MATRIX and DETERMINANT

$$\begin{array}{cc} \text{Pen} & \text{Pencil} \\ \end{array}$$

Matrix $\begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$ columns

2×2 → order

rows

Matrix → collection of elements
→ square
→ rectangular

value

Determinant $\begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix}$

→ always square

order = 2

(Because rows = columns in case of determinants)

$$\rightarrow \quad \Delta = \begin{vmatrix} 3 & 4 & 5 \\ 6 & 2 & 1 \\ 5 & 4 & 9 \end{vmatrix}$$

order = 3

DETERMINANT

1) Order = 2

$$\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

$$= 2 \times 5 - 3 \times 4$$

$$= -2$$

$$\Delta = \begin{vmatrix} 3 & -4 \\ 5 & 6 \end{vmatrix}$$

$$= 3 \times 6 - 5 \times (-4)$$

$$= 38$$

2) Order = 3

$$\Delta = \begin{vmatrix} \overset{+}{2} & \overset{-}{3} & \overset{+}{4} \\ 3 & 4 & 2 \\ 1 & 2 & 5 \end{vmatrix}$$

$$= 2(4 \times 5 - 2 \times 2) - 3(3 \times 5 - 2 \times 1) + 4(3 \times 2 - 4 \times 1)$$

$$= 32 - 39 + 8 = 1$$

3) Order = 4

$$\Delta = \begin{vmatrix} \overset{+}{3} & \overset{-}{4} & \overset{+}{1} & \overset{-}{2} \\ 3 & 2 & 1 & 5 \\ 2 & 2 & 1 & 3 \\ 3 & 3 & 1 & 5 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 2 & 1 & 5 \\ 2 & 1 & 3 \\ 3 & 1 & 5 \end{vmatrix} - 4 \begin{vmatrix} 3 & 1 & 5 \\ 2 & 1 & 3 \\ 3 & 1 & 5 \end{vmatrix} + 1 \begin{vmatrix} 3 & 2 & 5 \\ 2 & 2 & 3 \\ 3 & 3 & 5 \end{vmatrix} - 2 \begin{vmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 234 & 921 & 135 \\ 789 & 461 & 642 \\ 456 & 542 & 789 \end{vmatrix} = ?$$

PROPERTIES OF DETERMINANT

1) for changing between rows and columns

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= - \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= (-) - \begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \end{vmatrix} = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \end{vmatrix}$$

2) If any two rows / columns are same, $\Delta = 0$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 4 & 3 \\ 5 & 2 & 5 \end{vmatrix} = 0$$

3)
$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 0 & a_1 & b_1 \\ 0 & a_2 & b_2 \\ 0 & a_3 & b_3 \end{vmatrix} = 0$$

principal diagonal

4) $$\Delta = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc \quad , \quad \Delta = \begin{vmatrix} 0 & 0 & a \\ 0 & b & 0 \\ c & 0 & 0 \end{vmatrix} = -abc$$

$$\Delta = \begin{vmatrix} a & a_1 & b_1 \\ 0 & b & c_2 \\ 0 & 0 & c \end{vmatrix} = abc$$

$$\Delta = \begin{vmatrix} a & 0 & 0 \\ a_1 & b & 0 \\ b_1 & c_1 & c \end{vmatrix} = abc$$

5) $$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 12 \quad (\text{let's say})$$

$$= \frac{1}{k} \begin{vmatrix} k \cdot a_1 & k\,a_1 & k \cdot c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \cancel{12k}$$

$$= \frac{1}{k^2} \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \frac{1}{k^3} \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix}$$

Q) If in a third-order determinant, all the rows are multiplied by k and all the columns are multiplied by k', then the value of the determinant is changed by $\longrightarrow (kk')^3$

Q) $\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

(common from row) $= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

(common from column) $= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= 4a^2b^2c^2$

6) METHOD to operate

You can add/subtract rows/columns

$\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -2$

$\Delta = \begin{vmatrix} 6 & 8 \\ 4 & 5 \end{vmatrix} = -2$

Q) $\Delta = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$

$= \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0$ (identical columns)

Q) $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$

$= a+b+c \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0$

(Back to properties)

7) $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Transpose

$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

i) $\Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

Transpose $\rightarrow \Delta^T = \begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

Take (-1) common from all rows

$\rightarrow - \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

$\Delta^T = -\Delta$

$\Delta^T + \Delta = 0$

We know, $\Delta = \Delta^T$

$2\Delta = 0$

$\boxed{\Delta = 0}$

8) (i) $\Delta = \begin{vmatrix} a_1+\alpha & b_1 & c_1 \\ a_2+\beta & b_2 & c_2 \\ a_3+\gamma & b_3 & c_3 \end{vmatrix}$

$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha & b_1 & c_1 \\ \beta & b_2 & c_2 \\ \gamma & b_3 & c_3 \end{vmatrix}$

ii) $\Delta = \begin{vmatrix} a_1+\alpha_1 & b_1+\alpha_2 & c_1 \\ a_2+\beta_1 & b_2+\beta_2 & c_2 \\ a_3+\gamma_1 & b_3+\gamma_2 & c_3 \end{vmatrix}$

$= \begin{vmatrix} a_1 & b_1+\alpha_2 & c_1 \\ a_2 & b_2+\beta_2 & c_2 \\ a_3 & b_3+\gamma_2 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1+\alpha_2 & c_1 \\ \beta_1 & b_2+\beta_2 & c_2 \\ \gamma_1 & b_3+\gamma_2 & c_3 \end{vmatrix}$

$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & \alpha_2 & c_1 \\ a_2 & \beta_2 & c_2 \\ a_3 & \gamma_2 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \beta_1 & b_2 & c_2 \\ \gamma_1 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & \alpha_2 & c_1 \\ \beta_1 & \beta_2 & c_2 \\ \gamma_1 & \gamma_2 & c_3 \end{vmatrix}$

Q) In a third-order determinant, if the first column consist of sum of 2 elements, II column - sum of 3 elements, III column - sum of four elements. No of determinants that can be formed $= 2 \times 3 \times 4 = 24$

Q) Prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

$\Delta^T = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

$= \begin{vmatrix} b & c+a & a+b \\ q & r+p & p+q \\ y & z+x & x+y \end{vmatrix} + \begin{vmatrix} c & c+a & a+b \\ r & r+p & p+q \\ z & z+x & x+y \end{vmatrix}$

$= \begin{vmatrix} b & c & a+b \\ q & r & p+q \\ y & z & x+y \end{vmatrix} + \underbrace{\begin{vmatrix} b & a & a+b \\ q & p & p+q \\ y & x & x+y \end{vmatrix}}_{0} + \overbrace{\begin{vmatrix} c & c & \cdot \\ r & r & \cdot \\ z & z & \cdot \end{vmatrix}}^{0} + \begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$

row + column = even $\longrightarrow$ +ve
row + column = odd $\longrightarrow$ −ve

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COFACTORS AND MINORS

$$\Delta = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix}$$

column, row

$c_{11} = 6$	$m_{11} = 6$
$c_{12} = -5$	$m_{12} = 5$
$c_{21} = -4$	$m_{21} = 4$
$c_{22} = 3$	$m_{22} = 3$

$$\Delta = \begin{vmatrix} 3 & 4 & 2 \\ 1 & 2 & 5 \\ 6 & -2 & -3 \end{vmatrix}$$

$c_{11} = \begin{vmatrix} 2 & 5 \\ -2 & -3 \end{vmatrix}$	$c_{21} = -$	$c_{31} =$
$c_{12} = -\begin{vmatrix} 1 & 5 \\ 6 & -3 \end{vmatrix}$	$c_{22} =$	$c_{32} = -$
$c_{13} =$	$c_{23} = -$	$c_{33} =$

Q) $\cancel{7}$ 14 $\begin{vmatrix} -2 & -5 & 4/2 \\ -\cancel{7}1 & -\cancel{7}1 & +\cancel{4}1 \\ -3 & -11 & +\cancel{6}3 \end{vmatrix}$

Multiply the elements of Ist row (or II / III) with the elements of Ist row of Δ^c, and add them you'll get Δ.

$$\boxed{\Delta^c = \Delta^{n-1}}$$

Property - 9 (Multiplication theorem)
(for multiplication of determinants)

$$\Delta_1 \Delta_2 = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} \begin{vmatrix} 3 & 4 \\ 6 & 2 \end{vmatrix}$$

4X 85

$$= \begin{vmatrix} 18 & 18 \\ 32 & 34 \end{vmatrix} = 36$$

Q)

$$\Delta = \begin{vmatrix} a_1x_1 + b_1y_1 & a_1x_2 + b_1y_2 & a_1x_3 + b_1y_3 \\ a_2x_1 + b_2y_1 & a_2x_2 + b_2y_2 & a_2x_3 + b_2y_3 \\ a_3x_1 + b_3y_1 & a_3x_2 + b_3y_2 & a_3x_3 + b_3y_3 \end{vmatrix}$$

$$= \underbrace{\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix}}_{\to 0} \underbrace{\begin{vmatrix} x_1 & y_1 & 0 \\ x_2 & y_2 & 0 \\ x_3 & y_3 & 0 \end{vmatrix}}_{\to 0}$$

$\therefore \quad \Delta = 0$

10) Differentiation

$$\Delta = \begin{vmatrix} f_1(x) & f_2(x) \\ f_3(x) & f_4(x) \end{vmatrix}$$

$$= f_1(x)\cdot f_4(x) - f_2(x)\cdot f_3(x)$$

$$= f_1(x)\cdot f_4'(x) + f_4(x) f_1'(x) - f_2(x)\cdot f_3'(x) - f_3(x) f_2'(x)$$

(i) or $= \begin{vmatrix} f_1'(x) & f_2'(x) \\ f_3(x) & f_4(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) \\ f_3'(x) & f_4'(x) \end{vmatrix}$

$$\Delta = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7(x) & f_8(x) & f_9(x) \end{vmatrix}$$

$$= \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7(x) & f_8(x) & f_9(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4'(x) & f_5'(x) & f_6'(x) \\ f_7(x) & f_8(x) & f_9(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7'(x) & f_8'(x) & f_9'(x) \end{vmatrix}$$

Q) (α, β) $ax^2 + bx + c = 0$

$S_n = \alpha^n + \beta^n$

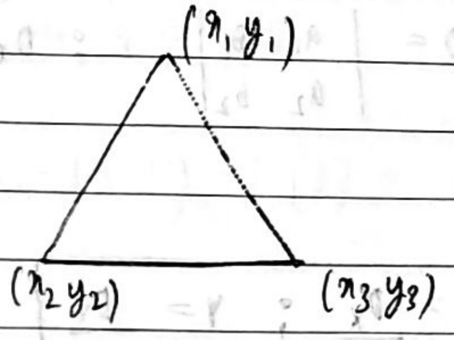
$$\Delta = \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix} = \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$$= (\alpha-1)^2 (\beta-1)^2 (\beta-\alpha)^2$$

APPLICATION I

1) To find the area of a Δ

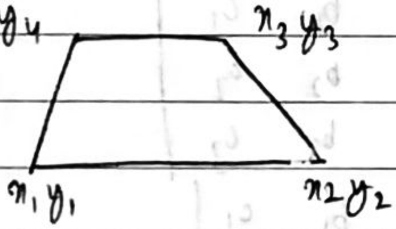


$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}\right]$$

If $A = 0 \Rightarrow$ points are collinear

→ ~~Polygon~~ Quad ?



$$A = \left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + \begin{vmatrix} x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}\right]$$

2) To solve linear equations (cramer's rule)

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Cramer's rule :

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} ; \quad D_x = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$\boxed{x = \frac{D_x}{D} ; \quad y = \frac{D_y}{D}}$$

ii) $a_1 x + b_1 y + c_1 z = k_1$

$a_2 x + b_2 y + c_2 z = k_2$

$a_3 x + b_3 y + c_3 z = k_3$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$$

$$x = \frac{D_x}{D} ; \quad y = \frac{D_y}{D} ; \quad z = \frac{D_z}{D}$$

Q) $2x + y - z = 1$

$x - y + z = 2$

$x + y + z = 6$

$$D = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2(-2) - 1(0) - 1(2) = -4 - 2 = -6$$

$$D_x = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 6 & 1 & 1 \end{vmatrix} = 1(-2) - 1(-4) - 1(8) = -2 + 4 - 8 = -6$$

$$D_y = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 6 & 1 \end{vmatrix} = 2(-4) - 1(0) - 1(4) = -8 - 4 = -12$$

$$D_z = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix} = 2(-8) - 1(4) + 1(2) = -16 - 4 + 2 = -18$$

$$x = \frac{-\cancel{18}6}{-6} = \cancel{3}1 ; \quad y = \frac{-12}{-6} = 2 ; \quad z = 3$$

3) Consistency for non-homogenous linear inequations

$$a_1x + b_1y + c_1z = k_1$$
$$a_2x + b_2y + c_2z = k_2$$
$$a_3x + b_3y + c_3z = k_3$$

i) If $D \neq 0$, system $\rightarrow$ consistent
we'll get a unique solution

ii) If $D = 0$, $D_x = D_y = D_z = 0$, system $\rightarrow$ consistent
$\rightarrow$ infinite no of solutions

iii) If $D = 0$ and one of $D_x, D_y, D_z \neq 0$, system $\rightarrow$ inconsistent
$\rightarrow$ No solutions (Dependent solution)

HOMOGENOUS $\rightarrow$

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

1) If $D \neq 0$, then the system of eq will have trivial solution ($x=0$, $y=0$, $z=0$)

2) If $D=0$, $\rightarrow$ non trivial solution

infinite (a non-zero solution)

Q) $ax + yb + z = 0$

$x + by + z = 0$

$x + y + cz = 0$

$$D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$$