LINEAR ALGEBRA

ASSIGNMENT

1.) $$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_2 - 2R_1$
$R_3 \longleftrightarrow R_3 - 3R_1$
$R_4 \longleftrightarrow R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \leftrightarrow R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \leftrightarrow R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Ans}$$

2.) $W \to$ V.S of sym. matrix, then it can represented as

$$\begin{bmatrix} x_1 & x_3 \\ x_3 & x_2 \end{bmatrix} \quad \therefore \text{Dim}^n(W) = 3$$

Now, nullity = Dim^n of null space, means subspace of N which gives 0 as output from P_2

For zero polynomial

$a - b = 0 \longrightarrow a = b$

$b - c = 0 \rightarrow b = c$

$c - a = 0 \rightarrow c = a$

$\therefore$ Matrix can be written by one variable

$k \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ → Null Space

$\therefore$ Nullity $=$ Dim^n(Nullspace)
$= 1$ Ans

3.) fo

3) for A^{-1}, let $B = A^{-1}$

$\therefore B = \frac{1}{|A|}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

Now, $(B - \lambda I)x = 0$

$\therefore \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ 1/3 & 2/3-\lambda \end{bmatrix} = 0$

$\Rightarrow \left(\frac{2}{3}-\lambda\right)^2 - \frac{1}{9} = 0$

$\Rightarrow \left(\frac{2}{3}-\lambda\right) = \pm\frac{1}{3}$ $\lambda = 1/3$, $\lambda = 1$

Now, for $\lambda = 1$

$\begin{bmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

we get, $-\frac{1}{3}x_1 + \frac{1}{3}x_2 = 0$

$\Rightarrow x_1 = x_2 = k$ [let]

$\therefore$ Eigen Vector $\Rightarrow k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans

Now,

for $\boxed{\lambda = \frac{1}{3}}$,

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow \frac{1}{3}x_1 + \frac{1}{3}x_2 = 0$$

$$x_1 = -x_2 = k \quad [\text{Let}]$$

$\therefore$ Eigen Vector $= k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ Ans

for $A+4I$, Let $C = A+4I$

$$\Rightarrow C = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

Now, $[C - \lambda I]\, x = 0$

$$\Rightarrow \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix} = 0$$

$(6-\lambda)^2 - 1 = 0$

$\Rightarrow \quad 6-\lambda = \pm 1$

$\boxed{\lambda = 5 \mid \lambda = 7} \longrightarrow$ Eigen Values of $A + 4I$.

Now for $\lambda = 5$,

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

~~$\Rightarrow x_1 = x_2$~~

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow \quad x_1 = x_2 = K$ (let)

Eigen Vector $\Rightarrow K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans.

for $\lambda = 7$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\Rightarrow -x_1 - x_2 = 0 \Rightarrow x_1 = -x_2 = k$ (let)

$\therefore$ Eigen Vector $\Rightarrow k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ Ans.

4) Iter. 1 → $x = y = z = 0$ (initially)

$3x = 7.85$

$\Rightarrow x = \cancel{2.16}\ 2.6167$

Now, $0.1x + 7y - 0.3\cancel{z}^{0} = -19.3$

$\Rightarrow 7y = -19.03$

$\Rightarrow y = -2.7945$

Now,

$0.3 \times 2.6167 - 0.2 \times (-2.7945) + 10 \times z = 71.4$

$\Rightarrow 10z = 70.05$

$\Rightarrow z = 7.0056$

Iteration 2 → $x = \cancel{2.16}\ 2.6167$

$y = -2.7945$

$z = 7.0056$

Now, leaving x & putting other ~~value~~ two,

$3x - 0.1 \times (-2.7945) - 0.2 \times (7.0056) = 7.85$

$\Rightarrow x = 2.990$

Now, $0.1 \times 2.99 + 7y - 0.3 \times 7.0056$
$= -19.3$

$\Rightarrow y = 2.49996$

Now, $0.3 \times 2.99 - 0.2 \times (-2.4996) + 10z$
$= 71.4$

$\Rightarrow 10z = 70.00308$

$\Rightarrow z = 7.0003$

Iteration 3 $\rightarrow$ $x = 2.99$, $y = (-2.4996)$
$z = (7.0003)$

Now, $3x - 0.1 \times 2.4996 - 0.2 \times 7.003$
$= 7.85$

$x = 3$ (approx)

Now, $0.1 \times 3 + 7y - 3 \times 7.0083 = -19.3$

$\Rightarrow 7y = -17.4999$

$\Rightarrow y = -2.5$ (approx)

Now, $0.3 \times 3 - 0.2 \times (-2.5) + 10z = 71.4$

$\Rightarrow 10z = 70$

$\Rightarrow z = 7$

Hence, $x = 3$, $y = -2.5$, $z = 7$ Ans

5.) Consistent System of eq^n -

$\Rightarrow$ having unique or many sol^n

Condition :- $\rho(A) = \rho(A:B)$ $\forall$ $AX = B$ sys. of eq^n.

Inconsistent System of eq^n.

$\Rightarrow$ having no sol^n

Condition :- $\rho(A) \neq \rho(A:B)$ $\forall$ sys. of eq^n $AX = B$.

$$\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

As it is a homogenous eq^n, it is always consistent as $\rho(A)$ will always equal to $\rho(A:B)$.

Now, Echlon form of A

$R_2 \leftrightarrow R_2 - 2R_1$ $R_3 \leftrightarrow R_3 - 3R_1$	$R_4 \leftrightarrow R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & 2 \\ 0 & 14 & 2 \end{bmatrix} \xrightarrow[R_4 \leftrightarrow R_4 + 2R_2]{R_3 \leftrightarrow R_3 - 2R_2}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore \rho(A) = 2$$

(no. of non-zero rows is 2)

But $\rho(A) \neq 3$ (dim^n)

$\therefore$ ∞-many sol^n possible.

Now, $x + 3y + 2z = 0$

$-7y - z = 0 \Rightarrow z = -7y = k$ (let)

Now, $x + 3\left(-\frac{k}{7}\right) + 2k = 0$

$\Rightarrow x + \frac{11}{7}k \Rightarrow x = -\frac{11k}{7}$

Ans $\rightarrow$ $k\begin{bmatrix} -11/7 \\ -1/7 \\ 1 \end{bmatrix}$

6.) Conditions of Linear Transformation

① $T(a+b) = T(a) + T(b)$

② $T(cU) = cT(U)$

for ①, lets take two vector as

$a_1 + b_1 x + c_1 x^2$ &

$a_2 + b_2 x + c_2 x^2$

$\therefore T\left((a_1 + b_1 x + c_1 x^2) + (a_2 + b_2 x + c_2 x^2)\right)$

$\Rightarrow T\left((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2\right)$

$(a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$

$= (a_1 + 1) + (b_1 + 1)x + (c+1)x^2 + a_2 + b_2 x + c_2 x^2$

$= T(a_1 + b_1 x + c_1 x^2) + a_2 + b_2 x + c_2 x^2$

$= T(a_1 + b_1 x + c_1 x^2) + \left((a_2 + 1) + (b_2 + 1)x + (c_2 + 1)x^2\right) - 1 - x - x^2$

$\neq T(a_1 + b_1 x + c_1 x^2) + T(a_2 + b_2 x + c_2 x^2)$

Hence, it is not a linear transformation

7.) $S = \{(1,2,3), (3,1,0), (2,1,3)\}$

Now, if $V_3(R)$ can be written as $c_1 V_1 + c_2 V_2 + c_3 V_3$ then

S spans $V_3(R)$.

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So for spanning

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$\Rightarrow c_1(1,2,3) + c_2(3,1,0) + c_3(-2,1,3) = 0$

$\cancel{c_1 - 3c_2 - 2c_3 =}$

$$\begin{cases} c_1 + 3c_2 - 2c_3 = 0 \\ 2c_1 + c_2 + c_3 = 0 \\ 3c_1 + 0c_2 + 3c_3 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - \frac{9}{5} R_1$

$R_3 \to R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{9}{5} R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

As rank $= 2 \neq$ Dim^n (3)

$\therefore$ Linear Dependent

$\therefore$ Not a Basis

Now, Dim^n of S = No. of Unique Vectors

$= 3$

A Basis of Subspace Spanned by of

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{Ay}$$

8.) Iteration 1 → $x = \dfrac{23+6(1)-2(1)}{3}$

$x = 9$

~~Iteration~~ $y = -15+4(1)+1 = -10$

$z = \dfrac{16-1+2}{7} = 2.57$

Iteration 2 →

$x = \dfrac{23+6(-10)-2(2.57)}{3}$

$x = -14.04$

$y = -15+4(9)+2.57$

$y = 23.57$

$z = 16-9+3x-10 = -3.98$

9.)

Iteration 3 →

$$x = \frac{23 + 6\times 23.57 - 2\times(-3.28)}{3}$$

$x = 57$

$y = -1\cancel{50} + 4(-14\cancel{0}.04) + (-3.28$

$y = -74.4$

$\cancel{\oplus}$ $z = 16-(-14.04)+3\times(23.57)$

$z = 100.75$

9.) One application of matrix operation in image processing is convolution, where a small matrix is applied to each pixel in the image to ~~fro~~ perform operatoory like binary, sharpening or edge detection

for ex! - A blur filter kernel could be used to blur an image by ~~over~~ ~~to~~ averaging pixel values in the neighbourhood, thus process is applied to every pixel.

Let's say we have a grayscale image represented by matrix of pixel values

We want to apply simple 3x3 blur 3x3 filter to image.

So, Blur Kernel $= \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

10.) Rotating a 2-D image involves applying a linear transformation known as rotation matrix. This matrix ~~oper~~ operates on the coordinates of each ~~of~~ pixels in the image, transforming them to new coordinates that represent the rotated image

The rotation matrix for a 2-D rotation look like

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

where θ represents the angle of ~~ope~~ rotation in radians. This matrix describes how each ~~out~~ point in the original image is mapped to a new position after rotation.

1. Iterate over each pixel in the original image.
2. For each pixel, apply the rotation matrix to its coordinates.
3. Round the resulting coordinates to obtain the nearest integer values [since pixels are discrete]
4. Copy the pixel value from the original image to the ~~near~~ new position in the rotated ~~array~~ image.

By applying this process to ~~a~~ every pixel in the original image, you obtain a rotated version of the image.