MA-101/1841

B.Tech. (Semester-I)Examination–2013

Mathematics-I

Time: Three Hours

Maximum Marks: **100**

Note: Attempt questions from all the sections.

Section–A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks. (4x10=40)

1. If $y = e^{\tan^{-1} x}$ prove that
$(1 + x^2)y_{n+2} + [2(n + 1)x - 1]y_{n+1} + n(n + 1)y_n = 0$
(2n+2)n-1 (2nn+2n-1)

2. Trace $y^2(a^2 + x^2) = x^2(a^2 - x^2)$

3. Expand $sin(xy)$ in powers of $(x - 1)$ and $(y - z/2)$ as far as the terms of second degree.

4. Discuss the maximum and minimum of
$x^2 + y^2 + 6x + 12$

5. Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $x^2/_{a^2} + y^2/_{b^2} + z^2/_{c^2} = 1$

6. The power P required topropel a steamer of length 'l' at a speed 'u' is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% & l is decreased by 1% find the corresponding increase in 'P'

4 7. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$$

4 8. Is the system of vectors $X_1 = (2,2,1)^T$, $X_2 = (1,3,1)^T$, $X_3 = (1,2,2)^T$ are linearly dependent

4 9. If $A = \begin{bmatrix} 2+3i & 1-2i & 2+4i \\ 3-4i & 4+3i & 2-6i \\ 5 & 5+6i & 3 \end{bmatrix}$

Find A^{θ} matrix

10. Evaluate $\int_0^2 \int_1^{e^x} dx\, dy$ by charging the order of integration.

11. Find the volume of the solid bounded by the parabolic $y^2 + z^2 = 4x$ and the plane $x = 5$.

12. Prove that $\lceil n \lceil 1-n = \pi/\sin n\pi (o < n < 1)$

4 13. Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point (1,1,1).

4 14. Find the divergence of $\vec{V}$ at (2,-1,1) where $\bar{V} = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 - y^2z)K$

4 15. Prove that $\bar{F}$ is irrational

Where $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xy + 2z)\hat{K}$

Section–B

(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. **(20x3=60)**

1. Verify Euler's theorem for $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$ and also prove that $\partial^2 u/\partial x dy = (x^2 - y^2)/(x^2 + y^2)$

15 10

15

2. If $x = r\, sin\theta cos\phi$

$y = rsin\theta sin\,\phi$

$z = rcos\,\theta$

Show that $\partial(x,y,z)/\partial(r,\theta,\phi) = r^2 sin\theta$.

3. Test for consistency the following system of equations & if consistent solve

$x_1 + 2x_2 - x_3 = 3$

$3x_1 - x_2 + 2x_3 = 1$

$2x_1 - 2x_2 + 3x_3 = 2$

$x_1 - x_2 + x_3 = -1$

$-1+8-4=3$

$x_3 = 4$

$4+8-x_3=3$

4. Find the characteristics equation of

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$x_1 - 4 + 4 = -1$

Verify Cayley Hamilton theorem and hence find A^{-1}.

5. Evaluate $\int\int\int x^2yz\, dx\, dy\, dz$ throughout the volume bounded by the planes

$x = 0, y = 0, z = 0\, x/a + y/b + z/c = 1$.

6. Verify divergence theorem, given that

$\bar{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface of the cube bounded by the planes

$x = 0, x = 1, y = 0, y = 1\; z = 0, z = 1$.

MA-101/1841-T-520

$2nx+2x$

$2(n+1)x$

A

Mathmatics (... Civil)

MA-101/1841

B. Tech. (Semester-I) Examination- 2012

Math-I

Time: Three Hours

Maximum Marks. 100

Note: Attempt questions from all the sections.

Section –A

Note: Attempt any ten questions. Each question carries four marks. (4x10=40)

1. Trace the curve $r^2 = a^2 \, Cos 2\theta$. Page 125

2. If $u = Sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$ Page 48

   Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -3\tan u$

3. If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that Page 54

   $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$

4. If u, v are functions of r, s where r, s are functions of x, y then prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$ Pg 17

5. Compute an approximate value of $(1.04)^{3.01}$

6. Find the extreme values of function $x^3 + y^3 - 3axy$ where $a < 0$. Page 208

7. Verify Cayley-Hamilton theorem for matrix

$$A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

8. Find the inverse of matrix by Employing Elementary transformation

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ Page. 252

9. Reduce matrix to normal form and find rank.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$$ Page - 258

10. Evaluate $\iint_R xy\,dxdy$ over the region bounded by $x = 0,\ y = 0,\ x + y = a$

11. Evaluate by changing into polar coordinates

$$\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$ Page 418

12. Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$ Page 512

13. State Gauss divergence theorem. Page 583

14. What is greatest rate of increase of $u = xyz^2$ at the point $(1,0,3)$?

15. For any closed surface S, prove that $\iint_s curl\,\vec{F}.\hat{n}\,ds = 0$ Page 58

Section -B

Note: Attempt any three questions. Each question carries twenty marks. **(20x3=60)**

1. (a) If $y = e^{m\cos^{-1}x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and calculate $y_n(0)$. Page 27

(b) Expand $\tan^{-1}\frac{y}{x}$ about $(1,1)$. Hence compute $f(1.1, 0.9)$ approximately.

2. (a) If u, v, w are the roots of cubic $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ Page 15

(b) The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of unit sphere $x^2 + y^2 + z^2 = 1$

(a) Determine the values of λ and μ such that system of equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$ has Page 282

(i) no solution

(ii) unique solution

(iii) infinite no. of solutions

(b) Find the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ [illegible] & [illegible] & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ Page 312

Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ Page 501

Evaluate by changing the order of integration

$\int [illegible] \int [illegible] \, e^{y} \, dx$ Page 408

[illegible]

[illegible] Page – 5[illegible]

Verify Gauss divergence theorem for [illegible] taken over the rectangle box [illegible]

[illegible] Page 577

H

B

MA-101

Mathematics - I

B. Tech. (Ist Semester) Exam. Dec.-2010

Time: Three Hours

Maximum Marks: 100

Note: Attempt all sections.

Section – A

Note: Attempt any ten questions. Each question carries four marks. **(4x10=40)**

1. Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.

399

2. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

3. Discuss the consistency of the following system of equations $2x+3y+4z=11$

$x+5y+7z=15,\ 3x+11y+13z=25$

4. If $y=\sin(m\sin^{-1}x)$, prove that

$(1-x^2)\,y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)\,y_n=0$

5. Find the expansion of the function $e^x\log(1+y)$ in a Taylor series in the neighborhood of the point (0,0).

6. Trace the curve $x^3+y^3=3axy$.

7. A balloon is in the form of right circular cylinder of radios 1.5 m & length 4 m & is surmounted by hemispherical ends. If the radius is increased by 0.01m & the length by 0.05 m. Find the percentage change in the volume of the balloon.

8. The temperature T at any point (x, y, z) in space is $T=400\,xyz^2$. Find the highest temp at the surface of a unit sphere $x^2+y^2+z^2=1$.

9. If $x+y+z=u$, $y+z=uv$, $z=uvw$ then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$

10. Find the volume of the cylindrical Column standing on the area common to the Parabolas $y^2=x$, $x^2=y$ and cut off by the surface $z=12+y-x^2$.

H. K. Dass (460) (17)

11. Evaluate H. K. Dass page (460)

Ex (10)

$$\int_0^{\pi}\int_0^{a(1-Cos\theta)} r^2\, Sin\theta\, dr\, d\theta$$

Ans $= \frac{4}{3} a^3$

12. Prove that H. K. Dass (465) (17) Ex

$$\iint_D x^{l-1} y^{m-1}\, dx\, dy = \frac{\Gamma l\, \Gamma m}{\Gamma l+m+1} h^{l+m}$$

When D is the domain $x \geq o, y \geq o \;\&\; x + y \leq h$

N. P. Bali 588

13. The vector field $\vec{F} = x^2\hat{\imath} + z\hat{\jmath} + yz\hat{k}$ is defined over the volume of the cuboid given by $o \leq x \leq a, o \leq y \leq b, o \leq z \leq c$ enclosing the surface S, evaluate by Gauss divergence theorem the surface integral $\iint_S \vec{F}.\vec{ds}$

Ans $= abc\left(a + \frac{b}{2}\right)$

H.K

14. A fluid motion is given by

668

$\vec{V} = (y+z)\hat{\imath} + (z+x)\hat{\jmath} + (x+y)\hat{k}$ show that the motion is irrotational.

15. Apply Green's theorem to evaluate

613

$\int_c [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C in the boundary of the area enclosed by the x axis and the upper half of circle $x^2+y^2=a^2$.

Ans $= \frac{4}{3}a^3$

Section - B

Note: Attempt any three questions. Each question carries twenty marks. (20x3=60)

1. Find the characteristic equation, Verify Clayey Hamilton theorem & then find A^{-1} of the matrix.

$$\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

2. If $u = \log(x^3+y^3+z^3-3xyz)$ then show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{-9}{(x+y+z)^2}$$

3. In a plane triangle ABC find the maximum value of Cos A Cos B Cos C.

4. By changing the order of integration, Evaluate.

$$\int_0^a \int_{y^2/a} \frac{y}{(a-x)\sqrt{ax-y^2}}\, dx\, dy.$$

5. Define gradient, divergence & curl. Find the divergence and curl of the vector function.

$$\vec{F}(x,y,z) = e^{xyz}(xy^2\hat{\imath} + yz^2\hat{\jmath} + zx^2\hat{k})$$ at the point (1, 2, 3).

6. Define the Eigen value & eigen vector of a matrix. Find the eigen values & eigen vectors of the matrix.

$$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

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A

MA-101/1841

B. Tech. (Semester-I) Examination-2014

Mathematics- I

Time: Three Hours] *[Maximum Marks:* **100**

Note: Attempt questions from all the sections.

Section-A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks (4x10=40)

1. Show that $A = \begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$ is skew Hermition matrix

2. Find the rank of matrix

$$\begin{bmatrix} 2 & 4 & 3 & 2 \\ -3 & -2 & -1 & 4 \\ 6 & -1 & [illegible] & [illegible] \end{bmatrix}$$

3. Using matrix method show that the equations $3x + 3y + 3z = 1$, [illegible] $2y = 1$, $10y + 3z = -2$ & $2x - 3y$ [illegible] are consistent

If $y = a\,Cos(log x) + b\,Sin(log x)$ then show that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$

If $\mu = e^{xyz}$ find the value of $\frac{\partial^3 \mu}{\partial x\, \partial y\, \partial z}$

6. Trace the curve $y^2(2a - x) = x^3$.

7. In estimating the number of bricks in a pile which is measured to be ($5m \times 10m \times 5m$) count of bricks is taken as 100 bricks per m^3. Find the error in the cost when the tope is stretched 2% beyond into standard length the cost of bricks is Rs 2000 per thousand bricks

8. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

9. Calculate the volume of the solid bounded by surface $x = 0, y = 0, x + y + z = 1$ & $z = 0$

10. Show that $\int_0^{\pi/2} (\sqrt{\cot\theta})\, d\theta = \frac{1}{2} \sqrt{1/4}\,\sqrt{3/4}$

11. Using Green's theorem evaluate $\int_C (x^2 y\, dx + x^2 dy)$ where C is the boundary described counter clock wise of the triangle with vertices (0,0) (1,0) (1,1)

12. Evaluate $\iint_S (yz\,i + zx\,j + xy\,k)\,ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.

13. Evaluate $\int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}\, dx\, dy$ by changing the order of integration.

14. Find the divergence of the vector field $\vec{V} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$

15. Use Lagrange's method of undetermined multipliers to find the minimum value of $x^2 + y^2 + z^2$ subject to the conditions $x + y + z = 1; xyz + 1 = 0$

Section-B

(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. **(20x3=60)**

1. Find the characteristics equation verify Cayley Hamilton theorem & hence find A^{-1} of matrix A $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

Find the force on all arms of the given strip (Beam) of the Bridge

2. Expand $x^2y + 3y - 2$ in powers of $(x - 1)(y + 2)$ using Taylor's theorem.

3. If $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$ show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.

4. Evaluate $\int_0^{\log x} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dz\, dy\, dx$

5. State the Gauss divergence theorem, verify the theorem for $\bar{F} = (x^3 - yz)i + (y^3 - zx)j + (z^3 - xy)\hat{k}$ taken over the cube found by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

6. Find the eigen values & eigen vectors of the matrix:

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Find the strength of the bridge

M.A.-101

B.Tech. (Semester-I) Examination – 2011

Math - I

Time: Three Hours

Maximum Marks: 100

Note: Attempt all the sections

Section-A

Note: Attempt any ten questions. Each question carries 4 marks. **(4x10=16)**

1. If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2y_2 + xy_1 + y = 0$$

2. Give the statement of Leibnitz's theorem and Euler's theorem.

3. Write the main steps involved during curve tracing of Cartesian equations.

4. Expand, $y = e^x$ by Taylor's theorem.

5. If $x^3 + 3x^2y + 6xy^2 + y^3 = 1$ find $\frac{dy}{dx}$.

6. Define skew symmetric matrix with example.

7. Show that the set of three vectors
$x_1 = (l,0,0), x_2(0,1,0), x_3(0,0,1)$
is linearly independent.

8. Prove that div $\left(\frac{\vec{r}}{r^3}\right)=0$

9. Show that the victor
$v = (\sin y + z)\, i + (x\cos y - z)\, j + (x - y)\, k$ is irrotational.

10. Write statement of Green's theorem.

11. If x=c cos u cosh v , y=c sin u sinh v
Prove that $\frac{\delta(x,y)}{\delta(u,v)} = \frac{1}{2}\, c^2 (\cos 2u - \cosh 2v)$

12. Evaluate
$$\int_0^2 \int_0^x \int_0^{x+y} ex\,(y + 2z)\,dx\,dy\,dz$$

13. Prove that $\sqrt{1/2} = \sqrt{\pi}$

14. Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

15. Define Beta and Gamma function.

Section-B

Note: Attempt any three questions. Each question carries 20 marks. **(20x3=60)**

MA-101-N-500

1. (a) Investigate the valu[illegible] system.

(637)
$$2x + 3y + 5z = [illegible]$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

has (i) unique [illegible]
(ii) No solu[illegible]
(iii) an infinit[illegible]

(b) Verify cayley Hamilton theore[illegible]

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$$

Also find A^{-1}.

2. (a) Trace the curve $r = a\cos 2Q$
(b) Trace the curve $9ay^2 = x(x - [illegible]$

3. (a) The period T of simple pendulum of len[illegible] given by $T = 2\pi \left(\frac{l}{g}\right)^{1/2}$ (20)

find (i) the error
(ii) percent error made in comput[illegible] T by using $l = 2ft$ and $g = 32ft/sec^2$
If the true values are $l = 1.95ft$ and $g = 32.2\ ft/sec^2$

(b) Find the n^{th} different[illegible] $x^{n-1}\log x$

... that (10+10)

(a) $\nabla^2 f(r) = f^{11}(r) + \frac{2}{r} f^1(r)$

(b) Show that

$$\int \text{Sin}^p\theta \, \text{Cos}^q\theta \, d\theta = \frac{\sqrt{\left(\frac{p+1}{2}\right)}\sqrt{\left(\frac{q+1}{2}\right)}}{2\sqrt{\frac{p+q+2}{2}}}$$

9.4.3

Expand by Maclaurin's theorem $e^x \log(1+x)$

(10+10)

Test the consistency of the following system and solve it.

$x - 2y - z = 5$

$x + 8y - 3z = -1$

$2x + y - 3z = 7$

N.P

Verify stroke's theorem for the vector (15+5)

$A = 3y\vec{i} - xz\,\vec{j} + yz^2\,\vec{k}$, Where

S is the surface of the paraboloid

$2z = x^2 + y^2$ bounded by $z = 2$ and c is boundary.

Evaluate

$$\int_0^{\log 2}\int_0^x\int_0^{x+y} e^{x+y+z}\,dxdydz$$

Roll No. .... [Total No. of Pages : 05

MA–101/1841

B. Tech. (EI) (First Semester)

EXAMINATION, 2019

MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : Attempt questions from both Sections as directed.

Section—A

(Short Answer Type Questions)

Note : Attempt any *ten* questions. Each questions carries 4 marks. 10×4=40

1. If $z = \dfrac{(x^2 + y^2)}{x + y}$, show that :

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

2. Use Maclaurin's theorem to prove :

$$e^x \sec x = 1 + x + \frac{2x^2}{2!} + \frac{4x^3}{3!} + ...$$

3. Find the points of inflexion of the curve :

$$y(a^2 + x^2) = x^3$$

4. Show that every square matrix can be uniquely expressed as a sum of a Hermitian and a skew Hermitian matrice.

5. Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$

6. Find whether the set of vectors :

$$\{(3, 2, 4), (1, 0, 2), (1, -1, -1)\}$$

are linearly independent.

7. If $x = r\cos\theta,\ y = r\sin\theta$, show that :

$$\frac{\partial(r, \theta)}{\partial(x, y)} = \frac{1}{r}$$

8. If

$$\mu = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right),$$

prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$$

9. Find the stationary points of the functions :

$$f(x, y) = xy(1 - x - y)$$

10. Evaluate $\iint x^2 y^2\, dx\, dy$ over the region $x^2 + y^2 \le 1$.

11. Evaluate the triple integral :

$$\iiint x^{l-1} y^{m-1} z^{n-1}\, dx\, dy\, dz$$

where $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \le 1$.

12. Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} = \pi$$

13. Show that :

$$\nabla^2\left(\frac{1}{r}\right)=0$$

14. Evaluate :

$$\int_C \vec{F}.d\vec{r},$$

where $\vec{F}=x^2y^3i+5yi$

and curve C is $y^2=4x$ in the *xy*-plane

point (0, 0) to (4, 4).

15. Show that :

$$F=(\sin y+z)i+(x\cos y-z)j+(x-y)k$$

is a conservative field.

Section—B

(Long Answer Type Questions)

Note : Attempt any *three* questions. Each ques

carries 20 marks. 3×20

1. If $y=e^{a\sin^{-1}x}$, show that :

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=$$

2. Find out for what value of λ , the equations :

$$x+y+z=1$$

$$x+2y+4z=\lambda$$

$$x+4y+10z=\lambda^2$$

have a solution and solve completely in each case.

3. Transform the following integral :

$$\int_0^a\int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dx\,dy$$

by changing to polar coordinates and hence deduce it.

4. Prove that :

$$\int_S (x^2i+y^2j+z^2k).nd\text{S}$$

vanishes, where S is the surface of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

5. Prove that every square matrix satisfies its own characteristic equation. Using it calculate :

$$2A^5 - 3A^4 + A^2 - 4I,$$

where $A = \begin{bmatrix} -3 & 1 \\ -1 & 2 \end{bmatrix}$.

6. Find the maximum value of $u = x^m y^n z^p$ subject to conditions $x + y + z = a$.

MA-101/1841

(A)

B.Tech (Semester-I) Exam.—2016

Mathematics-I

Time : Three Hours *Maximum Marks : 100*

Note : Attempt questions from all sections.

SECTION - A

(Short-answer Type Questions)

Note : Attempt any ten questions. Each question carries 4 marks. 10×4=40

1. Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.

2. Find the rank of the matrix

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

3. Show that the set/matrix

$$S = \{(1,0,0), (1,1,0), (1,1,1), (0,1,0)\}$$

is linearly dependent.

4. Show that the matrix

$$A = \begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$$

is skew Hermitran matrix

[P. T. O.

5. Trace the curve $x^3 + y^3 = 3axy$.

6. If $u = \tan^{-1}\left[\frac{y^3+y^3}{(x-y)}\right]$ prove that $x\frac{du}{dx} + y\frac{du}{dy} = \sin 2u$

7. Disuss the maximum and minimum of $x^2+y^2+6x+12$.

8. Find the expansion of the function $e^x\log(1+y)$ in a Taylor series in the neighbourhood of the point (0,0).

9. Find by triple integration, the volume of the paraboloid of revolution $x^2+y^2=4z$. Cut off by the plane $z = 4$

10. Prove that $B(m.n) = B(m+1,n) + B(m,n+1)$.

11. Evaluate $\int_2^0 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)dy.dx$

12. Evaluate $\iiint_R (x+y+z)\, dx\, dy\, dz$ where $R : 0 \le x \le 1,\ 1 \le y \le 2,\ 2 \le z \le 3$

13. Find the divergence of the vector field $\vec{V} = (x^2-y^2)\hat{i} + 2xy\hat{j} + (y^2-xy)\hat{k}$

14. Use Green's Theorem to evaluate $\int_C (x^2+xy)dx\ (x^2+y^2)\, dy$ Where C is the square formed by the lines $y = \pm 1$, $x = \pm 1$.

15. Given that $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ at $t = 2$ and $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ at $t = 3$ find $\int_2^3 \vec{r}\cdot\frac{d\vec{r}}{dt}\, dt$.

SECTION - B

(Long Answer type questions)

Note : Attempt any three questions. Each question carries 20 marks. 20x3=60

1. Find the characteristic equation, verify Cayley Hamilton Theorem & hence find A^{-1} of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$.

2. A square matrix A is defined by $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ Find the modal matrix P and the resulting diagonal matrix D of A.

[P. T. O.

3. (a) If $u(x+y) = x^2 + y^2$ then prove that $(du/dx - du/dy)^2 = 4\,(1 - du/dx - du/dy)$

(b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find the Jacobian of x,y,z with respect to r, θ, ϕ.

4. State Stoke's Theorem & Verify this theorem for

$$F = (x^2+y^2)\hat{i} - 2xy\hat{j}$$

taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$

5. Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int^{\sqrt{1-x^2y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\, dz\,dy\,dx$

6. Change the order of integration and evaluate.

$$\int_0^{\infty} \int_x^{\infty} (e^{-y}/y)\,dy\,dx$$

-0-

$$\int_0^{\infty} e^{-x} x^{n-1}\, dx = \Gamma n$$

$$\beta(l, m) = \frac{\Gamma l\, \Gamma m}{\Gamma m+l}$$

Abhijeet Singh

Singh

$A^2 = I$ Invol

$A^2 = A$ = Indem

$\bar{A} \cdot A = I$

$A \cdot A' = I$

$A = \bar{A}'$

MA-101/1841

B.Tech. (Semester-I) Exam–2017

Mathematics-I

Time: Three Hours

Maximum Marks: **100**

Note: Attempt questions from all the sections.

Section-A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks. **(4x10=40)**

1. Find the rank of the matrix:

$$\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ find two non-singular matrices P and Q such that PAQ=I.

3. Find the characteristic equation of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

4. If $x = \sin\left(\frac{\log y}{a}\right)$ then evaluate the value :
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

5. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ show that: 9 m8

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$.

6. Trace the curve:
$9ay^2 = x(x-3a)^2$, a>0.

7. Expand $e^x \sin y$ in powers of $x < y$, $x = 0$, $y = 0$ as far as terms of third degree by using Taylor's theorem.

8. If $x^2+y^2+u^2-v^2=0$ & $uv+xy=0$ then prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2-y^2}{u^2+v^2}$.

9. Evaluate: $\int_0^\infty \int_0^\infty e^{-x^2(1+y^2)} x\,dx\,dy$.

10. Evaluate: $\iint_A xy\,dx\,dy$ where A is the domain bounded by x-axis, ordinate $x = 2a$ & the curve $x^2 = 4ay$.

11. Prove that $\beta(l,m) = \frac{\sqrt{l}\sqrt{m}}{\sqrt{m+l}}$.

12. Find the directional derivative of $\varphi(x,y,z) = x^2yz + 4xz^2$ at (1, -2,1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Find the greatest rate of increase of φ.

13. If $\bar{A} = (3x^2+6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, Evaluate the line integral $\oint \bar{A}.d\bar{r}$ from (0,0,0) to (1,1,1) along the curve C. $x = t$, $y = t^2$, $z = t^3$. = 5

14. Find the divergence of :
$\bar{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at (2,-1,1).

15. In Estimating the number of bricks in a pile which is measured to be $(5m \times 10m \times 5m)$, Count of bricks is taken as 100 bricks per m^3. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is ₹2000/- per thousand bricks.

Section-B
(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. (20x3=60)

1. For what values of K the set of equations:
$2x - 3y + 6z - 5t = 3$
$y - 4z + t = 1$
$4x - 5y + 8z - 9t = K$
has (i) no solution (ii) infinite number of solutions.

2. If $u = \log(x^3+y^3+z^3-3xyz)$, show that :
$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

3. Change the order of integration and evaluate:

$$\int_0^a \int_0^y \frac{x\,dx\,dy}{\sqrt{(a^2 - x^2)(a - y)(y - x)}}$$

4. Evaluate: $\iiint \frac{dx\,dy\,dz}{(x + y + z + 1)^3}$ if the region of integration is bounded by the coordinate planes and the plane $x + y + z = 1$.

5. Verify the Gauss divergence theorem for $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

6. Find the Eigen values and Eigen vectors of the matrix given below:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Roll No. .... [Total No. of Pages : 6

MA–1841

B. Tech. (EI) (First Semester)

EXAMINATION, 2020

MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : Attempt questions from both Sections as directed.

Section—A

(Short Answer Type Questions)

Note : Attempt any *ten* questions. Each question carries 4 marks. $10 \times 4 = 40$

1. Find nth differential coefficient of $x^3 \cos x$.

2. If :

$$u = f\left(\frac{y}{x}\right),$$

show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

3. Explain $\log(1+x)$ by Maclaurin's theorem.

4. Prove that :

$$\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1.$$

5. The function :

$$u = x^2 + y^2 + 6x + 12$$

is minimum at what value ?

6. Transform :

$$\iint f(x,y)\,dx\,dy$$

to polar coordinate.

7. Find the whole area of the curve :

$$a^2y^2 = x^3(2a - x)$$

8. Evaluate :

$$\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$$

9. If $r = |\bar{r}|$, where :

$$\bar{r} = xi + yj + zk$$

prove that :

$$\nabla r^n = nr^{n-2}\bar{r}$$

10. Determine constant a so that vector :

$$V = (x+3y)\,i + (y-2z)\,j + (x+az)\,k$$

is solenoidal.

11. Evaluate :

$$\int_C F.dr$$

where $F = x^2 i + y^3 j$ and curve C is $y = x^2$ in xy-plane from (0, 0) to (1, 1).

12. Prove that matrix :

$$A = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$$

is unitary.

13. Find the rank of matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -4 & 1 \\ 3 & 6 & 5 \end{bmatrix}$$

14. Solve by Cramer's rule :

$$x + 2y = 2$$
$$3x - 4y = 11.$$

15. Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} = \pi$$

Section—B

(Long Answer Type Questions)

Note : Attempt any *three* questions. Each question carries 20 marks. 3×20=60

1. Verify Stokes' theorem for :

$$F = (x^2 + y^2)i - 2xyj$$

taken round the rectangle bounded by :

$$x = \pm a$$
$$y = 0$$
$$y = b.$$

2. Find the characteristic roots and corresponding characteristic vector of :

$$A = \begin{bmatrix} 4 & 1 & -2 \\ -1 & 2 & -5 \\ 1 & 1 & -5 \end{bmatrix}$$

3. Trace the curve :

$$y^2(a^2 + x^2) = x^2(a^2 - x^2)$$

4. Find the volume of the solid surrounded by the surface :

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1.$$

5. Show that the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is $\frac{8abc}{3\sqrt{3}}$.

6. If :

$$u = \sin^{-1} \frac{x + y}{\sqrt{x} + \sqrt{y}}$$

prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u .$$

360

A

MA-101/1841

B.Tech. (Semester-I) Exam.--2015

Mathematics - I

Time : Three Hours

Maximum Marks : 100

Note : Attempt questions from all sections.

SECTION - A

(Short-answer Type Questions)

Note : Attempt **any three** questions. Each question carries 20 marks. 20x3=60

1. Find the n^{th} derivative of $x^3 \cos x$.

X 2. Apply ~~machanism's~~ theorem to prove that

$$\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \text{----}$$

3. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that

$$x\frac{du}{dx} + y\frac{du}{dy} = \tan u$$

[P. T. O.

4. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

5. Test the consistency and solve

$5x+3y+7z=4$; $3x+26y+2z=9$; $7x+2y+10z=5$.

6. Evaluate $\int_0^3 \int_1^2 xy\,(1+x+y)\,dx\,dy$

7. To prove that $\beta\,(m.n) = \dfrac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$; $(m,n>0)$.

8. Find the percentage error in the area of an ellipse if 1% error is made in measuring the major and minor axis.

9. If $\vec{r} = xi + yj + zk$, show that grad $r^n = nr^{n-2}\,\vec{r}$

10. If $\vec{F} = (x+y+1)i + j - (x+y)\,k$, prove that $\vec{F}\cdot$ curl $\vec{F} = 0$

11. Prove that divgrad $r^m = m(m+1)\ r^{m-2}$

12. Show that $\nabla^2 f(r) = f'' + \dfrac{2}{r} + f'(r)$

13. Evaluate $\int_0^2 \int_0^x \int_0^{x+y} e^x\,(y+2z)\,dx\,dy\,dz$

14. If $\mu = e^{xyz}$ find the value of $\dfrac{d^3\mu}{dx\,dy\,dz}$

15. Trace curve $y^2\,(2a-x) = x^3$.

SECTION - B

(Long-answer Type Questions)

Note : Attempt **any three** questions. Each question carries 20 marks. 20x3=60

1. To prove that $\overline{|m}\ \overline{|m+\tfrac{1}{2}} = \dfrac{\sqrt{\pi}}{2^{m-1}}\,\overline{|2m}$ where $m>0$.

2. Evaluate $\iint_S \vec{A}\cdot\hat{n}\,ds$ where $\vec{A} = 18zi - 12j + 3yk$ and S is part (i.e. surface) of the plane $2x+3y+6z = 12$ which is in the first octant.

3. Find the eigen values and eigen vector, of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

4. Use Tarylor's theorem to prove that

$\tan^{-1}(x+h) = \tan^{-1}x + h\sin\theta\,\dfrac{\sin\theta}{1}\ (h\sin\theta)^2\,\dfrac{\sin 2\theta}{2} + (h\sin\theta)^3\,\dfrac{\sin 3\theta}{3}$ Where $\theta = \cot^{-1} x$

[P. T. O.

5. Find the maxima and minima of $u = x^2 + y^2 + z^2$ subject to the condition's $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$.

6. State divergence theorem and apply it to show that $\int\int_s \nabla (x^2+y^2+z^2) . ds = 6v$; where s is any closed surface enclosing volume v.

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