Theoretical Assignment 1 Solutions

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1 Problem 1. Solution

1.1 Problem 1(a).

LHS: $\frac{d}{d\vec{x}}[A\vec{x}]$. So,we can write $A\vec{x}$ as:

$$A\vec{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

Now, via differentiation $w.r.t\ \vec{x}$:

$$\frac{d\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}}{d\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}$$

$$\begin{bmatrix} \frac{d(a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n)}{dx_1} & \frac{d(a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n)}{dx_2} & \cdots + \frac{d(a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n)}{dx_n} \\ \frac{d(a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n)}{dx_1} & \frac{d(a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n)}{dx_2} & \cdots + \frac{d(a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n)}{dx_n} \\ \vdots & \vdots & \vdots \\ \frac{d(a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n)}{dx_1} & \frac{d(a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n)}{dx_1} & \cdots + \frac{d(a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n)}{dx_n} \end{bmatrix}$$

it then reduces to:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = A$$

H.P

1.2 Problem 1(b).

LHS: $\frac{d}{d\vec{x}}[\vec{x}^T A\vec{x}]$

$$\vec{x}^T A\vec{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\begin{bmatrix} (x_1a_{11} + x_2a_{21} + \cdots + x_na_{n1}) + \cdots + (x_1a_{1n} + x_2a_{2n} + \cdots + x_na_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$x_1 \sum_{i=1}^{n} x_i a_{i1} + x_2 \sum_{i=1}^{n} x_i a_{i2} + \cdots + x_n \sum_{i=1}^{n} x_i a_{in} = \sum_{j=1}^{n} \sum_{i=1}^{n} x_j x_i a_{ij} = F$$

It just reduces to Scalar $w.r.t$ Vector type differentiation:

$$\begin{bmatrix} \frac{dF}{dx_1} & \frac{dF}{dx_2} & \cdots & \frac{dF}{dx_n} \end{bmatrix}$$

$$\begin{bmatrix} 2x_1a_{11} + x_2(a_{21} + a_{12}) + \cdots + x_n(a_{n1} + a_{1n}) & \cdots & x_1(a_{1n} + a_{n1}) + x_2(a_{2n} + a_{n2}) + \cdots + 2x_na_{nn} \end{bmatrix}$$

$$\begin{bmatrix} x_1a_{11} + \cdots + x_na_{n1} & \cdots & x_1a_{1n} + \cdots + x_na_{nn} \end{bmatrix} + \begin{bmatrix} x_1a_{11} + \cdots + x_na_{1n} & \cdots & x_1a_{n1} + \cdots + x_na_{nn} \end{bmatrix}$$

it further reduces to :

$$\vec{x}^T(A + A^T)$$

H.P

2 Problem 2. Solution

Given: A Matrix M_{mxn} :

$$M = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\frac{d}{d\vec{x}}[M], \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}$$

So, its a Matrix *w.r.t* Vector differentiation :

$$\begin{bmatrix} \frac{d\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}}{d\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}} & \frac{d\begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix}}{d\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}} & \cdots & \frac{d\begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}}{d\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}} \end{bmatrix}$$

Now,it reduces to Vector *w.r.t* Vector type differentiation.Therefore, each differentiation will create a mxk matrix and overall there will be 'n' number of these mxk matrices.
Hence, overall order will be $m \times (nk)$.

3 Problem 3. Solution

3.1 3(a).

$$\frac{d\begin{bmatrix} 2\sin^2(x)\cos(y) \\ x^2 + 3e^y \end{bmatrix}}{d\begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} 2\sin(2x)\cos(y) & -2\sin^2(x)\sin(y) \\ 2x & 3e^y \end{bmatrix}$$

3.2 3(b).

$$\frac{d\begin{bmatrix} 3x^2y + xyzw \\ \sin(x^2 + yw - z) \end{bmatrix}}{d\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}} = \begin{bmatrix} \cos(x^2 + yw - z)(2x) & 6xy + yzw \\ \cos(x^2 + yw - z)(w) & 3x^2 + xzw \\ \cos(x^2 + yw - z)(-1) & xyw \\ \cos(x^2 + yw - z)(y) & xyz \end{bmatrix}^T$$

4 Problem 4. Solution

Given: $\beta = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ we have to find :

$$\frac{d}{d\vec{x}}[e^{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}] = \frac{d}{d\vec{x}}[e^{\begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}] = \frac{d}{d\vec{x}}[e^{b_1 x_1 + b_2 x_2 + \cdots + b_n x_n}]$$

It reduces to Scalar *w.r.t* Vector differentiation :
Let, $e^{b_1 x_1 + b_2 x_2 + \cdots + b_n x_n}$ be F

$$\begin{bmatrix} \frac{dF}{dx_1} & \frac{dF}{dx_2} & \cdots & \frac{dF}{dx_n} \end{bmatrix}$$

$$\begin{bmatrix} b_1 F & b_2 F & b_3 F & \cdots & b_n F \end{bmatrix}$$

So, the above matrix is the final result. We can expand F as usual!!