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Homework 3: PHYS512
Problem 1:

In order for this paraboloid equation to be linear, a change to cylindrical coordinates is necessary.

$$x' = rcos\theta, \qquad y' = rsin\theta, \qquad z = z$$

This leaves a paraboloid equation as; $z = z_0 + ar^2$ where $r^2 = x^2 + y^2$. If we assume that the paraboloid passes through points (0,0,0) then the initial (x_0, y_0, z_0) can be ignored. If it does not, then the *x'* and *y'* values will contain the original format of $(x - x_0)$ and $(y - y_0)$ and the z_0 value will remain in the new paraboloid equation. For this problem we will assume that the paraboloid passes through the origin as that will allow for the use of the focal length equation included in the assignment. This leaves a very simple linear equation where the only unknown value left is *a.* The parameters used for the fit was then simply *r* and *z*. The *r* value was constructed based on the *x* and *y* terms as mentioned above therefore, those values remained the same in the fit. The value for *z* is where the noise was incorporated. The noise for the prediction of *z* was calculated at 806.353 and from that the uncertainty in *a* was estimated to be $2.1308x10^{-6}$.

The focal length was calculated using the adjusted equation for the parameters used in this problem, meaning: $f = \frac{r^2}{4z}$. The average number for the focal length was 6.148 meters which is quite a bit higher than the expected 1.5m. This could indicate that the change in parameters used was not the right choice. The error bar was calculated to be 0.00239 which clearly is not accurate since we are expecting a much smaller number.

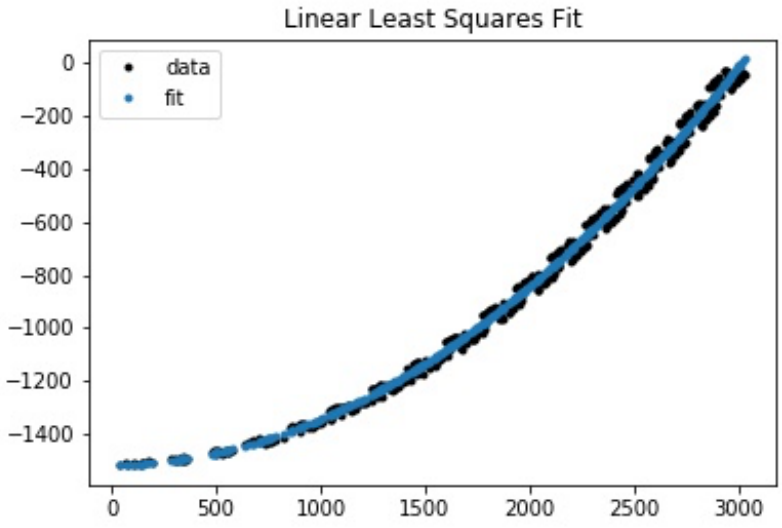


Figure 1: Least squares fit for the dish data.

Problem 2:

The equation used to solve this problem is; $\chi^2 = \sum \frac{(x-m)^2}{\sigma^2}$. Where x is the data from the WMAP file, m is the calculated spectra from CAMB and σ^2 is the error given in the WMAP 3rd column. The final answer was 1588.23764 for χ^2.

Problem 3:

As the derivatives are required in order to us the Newton's method, a numerical derivative was performed using the central differencing equation;

$$\frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

The spectrum was calculated at two separate values, the first with the parameters plus a step of 1% the value and a second with the parameters minus the 1%. The calculated values were then input into the derivatives equation in order to create the derivatives matrix. When plotting this matrix, the plots needed to be separated into each parameter as the difference in values made it impossible to see the smaller value's plots. All the plots [figure 2] have relatively the same shape and has wiggles. This makes sense since the derivative is being performed in steps and as the value for the spectrum changes drastically, we expect a change in the derivative step relative to the spectrum. When running this script with only Newton's method, the values begin to diverge rather quickly, meaning a Lavenberg Marquadt minimizer was required as well. The method suggested in class was used, as in λ was started at zero and increase by two for each step with an increase in χ^2 and decreased by $\lambda/\sqrt{2}$ for each decrease in χ^2. When running the algorithm for tau held at 0.05 the values received after 10 iterations for the best fit are 63.25278713574684 for H_0, 0.022052868015065476 for $\omega_b h^2$, 0.12185338008847825 for $\omega_c h^2$, $2.0807797203976893x10^{-9}$ for A_S and 0.9499219480097826 for the primordial power law. The errors for these parameters are $4.51561009\ x10^{-2}$, $9.43917843\ x10^{-4}$, $1.98620368\ x10^{-3}$, $2.90657937\ x10^{-7}$ and $6.86199204\ x10^{-3}$ for H_0, $\omega_b h^2$, $\omega_c h^2$, A_S and the primordial power law respectively. When the algorithm is run now with τ floating and the other values fixed, τ very quickly diverges and breaks. It very quickly goes towards negative τ values with cause the subsequent derivatives to be zero. The errors increase a bit with a run with τ floating so it would most likely continue to get bigger if more iterations were run before it breaks the chain. The new errors for the terms are $6.17691216\ x10^{-2}$, $1.29144678\ x10^{3}$, 2.71917428e-03 $3.97802270\ x10^{-7}$, $9.37905066\ x10^{-3}$ and $2.43819351\ x10^{-2}$ for H_0, $\omega_b h^2$, $\omega_c h^2$, A_S, the primordial power law and τ respectively. When forcing the derivative for τ to be positive and rerunning everything else the same for a few more iterations, the value for τ still diverges to a much larger number than expected (it was approximately 7 at iteration 2) and breaks after only 2 iterations. Therefore, the iterations cannot even be completed when tau is taken into account for the Newton's method.

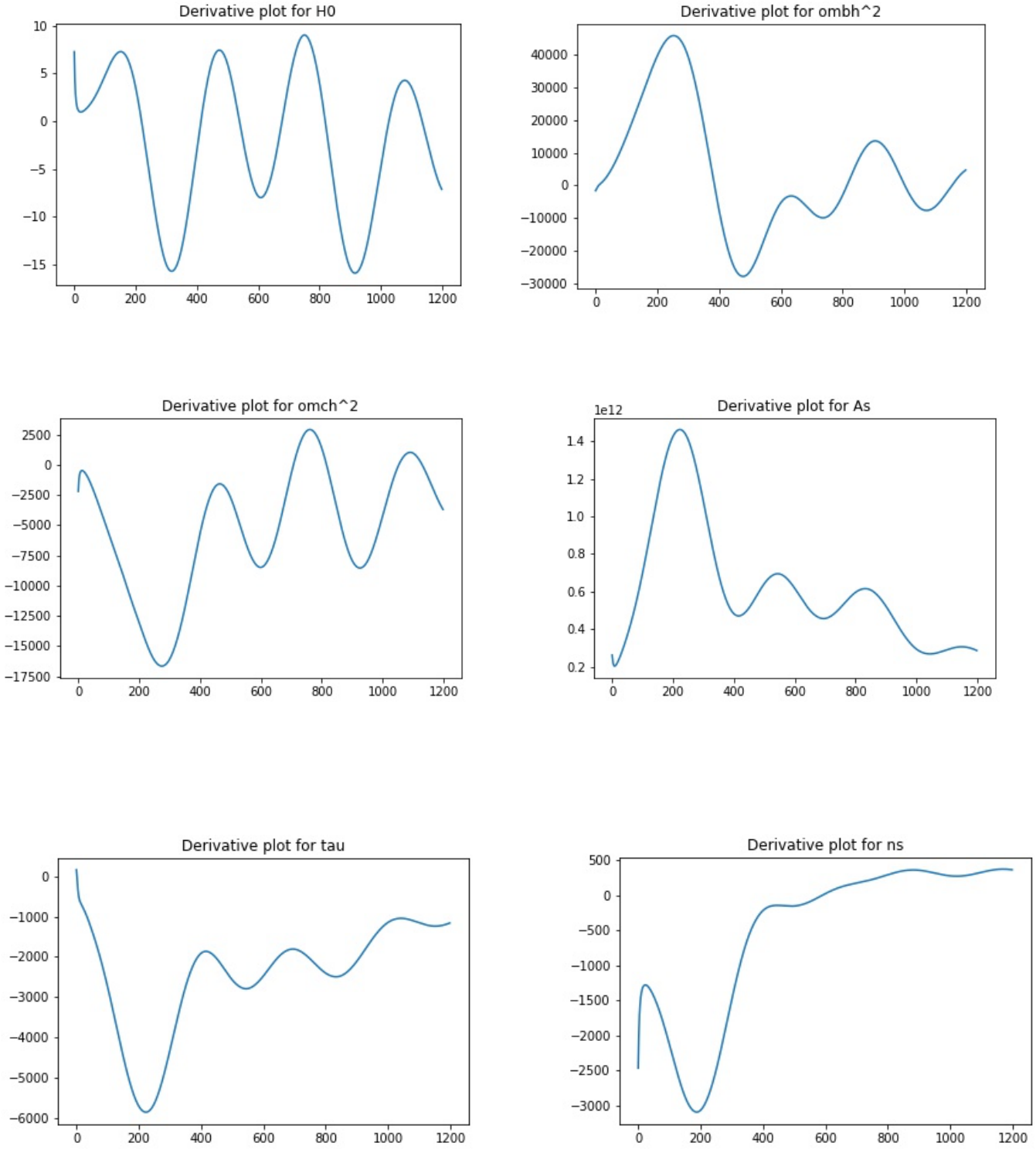


Figure 2: Derivative plots for all six parameters.

Problem 4:

The Markov Chain-Monte Carlo was run using the curvature matrix and covariance matrix from Newton's method in problem 3. The curvature matrix was used to calculate the appropriate step size that was used to run the first chain and the covariance matrix was used to calculate the sampling as well as eliminate the negative tau values. The covariance matrix was set up by first taking the inverse of the curvature matrix and then running a Cholesky decomposition on it. The negative tau values were discarded by adding a loop which ignored all values of the random gaussian draw that sampled a negative value in the column for tau. Coupled with the acceptance condition, it took quite a while to run the MCMC with 20 000 steps therefore I added a scale factor of 0.7 to the Cholesky matrix in order to increase my acceptance and decrease the step size. The first chain had parameter values of 63.3817115, 0.0193605187, 0.103169985, $2.01390894x10^{-9}$ and 0. 105457141 for H_0, $\omega_b h^2$, $\omega_c h^2$, A_S, the primordial power law and τ respectively. What was interesting was all the parameter values were very close to what was expected except for τ. The errors 0.763252386, $5.03818394x10^{-4}$, $1.38748920x10^{-3}$, $2.41395782x10^{-11}$, $7.98600460x10^{-3}$ and 0.0319062288 for H_0, $\omega_b h^2$, $\omega_c h^2$, A_S, the primordial power law and τ respectively. In order to check for convergence, a plot of the chain and the Fourier transform of It was required. Due to extremely long computing time, the plots were taken for only H_0. As mentioned in class, a converged chain would have a lot of white noise for the chain plot and the Fourier transform should have a flat section which would indicate that the power spectrum is leveling off. Unfortunately, that is not what occurs in the plots below [Figure 3] indicating that the Markov chain was not run sufficiently long. The Fourier transform appears to just decrease with no sign of a leveling off section anywhere in the spectrum. It is possible that this problem required closer to 50 000 steps, whereas only 20 000 were done here. It appears it takes approximately 5000 steps before it's forgotten its past and being that that is quite a large amount it is no surprise that 20 000 steps was not enough as it appears to stabilize towards the 12 500th step and on.

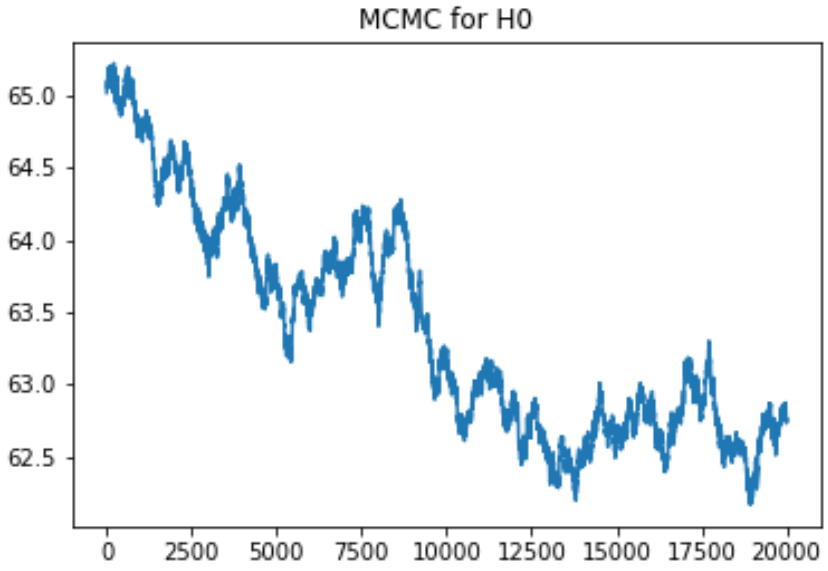
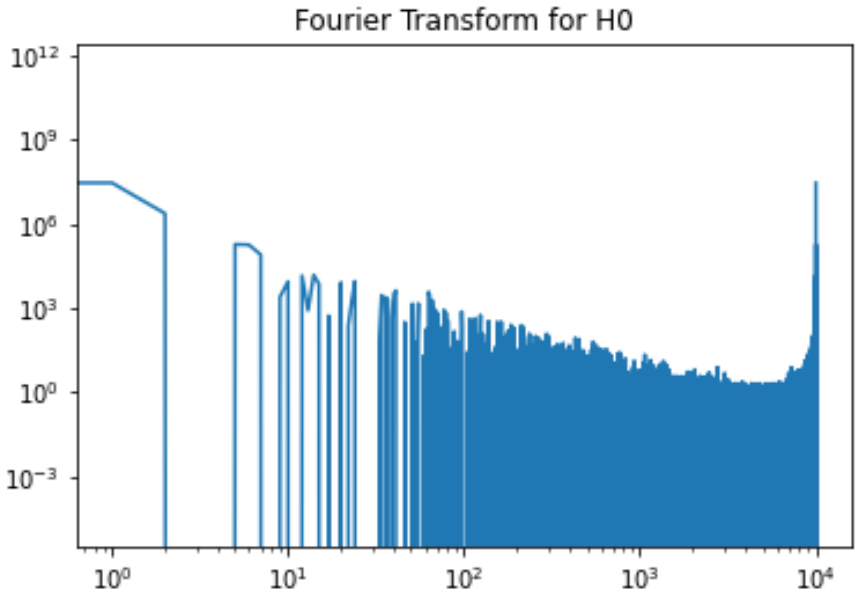


Figure 3: (Above) Plot of Chain for parameter H0. (Below) Fourier Transform of chain for parameter H0.



Problem 5:

The prior weighting was included via the likelihood equations. The value for τ was used to create a gaussian likelihood equation: $\frac{(\tau-0.0544)^2}{0.0073^2}$. This was then included by multiplying the old likelihood by the gaussian one. This routine was also only run for 20 000 steps meaning it did not converge either. The values for the parameters were 64.5575772, 0.0199079650, 0.101254343, $1.80130258x10^{-9}$, 0.953943741 and 0.00769398404 with errors of 0.374010646, $5.35470412x10^{-5}$, $8.33220086\ x10^{-4}$, $6.88140166\ x10^{-11}$, $3.06989641\ x10^{-3}$ and 0.0162557806 for H_0, $\omega_b h^2$, $\omega_c h^2$, A_S, the primordial power law and τ respectively. What was interesting about this run was that τ was still not close to the expected value even with prior weighting. Based on the graph it is apparent that steps in the wrong direction were taken at first and required a big decrease for a while. This appears to be caused by an error CAMB reported: "Warning: xe at redshift zero is < 1; Check input parameters an Reionization_xe function in the Reionization module". Clearly the steps in the section with the spike on Figure 4 was leading to this error in the reionization function, which it then attempted to correct but did take up a large portion of the steps. Knowing this fact and that 20000 did not lead to convergence in the previous run it is not surprising that this chain also did not lead to convergence.

Figure 4: Prior weighted chain for parameter H0
(Above), and Fourier Transform for H0 (Below)