PHYS 512 - Homework 7
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Problem 1:

The leapfrog scheme is given by:

$$\frac{f(t + dt, x) - f(t - dt, x)}{2dt} = -\frac{v(f(t, x + dx) - f(t, x - dx))}{2dx}$$

By plugging in a solution of $f(x, t) = \xi^t e^{ikx}$ we can check to see if the CFL condition will be meet. The factor of ξ^t is assumed to be complex and a function of k meaning it also has no dependence on x, only t. Plugging in, and isolating leads to:

$$\frac{e^{ikx}[\xi^{t+dt} - \xi^{t-dt}]}{\xi^t} = -\frac{vdt}{dx} e^{ikx}(e^{ikdx} - e^{-ikdx})$$

Where $(e^{ikdx} - e^{-ikdx}) = isin(dxk)$ and the factor of e^{ikx} can be cancelled on both sides. This results in,

$$\frac{[\xi^{t+dt} - \xi^{t-dt}]}{\xi^t} = -\frac{vdt}{dx} isin(dxk)$$

Now the CFL condition says that in order for it to be stable $\frac{vdt}{dx} \leq 1$. Looking at the magnitude of the right-hand side, it will obey the condition as long as $-\frac{vdt}{dx}$ is smaller than one. On the left hand-side, the factor is dependent on k so as long as the numerator remains small, which should be the case being that the function is complex meaning it will most likely lead to a sine or cosine solution as with the right hand side, we can assume that the left hand side will also remain smaller than one. This means the only factor that can violate the CFL condition is the spatial derivative itself, therefore the leapfrog scheme conserves energy and is stable as long as the condition is met.

Problem 2:

Unfortunately, the correct answer was not obtained for section A, where we are setting up Green's function, meaning the remain graphs are not perfectly correct either. For reference, I was receiving a value of 0.98556 for V[5,0] rather than the expected 1.05 approximately. This lead to a value of 0.99514 for V[1,0] and 0.9927476 for V[2,0]. This obviously means that all my values obtained with Green's function are incorrect leading to issues when the charge density and potential are calculated later. Checking the values of the potential calculated in part C for inside the box it is not clear that the values are constant, where the values hover around 500 ± 150. The graph [Figure 1] is not really what was expected, and it could be due the error in Green's function or possibly the convolution was not done correctly. The charge density on one side of the box is plotted below [Figure 2]. The straight lines down to zero occur at the edges but the charge along the edge is the curve alone. An oddity encountered when calculating the conjugate gradient was the fact that the residual was actually increasing with each iteration rather than decreasing as it should have been. It is hard to tell if the issue is coming from anything other than the Green's function since that function is what we use to create the original Ax which directly leads to the calculation of the residual. It is safe

to say that the rho calculated then is not correct since the conjugate gradient should only be cut off once the value for the residual becomes small enough, and this shows no sign of shrinking.
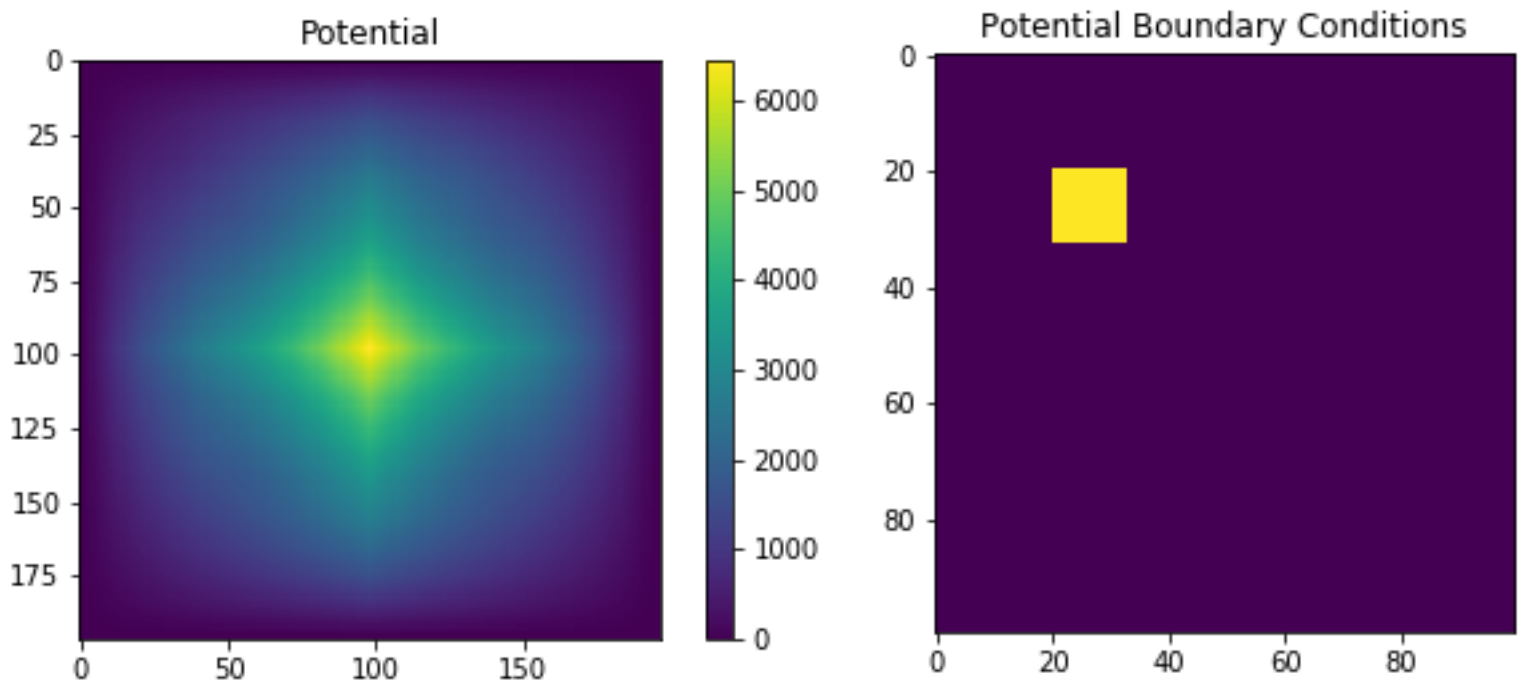

Figure 1: (Left) The graph of the potential energy calculated in part C. (Right) For reference, the boundary conditions used for the square being held at constant potential.
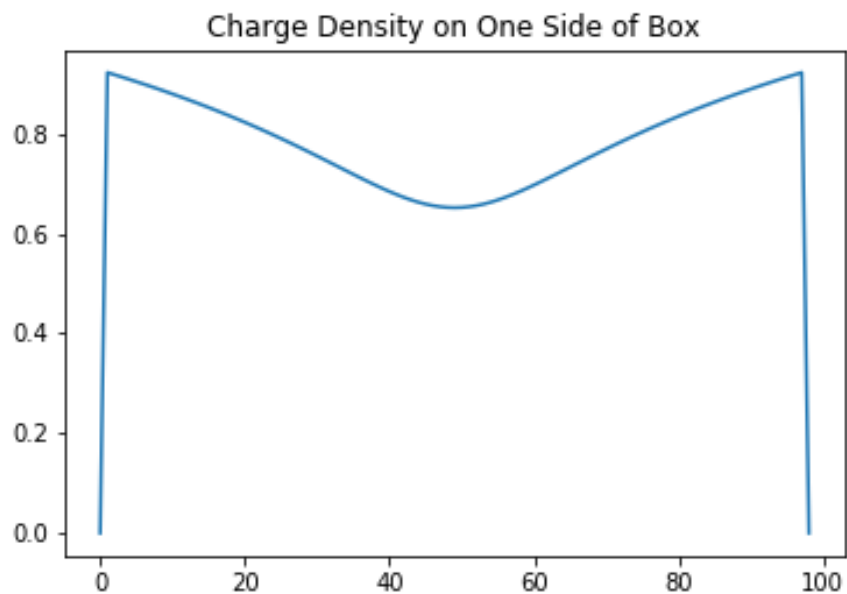

Figure 2: Graph of charge density for one side of the box.