Wednesday, December 22, 2021 1:37 PM

D'Alembert's Ratio test

If $\sum u_n$ is a positive term series such that

$$\underset{n\to\infty}{Lt} \frac{u_{n+1}}{u_n} = k$$

a) The series is convergent if $k < 1$

b) The series is divergent if $k > 1$

$\boxed{k=1}$ ratio test fails,

$$\underset{n\to\infty}{Lt} \frac{u_n}{u_{n+1}} = k \text{ (finite)}$$

a) convergent if $k > 1$

b) divergent if $k < 1$

Problems

① Test for convergence the series whose n^{th} term is $\frac{n^2}{2^n}$

Sol

$$u_n = \frac{n^2}{2^n}$$

$$u_{n+1} = \frac{(n+1)^2}{2^{n+1}}$$

$$\underset{n\to\infty}{Lt} \frac{u_{n+1}}{u_n} = \underset{n\to\infty}{Lt} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2}$$

$$= \underset{n\to\infty}{Lt} \frac{\cancel{n^2}\left(1+\frac{1}{n}\right)^2 \cancel{2^n}}{\cancel{2^n}.2 \cdot \cancel{n^2}}$$

$$= \frac{1}{2} < 1$$

By Ratio test,

The series is convergent

② Test for convergence the series whose n^{th} term is $\frac{2^n}{n^3}$.

Home work

Sol

$$u_n = \frac{2^n}{n^3}$$

$$u_{n+1} = \frac{2^{n+1}}{(n+1)^3}$$

$$\underset{n\to\infty}{Lt} \frac{u_{n+1}}{u_n} = \underset{n\to\infty}{Lt} \left(\frac{2^{n+1}}{(n+1)^3} \cdot \frac{n^3}{2^n} \right)$$

$$= \underset{n\to\infty}{Lt} \left(\frac{\cancel{2^n}.2}{\cancel{n^3}\left(1+\frac{1}{n}\right)^3} \cdot \frac{\cancel{n^3}}{\cancel{2^n}} \right)$$

$$= 2 > 1$$

The series is divergent.