Introduction to Financial Engineering

Assignment - 4

Group 5

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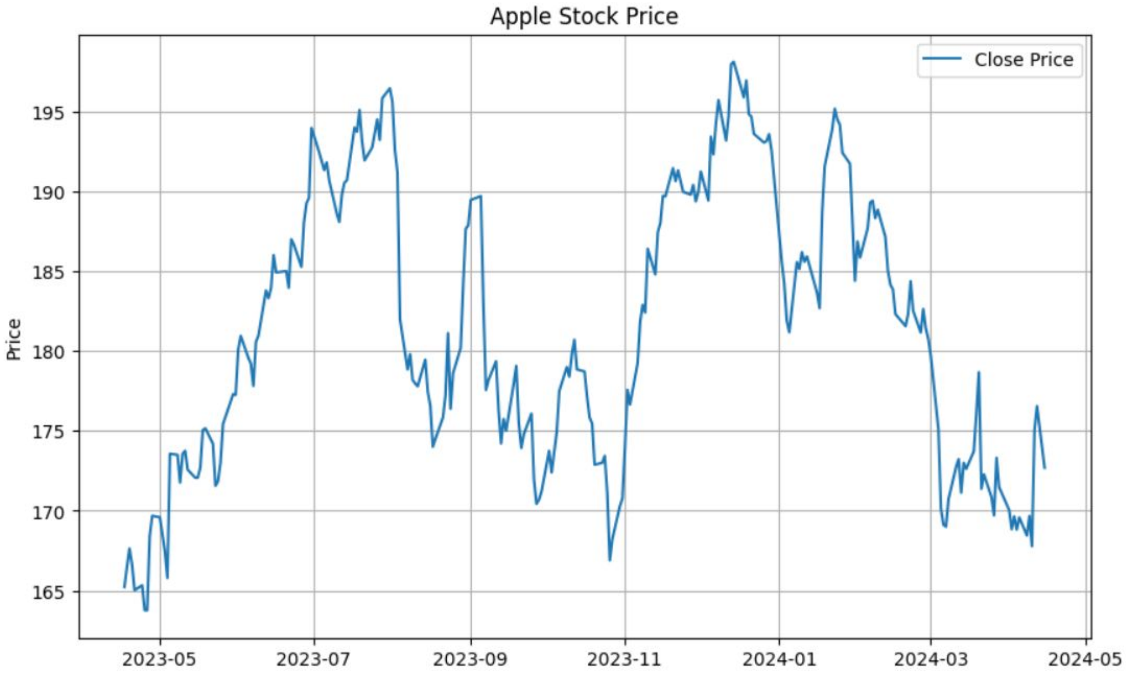
Stock Chosen

- Apple Stock chosen for the purpose of this assignment.

- Data was collected from yahoo Finance for the 1 year start="2023-04-17", end="2024-04-16" . Therefore Data consists of 252 entries.

Variation in Apple Stock Price



Calculation of Annual Volatility

Daily return is calculated & Standard Deviation is calculated

$$\text{Annual Volatility} = \sigma \times \sqrt{252}$$

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where:

- r_t = Daily return for day t
- P_t = Closing price on day t
- P_{t-1} = Closing price on the previous day

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(r_i - \bar{r})^2}{n-1}}$$

Where:

- σ = Standard deviation
- r_i = Daily return for day i
- $\bar{r}$ = Mean of the daily returns
- n = Total number of observations (usually the number of trading days in a year, typically around 252 for stock markets)

US Treasury Rate

It is risk free interest rates (r)

which is `risk_free_rate = 0.0452`

Binomial Model

The binomial model is a mathematical model used in finance to price options by considering the possible future price movements of an underlying asset. The most common application of the binomial model is in pricing options such as American and European options.

$$C = \sum_{j=0}^{N} \left(\binom{N}{j} \times p^j \times (1-p)^{N-j} \times \max(S - X, 0) \right)$$

Where:

- C = Option price (often the call option price)
- N = Number of time steps or periods in the model
- S = Current price of the underlying asset
- X = Strike price of the option
- p = Probability of an up movement in the asset price from one period to the next
- $(1-p)$ = Probability of a down movement in the asset price from one period to the next
- $\binom{N}{j}$ = Binomial coefficient, which represents the number of ways to choose j successes out of N trials

For different strike prices and time of maturity to evaluate the call/ put option price by using Binomial model

```
Strike Price 170
Time of maturity 0.5
Call Option Price: 13.070615361165098
Put Option Price: 6.716711744061004
Strike Price 175
Time of maturity 0.75
Call Option Price: 13.521722241204538
Put Option Price: 10.133658399877637
Strike Price 180
Time of maturity 1
Call Option Price: 14.21686559285744
Put Option Price: 13.707006854830881
Strike Price 185
Time of maturity 1.25
Call Option Price: 14.67822612037065
Put Option Price: 16.9605327519727
Strike Price 190
Time of maturity 1.5
Call Option Price: 14.97609154084787
Put Option Price: 19.966093968940783
Strike Price 192
```

Black Scholes Model

The Black-Scholes model is a widely used mathematical model for pricing European options. It provides a theoretical estimate of the price of options over time, taking into account various factors such as the current price of the underlying asset, the option's strike price, the time until expiration, the risk-free interest rate, and the volatility of the underlying asset's returns.

$$C = S_0 N(d_1) - X e^{-rT} N(d_2)$$

Where:

- C = Call option price
- S_0 = Current price of the underlying asset
- X = Strike price of the option
- r = Risk-free interest rate
- T = Time to expiration (in years)
- $N(d_1)$ and $N(d_2)$ = Cumulative distribution functions of the standard normal distribution, calculated as:
  $N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\frac{x^2}{2}} dx$
  d_1 and d_2 are calculated as:
  $d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$
  $d_2 = d_1 - \sigma\sqrt{T}$
- σ = Volatility of the underlying asset's returns

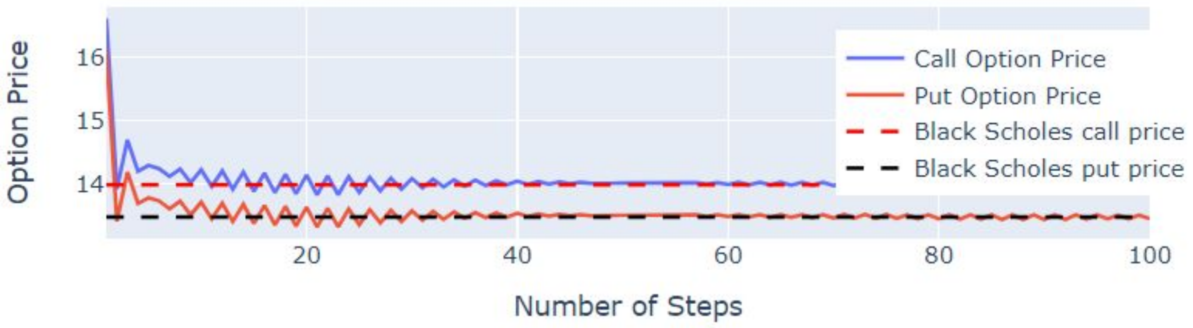
Evaluate the option price by using Black Scholes Formula

```
Strike Price 170
Time of maturity 0.5
Call Option Price (Black-Scholes): 13.05855728808919
Put Option Price (Black-Scholes): 6.704653670984968
Strike Price 175
Time of maturity 0.75
Call Option Price (Black-Scholes): 13.56258261303509
Put Option Price (Black-Scholes): 10.174518771708165
Strike Price 180
Time of maturity 1
Call Option Price (Black-Scholes): 13.981104381843181
Put Option Price (Black-Scholes): 13.471245643816701
Strike Price 185
Time of maturity 1.25
Call Option Price (Black-Scholes): 14.347978270859869
Put Option Price (Black-Scholes): 16.630284902462037
Strike Price 190
Time of maturity 1.5
Call Option Price (Black-Scholes): 14.683150074281485
Put Option Price (Black-Scholes): 19.673152502374492
```

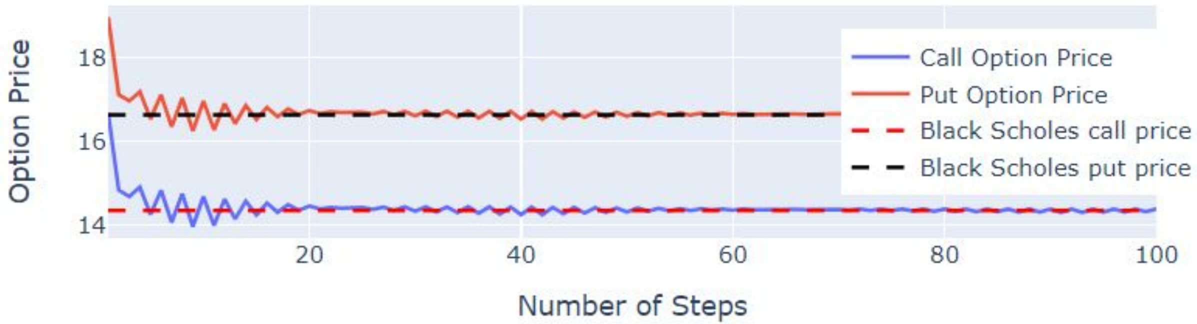
Increase the number of steps in Binomial model to verify that the price converges

```
Strike Price 170
Time of maturity 0.5
Call Option Price (Binomial Model with 100 steps): 13.08103829408337
Put Option Price (Binomial Model with 100 steps): 6.727134676978432
Strike Price 175
Time of maturity 0.75
Call Option Price (Binomial Model with 100 steps): 13.588281057593095
Put Option Price (Binomial Model with 100 steps): 10.200217216264276
Strike Price 180
Time of maturity 1
Call Option Price (Binomial Model with 100 steps): 13.95793892913785
Put Option Price (Binomial Model with 100 steps): 13.448080191110945
Strike Price 185
Time of maturity 1.25
Call Option Price (Binomial Model with 100 steps): 14.379612859347409
Put Option Price (Binomial Model with 100 steps): 16.661919490948748
Strike Price 190
Time of maturity 1.5
Call Option Price (Binomial Model with 100 steps): 14.643505865152951
Put Option Price (Binomial Model with 100 steps): 19.633508293244795
```

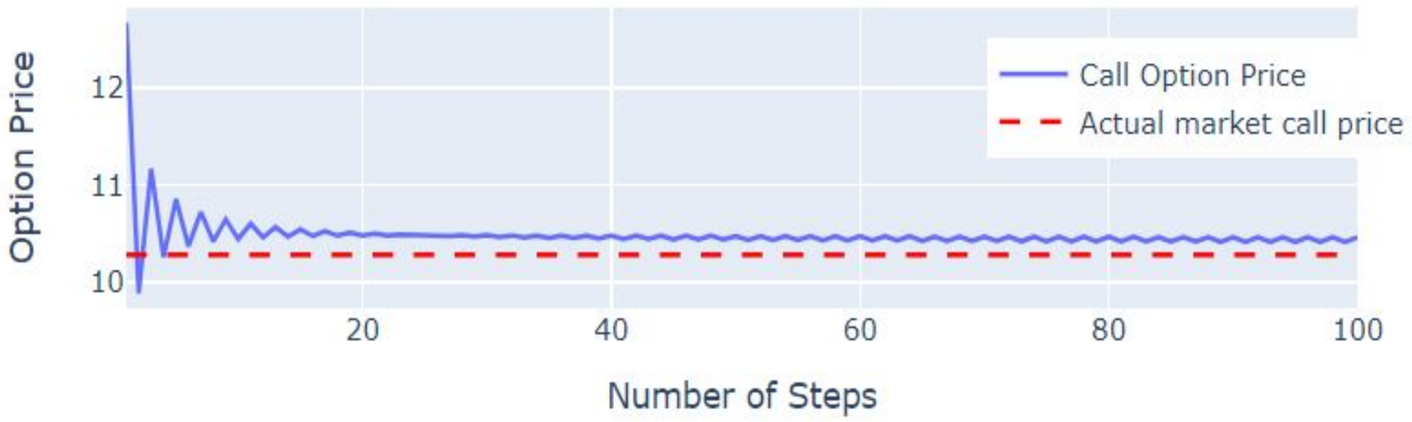
Convergence of Option Price for K = 180 and T = 1



Convergence of Option Price for K = 185 and T = 1.25



Calculated Option Price vs Actual Market Values for K = 175 and T = 0.5 year



Delta Neutral Portfolio

A delta-neutral portfolio is a portfolio of options and/or other financial instruments that has a delta value of zero, or very close to zero. Delta measures the sensitivity of the option price to changes in the price of the underlying asset.

Strike (K)	Time to Maturity (T in days)	Risk-Free Rate (r)	Volatility (σ)	Call Delta	Portfolio (number of put options)
30.0	23	0.0452	0.2	0.999653	28814.072628
55.0	36	0.0452	0.2	0.998198	5540.714356
60.0	23	0.0452	0.2	0.996200	2621.532644
70.0	37	0.0452	0.2	0.996714	3033.190207
90.0	65	0.0452	0.2	0.998938	9405.599308
255.0	36	0.0452	0.2	0.948733	185.056863
270.0	22	0.0452	0.2	0.852025	57.578943
285.0	37	0.0452	0.2	0.941057	159.655609
290.0	57	0.0452	0.2	0.984663	642.026649

Implied Volatility

Implied volatility (IV) is a measure of the market's expectation for future volatility of the underlying asset's price, as implied by the prices of options on that asset. It represents the level of volatility that is implied by the current option prices in the market.

K	C	T	S0	r	Implied Volatility(%)
30.0	142.65	23	173.309998	0.0452	0.000000
55.0	118.80	36	173.000000	0.0452	0.000000
60.0	112.60	23	173.309998	0.0452	0.000000
70.0	101.60	37	171.130005	0.0452	122.382022
90.0	94.81	65	184.149994	0.0452	0.000000
255.0	0.02	36	173.000000	0.0452	43.418295
270.0	0.01	22	171.479996	0.0452	60.461879
285.0	0.01	37	171.130005	0.0452	51.741371
290.0	0.01	57	184.369995	0.0452	37.041701