Modern Portfolio Theory (MPT)

- Modern portfolio theory (MPT) is a theory on how risk-averse investors can construct portfolios to **maximize** expected return based on a given level of market **risk**.
- Harry Markowitz pioneered this theory in his paper 'Portfolio Selection,' published in 1952. He was later awarded a Nobel Prize for his work on modern portfolio theory.

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Core Concepts of MPT

- MPT argues that an investment's risk and return should be evaluated based on its effect on the overall portfolio.
- Investors can construct portfolios that maximize returns for a given level of risk.
- Based on variance and correlation, MPT focuses on portfolio-level performance, not individual assets.

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Assumptions and Expected Return

- MPT assumes investors are risk-averse — preferring lower risk for a given return.
- Expected portfolio return = Weighted sum of individual assets' returns.

Example: (4%×25%) + (6%×25%) + (10%×25%) + (14%×25%) = 8.5%

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Markowitz Optimization and the Efficient Frontier

Objective:
Minimise portfolio risk for a given level of expected return (or maximise return for a given risk).

Optimization Problem:
Minimise: $w^T\Sigma w - q \times R^T w$
Subject to: $\Sigma w_i = 1$ (Weights w_i can be negative for short selling)

Where:
- w: Vector of portfolio weights
- Σ: Covariance matrix of asset returns
- R: Vector of expected returns
- $q \geq 0$: Risk tolerance factor
  - $q = 0 \rightarrow$ Minimal risk portfolio
  - $q \rightarrow \infty \rightarrow$ Max return with high risk
- $w^T\Sigma w$: Portfolio variance (risk)
- $R^T w$: Expected portfolio return

Interpretation:
Varying q traces the Efficient Frontier, representing optimal portfolios balancing risk vs. return.

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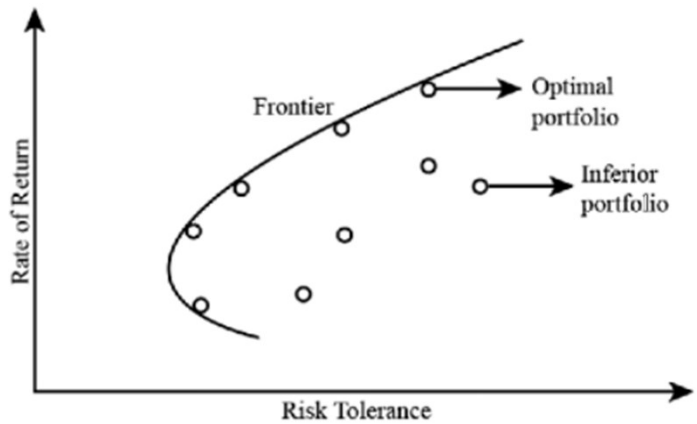
Efficient Frontier



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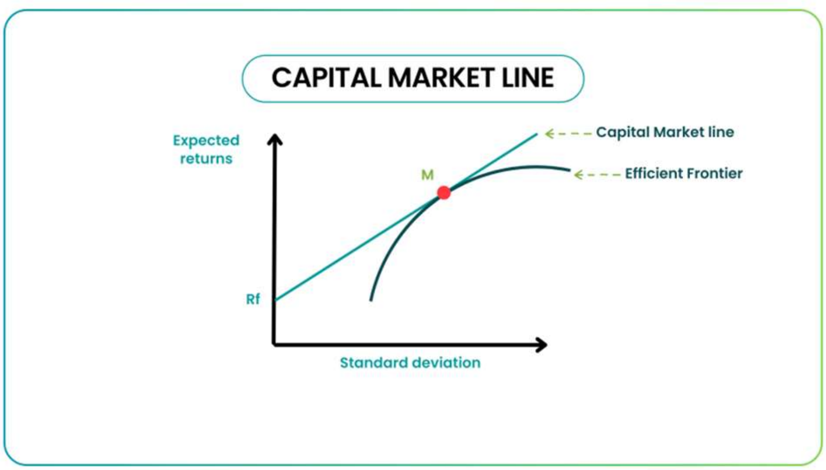
Capital Market Line (CML):

The Capital Market Line (CML) represents the risk-return trade-off for efficient portfolios that include a risk-free asset and risky assets. It is a key concept in Modern Portfolio Theory (MPT) and builds upon the efficient frontier by incorporating risk-free borrowing and lending.



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1. **Risk-Free Asset Inclusion**

- **Definition of Risk-Free Asset**:
  A risk-free asset is an investment with a guaranteed return and no risk of default, often represented by government securities like Treasury bills.
  - **Risk-Free Return** (r_f): The rate of return on the risk-free asset.
  - **Standard Deviation** (σ_{r_f}): Always zero, as there is no variability in returns.
- **Impact on Portfolio**:
  By including a risk-free asset:
  - Investors can adjust their portfolio's risk by varying allocations between the risk-free asset and a portfolio of risky assets.
  - Borrowing at the risk-free rate allows investors to achieve portfolios with returns above the efficient frontier.

2. **Tangency Portfolio and Optimal Allocation**

- **Tangency Portfolio**:
  - The **tangency portfolio** (or market portfolio) is the portfolio on the efficient frontier that maximizes the Sharpe ratio (risk-adjusted return).
  - It is the point where the CML is tangent to the efficient frontier.
  - This portfolio contains the optimal combination of risky assets and provides the highest return per unit of risk.

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- **Mathematical Representation**:
  The expected return $E[R]$ of a portfolio on the CML is: $E[R_p] = r_f + \boxed{\frac{E[R_m]-r_f}{\sigma_m}} \cdot \sigma_p$ Sharpe ratio
  - $E[R_p]$: Expected return of the portfolio.
  - r_f: Risk-free rate.
  - $E[R_m]$: Expected return of the market (tangency portfolio).
  - σ_m: Standard deviation of the market portfolio.
  - σ_p: Standard deviation of the portfolio.
- **Optimal Allocation**:
  - **Risk-Free Asset Allocation** (w_f): Proportion of the portfolio invested in the risk-free asset.
  - **Risky Portfolio Allocation** (w_m): Proportion of the portfolio invested in the tangency portfolio.
  - Portfolio combinations are created by adjusting (w_f) and (w_m).
    - ($w_f > 0$): Lending at the risk-free rate (low-risk portfolios).
    - ($w_f < 0$): Borrowing at the risk-free rate (leveraged portfolios).

Applications of CML

1. **Portfolio Optimization**: Helps investors choose the best risk-return trade-off based on their risk tolerance.
2. **Risk Management**: Guides portfolio managers in balancing risk-free and risky assets.
3. **Leveraging and Hedging**: Demonstrates the effects of borrowing or lending at the risk-free rate.

The inclusion of the risk-free asset and the tangency portfolio's role make the CML a powerful tool for optimizing investment portfolios in practice.

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Benefits of Modern Portfolio Theory (MPT)

- Helps investors build diversified portfolios.
- Exchange Traded Funds (ETFs) make diversification easier.
- Reduces risk by allocating small portions to government bonds.
- Can lower volatility by mixing asset classes (e.g., small-cap stocks and bonds).

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Efficient Frontier

- Efficient portfolios can be plotted with risk (X-axis) vs. return (Y-axis).
- The efficient frontier represents optimal portfolios offering maximum returns for a given risk.
- Portfolios below the curve are sub-optimal.

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Criticism of Modern Portfolio Theory

- MPT uses variance instead of downside risk to evaluate portfolios.
- Two portfolios with same variance but different loss patterns are treated equally.
- Post-modern portfolio theory (PMPT) improves MPT by minimizing downside risk instead.

Sortino Ratio:
A variation of the Sharpe ratio that uses downside deviation instead of total standard deviation:

$$\text{Sortino } Ratio = \frac{R_p - R_f}{Downside\ Deviation}$$

where R_p is portfolio return and R_f is risk-free rate.

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Modern Portfolio Theory (MPT): Overview

Developed by Harry Markowitz (1952).

- Goal: Construct portfolios that maximize return for a given risk or minimize risk for a given return.
- Based on diversification and correlation among assets.
- Foundation for CAPM, Black–Litterman, and other models.
- Formulas:
- $E(Rp) = \Sigma\ wi\ E(Ri)$
- $\sigma p = \sqrt{[\Sigma\ wi^2\ \sigma i^2 + \Sigma\Sigma\ wi\ wj\ Cov(Ri, Rj)]}$

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Risk and Return

- Expected Return: Weighted average of possible outcomes.
- Variance / Standard Deviation: Measures total risk (volatility).
- Covariance & Correlation:
  - Covariance → direction of co-movement.
  - Correlation (ρ) → strength of co-movement (−1 to +1).
- Formulas:
  - $\rho ij = Cov(Ri, Rj) / (\sigma i\ \sigma j)$
    - $\sigma p^2 = \Sigma\Sigma\ wi\ wj\ Cov(Ri, Rj)$

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Efficient Frontier & Optimal Portfolios

- Efficient Frontier: Set of portfolios with max return for each risk level.
- Minimum Variance Portfolio (MVP): Lowest-risk portfolio on the frontier.
- Risk-Free Asset: Zero variance (e.g., Treasury Bills).
- Capital Market Line (CML): Optimal combinations of risky & risk-free assets.
- Tangency Portfolio: Point of maximum Sharpe Ratio (best risk-adjusted return).
- Formula: $E(Rp) = Rf + [(E(Rm) - Rf)/\sigma m] * \sigma p$

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Models Extending MPT

- CAPM: $E(R_i) = R_f + \beta_i [E(R_m) - R_f]$
  - β: Asset's sensitivity to market.
  - α: Excess return beyond CAPM prediction.
- Black–Litterman Model:
  - Combines market equilibrium returns with investor views.
  - Uses Bayesian adjustment to compute posterior expected returns.
- Arbitrage Pricing Theory (APT): Multi-factor alternative to CAPM.

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Performance Ratios

- Sharpe Ratio = $(R_p - R_f) / \sigma_p$ → Return per total risk.
- Treynor Ratio = $(R_p - R_f) / \beta_p$ → Return per systematic risk.
- Jensen's Alpha = $R_p - [R_f + \beta_p (R_m - R_f)]$ → Excess over CAPM return.
- Sortino Ratio = $(R_p - R_f)$ / Downside Deviation → Return per downside risk.
- Information Ratio = $(R_p - R_b) / \sigma(p-b)$ → Active return vs. benchmark.

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Risk Types & Advanced Measures

- Systematic Risk: Market-wide, non-diversifiable.
- Unsystematic Risk: Asset-specific, diversifiable.
- Value at Risk (VaR): Maximum loss at a confidence level.
- Expected Shortfall (CVaR): Average loss beyond VaR.
- Mean–Variance Optimization: Mathematical process for efficient portfolios.
- Utility Function: $U = E(Rp) - \frac{1}{2}A\sigma p^2$ (represents investor’s risk–return preference).

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Thank you

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