Assignment 2
Introduction to Financial Engineering (MAL4330)

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Problem Statement

Choose any 10 risky assets from the market. These could be stocks, bonds, ETFs, or any other investable assets with readily available price data. Gather the closing prices for your chosen assets over the past 3 months. Calculate the simple/log returns for each asset over the chosen period. Apply Markowitz's mean-variance optimization to construct the efficient frontier. Choose two points on the efficient frontier representing two different risk tolerance levels. For each point, calculate the corresponding weights for each asset to construct a portfolio that maximizes expected return for that given level of risk.

Data Set

We have chosen the stock price of 10 companies that are listed on the Bombay Stock Exchange (BSE). These companies' data was taken for a 3-month period, starting from July 1st, 2023, to September 30th, 2023. In this data set, we have six different pieces of information: closing price, opening price, high price, low price, market volume, and percentage change. Out of these, the closing price was used for modeling purposes. In this data set, we have a total of 63 entries, and the selected ten companies are: HCLTech, TCS, INFOSYS, SBI, HDFC Bank, ICICI Bank, Hindustan Unilever, Bajaj Finance, Adani Enterprises, and Larsen & Toubro.

Markowitz's portfolio optimization Theory

Markowitz portfolio optimization is a practical method for selecting investments in order to maximize their overall returns within an acceptable level of risk. This mathematical framework is used to build a portfolio of investments that maximize the amount of expected return for the collective given level of risk. Harry Markowitz pioneered this theory in his paper "Portfolio Selection," which was published in the Journal of Finance in 1952.

A key component of the MPT theory is diversification. Most investments are either high risk and high return or low risk and low return. Markowitz argued that investors could achieve their best results by choosing an optimal mix of the two based on an assessment of their individual tolerance to risk.

A portfolio constructed from n different securities can be described in terms of their weights

$$w_i = \frac{x_i S_i(0)}{V(0)}, \quad i = 1, \ldots, n,$$

where x_i is the number of shares of type i in the portfolio, $S_i(0)$ is the initial price of security i, and $V(0)$ is the amount initially invested in the portfolio. It will prove convenient to arrange the weights into a one-row matrix

$$\boldsymbol{w} = \left[\begin{array}{cccc} w_1 & w_2 & \cdots & w_n \end{array} \right].$$

Just like for two securities, the weights add up to one, which can be written in matrix form as

$$1 = \boldsymbol{u}\boldsymbol{w}^T,$$

where

$$\boldsymbol{u} = \left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array} \right]$$

is a one-row matrix with all n entries equal to 1, $\boldsymbol{w}^T$ is a one-column matrix, the transpose of $\boldsymbol{w}$, and the usual matrix multiplication rules apply. The *attainable set* consists of all portfolios with weights $\boldsymbol{w}$ satisfying (5.14), called the *attainable portfolios*.

Suppose that the returns on the securities are $K_1, \ldots, K_n$. The expected returns $\mu_i = E(K_i)$ for $i = 1, \ldots, n$ will also be arranged into a one-row matrix

$$\boldsymbol{m} = \left[\begin{array}{cccc} \mu_1 & \mu_2 & \cdots & \mu_n \end{array} \right].$$

The covariances between returns will be denoted by $c_{ij} = \mathrm{Cov}(K_i, K_j)$. They are the entries of the $n \times n$ *covariance matrix*

$$\boldsymbol{C} = \left[\begin{array}{cccc} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{array} \right].$$

The expected return $\mu_V = E(K_V)$ and variance $\sigma_V^2 = \mathrm{Var}(K_V)$ of a portfolio with weights $\boldsymbol{w}$ are given by

$$\mu_V = \boldsymbol{m}\boldsymbol{w}^T,$$
$$\sigma_V^2 = \boldsymbol{w}\boldsymbol{C}\boldsymbol{w}^T.$$

We need to find the minimum of risk(variance of portfolio) subject to the constraint that sum of weights is unity . To this end we can use the method of Lagrange multipliers. Let us put

$$F(\boldsymbol{w}, \lambda) = \boldsymbol{w}\boldsymbol{C}\boldsymbol{w}^T - \lambda \boldsymbol{u}\boldsymbol{w}^T,$$

where λ is a Lagrange multiplier. Equating to zero the partial derivatives of F with respect to the weights w_i we obtain $2\boldsymbol{w}\boldsymbol{C} - \lambda\boldsymbol{u} = 0$, that is,

$$\boldsymbol{w} = \frac{\lambda}{2}\boldsymbol{u}\boldsymbol{C}^{-1},$$

which is a necessary condition for a minimum. Substituting this into constraint (5.14) we obtain

$$1 = \frac{\lambda}{2}\boldsymbol{u}\boldsymbol{C}^{-1}\boldsymbol{u}^T,$$

where we use the fact that $\boldsymbol{C}^{-1}$ is a symmetric matrix because $\boldsymbol{C}$ is. Solving this for λ and substituting the result into the expression for $\boldsymbol{w}$ will give the asserted formula.

Results :

Covariance matrix of assets are as follows

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cov_matrix
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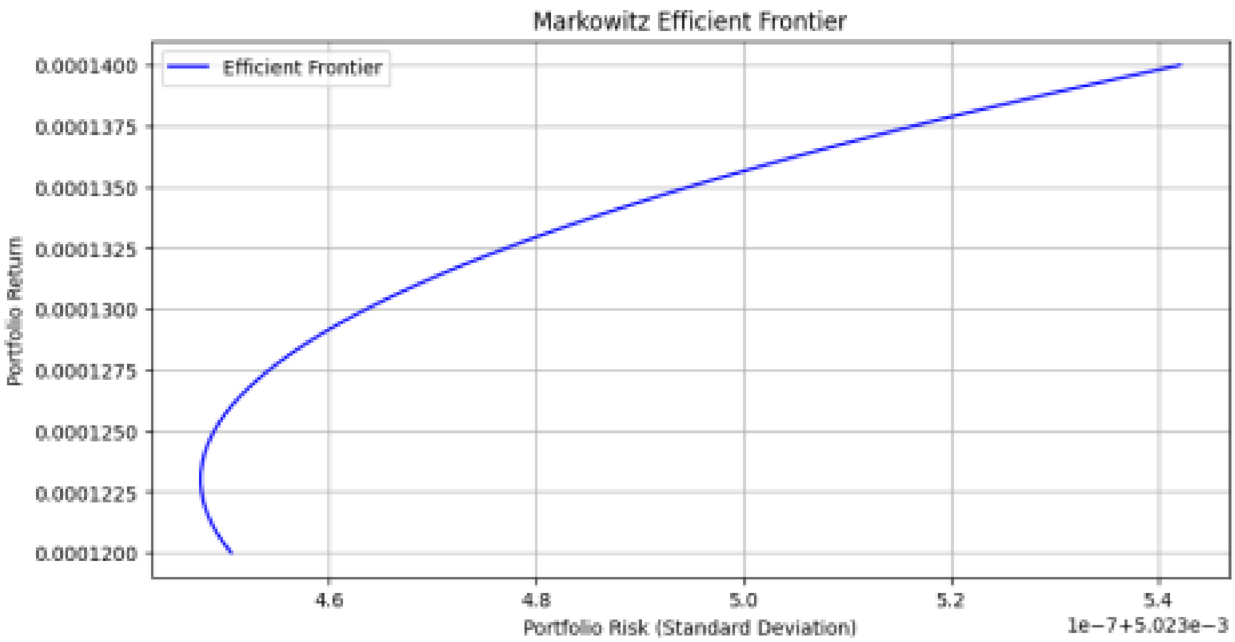
	Return_1	Return_2	Return_3	Return_4	Return_5	Return_6	Return_7	Return_8	Return_9	Return_10
Return_1	0.000293	0.000007	0.000022	0.000002	0.000003	0.000029	-0.000020	-0.000019	-0.000027	0.000014
Return_2	0.000007	0.000215	0.000030	-0.000003	0.000023	0.000006	0.000021	-0.000012	0.000036	0.000014
Return_3	0.000022	0.000030	0.000102	0.000028	0.000019	0.000040	0.000027	0.000026	0.000020	-0.000008
Return_4	0.000002	-0.000003	0.000028	0.000247	-0.000046	0.000115	0.000123	-0.000021	0.000010	0.000066
Return_5	0.000003	0.000023	0.000019	-0.000046	0.000124	0.000005	-0.000008	0.000031	0.000029	-0.000002
Return_6	0.000029	0.000006	0.000040	0.000115	0.000005	0.000171	0.000099	0.000014	0.000010	0.000031
Return_7	-0.000020	0.000021	0.000027	0.000123	-0.000008	0.000099	0.000134	0.000011	0.000025	0.000022
Return_8	-0.000019	-0.000012	0.000026	-0.000021	0.000031	0.000014	0.000011	0.000149	0.000021	-0.000009
Return_9	-0.000027	0.000036	0.000020	0.000010	0.000029	0.000010	0.000025	0.000021	0.000073	0.000015
Return_10	0.000014	0.000014	-0.000008	0.000066	-0.000002	0.000031	0.000022	-0.000009	0.000015	0.000085

In the modeling process, The numbering of the stocks are as follows: Adani Enterprises, Bajaj Finance, HDFC Bank, Infosys, SBI, HCL Tech, TCS, Larsen & Toubro, ICICI

Bank, and Hindustan Unilever. And corresponding to these we got the weights for minimum risk portfolio.

The weights for minimum risk portfolio are:
[0.09521674 0.0439849 0.11505803 0.01426402 0.11790629 0. 0.07855369 0.11885617 0.17216998 0.24399019]



Limitations of markowitz optimizations & its real world applications

Assumption of Normal Distribution: Markowitz optimization assumes that asset returns follow a normal distribution. However, in reality, asset returns often exhibit non-normal distributions, including fat tails and skewness. This assumption can lead to suboptimal portfolios or increased vulnerability during periods of extreme market volatility.

Estimation Error: Markowitz optimization relies heavily on historical data to estimate parameters such as expected returns, variances, and correlations. However, these parameters are subject to estimation error, especially when historical data is limited or when market conditions change. Small changes in these inputs can lead to significant changes in portfolio allocations, potentially resulting in suboptimal performance.

Sensitivity to Input Parameters: Markowitz optimization is highly sensitive to input parameters such as expected returns, variances, and correlations. Small changes in these parameters can lead to large changes in portfolio allocations. As a result, the optimal portfolio may be highly sensitive to the choice of input parameters, which can be difficult to estimate accurately.

Diversification Benefits May Plateau: Markowitz optimization emphasizes diversification as a means of reducing portfolio risk. However, there may be diminishing marginal benefits to diversification, especially when correlations between assets are high or when including additional assets with similar risk-return profiles. In such cases, the benefits of further diversification may be limited.

Transaction Costs and Constraints: Markowitz optimization typically does not account for transaction costs or portfolio constraints, such as liquidity constraints, short-selling constraints, or regulatory constraints. Ignoring these factors can lead to portfolios that are not feasible or practical to implement in real-world settings.

Dynamic Nature of Markets: Markets are dynamic and constantly evolving, with changes in economic conditions, market sentiment, and geopolitical factors. Markowitz optimization assumes static input parameters over the investment horizon, which may not accurately capture changing market dynamics. As a result, portfolios optimized using historical data may not perform as expected in future market environments.

Real-world applications of Markowitz optimization include:

Portfolio Construction: Asset managers and investors use Markowitz optimization to construct diversified portfolios that balance risk and return according to their investment objectives and risk preferences.
Asset Allocation: Institutional investors, such as pension funds and endowments, use Markowitz optimization to allocate capital across different asset classes, such as stocks, bonds, and alternative investments, to achieve

optimal risk-adjusted returns.

Risk Management: Financial institutions and corporations use Markowitz optimization to manage portfolio risk by identifying and hedging against potential sources of risk, such as market volatility, interest rate risk, and currency risk.

Performance Evaluation: Investors and fund managers use Markowitz optimization to evaluate the performance of investment portfolios relative to benchmark indices and peer groups, identifying opportunities for portfolio optimization and rebalancing.

REFERENCES:

The data is taken from https://www.investing.com/

To learn about the theory, we used https://www.investopedia.com/ and also the Book "Mathematics for Finance" was used.