Section 4.1

1) The symmatric difference of two set is a new set that contains every element that is in either set except the element that are in both set.

The symmatric difference of two set S_1 and S_2 be defined as

$$S_1 \ominus S_2 = \{ x : x \in S_1 \text{ or } x \in S_2 \text{ but } x \text{ is not in both } S_1, S_2 \}$$

$$\therefore S_1 \ominus S_2 = (S_1 \cap \bar{S_2}) \cup (S_2 \cap \bar{S_2})$$

we know regular sets are closed under union, intesection and complement.

$\therefore$ if S_1 and S_2 regular then $S_1 \cap \bar{S_2}$ regular and $S_2 \cap \bar{S_1}$ also regular.

$\therefore$ $S_1 \ominus S_2 = (S_1 \cap \bar{S_2}) \cup (S_2 \cap \bar{S_1})$ is also regular.

Hence, the family of regular language is closed Under symmetric difference.

Section 4-3

(2) Language $L = \{w : n_a(w) = n_b(w)\}$ is not regular because we used to first store that count of a and then compare it again count of b, but finite automation has a limited storage, but in L there can be infinite number of a's comming in which can't be stored.

we can prove that non-regular languages by pumping lemma by contradictions. we can say that languages is regular or not.
if $L = \{w : n_a(w) != n_b(w)\}$ is regular then compliment
$(L)' = \{w : n_a(w) = n_b(w)\}$ is regular.

Note $L(a^* b^*)^n$ compliment $+(L) = L(a^n b)$

if we want to prove
$L = \{w : n_a(w) = n_b(w)\}$ is not regular

we only used to prove $L(a^n b^n)$
is not regular.

Give m, we pick $w = a^m b^m$ with
$|w| \geq m$
let $w = xyz$ where $|xy| \leq m$ and $|y| \geq 1$

Then $y = a^k$ where $1 \leq k \leq m$

Now we consider the pumping strings

$w_i = a^{m + (i-1)k} b^m$

when $i = 0$

$w_0 = a^{m-k} b^m \notin L(a^n b^n)$

According to the pumping lemma,
$L(a^n b^n)$ is not regular.
Hence, $L = \{ w : n_a(w) \neq n_b(w) \}$ is
not regular.

3) Assume that L is regular and so the pumping lemma must hold for any string wing. let p be the smallest integer such that.

$2P > m$

choose and they $y = a^k$ for some $1 \leq k \leq m$

The pumped string will be for $i = 0, 1, 2 \ldots n$

w_0 is the string that doesn't contain y.

But because $2p < 2p+k \leq 2p+m < 2p+2p = 2p+1$ i.e $|w_2|$ is between two consecutive power of 2.

This is a contradiction. So, L is not regular.

So, $L = \{a^n : n \text{ is power of } 2\}$ is not regular.