1 Find cofactors, determinant, adjoint and inverse of a matrix.

```
m: matrix(
 [4,5,7],
 [8,1,2],
 [0,6,4]
);
```

$$\begin{pmatrix} 4 & 5 & 7 \\ 8 & 1 & 2 \\ 0 & 6 & 4 \end{pmatrix}$$

1.1 adjoint

```
a:adjoint(m);
```

$$\begin{pmatrix} -8 & 22 & 3 \\ -32 & 16 & 48 \\ 48 & -24 & -36 \end{pmatrix}$$

1.2 cofactor

```
t:transpose(a);
```

$$\begin{pmatrix} -8 & -32 & 48 \\ 22 & 16 & -24 \\ 3 & 48 & -36 \end{pmatrix}$$

1.3 determinant

```
d:determinant(m);
```

$$144$$

1.4 inverse

```
i=(1/determinant(m))·adjoint(m);
```

$$i = \begin{pmatrix} -\left(\frac{1}{18}\right) & \frac{11}{72} & \frac{1}{48} \\ -\left(\frac{2}{9}\right) & \frac{1}{9} & \frac{1}{3} \\ \frac{1}{3} & -\left(\frac{1}{6}\right) & -\left(\frac{1}{4}\right) \end{pmatrix}$$

2 Convert the matrix into echelon form

and find its rank.

```
b:matrix([2,3,4],[5,6,7],[2,9,1]);
```

$$\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 2 & 9 & 1 \end{pmatrix}$$

```
ec:echelon(b);
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

```
rank(b);
```

$$3$$

```
b;
```

$$\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 2 & 9 & 1 \end{pmatrix}$$

```
b[1]:b[1]·1/2;
```

$$\left[1,\frac{3}{2},2\right]$$

```
b;
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 5 & 6 & 7 \\ 2 & 9 & 1 \end{pmatrix}$$

```
b[2]:b[2]−5·b[1];
```

$$\left[0,-\left(\frac{3}{2}\right),-3\right]$$

```
b;
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 0 & -\left(\frac{3}{2}\right) & -3 \\ 2 & 9 & 1 \end{pmatrix}$$

```
b[3]:b[3]-2·b[1];
```

$$[0,6,-3]$$

```
b;
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 0 & -\left(\frac{3}{2}\right) & -3 \\ 0 & 6 & -3 \end{pmatrix}$$

```
b[2]:b[2]·(-2/3);
```

$$[0,1,2]$$

```
b;
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 6 & -3 \end{pmatrix}$$

```
b[3]:b[3]-6·b[2];
```

$$[0,0,-15]$$

```
b;
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -15 \end{pmatrix}$$

```
b[3]:b[3]·(-1/15);
```

$$[0,0,1]$$

```
b;
```

$$\begin{pmatrix} 1 & \frac{3}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

3 Solve a system of equations using Gauss elimination method.

```
ab:augcoefmatrix([x+y+2·z=8,−x−2·y+3·z=1,3·x−7·y+4·z=10],[x,y,z]);
```

$$\begin{pmatrix} 1 & 1 & 2 & -8 \\ -1 & -2 & 3 & -1 \\ 3 & -7 & 4 & -10 \end{pmatrix}$$

3.1 REF FORM:

```
echelon(ab);
```

$$\begin{pmatrix} 1 & 1 & 2 & -8 \\ 0 & 1 & -5 & 9 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

RANK OF AB=3 & RANK OF A =3, HENCE AB MATRIX IS CONSISTENT

```
linsolve([x+y+2·z=8,y−5·z=−9,z=2],[x,y,z]);
```

$$[x=3,y=1,z=2]$$

IT HAS UNIQUE SOLUTIONS.

4 Solve a system of equations using the Gauss Jordan method.

```
ab:augcoefmatrix([w+x+2·y=1,2·w−x+z=−2,w−x−y−2·z=4,2·w−x+2·y−z=0],[w,x,y,z]);
```

$$\begin{pmatrix} 1 & 1 & 2 & 0 & -1 \\ 2 & -1 & 0 & 1 & 2 \\ 1 & -1 & -1 & -2 & -4 \\ 2 & -1 & 2 & -1 & 0 \end{pmatrix}$$

changing it into RREF form through Row operations.

d:ab;

$$\begin{pmatrix} 1 & 1 & 2 & 0 & -1 \\ 2 & -1 & 0 & 1 & 2 \\ 1 & -1 & -1 & -2 & -4 \\ 2 & -1 & 2 & -1 & 0 \end{pmatrix}$$

d[2]:d[2]−2·d[1];

$$[0,-3,-4,1,4]$$

d[3]:d[3]−d[1];

$$[0,-2,-3,-2,-3]$$

d[4]:d[4]−2·d[1];

$$[0,-3,-2,-1,2]$$

d;

$$\begin{pmatrix} 1 & 1 & 2 & 0 & -1 \\ 0 & -3 & -4 & 1 & 4 \\ 0 & -2 & -3 & -2 & -3 \\ 0 & -3 & -2 & -1 & 2 \end{pmatrix}$$

d[2]:d[2]/(−3);

$$\left[0,1,\frac{4}{3},-\left(\frac{1}{3}\right),-\left(\frac{4}{3}\right)\right]$$

d[3]:d[3]+2·d[2];

$$\left[0,0,-\left(\frac{1}{3}\right),-\left(\frac{8}{3}\right),-\left(\frac{17}{3}\right)\right]$$

d[4]:d[4]+3·d[2];

$$[0,0,2,-2,-2]$$

d[3]:d[3]·(−3);

$$[0,0,1,8,17]$$

d[1]:d[1]−d[2];

$$\left[1,0,\frac{2}{3},\frac{1}{3},\frac{1}{3}\right]$$

d[4]:d[4]−2·d[3];

$[0,0,0,-18,-36]$

```
d[2]:d[2]-(4/3)·d[3];
```

$[0,1,0,-11,-24]$

```
d[1]:d[1]-(2/3)·d[3];
```

$[1,0,0,-5,-11]$

```
d[4]:d[4]/(-18);
```

$[0,0,0,1,2]$

```
d[3]:d[3]-8·d[4];
```

$[0,0,1,0,1]$

```
d[2]:d[2]+11·d[4];
```

$[0,1,0,0,-2]$

```
d[1]:d[1]+5·d[4];
```

$[1,0,0,0,-1]$

```
d;
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

rank of aug ab = 3 and rank of b = 3 hence it is consistent.

```
linsolve([w=1,x=2,y=2,z=-2],[w,x,y,z]);
```

$[w=1,x=2,y=2,z=-2]$

Hence it has unique solutions.

5 Verify the linear dependence of vectors. Generate a linear combination of given vectors of R^n/ matrices of the same size.

```
v1:[1,2,1];
v2:[2,1,−4];
v3:[3,−2,1];
```

[1 ,2 ,1]   [2 ,1 ,− 4]   [3 ,− 2 ,1]

```
load("C:\\Users\\Anmol Goel\\Desktop\\New folder\\mbe5.mac");
```

(%o5) "C:\\Users\\Anmol Goel\\Desktop\\New folder\\mbe5.mac"

```
display2d:true;
m:mcombine([matrix(v1),matrix(v2),matrix(v3)]);
```

true $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{pmatrix}$

```
length(transpose(m));
```

3

```
rank(m);
```

3

```
s:columnspan(m);
```

$\operatorname{columnspan}\left(\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{pmatrix}\right)$

```
[s1,s2,s3]:5,span="[";
```

[5,5,5]

```
v:[7,1,9];
```

[7,1,9]

```
t:transpose(v);
```

$\begin{pmatrix} 7 \\ 1 \\ 9 \end{pmatrix}$

```
arraym:addcol(m,t);
```

$\begin{pmatrix} 1 & 2 & 3 & 7 \\ 2 & 1 & -2 & 1 \\ 1 & -4 & 1 & 9 \end{pmatrix}$

```
rank(m);
```

3

```
rank(arraym);
```

3

6 Check the diagonalizable property of matrices and find the corresponding eigenvalue and verify the CayleyHamilton theorem

```
B:matrix([2,2,1],[1,3,1],[1,2,2]);
```

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

```
[evals, evecs]:eigenvectors(B);
```

$$\left[[[5,1],[1,2]],\left[[[1,1,1]],[[1,0,-1],[0,1,-2]]\right]\right]$$

```
load("C:/Users/PC1/Downloads/mbe5.mac")$
display2d:true;
```

true

```
diagp(B);
```

true

```
[v1]:jordan_chain(B,5);
```

[[1,1,1]]

```
[v2,v3]:jordan_chain(B,1);
```

[[1,0,−1],[0,1,−2]]

```
P:mcombine([matrix(v1),matrix(v2),matrix(v3)]);
```

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{pmatrix}$$

```
P1:invert(P);
```

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{3}{4} & -\left(\frac{1}{2}\right) & -\left(\frac{1}{4}\right) \\ -\left(\frac{1}{4}\right) & \frac{1}{2} & -\left(\frac{1}{4}\right) \end{pmatrix}$$

D:P1.B.P;

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

charpoly(B,x);

$$x+((2-x)(3-x)-2)(2-x)-2(1-x)-1$$

expand(charpoly(B,x));

$$-x^3+7x^2-11x+5$$

−B^^3+7·B^^2−11·B+5·ident(3);

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

7 Compute Gradient of a scalar field, Divergence and Curl of a vector field.

load(vect);

C:/maxima−5.47.0/share/maxima/5.47.0/share/vector/vect.mac

express(grad(x^2+y^2+z^2));

$$\left[\frac{d}{dx}(z^2+y^2+x^2),\frac{d}{dy}(z^2+y^2+x^2),\frac{d}{dz}(z^2+y^2+x^2)\right]$$

ev(express(grad(x^2+y^2+z^2)),diff);

$$[2x,2y,2z]$$

express(div([x^2,y^2,z^2]));

$$\frac{d}{dz}z^2+\frac{d}{dy}y^2+\frac{d}{dx}x^2$$

ev(express(div([x^2,y^2,z^2])),diff);

$$2z+2y+2x$$

```
express(curl([x^2,y^2,z^2]));
```

$$\left[\frac{d}{d\,y}z^2-\frac{d}{d\,z}y^2,\frac{d}{d\,z}x^2-\frac{d}{d\,x}z^2,\frac{d}{d\,x}y^2-\frac{d}{d\,y}x^2\right]$$

```
ev(express(curl([x^2,y^2,z^2])),diff);
```

$$[0,0,0]$$

```
express(laplacian(x^2·y^2·z^2));
```

$$\frac{d^2}{d\,z^2}(x^2\,y^2\,z^2)+\frac{d^2}{d\,y^2}(x^2\,y^2\,z^2)+\frac{d^2}{d\,x^2}(x^2\,y^2\,z^2)$$

```
ev(express(laplacian(x^2·y^2·z^2)),diff);
```

$$2\,y^2\,z^2+2\,x^2\,z^2+2\,x^2\,y^2$$