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1. $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{40.320}{720} = 56$

Letra B.

2. $\binom{200}{198} = \frac{200!}{198!(200-198)!} = \frac{200.199.198!}{198!2!} = \frac{39.800}{2} = 19.900$

Letra A.

3. $\binom{n-1}{2} = \binom{n+1}{4}$

$\frac{n-1!}{2!(n-1-2)!} = \frac{n+1!}{4!(n+1-4)!}$

$\frac{n-1!}{2!n-3!} = \frac{n+1!}{4!n-3!}$

$\frac{n-2.n-2.n-3!}{2.n-3!} = \frac{n+1.n+n-1.n-2.n-3!}{24.n-3!}$

$\frac{n^2-3n+2}{2} = \frac{n^4-2n^3-n^2+2n}{24}$

$24n^2-72n+48 = 2n^4-4n^3-2n^2+4n$

$-2n^4+4n^3+26n^2-76n+48=0$

$n=1$

$-2 \cdot 1^4 + 4 \cdot 1^3 + 26 \cdot 1^2 - 76 \cdot 1 + 48 = 0$

$-2 + 4 + 26 - 76 + 48 = 0$

$78 - 78 = 0$

$0 = 0$

$n=2$

$-2 \cdot 2^4 + 4 \cdot 2^3 + 26 \cdot 2^2 - 76 \cdot 2 + 48 = 0$

$-32 + 32 + 104 - 152 + 48 = 0$

$152 - 152 = 0$

$0 = 0$

$n=3$

$-2 \cdot 3^4 + 4 \cdot 3^3 + 26 \cdot 3^2 - 76 \cdot 3 + 48 = 0$

$-162 + 108 + 234 - 228 + 48 = 0$

$390 - 390 = 0$

$0 = 0$

$V = \{1, 2, 3\}$

4. $\binom{20}{13} + \binom{20}{14}$

↓

2 consecutivos da Linha 20 $\Rightarrow$ R: $\binom{21}{14}$

5. $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} \cdots \binom{n}{n}$

soma na linha $n \to 2^n$

6. a. $\sum_{p=0}^{10} \binom{10}{p} \to \binom{10}{0} + \binom{10}{1} + \binom{10}{2} \cdots \binom{10}{10}$

soma da linha $10 \to 2^{10} = 1024$

b. $\sum_{p=0}^{9} \binom{10}{p} \to \binom{10}{0} + \binom{10}{1} + \binom{10}{2} \cdots \binom{10}{9}$

$\binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \binom{10}{4} + \binom{10}{5} + \binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9}$

$\binom{10}{0} = \frac{10!}{0!10!} = 1 \quad \binom{10}{1} = \frac{10!}{1!9!} = 10 \quad \binom{10}{2} = \frac{10 \cdot 9!}{2!8!} = 45$

$\binom{10}{3} = \frac{10 \cdot 9 \cdot 8!}{6 \cdot 7!} = 120 \quad \binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{24 \cdot 6!} = 210$

$\binom{10}{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{120 \cdot 5!} = 252$

$1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 = 1023$

c. $\sum_{p=2}^{9} \binom{9}{p}$ $\binom{9}{2}$ $\binom{9}{7}$ $\binom{9}{3}$ $\binom{9}{6}$ $\binom{9}{4}$ $\binom{9}{5}$

complementares

$\binom{9}{2} = \frac{9.8.7!}{2.7!} = 36$ $\binom{9}{3} = \frac{9.8.7.6!}{6.6!} = 84$ $\binom{9}{4} = \frac{9.8.7.6.5!}{24.5!} = 126$

$\binom{9}{8} = \frac{9.8!}{8!.1} = 9$ $\binom{9}{9} = \frac{8!}{9!} = 1$

$36+84+126+126+84+36+9+1 = 502$

d. $\sum_{p=4}^{10} \binom{p}{4}$ $\binom{4}{4} = \frac{4!}{4!0!} = 1$ $\binom{5}{4} = \frac{5.4!}{4!.1} = 5$

$\binom{6}{4} = \frac{6.5.4!}{4!.2} = 15$ $\binom{7}{4} = \frac{7.6.5.4!}{4!.6} = 35$ $\binom{8}{4} = \frac{8.7.6.5.4!}{24.4!} = 70$

$\binom{9}{4} = \frac{9.8.7.6.5!}{24.5!} = 126$ $\binom{10}{4} = \frac{10.9.8.7.6!}{24.6!} = 210$

$1+5+15+35+70+126+210 = 462$

e. $\sum_{p=5}^{10} \binom{p}{5}$

$\binom{5}{5} = \frac{5!}{5!.0!} = 1$ $\binom{6}{5} = \frac{6.5!}{5!.1} = 6$ $\binom{7}{5} = \frac{7.6.5!}{5!.2} = 21$

$\binom{8}{5} = \frac{8.7.6.5!}{5!.2} = 56$ $\binom{9}{5} = \frac{9.8.7.6.5!}{5!.24} = 126$

$\binom{10}{5} = \frac{10.9.8.7.6.5!}{120.5!} = 252$

$1+6+21+56+126+252 = 462$

7. $\sum_{K=0}^{m} \binom{m}{K} = 512$

$\binom{m}{0} + \binom{m}{1} + \binom{m}{2} \dots \binom{m}{m} \rightarrow$ soma da linha $m \rightarrow 2^m$

$m = 9$

$512 = 2^9$

Letra E.