

$P\left(z > \frac{75-\mu}{\sigma}\right) = 0.05$

$0.5 - P\left(0 < z < \frac{75-\mu}{\sigma}\right) = 0.45.$



Unit-3

Testing of Hypothesis

Population → Collection of Individuals

sample → A finite subset of population

Parameter and statistics

→ statistical measures calculated on the basis of population are called parameter [Mean − μ, Varience − σ^2]

→ " " " on the basis of sample are called statistics [Mean → $\bar{x}$, Varience − s^2]

A sample statistic is denoted by 't'

Sampling distribution

The proby distn of a statistic 't'

standard error:

The standard deviation of the sampling distn of a statistics

Null hypotesis (H_0)

A hypothesis of no differnce (i.e, no diff b/w pop and sample)

Alternate hypothesis (H_1)

A hypothesis which is different from H_0.

→ A procedure to accept or reject null hypothesis is called testing of hypothesis

Reject H_0 when it is true [Type 1]

Accept H_0 " " false [Type 2]

One Tail and Two tail test :

set $H_0 = \theta = \theta_0$

suppose $H_1 : \theta \neq \theta_0$ [$\theta > \theta_0$ (or) $\theta < \theta_0$ Two tail]

$H_1 = \theta > \theta_0$ [right tail sample > pop]

$H_1 = \theta < \theta_0$ [left tail sample < pop].

Critical region

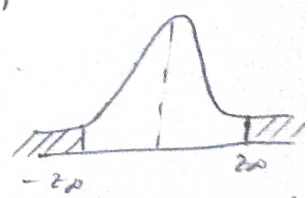
A region where we reject H_0 is called Critical region (or) region of rejection.

→ The region complementary to CR is Acceptance region.

Critical value [z_α]

The value of a statistic z for which the critical and acceptance region are seperated [α → level of significance].

Nature	1%	2%	5%	10%
2 tail	2.58	2.33	1.96	1.645
Right tail	2.33	2.055	1.645	1.28
Left tail	-2.33	-2.055	-1.645	-1.28.



Large Sample

When size of sample is greater than 30 is called large sample

otherwise it is small sample.

Procedure (for testing of hypothesis)

1) Set H_0

2) Set H_1 [check whether it is 1 tail or 2 tail]

3) Find $|z|$ [test statistic] and z_α

4) If $|z| < |z_\alpha|$ [Accept H_0]

$|z| > |z_\alpha|$ [Reject H_0]

Test 1

[Test of significance between sample proportion and population proportion]

test statistic $z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}}$

where

$p \rightarrow$ sample proportion

$P \rightarrow$ population "

$Q = 1 - P$

$n \Rightarrow$ size of sample.

95% confidence limits are

$$\frac{|p-P|}{\sqrt{PQ/n}} \leq 1.96 \rightarrow \text{from table. } 5\%$$

$$= \left(p - 1.96\sqrt{PQ/n}\ ,\ p + 1.96\sqrt{\frac{PQ}{n}}\right)$$

* 20% of manufactured product is of top quality. In one day production of 400 articles only 50 are of top quality. Verify the hypothesis and also find 95% confidence limit.

Sol:- $P = 20\% = 0.2$

$p = \frac{50}{400} = 0.125$, $n = 400$

$Q = 1 - P = 0.8$

$H_0 : P = p$ $[P = 0.2]$

pop sample

$H_1 : P \neq p$ [two tail]

let the level of significance be 5%

$Z_\alpha = 1.96$.

$$z = \frac{p - P}{\sqrt{PQ/n}} = -3.75$$

$|z| > |z_\alpha|$ Reject H_0

there is a sg diff

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3) $P = 17.26\% = 0.1726$.

$n = 640$

$p = \frac{63}{640} = 0.0984$

$H_0 : P = p$ (Hospital is not sufficient)

$H_1 : p < P$ (Hospital is sufficient)

(left tail) (sample < pop).

$$z = \frac{p - P}{\sqrt{PQ/n}}$$

$Q = 1 - P$
$= 0.8274$

$= -4.96$.

let the level of significance be 5%

$Z_\alpha = -1.645$ (Table value)

$|z| > |z_\alpha|$ Reject H_0

Hospital is sufficient.

4) $P = 60\% = 0.6$.

$n = 50$

$p = \frac{35}{50} = 0.7$

Claim, Comparative - 1 tail

$H_0: p = P$

$H_1: p > P$ [sample > pop, right tail].

$Q = 1-P \Rightarrow Q = 0.4$

Let the LOS be 5%.

$z = \frac{p-P}{\sqrt{PQ/n}} = 1.443$.

$z_\alpha = 1.645$.

$|z| < |z_\alpha|$ Accept H_0.

$\Rightarrow$ 60% of the shoppers entering the store leaves without making the purchase.

Test - 2

Test of significance of difference between two sample propn.

$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$P \rightarrow$ Pop propn

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}.$$

Problem

1, In a city, 20% of random sample of 900 boys had a slight physical defect. In another city 18.5% of a random sample of 1600 school boys had the defect. Is the differences ~~bet~~ b/w the proportions significance.

Sol:

$n_1 = 900$ $n_2 = 1600$

$p_1 = 20\% = 0.2$ $p_2 = 18.5\%$

$= 0.185$.

$H_0 \Rightarrow p_1 = p_2$

$H_1 \Rightarrow p_1 \neq p_2$ (Two tail)

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.1904$$

$$Q = 1 - P = 0.8096$$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 0.92.$$

Let the loss be 5%. $Z_\alpha = 1.96$.

$|Z| < |Z_\alpha|$ Accept H_0

no significant diff.

2, 15.5% of a random sample of 1600 UG were smokers wher as 20% of random sample of 900 PG were smokers is a state. Can we condeclude that less nol- of UG are smoker the PG.

A. $n_1 = 1600$ $n_2 = 900$

$P_1 = 15.5\%$ $P_2 = 20\%$

$= 0.155$ $= 0.2$

$H_0 : p_1 = p_2$

$H_1 : p_1 < p_2$ (one tail / lef tail)

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.1712$$

$$Q = 1 - P = 0.8288.$$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -2.87$$

Let the los be 5% $Z_\alpha = -1.645$

$|Z| > |Z_\alpha|$

Reject H_0

no. of ↑ smokers in UG < PG ~~smokers~~

3) Before an ↑ in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After ↑ in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant ↓ in the consumption of tea after the ↑ in duty.

Sol: $p_1 = \frac{800}{1000} = \frac{4}{5}$ $\quad n_1 = 1000$

$n_2 = 1200$

$p_2 = \frac{800}{1200} = \frac{2}{3}$

$H_0 : P_1 = P_2$

$H_1 : p_1 > p_2$ (right tail)

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.7236$$

$$Q = 1 - P =$$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 6.82$$

let the LOS be 5%. $\quad Z_\alpha = -1.645$.

$|Z| > |Z_\alpha|$

Reject H_0

Test 3

Test of significance between sample mean & pop mean.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad (\text{or}) \quad = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$\bar{x} \rightarrow$ sample mean,

$\mu =$ pop "

$\sigma =$ pop s.D

$s =$ sample s.D

95% Confidence interval $\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right)$

99% Confidence interval. $\left(\bar{x} - 2.58\frac{\sigma}{\sqrt{n}}, \bar{x} + 2.58\frac{\sigma}{\sqrt{n}}\right)$

Q. A sample of 100 students is taken. the mean height of this sample is 160cm. Can it reasonable regarded that it in the population the mean height is 165 and SD is 10cm at 1% LOS.

Sol :- $n = 100$ $\quad \mu = 165$

$\bar{x} = 160$ $\quad \sigma = 10$

$H_0 : \bar{x} = \mu$

$H_1 : \bar{x} \neq \mu$ (2 tail)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -5.$$

LOS be 1%

$z_\alpha = 2.58$

$|z| > |z_\alpha|$

Reject H_0.

Q, The mean life time of sample of 45 balls is 1550 hrs and S.D of 120 hrs. The company manufacturing the bulbs claims claims that the average life of bulbs is 1600 hrs. Is the claim acceptable at 5% LOS

$n = 45$

$\bar{x} = 1550$

$s = 120$

$\mu = 1600$

$H_0 : \bar{x} = \mu.$

$H_1 : \bar{x} < \mu.$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = -2.79$$

$z_\alpha = -1.645$

$|z| > |z_\alpha|$

Reject H_0.

Test 4

test of significance of difference b/w 2 means

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

If $\sigma_1 = \sigma_2 = \sigma$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

$\bar{x}_1 \rightarrow$ 1st sample mean

$\bar{x}_2 \rightarrow$ 2nd sample "

$\sigma_1 \rightarrow$ S.D of pop 1st

$\sigma_2 \rightarrow$ " " 2nd

$S_1 \rightarrow$ SD of 1st sample

$S_2 \rightarrow$ " " 2nd sample.

1) $n_1 = 1000$ $n_2 = 2000$

$\bar{x}_1 = 67.5$ $\bar{x}_2 = 68$

$\sigma = 2.5$

$H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 \neq \bar{x}_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = 5.96$$

let the LOS be 5%

$Z_\alpha = 1.96$

$|Z| > |Z_\alpha|$

Reject H_0.

2, 3) $\rightarrow$ Refer pdf

4). 1st sample (English man)

$n_1 = 6400$

$\bar{x}_1 = 170$

$S_1 = 6.4$

2nd sample (American)

$n_2 = 1600$ $\bar{x}_2 = 172.2$

$S_2 = 6.3$

$H_0 : \bar{x}_1 = \bar{x}_2$ $H_1 : \bar{x}_1 < \bar{x}_2$ (One tail)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = 11.3$$

let the LOS be 5%

$Z_\alpha = 2.33$

$|Z| > |Z_\alpha|$

Reject H_0.

Americans are taller than english man.

5, $n_1 = 32$ $n_2 = 36$

$\bar{x}_1 = 22$ $\bar{x}_2 = 20$

$s_1 = 8$ $s_2 = 6$

H_0 $\bar{x}_1 = \bar{x}_2$

H_1 = $\bar{x}_1 > \bar{x}_2$ (Right tail)

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} =$$

Let the los be 1%

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Small sample → $n < 30$

Test 1

Test of significance of sample and population mean

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$\bar{x}$ → Sample mean

μ → population mean

s → Sample S.D

degree of freedom $\gamma = n-1$

Problem:

1, A machine is designed to produce insulting for electrical devices of average thickness of 0.025 cm. A random sample of 10 washes was found to have a thickness of 0.024 cm with a standard deviation of 0.02 cm. Test the significance of deviation.

2 tail	0.05
1 tail	0.10

Sol :- $\mu = 0.025$

$n = 10$

$\bar{x} = 0.024$

$s = 0.02$

$H_0 = \bar{x} = \mu$

$H_1 : \bar{x} \neq \mu$ [2 tail]

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = -1.5$$

dof $\nu = n-1 = 9$.

from table pg. 12 (pdf)

row - 0.005 ($\because$ 2 tail)

Column - 9.

tab $t = 2.262$.

$|t| < |tab\,t|$

Accept H_0

$\therefore$ The difference between $\bar{x}$ and μ is not significa

2) The mean weakly sales of soap bars departmental stores was 146.3 bars. After Advertaisement, the sales in 22 stores was increased to 153.7 with S.D of 17.2. was the Advertisement successful.

Sol :- $\mu = 146.3$

$n = 22$, $\bar{x} = 153.7$

$s = 17.2$

$H_0 : \bar{x} = \mu$

$H_1 : \bar{x} > \mu$ (1 tail)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \cancel{1.97}\ \frac{153.7 - 146.3}{17.2/\sqrt{21}} = 1.97.$$

dof $= \nu = n-1$
$= 21$

1 tail $\Rightarrow$ column ~~0.10~~ 21

row ~~21~~ 0.10

tab $t = 1.72$

$|t| > |tab\,t|$

Reject H_0.

* A certain injection given to 12 patients resu... increase of BP 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 can it be conclu... concluded that the injection increases the BP

Sol:: $\bar{x} = \frac{\Sigma x}{n} = \frac{31}{12} = 2.58$.

$s^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= \frac{185}{12} - 6.6564$.

$= 8.76$

$s = \sqrt{8.76} = 2.95$

$H_0 : \bar{x} = \mu$

$H_1 : \bar{x} > \mu$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$

$= \frac{2.58 - 0}{2.95/\sqrt{11}} = 2.85$

dof = 11 $|t| >$ tab...

tab $t = 1.8$ Reject H_0

→ when mean & SD not given directly

* A random sample of 10 boys had the following I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q's of 100 and find the range.

Sol:: $A = \frac{120 + 70}{2} = 95 \simeq 100$

x_i	$d_i = x_i - A$ $= x_i - 100$	d_i^2
70	-30	900
120	20	400
110	10	100
101	1	1
88	-12	144
83	-17	289
95	-5	25
98	-2	4
107	7	49
100	0	0
	-28	1912

$\bar{d} = \frac{\Sigma d_i}{n}$

$= \frac{-28}{10}$

$= -2.8$.

$x_i = d_i + A$

$\bar{x} = \bar{d} + A$

$= -2.8 + 100$

$= 97.2$

$s^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2$

$= \frac{1912}{10} - (-2.8)^2$

$= 183.36$

$s = 13.54$

$H_0 : \bar{x} = \mu.$

$H_1 : \bar{x} \neq \mu$

$\mu = 100$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = -0.62$$

dof $\nu = 9$

$t_{tab} = 2.26$

$|t| < t_{tab}$

Accept H_0.