CSE 571 Fall 2022 HW4

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Exercise 1.1

a. We know that there is a stench in location [1, 2] and there is a pit in [2,1]. In locations [1, 3] [2, 2] [3, 1] we know that the Wumpus can be at none of these locations or at any single one of them. The wumpus's possible states are 4. In the locations mentioned the pit can be in either one, two, three, or none of them. We get the possible state of pits to be 8.
We get the possible worlds out to be $8*4 = 32$

POS [1,3]	POS [3,1]	POS [2,2]
PIT	-	-
-	-	PIT
-	PIT	-
PIT	-	PIT
-	PIT	PIT
PIT	PIT	
PIT	PIT	PIT
WUMPUS	-	-
-	-	WUMPUS
-	WUMPUS	-
WUMPUS/ PIT	-	-
-	-	WUMPUS / PIT
-	WUMPUS / PIT	-
WUMPUS	-	PIT
WUMPUS	PIT	-
PIT	-	WUMPUS
-	PIT	WUMPUS
-	WUMPUS	PIT
PIT	WUMPUS	-
WUMPUS /PIT	-	PIT
PIT	-	WUMPUS / PIT
WUMPUS /PIT	PIT	-

PIT	WUMPUS / PIT	-
-	PIT	WUMPUS / PIT
-	WUMPUS / PIT	PIT
WUMPUS	PIT	PIT
PIT	PIT	W
PIT	WUMPUS	PIT
WUMPUS /PIT	PIT	PIT
PIT	PIT	WUMPUS / PIT
PIT	WUMPUS / PIT	PIT
-	-	-

b. We know that the Wumpus can be at [1, 3], [1, 2] as the stench is present in [1, 2]
We know that since there is a breeze at [1, 2], the pits could be at [2, 3], [1,3], or both.
Should the Wumpus be present at [2, 2] then there would exist a stench at [1,2] but this is not possible as per the Knowledge Base. The same could be applied to pit and no breeze at [2, 2] and [1, 2] respectively, this implies there won't be a pit at [2,2].
$\alpha 2$ is True when there is no pit at [2,2]
$\alpha 3$ is True when there is Wumpus at [1, 3]

From the Truth table given below, we can see that,
$KB \vdash \alpha 2$ and $KB \vdash \alpha 3$ are constrained to KB being true when both $\alpha 2$ and $\alpha 3$ is true.

POS[1,3]	POS [3,1]	POS [2,2]	α_2	α_3	KB
PIT	-	-	TRUE	FALSE	FALSE
-	-	PIT	FALSE	FALSE	FALSE
-	PIT	-	TRUE	FALSE	FALSE
PIT	-	PIT	FALSE	FALSE	FALSE
-	PIT	PIT	FALSE	FALSE	FALSE
PIT	PIT		TRUE	FALSE	FALSE
PIT	PIT	PIT	FALSE	FALSE	FALSE
WUMPUS	-	-	TRUE	TRUE	FALSE
-	-	WUMPUS	TRUE	FALSE	FALSE
-	WUMPUS	-	TRUE	FALSE	FALSE

WUMPUS/ PIT	-	-	TRUE	TRUE	FALSE
-	-	WUMPUS / PIT	FALSE	FALSE	FALSE
-	WUMPUS / PIT	-	TRUE	FALSE	FALSE
WUMPUS	-	PIT	FALSE	TRUE	FALSE
WUMPUS	PIT	-	TRUE	TRUE	TRUE
PIT	-	WUMPUS	TRUE	FALSE	FALSE
-	PIT	WUMPUS	TRUE	FALSE	FALSE
-	WUMPUS	PIT	FALSE	FALSE	FALSE
PIT	WUMPUS	-	TRUE	FALSE	FALSE
WUMPUS /PIT	-	PIT	FALSE	TRUE	FALSE
PIT	-	WUMPUS / PIT	FALSE	FALSE	FALSE
WUMPUS /PIT	PIT	-	TRUE	TRUE	FALSE
PIT	WUMPUS / PIT	-	TRUE	FALSE	FALSE
-	PIT	WUMPUS / PIT	FALSE	FALSE	FALSE
-	WUMPUS / PIT	PIT	FALSE	FALSE	FALSE
WUMPUS	PIT	PIT	FALSE	TRUE	FALSE
PIT	PIT	WUMPUS	TRUE	FALSE	FALSE
PIT	WUMPUS	PIT	FALSE	FALSE	FALSE
WUMPUS /PIT	PIT	PIT	FALSE	TRUE	FALSE
PIT	PIT	WUMPUS / PIT	FALSE	FALSE	FALSE
PIT	WUMPUS / PIT	PIT	FALSE	FALSE	FALSE
-	-	-	TRUE	FALSE	FALSE

c. $B \lor C$ is TRUE if:
Either B or C or both should be TRUE
$B \lor C$ to be TRUE the values of A and D don't matter.
In the truth table 12 / 16 rows of the tables entail "$B \lor C$" because such entries are true when "$B \lor C$" is true.

$\neg A \lor \neg B \lor \neg C \lor \neg D$ is TRUE if :
A, B, C, D must not be TRUE. (Totals models = 15).

¬A ∨ ¬B ∨ ¬C ∨ ¬D (15 models out of 16 entail this)
These combinations are True as and when ¬A ∨ ¬B ∨ ¬C ∨ ¬D is true.

$(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$
Has 0 models as no combinations of ABCD will entail
$(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

Exercise 1.2

a. $p \rightarrow q \wedge r \rightarrow s \vdash p \vee r \rightarrow q \vee s$
LHS: We need to show that LHS is true so that we can prove RHS to be true to show entailment
$p \rightarrow q \wedge r \rightarrow s$
both $p \rightarrow q,\ r \rightarrow s$ == True, True
when does this happen?

For $p \rightarrow q$ To be true:
p, q = False, True
p, q = False, False
p, q = True, True

For $r \rightarrow s$ To be True:
r, s = False, True
r, s = False, False
r, s = True, True

Here LHS will be true for any of the above-given values where both $p \rightarrow q$ and $r \rightarrow s$ are true.

Also for the conditions where LHS is true and RHS also turns out to be true, we say that LHS entails RHS.

Let's see an example of this

If p, q, r, s = False, True, False, True
LHS: $p \rightarrow q \wedge r \rightarrow s = True$
RHS: $p \vee r \rightarrow q \vee s = False \vee True \vee True = True$
LHS = RHS, LHS entails RHS

b. $(p \vee (q \rightarrow p)) \wedge q \vdash p$

LHS: We need to show that LHS is true so that we can prove RHS to be true to show entailment $(p \lor (q \to p)) \land q$

If LHS == True
q and $(p \lor (q \to p)) \land q$ both should be true.
$\left(p \lor (q \to p)\right) = p \lor (\neg q \lor p) = p \lor \neg q$

To prove LHS, we want both q and $p \lor \neg q$ to be true.
For this condition both q and p has to be true.
Hence LHS is only true iff p, q are true.

RHS: p
Which is true

So for LHS to entail RHS we want them both to be True.

c. For p⟶(q ⋁r) ⋀ q⟶s ⋀ r⟶s ⊢ p⟶s,

p⟶(q ⋁r) ⋀ q⟶s ⋀ r⟶s == True for the following conditions:

1. P, Q, R, S = TRUE, TRUE, FALSE, TRUE
2. P, Q, R, S = TRUE, FALSE, TRUE, TRUE
3. P, Q, R, S = FALSE, Any value, Any value, Any value.

Here we see that **S should be true, if P is True**

Hence for P, S = True, FALSE,
This condition does not satisfy
p⟶s
Which is same for "p⟶(q ⋁r) ⋀ q⟶s ⋀ r⟶s"
while for all other combinations from 1, 2, 3
p⟶(q ⋁r) ⋀ q⟶s ⋀ r⟶s entails p⟶s.

Exercise 1.4

a. I) *Here a is the agent*
b is policies
c is a person

∃ a Agent(a) ∧ ∀ b, c Policy (b) ∧ Sells (a, b, c) ⟹ Person (c) ∧ ¬ Insured (c)

for some agent a:

$\exists$ a Agent(a)

For all policies b, and for all person b
there exists a policy of b:
$\wedge \forall$ b, c Policy (b)

a-> sells policy of b to c:
$\wedge$ Sells (a, b, c) $\Longrightarrow$

c ->buys policy but is not insured:
Person (c) $\wedge \neg$ Insured (c)

II) **$\forall$ x Politician(x) $\Longrightarrow$ {($\exists$ y $\forall$ t Person(y) $\wedge$ Fools(x, y, t)) $\wedge$ ($\exists$ t $\forall$ y Person(y) $\Longrightarrow$ Fools(x, y, t)) $\wedge$ $\neg$ ($\forall$ t $\forall$ y Person(y) $\Longrightarrow$ Fools(x, y, t))}**

politician x:
$\forall$ x Politician(x) $\Longrightarrow$ {

x can fool some of the people all the time.
($\exists$ y $\forall$ t Person(y) $\wedge$ Fools(x, y, t)) $\wedge$

x fools all the people some of the time.
($\exists$ t $\forall$ y Person(y) $\Longrightarrow$ Fools(x, y, t)) $\wedge$

x can't fool all the people all of the time.
$\neg$ ($\forall$ t $\forall$ y Person(y) $\Longrightarrow$ Fools(x, y, t)) }

b. W(x): Wumpus is at location x
equal_loc(x, y): Location x, y are the same
ADJ_LOC(x,y): Location x, y are the same
SMELLY_LOC(x): Location x is smelly
breeze(x): Breeze is at location x
pit(x): There is a pit at location x

I) $\exists$loc_a $\forall$ loc_y W(loc_a) $\wedge$ ($\neg$equal_loc (loc_a, loc_y)$\leftrightarrow\neg$W(loc_y))
II) $\forall$loc_a $\forall$ loc_y W(loc)$\longrightarrow$(ADJ_LOC(x,y)$\leftrightarrow$SMELLY_LOC(loc_y))
III) $\forall$loc_a $\exists$ loc_y Breeze(loc_a)$\longrightarrow$ (ADJ_LOC (loc,y) $\wedge$ Pit(loc_y))

Exercise 1.5

a. Let us consider the following symbols:
1. B: It is a baby

2. L: It is logical
3. M: It can manage a crocodile
4. D: It is despised

Now let's consider the given KB:

$B \longrightarrow \neg L$ -> We can simplify this to $\neg B \vee \neg L$ -------- (1)

$M \longrightarrow \neg D$ -> We can simplify this to $\neg M \vee \neg D$ --------- (2)

$\neg L \longrightarrow D$ -> We can simplify this to $L \vee D$ ------------ (3)

From 1 and 3 --→ $\neg B \vee D$ -----------(4)

From equations 4 and 2 -> $\neg B \vee ⟦\neg M⟧$

We finally get $B \longrightarrow \neg M$

b. Knight(x): x is a knight
Knave(x): x is a knave
Spy(x): x is a spy

from the question we know that a knight does not lies this would imply that Cody is not the knight.

For not let's assume that Alex could be the liar then this would imply Alex is a knave.

If cody is a Spy then:

KC = Knight(Cody)
KnC = Knave(Cody)
SC = Spy(Cody)

$\neg$ KC $\wedge$ SC $\neg$ KnC $\wedge$ $\Leftrightarrow$True
$\neg$ Knave(Cody) $\Leftrightarrow$True -> Knave(Cody)$\Leftrightarrow$False

Now if Cody is a spy and also Alex is a knave then we have:
$\neg$Knight(Alex) $\Leftrightarrow$True

Knave(Alex) $\Leftrightarrow$True
$\neg$Spy(Alex) $\Leftrightarrow$True

Which is:

$\neg$Knight(Alex) $\wedge$ Knave(Alex) $\wedge$ $\neg$Spy(Alex)$\Leftrightarrow$True
$\neg$Knight(Alex)$\Leftrightarrow$True => Knight(Alex) -> False

If we add Ben as the knight then it will nullify the above-given logic
If we try Ben as knave and Alex as Knight:
Knight(Alex)$\Leftrightarrow$True

If alex says that Cody is knave -> ben is not(knave).
Since Ben can't be a knight -> Ben is the spy && Cody is a knave.

Therefor their true identities are Alex is the knight, Cody is the knave and Ben is the Spy.

Exercise 1.4

a. First Assumption = No wumpus at [3,1]
   Assumption 1 = $\neg W3,1$

   Since [1,1] is a valid and safe position,
   [1,1] has no Wumpus and pit

   =>Fact 1: $\neg W1,1 \wedge \neg P1,1$

   We find no stench in [1,1]=>
   [1, 2], [2, 1] does not contain a Wumpus.
   Reasoning 1: $\neg S1,1$ => Fact 2: $\neg S1,1 \Leftrightarrow (\neg W1,2 \wedge \neg W2,1)$

   No stench in [1,2]=> No Wumpus in [1, 3] & [2, 2].
   Reasoning 2: $\neg S1,2$ => Fact 3: $\neg S1,2 \Leftrightarrow (\neg W1,3 \wedge \neg W2,2)$

   [2, 1] has a stench present=> Wumpus present in [2, 2], [3, 1]
   Reasoning 3: $S2,1$ => Fact 4: $S\,2,1 \Leftrightarrow (W3,1 \vee W2,2)$

KB is Fact1 $\wedge$ Fact2 $\wedge$ Fact3 $\wedge$ $\neg$WUMPUS3,1

Throught the above fact we know that:
1.No WUMPUS at Location 1,1

2. Stench presentat 2,1 hence WUMPUS could be at [2,2] or [3,1] or [1,1]
3. point 1 says no WUMPUS at 1,1 hence WUMPUS could be at [2,2] or [3,1]

Through point 3 we no stench at 1,2 => No WUMPUS at [2,2] OR [1,3] OR [1,1

From elimination of the facts and reasoning

$\neg W1,1$ and $\neg P1,1$

We can sum up that $\neg B(1,2) \leftrightarrow \neg P(2,2)$,

$S2,1 \leftrightarrow \neg W2,2$. From $S2,1 \leftrightarrow W3,1$, we can say that Wumpus is in [3,1].

Hence KB $\vdash W3,$