Problem 1:

(a) Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m*n}$$

and

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n*1}$$

$$A\vec{x} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}v_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}_{m*1}$$

Now,

$$\frac{d(A\vec{x})}{d\vec{x}} = \begin{pmatrix} \frac{d(a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n)}{dx_1} & \cdots & \frac{d(a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n)}{dx_n} \\ \vdots & \ddots & \vdots \\ \frac{d(a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n)}{dx_1} & \cdots & \frac{d(a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n)}{dx_n} \end{pmatrix}_{m*n}$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m*n} = A$$

Hence, proved.

(b) Make assumptions same as above but make m=n.
So, as done earlier

$$A\vec{x} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{pmatrix}_{n*1}$$

Now,

$$\vec{x}^T A\vec{x} =$$

$$\left(a_{11}x_1x_1 + a_{12}x_2x_1 + \cdots + a_{1n}x_nx_1 + a_{21}x_1x_2 + a_{22}x_2x_2 + \cdots + a_{2n}x_nx_2 + \cdots a_{n1}x_1x_n + a_{n2}x_2x_n + \cdots + a_{nn}x_nx_n\right)_{1*1}$$

Now, we have

$$\frac{d(\vec{x}^T A\vec{x})}{d\vec{x}} =$$

$$\begin{pmatrix} \frac{d(a_{11}x_1x_1+a_{12}x_2x_1+\cdots+a_{1n}x_nx_1+\cdots a_{n1}x_1x_n+a_{n2}x_2x_n+\cdots+a_{nn}x_nx_n)}{dx_1} & \cdots & \frac{d(a_{11}x_1x_1+a_{12}x_2x_1+\cdots+a_{1n}x_nx_1+\cdots a_{n1}x_1x_n+a_{n2}x_2x_n+\cdots+a_{nn}x_nx_n)}{dx_n} \end{pmatrix}_{1*n}$$

$$= \begin{pmatrix} (2*a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n)+ & (2*a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n)+ & \cdots & (2*a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n)+ \\ (a_{21}x_2 + a_{31}x_3 + \cdots + a_{n1}x_n) & (a_{12}x_1 + a_{32}x_3 + \cdots + a_{n2}x_n) & & (a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{(n-1)n}x_{n-1}) \end{pmatrix}_{1*n}$$

From RHS, we have

$$\vec{x}^T A = \begin{pmatrix} a_{11}x_1 + a_{21}x_2 + \cdots + a_{n1}x_n & a_{12}x_1 + a_{22}x_2 + \cdots + a_{n2}x_n & \cdots & a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{nn}x_n \end{pmatrix}_{1*n}$$

and

$$\vec{x}^T A^T = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \cdots & a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{pmatrix}_{1*n}$$

Adding both we get,

$$RHS = \begin{pmatrix} (a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n) & \cdots & (a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n) \\ +(a_{11}x_1 + a_{21}x_2 \cdots + a_{n1}x_n) & & +(a_{n1}x_1 + a_{1n}x_1 + \cdots + a_{(n-1)n}x_{n-1}) \end{pmatrix}_{1*n} = LHS$$

Hence, proved.

Problem 2:

Resulting matrix will be of order $m * (nk)$

Problem 3:

(a)

$$\begin{pmatrix} 4sin(x)cos(x)cos(y) & -2sin^2(x)sin(y) \\ 2x & 3e^y \end{pmatrix}$$

(b)

$$\begin{pmatrix} 6xy + yzw & 3x^2 + xzw & xyw & xyz \\ 2xsin(x^2 + yw - z) & wsin(x^2 + yw - z) & -sin(x^2 + yw - z) & ysin(x^2 + yw - z) \end{pmatrix}$$

Problem 4:

Let

$$\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}_{n*1}$$

and

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n*1}$$

So,

$$\beta^T \vec{x} = b_1 x_1 + b_2 x_2 + \cdots + b_n x_n$$

$$\frac{d(e^{\beta^T \vec{x}})}{d\vec{x}} = e^{\beta^T \vec{x}} \frac{d(\beta^T \vec{x})}{d\vec{x}} = e^{\beta^T \vec{x}} \begin{pmatrix} \frac{d(b_1x_1+b_2x_2+\cdots+b_nx_n)}{dx_1} & \frac{d(b_1x_1+b_2x_2+\cdots+b_nx_n)}{dx_2} & \cdots & \frac{d(b_1x_1+b_2x_2+\cdots+b_nx_n)}{dx_n} \end{pmatrix}_{1*n}$$

$$= e^{\beta^T \vec{x}} \begin{pmatrix} b_1 & b_2 & \cdots & b_n \end{pmatrix}_{1*n}$$

$$= \beta^T e^{\beta^T \vec{x}}$$

Hence,

$$\frac{d(e^{\beta^T \vec{x}})}{d\vec{x}} = \beta^T e^{\beta^T \vec{x}}$$