

Theoretical Machine Learning

THEORETICAL ASSIGNMENT 1

Problem 1. Prove the following :

(a) A is an $m \times n$ matrix. \vec{x} is an $n \times 1$ vector. Then,

$$\frac{d}{d\vec{x}}[A\vec{x}] = A$$

(b) A is an $n \times n$ matrix, \vec{x} is an $n \times 1$ vector. Then,

$$\frac{d}{d\vec{x}}[\vec{x}^T A \vec{x}] = \vec{x}^T (A + A^T)$$

Problem 2. Suppose you have a matrix of dimension $m \times n$, and you differentiating wrt a $k \times 1$ vector, what is the dimension of the final result?

Problem 3. Solve :

$$(a) \frac{\frac{d}{d\begin{bmatrix} x \\ y \end{bmatrix}} \begin{bmatrix} 2\sin^2(x)\cos(y) \\ x^2 + 3e^y \end{bmatrix}}{\frac{d}{d\begin{bmatrix} x \\ y \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}}$$

$$(b) \frac{\frac{d}{d\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}} \begin{bmatrix} 3x^2y + xyzw \\ \sin(x^2 + yw - z) \end{bmatrix}}{\frac{d}{d\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}}$$

Problem 4. BONUS : β is an $n \times 1$ vector. \vec{x} is an $n \times 1$ vector. Solve for : $\frac{d}{d\vec{x}}[e^{\beta^T \vec{x}}]$