Nome: Amanda chen zhen CTII 348

1. $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{40.320}{720} = 56$

Letra B.

2. $\binom{200}{198} = \frac{200!}{198!(200-198)!} = \frac{200.199.198!}{198!2!} = \frac{39.800}{2} = 19,900$

Letra A.

3. $\binom{n-1}{2} = \binom{n+1}{4}$

$\frac{n-1!}{2!(n-1-2)!} = \frac{n+1!}{4!(n+1-4)!}$

$\frac{n-1!}{2!n-3!} = \frac{n+1!}{4!n-3!}$

$\frac{n-1.n-2.\cancel{n-3!}}{2.\cancel{n-3!}} = \frac{n+1.n.n-1.n-2.\cancel{n-3!}}{24.\cancel{n-3!}}$

$\frac{n^2-3n+2}{2} = \frac{n^4-2n^3-n^2+2n}{24}$

$24n^2-72n+48 = 2n^4-4n^3-2n^2+4n$

$-2n^4+4n^3+26n^2-76n+48=0$

$n=1$

$-2.1^4+4.1^3+26.1^2-76.1+48=0$

$-2+4+26-76+48=0$

$78-78=0$

$0=0$

$n=2$

$-2.2^4+4.2^3+26.2^2-76.2+48=0$

$\cancel{-32}+\cancel{32}+104-152+48=0$

$152-152=0$

$0=0$

$n=3$

$-2.3^4+4.3^3+26.3^2-76.3+48=0$

$-162+108+234-228+48=0$

$390-390=0$

$0=0$

$V=\{1,2,3\}$

4. $\binom{20}{13} + \binom{20}{14}$

↓

2 consecutivos da Linha 20 → R: $\binom{21}{14}$

5. $\underbrace{\binom{n}{0} + \binom{n}{1} + \binom{n}{2} \dots \binom{n}{n}}$

soma na linha $n \rightarrow 2^n$

6. a. $\sum_{p=0}^{10} \binom{10}{p} \rightarrow \binom{10}{0} + \binom{10}{1} + \binom{10}{2} \dots \binom{10}{10}$

$\downarrow$

soma da linha $10 \rightarrow 2^{10} = 1024$

b. $\sum_{p=0}^{9} \binom{10}{p} \rightarrow \binom{10}{0} + \binom{10}{1} + \binom{10}{2} \dots \binom{10}{9}$

$\binom{10}{1} \binom{10}{9} \quad \binom{10}{2} \binom{10}{8} \quad \binom{10}{3} \binom{10}{7} \quad \binom{10}{4} \binom{10}{6}$

complementares

$\binom{10}{0} = \frac{10!}{0!10!} = 1 \qquad \binom{10}{1} = \frac{10.9!}{1.9!} = 10 \qquad \binom{10}{2} = \frac{10.9.8!}{2.8!} = 45$

$\binom{10}{3} = \frac{10.9.8.7!}{6.7!} = 120 \qquad \binom{10}{4} = \frac{10.9.8.7.6!}{24.6!} = 210$

$\binom{10}{5} = \frac{10.9.8.7.6.5!}{120.5!} = 252$

$1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 = 1023$

c. $\sum_{p=2}^{9}\binom{9}{p}$ $\quad\binom{9}{2}\binom{9}{7}\quad\binom{9}{3}\binom{9}{4}\quad\binom{9}{4}\binom{9}{5}$

complementares

$\binom{9}{2}=\frac{9.8.\cancel{7!}}{2.\cancel{7!}}=36\qquad\binom{9}{3}=\frac{9.8.7.\cancel{6!}}{6.\cancel{6!}}=84\qquad\binom{9}{4}=\frac{9.8.7.6.\cancel{5!}}{24.\cancel{5!}}=126$

$\binom{9}{8}=\frac{9.\cancel{8!}}{\cancel{8!}.1}=9\qquad\binom{9}{9}=\frac{\cancel{9!}}{\cancel{9!}}=1$

$36+84+126+126+84+36+9+1=502$

d. $\sum_{p=4}^{10}\binom{p}{4}$ $\quad\binom{4}{4}=\frac{\cancel{4!}}{\cancel{4!}0!}=1\qquad\binom{5}{4}=\frac{5.\cancel{4!}}{\cancel{4!}.1}=5$

$\binom{6}{4}=\frac{6.5.\cancel{4!}}{\cancel{4!}.2}=15\qquad\binom{7}{4}=\frac{7.6.5.\cancel{4!}}{\cancel{4!}.6}=35\qquad\binom{8}{4}=\frac{8.7.6.5.\cancel{4!}}{24.\cancel{4!}}=70$

$\binom{9}{4}=\frac{9.8.7.6.\cancel{5!}}{24.\cancel{5!}}=126\qquad\binom{10}{4}=\frac{10.9.8.7.\cancel{6!}}{24.\cancel{6!}}=210$

$1+5+15+35+70+126+210=462$

e. $\sum_{p=5}^{10}\binom{p}{5}$

$\binom{5}{5}=\frac{\cancel{5!}}{\cancel{5!}0!}=1\qquad\binom{6}{5}=\frac{6.\cancel{5!}}{\cancel{5!}1}=6\qquad\binom{7}{5}=\frac{7.6.\cancel{5!}}{\cancel{5!}.2}=21$

$$\binom{8}{5} = \frac{8.7.6.\cancel{5!}}{\cancel{5!}.2} = 56 \qquad \binom{9}{5} = \frac{9.8.7.6.\cancel{5!}}{\cancel{5!}.24} = 126$$

$$\binom{10}{5} = \frac{10.9.8.7.6.\cancel{5!}}{120.\cancel{5!}} = 252$$

$$1+6+21+56+126+252 = 462$$

7. $\sum_{k=0}^{m} \binom{m}{k} = 512$

$\binom{m}{0} + \binom{m}{1} + \binom{m}{2} \dots \binom{m}{m}$ → soma da linha m → 2^m

$m = 9$

$512 = 2^9$

Letra E.