Black-Litterman Model

An Alternative to the Markowitz Asset Allocation Model

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What is the Black-Litterman Model?

- The Black-Litterman Model is used to determine optimal asset allocation in a portfolio
- Black-Litterman Model takes the Markowitz Model one step further
  - Incorporates an investor's own views in determining asset allocations

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Two Key Assumptions

- Asset returns are normally distributed
  - Different distributions (t-dist. for fat tail behaviour) could be used, but using normal is the simplest
- Variance of the prior and the conditional distributions about the true mean are known
  - Actual true mean returns are not known
  - We can only **estimate** them using:
    - Market equilibrium (implied) data, or
    - Our own subjective views.

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Basic Idea

1. Find implied returns
2. Formulate investor views
3. Determine what the expected returns are
4. Find the asset allocation for the optimal portfolio

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Implied vs. Historical Returns

- Analogous to implied volatility
- CAPM (*E(Ri)=Rf+βi[E(Rm)−Rf]*) is assumed to be the true price such that given market data, implied return can be calculated
- Implied return will not be the same as historical return

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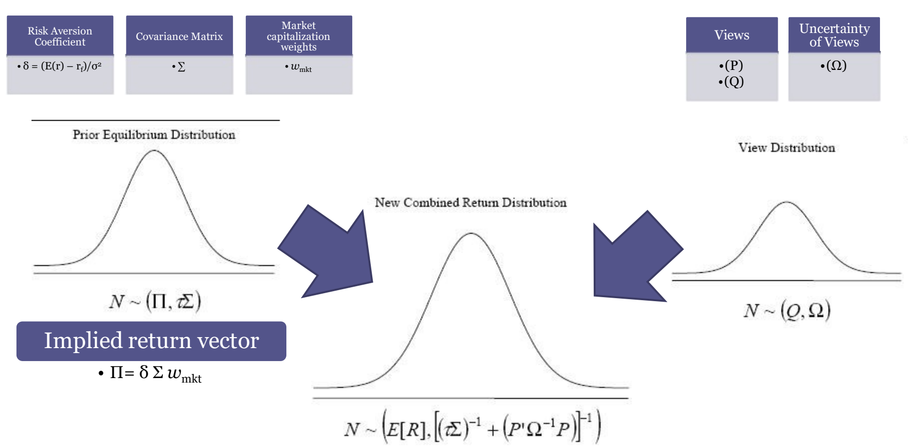
Implied Returns + Investor Views = Expected Returns



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Implied Returns:

$$\Pi = \delta \, \Sigma \, w_{mkt}$$

- Π = The equilibrium risk premium over the risk free rate (Nx1 vector)
- $\delta = (E(r) - r_f)/\sigma^2$, risk aversion coefficient
- Σ = A covariance matrix of the assets (NxN matrix)

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Views:

- P = A matrix with investors views; each row a specific view of the market and each entry of the row represents the portfolio weights of each assets (KxN matrix)
- Ω = A diagonal covariance matrix with entries of the uncertainty within each view (KxK matrix)
- Q = The expected returns of the portfolios from the views described in matrix P (Kx1 vector)

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Bayesian Theory

- Traditionally, personal views are used for the prior distribution
- Then observed data is used to generate a posterior distribution
- The Black-Litterman Model assumes implied returns as the prior distribution and personal views alter it

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

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Model Formula

$E(r - r_f) = E(R) = [(\tau\,\Sigma)^{-1} + P^T\,\Omega P]^{-1}\,[(\tau\,\Sigma)^{-1}\,\Pi + P^T\,\Omega Q]$

*Expected returns = Uncertainty of returns * Investor's views*

Where:
- Π = Implied (market equilibrium) excess returns
- Q = Investor's views on returns
- Σ = Covariance matrix of returns
- τ = Scaling factor for uncertainty in the prior
- Ω = Covariance matrix of uncertainty in views
- P = Matrix linking views to assets

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Concept of the Black-Litterman Model

- The Black–Litterman model combines market equilibrium returns (implied returns) with investor-specific views.

- It treats the implied returns as a prior distribution (market consensus) and the investor's views as new evidence.

- Using Bayesian updating, it produces a posterior distribution of expected returns.

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Example: One Stock (AMZN)

Market data:
- Implied excess return (Π) = 0.74% per month
- Variance = 2.015%

Investor's view:
- Expected excess return (Q) = 2.0% per month
- Uncertainty (Ω) = $(0.50\%)^2$

Assume: $\tau = 1$, $P = 1$

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Calculation of Posterior Expected Return

$E(r - r\_f) = [0.02015^{-1} + 0.005^{-1}]^{-1}$
$[0.02015^{-1} \times 0.0074 + 0.005^{-1} \times 0.02]$

$= [49.63 + 200]^{-1} \times [49.63 \times 0.0074 + 200 \times 0.02]$
$= 1.749\%$ per month

The posterior expected return (1.749%) lies between the market's 0.74% and your view of 2.0%, weighted by confidence.

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Interpretation

• The posterior return combines both the market and investor perspectives.
• Closer to your view if confidence is high (small Ω).
• Closer to market if your view is uncertain (large Ω).

This balance avoids extreme portfolio weights and produces stable allocations.

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Intuitive Analogy

Think of it as a weighted average:
Posterior = w_1(Market Implied) + w_2(Your View)

• The higher your confidence, the more influence your view has.
• If uncertain, the model relies more on the market consensus.

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Advantages and Disadvantages

- Advantages
  - Investor's can insert their view
  - Control over the confidence level of views
  - More intuitive interpretation, less extreme shifts in portfolio weights
- Disadvantages
  - Black-Litterman model does not give the best possible portfolio, merely the best portfolio given the views stated
  - As with any model, sensitive to assumptions
    - Model assumes that views are independent of each other

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Example
Model in Practice

- Example illustrated in Goldman Sachs paper
- Determine weights for countries
  - View: Germany will outperform the rest of Europe by 5%

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Statistical Analysis

Country Metrics

Country	Equity Index Volatility (%)	Equilibrium Portfolio Weight (%)	Equilibrium Expected Returns (%)
Australia	16.0	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9.0
Japan	21.0	11.6	4.3
UK	20.0	12.4	6.8
USA	18.7	61.5	7.6

Covariance Matrix

	AUS	CAN	FRA	GER	JAP	UK	USA
AUS	0.0256	0.01585	0.018967	0.02233	0.01475	0.016384	0.014691
CAN	0.01585	0.041209	0.033428	0.036034	0.027923	0.024685	0.024751
FRA	0.018967	0.033428	0.061504	0.057866	0.018488	0.038837	0.030979
GER	0.02233	0.036034	0.057866	0.073441	0.020146	0.042113	0.033092
JAP	0.01475	0.013215	0.018488	0.020146	0.0441	0.01701	0.012017
UK	0.016384	0.024685	0.038837	0.042113	0.01701	0.04	0.024385
USA	0.014691	0.029572	0.030979	0.033092	0.012017	0.024385	0.034969

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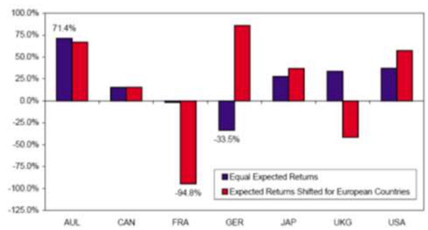
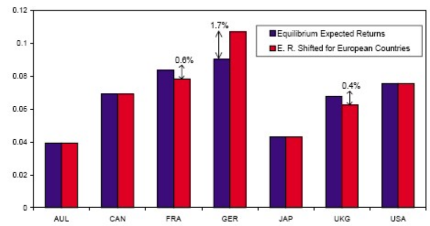
Traditional Markowitz Model

Portfolio Asset Allocation



Expected Returns



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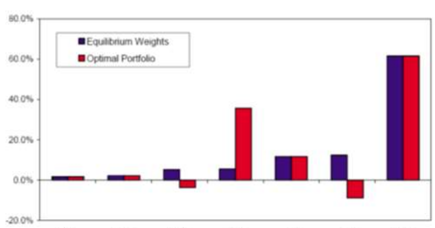
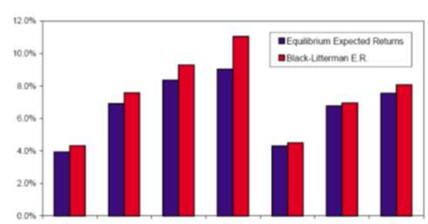
Portfolio Asset Allocation



Expected Returns



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Thank you

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