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1. $A=\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $B=\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ $In=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A.B = In$

$\begin{cases} 3x+y=1 \\ -x+2=0 \end{cases}$ $\begin{cases} 15+3y=0 \\ -5+6=1 \end{cases}$ $\det A = 6-5=1$

$\det A^{-1} = 6-5=1$

$\det A . \det A^{-1} = 1$

$-x+2=0$ $\quad 15+3y=0$

$-x=-2 \ (-1)$ $\quad 3y=-15$

$x=2$ $\quad y=-5$

$x+y=2-5 \rightarrow -3$

Letra C.

2. $A=\left(\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ k & 1 & 3 & k & 1 \\ 1 & k & 3 & 1 & k \end{array}\right)$ $\rightarrow k^2+3-1-3k \rightarrow k^2-3k+2$

$1+k+3 \quad 3+0+k^2$

$x' = \frac{3+1}{2} = \frac{4}{2} = 2$

$\Delta = b^2-4ac$ $\quad x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{3 \pm 1}{2}$

$x'' = \frac{3-1}{2} = \frac{2}{2} = 1$

$\Delta = (-3)^2 - 4 \cdot 1 \cdot 2$

$\Delta = 9-8$

$\Delta = 1$ $\quad k=2$ ou $k=1 \rightarrow \det A = 0$

Letra C.

3. $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} x & y \\ a & b \end{bmatrix}$ $In = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A \cdot B = In$

$3x + 5a = 1 \ .(-2)$ $\quad 3y + 5b = 0 \ .(-2)$

$2x + 4a = 0 \ .(3)$ $\quad 2y + 4b = 1 \ .(3)$

$2a = -2$ $\quad 2x - 4 = 0$ $\quad 2b = 3$ $\quad 2y + 6 = 1$

$a = -1$ $\quad 2x = 4$ $\quad b = \frac{3}{2}$ $\quad 2y = -5$

$x = 2$ $\quad y = \frac{-5}{2}$

Letra C.

4. $\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix} \rightarrow x^2 + 2x - 6x - 2 \cdot 3 \rightarrow x^2 - 5x + 6$

$2x + 2x + 3x \quad x^2 + 2x + 6$

$\Delta = b^2 - 4ac$ $\quad x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{5 \pm 1}{2}$

$\Delta = (-5)^2 - 4 \cdot 1 \cdot 6$ $\quad x' = \frac{5+1}{2} = \frac{6}{2} = 3$

$\Delta = 25 - 24$ $\quad x'' = \frac{5-1}{2} = \frac{4}{2} = 2$

$\Delta = 1$

Para que haja uma inversa dessa matriz $\rightarrow x \neq 3$ e $x \neq 2$.

Letra A.

5. $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} x & y & z \\ a & b & c \\ d & e & f \end{bmatrix}$

$-x - a + 2d = 1$ $\quad 2x + a - 2d = 0$ $\quad x + a - d = 0$

$-y - b + 2e = 0$ $\quad 2y + b - 2e = 1$ $\quad y + b - e = 0$

$-z - c + 2f = 0$ $\quad 2z + c - 2f = 0$ $\quad z + c - f = 1$

$x = 1$ $\qquad y = 1$ $\qquad z = 0$

$2 + a - 2d = 0$ $\quad a = 2d - 2 \rightarrow a = 0$

$2 + b - 2e = 1$ $\quad b = 2e - 1 \rightarrow b = -1$

$c - 2f = 0$ $\quad c = 2f \rightarrow c = 2$

$1 + 2d - 2 - d = 0$ $\quad d = 1$

$1 + 2e - 2 - e = 0$ $\quad e = 0$

$2f - f = 1$ $\quad f = 1$

$$A + A^{-1} = \begin{bmatrix} -1+1 & -1+1 & 2+0 \\ 2+0 & 1-1 & -2+2 \\ 1+1 & 1+0 & -1+1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

Letra B.

6. $(XA)^t = B$ $\quad A \cdot A^{-1} = I$

$(X^t \cdot A^t) = B^t$

$X \cdot A = B^t \cdot (A^{-1})$

$X = B^t \cdot A^{-1}$

Letra B.

7. $B = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ → ou seja, $a = 4$ $b = 5$ $c = 5$ $d = 6$

$A^{-1} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ $\quad In = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$4e + 5g = 1 \quad .(-5)$ $\quad 4f + 5h = 0 \quad .(-5)$

$5e + 6g = 0 \quad .(4)$ $\quad 5f + 6h = 1 \quad .(4)$

$-g = -5 \quad .(-1)$ $\quad -h = 4 \quad .(-1)$

$g = 5$ $\quad h = -4$

$4e + 25 = 1$ $\quad 4f - 20 = 0$

$4e = -24$ $\quad 4f = 20$

$e = -6$ $\quad f = 5$

Letra D.

8. $A = \begin{pmatrix} 2 & K \\ -2 & 1 \end{pmatrix}$ $K = -2$ $A^{-1} = \begin{pmatrix} -1 & K \\ -2 & -2 \end{pmatrix}$

4 2 4 2

$\det A = 2 - 4 = -2$ $\det A^{-1} = 2 - 4 = -2$

$\det A = \det A^{-1}$

Letra B.

9. a. $(A+B)\cdot(A-B)$

$A^2 - AB + BA - B^2$

b. $(A+B)^2 = A^2 + 2AB + B^2$

$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$

$AB + BA = 2AB$

$BA = 2AB - AB$

$BA = AB$

c. $\frac{\det(A)}{\det(-A)}$

$\det A \cdot \det A^{-1} = 1$

$\det A^{-1} = \frac{1}{\det A}$

$1 \cdot \det A^{-1} = \det A$

$1 = \frac{\det A}{\det A^{-1}}$

d. $\det A \cdot \det B = 1$

$\det B = \frac{1}{\det A}$

→ Propriedades das matrizes inversas