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Assignment 5
PHYS 512

Problem 1:

A – For the noise model, as suggested I used a window with an extended flat period. Specifically, a tapered cosine function was used via scipy's library for signals. To smooth/ whiten the data I first attempted to use a gaussian filter since it does not weigh all the pixels in the filter the same as with the boxcar filter. The issue with the gaussian filter compared to the boxcar one was the fact that it appeared to cut off a final spike in the data (see figure 1). This was concerning as that spike may have important data and cutting it out entirely did not seem like a good filter. The boxcar filter was used instead throughout.

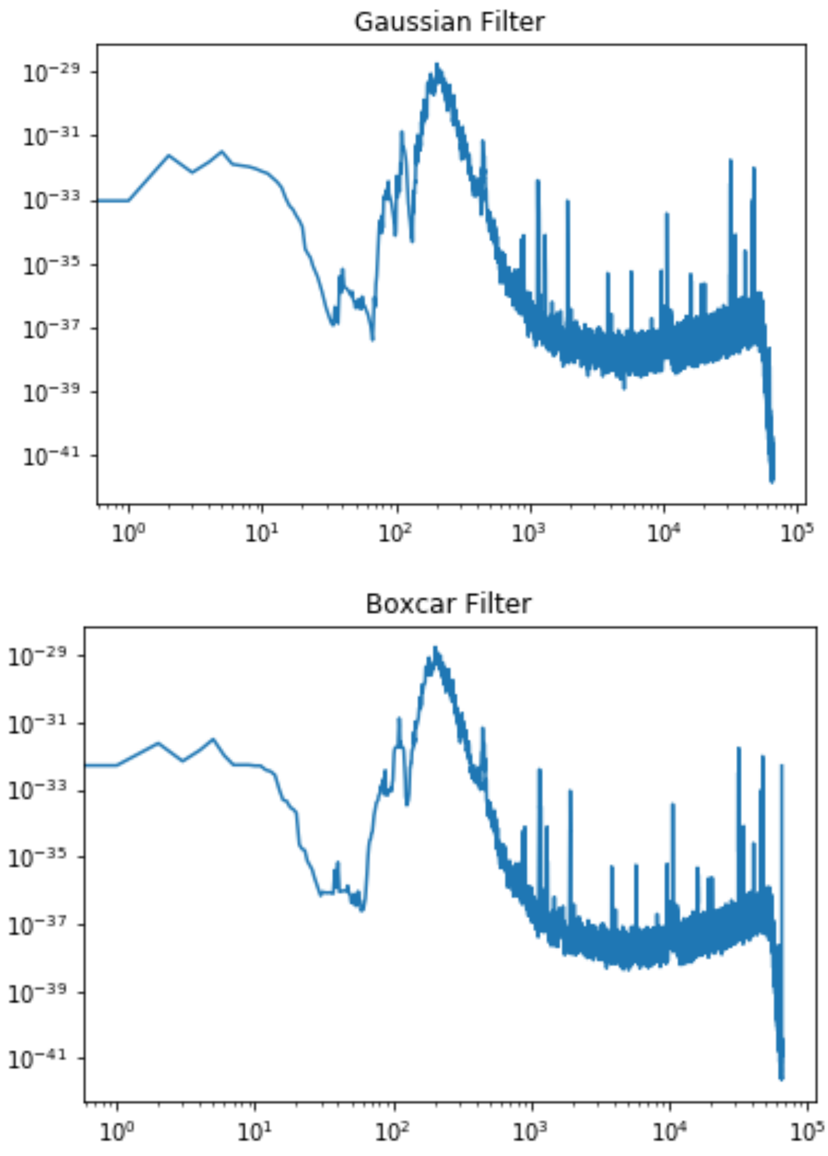


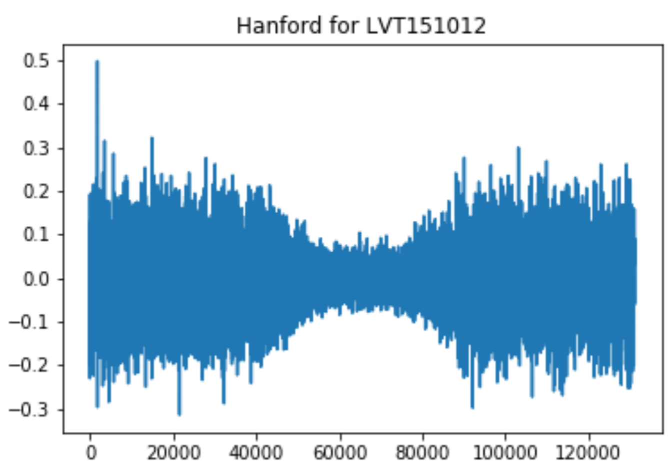
Figure 1: (Upper) Gaussian filter applied for smoothing compared to (Lower) Boxcar filter for same number of pixels. Visible spike towards end of curve missing from Gaussian filtered noise model. Event: LVT151012 at Hanford detector.

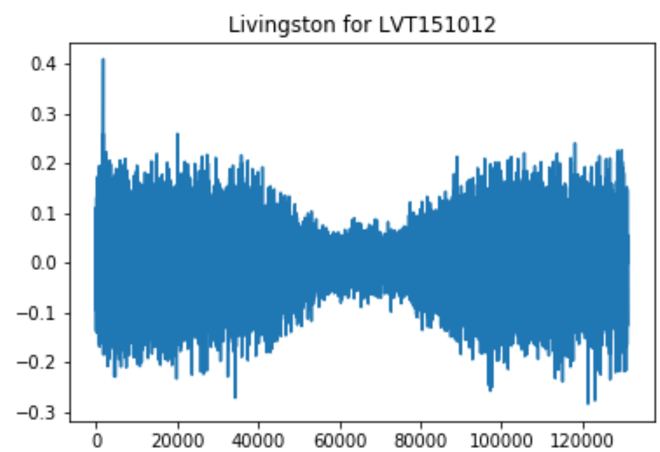
The lines were treated by using numpys maximum which returns the higher value between two arrays. The two arrays were the noise model with the whitened and windowed data and the original noise model simply given by the square of the Fourier transform of the strain data. This method is used just to ensure that lines which had a higher value in the original noise model were used instead in order to avoid over smoothing which dampens the noise too much. The final noise model was therefore a combination of the maximum, the window function and the whitening/smoothing filter.

B and C –

The noise was given by the standard deviation of the matched filter. The signal to noise ratio was given by the numpy max of the absolute value of the matched filter divided by the noise. The combined signal to noise ratio was found using a quadrature sum of the two separate ratios. Certain events had a very clear spike for the gravitational wave detection such as GW150914, whereas others had a bit more noise resulting in a lower signal to noise ratio (for example GW151226 at Livingston). The clearest event was GW150914 with a combined SNR of 24.11 which is very close to the approximately 25 that LIGO received.

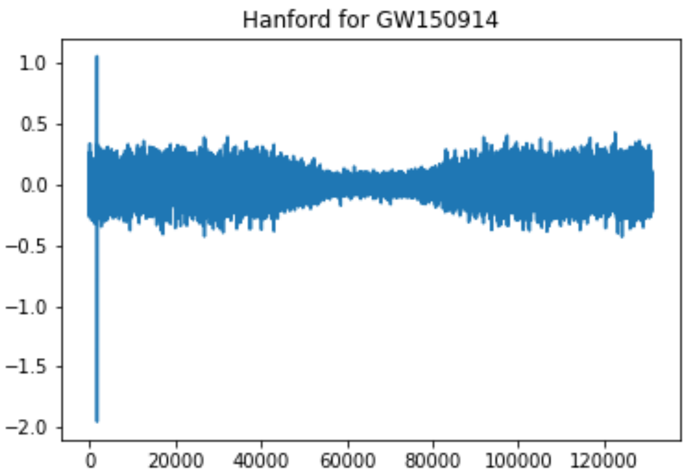
Event: LVT151012

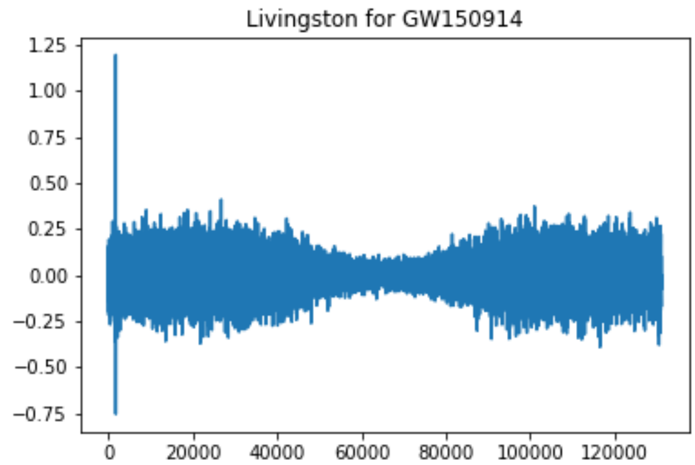




Noise for Hanford: 0.066092856663097593
Signal to Noise at Hanford: 7.5417326431105005
Noise for Livingston: 0.059750375231724794
Signal to Noise at Livingston: 6.853551677543717
Combine SNR: 10.190628138486892.

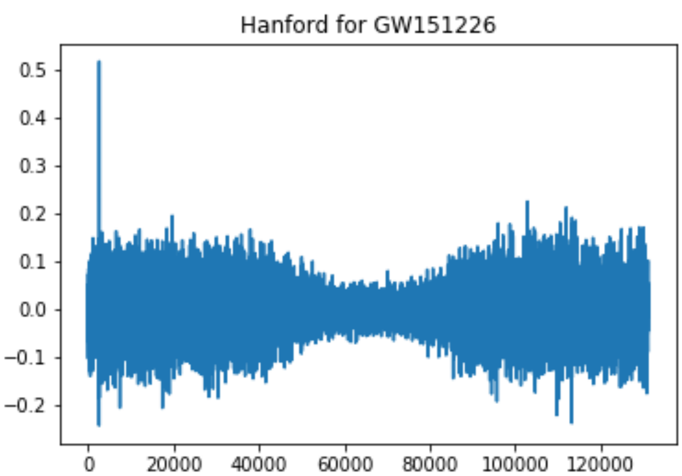
Event: GW150914

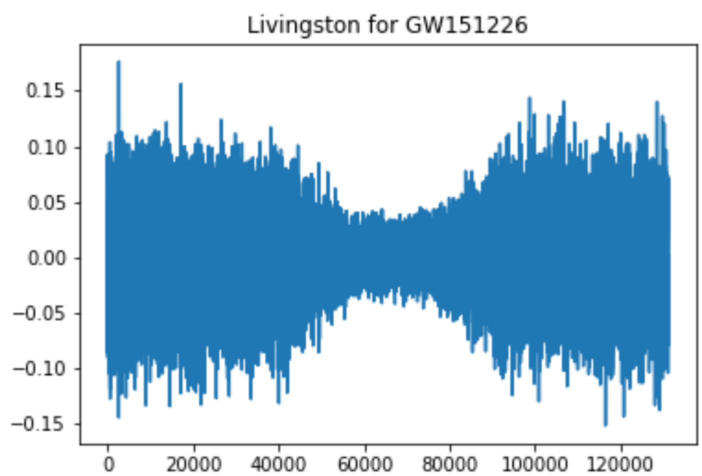




Noise for Hanford 0.09915485138771815
Signal to Noise at Hanford 19.709812444323163
Noise for Livingston 0.08604326867975191
Signal to Noise at Livingston 13.888082306793073
Combine SNR 24.11131553338919

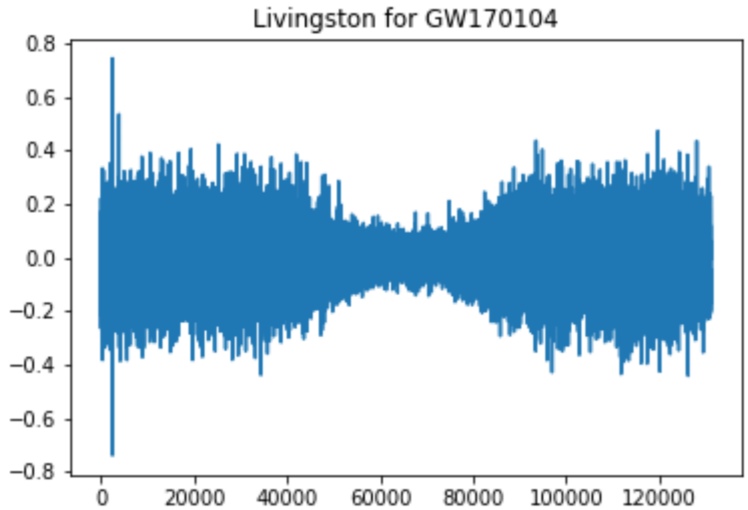
Event: GW151226

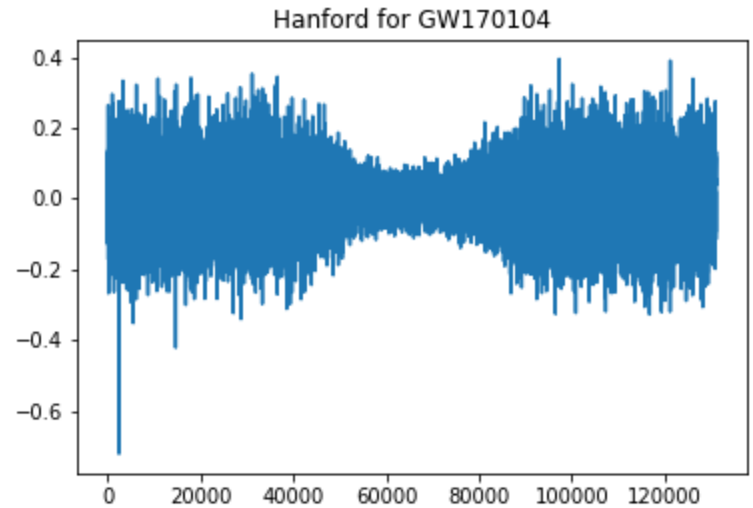




Noise for Hanford 0.04532381703803321
Signal to Noise at Hanford 11.37949187877167
Noise for Livingston 0.03168496161586664
Signal to Noise at Livingston 5.577059506362748
Combine SNR 12.672664603647549

Event: GW170104





Noise for Hanford 0.08197865197038329
Signal to Noise at Hanford 8.80701767995046
Noise for Livingston 0.10393919951225217
Signal to Noise at Livingston 7.176024968104373
Combine SNR 11.36040909288998 *6.690461609526588 5.4700293662505635*

D –

LVT151012:
SNR from Matched Filter: 7.5417326431105005 (H), 6.853551677543717 (L)
SNR from Noise Model: 5.724035543371799 (H), 5.209370762849318 (L)

GW150914:
SNR for Matched Filter: 19.709812444323163 (H), 13.888082306793073 (L)
SNR for Noise Model: 15.04100284472195 (H), 10.548073147911092 (L)

GW151226:
SNR for Matched Filter: 11.37949187877167 (H), 5.577059506362748 (L)
SNR for Noise Model: 8.619971678406507 (H), 4.229257231690014 (L)

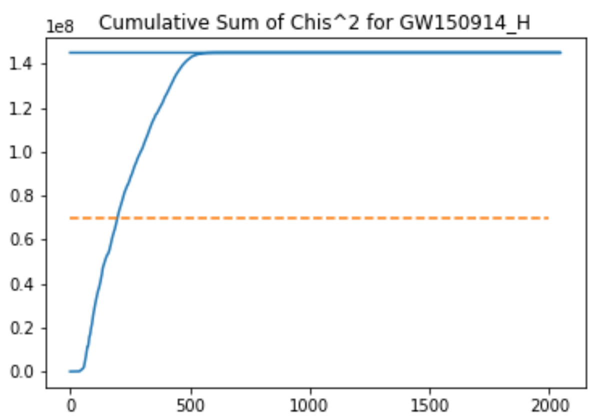
GW170104:
SNR for Matched Filter: 8.80701767995046 (H), 0.10393919951225217 (L)
SNR for Noise Model: 6.690461609526588 (H), 5.4700293662505635 (L)

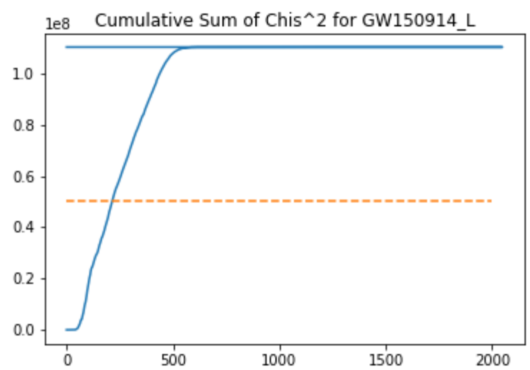
Based on the values for each of the events it is clear that the SNR for the noise model is always close but a little smaller than that for the matched filter. This is most likely due to the way in which we take the SNR for the noise model. The denominator in the noise model case is given by $\sqrt{\left(\frac{A}{\sqrt{N}}\right)^2}$ which includes a lot more noise than the standard deviation of the matched filter. Our noise model therefore takes in a lot more noise with larger and varying peaks and lines compared to the matched filter which results in a smaller signal to noise ratio for the noise model.

E –

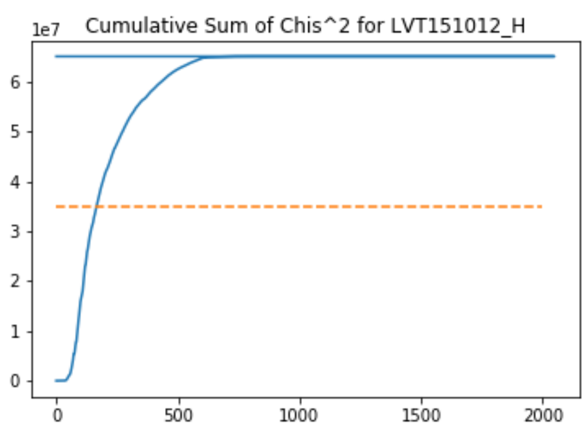
The equation for the power spectrum per frequency is: $\chi^2 = \left(\frac{FT(A)}{N^{\frac{1}{2}}}\right)^2$. By plotting the frequency versus the cumulative sum of χ^2 we can see approximately where the halfway point is in the plot and determine the frequency in question. The frequency for all events was found to be around 150-250 Hz approximately. The list of the frequencies per event is:

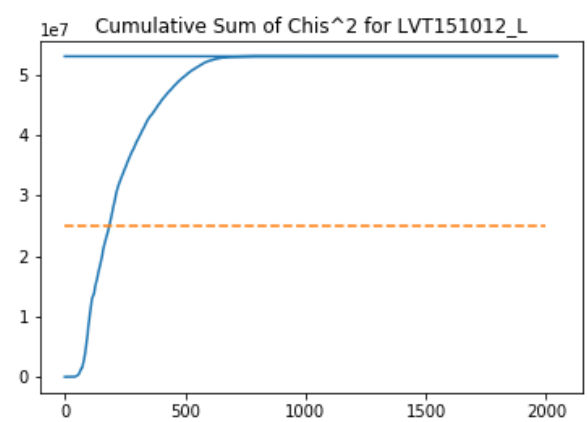
Event: GW150914
Hanford frequency: 144 Hz
Livingston frequency: 213 Hz





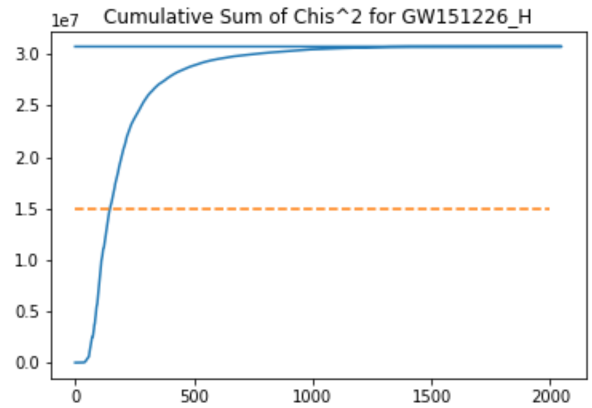
Event: LVT151012
Hanford frequency: 165 Hz
Livingston frequency: 184 Hz

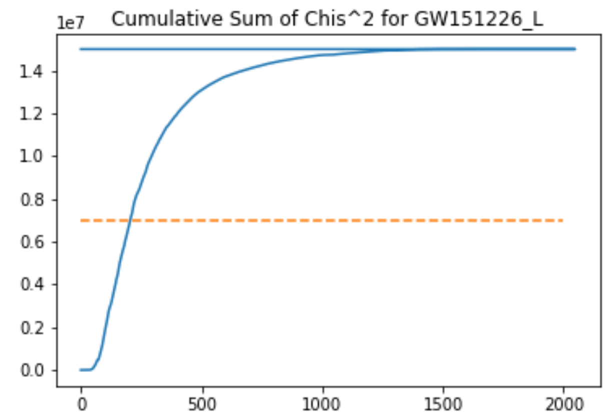




Event: GW151226

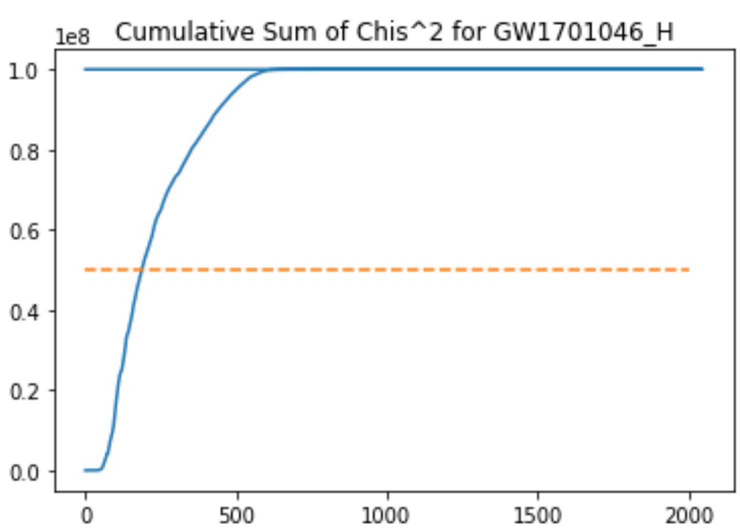
Hanford frequency: 146 Hz
Livingston frequency: 204 Hz

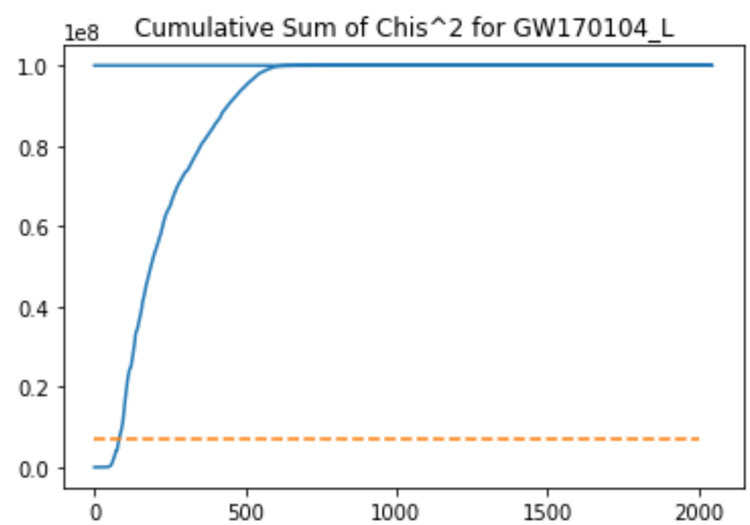




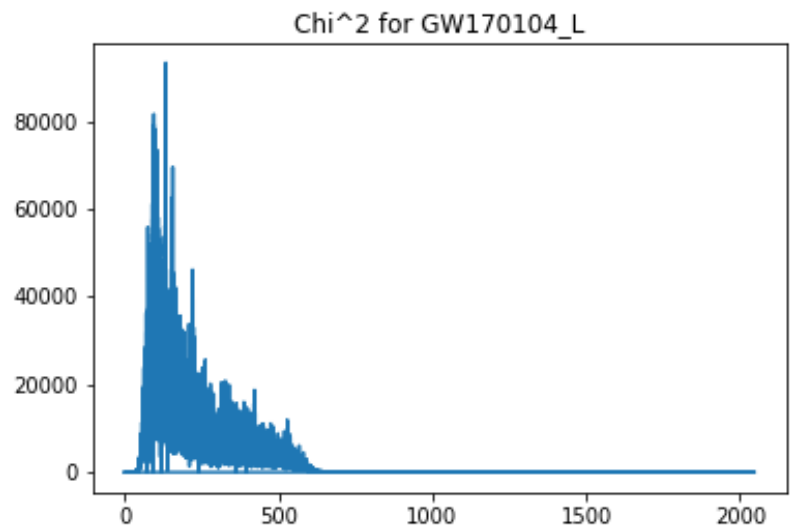
Event: GW170104

Hanford frequency: 186 Hz
Livingston Frequency: 186 Hz





As visible in all the graphs, there was an issue with the array received when taking the frequency which resulted in symmetric but negative frequencies at the end of the array (last 10 or so entries). I attempted to fix this issue by using fft.rfftfreq instead of fft.fftfreq but for some reason using the real version resulted in a frequency array half the size it should be meaning the two components of the plot could not be broadcasted together. The reason for this issue is still not understood as all values in which the Fourier transform, or the inverses Fourier transform was taken were all of the same length (this was checked using len for each variable in question). Everything had the exact same length up until the real frequency array. It is for that reason that the line remains. This line could be causing an error in the frequency, meaning my frequency may be a bit higher than expected due to the end of the array bring the plateau up but overall the frequencies are expected to be in the low range like those received so it should not have affected the results too much. A set of plots for χ^2 are displayed below [figure 2]. Since they are all very similar and the cumulative plot is what we are interested in, only one event was included below.



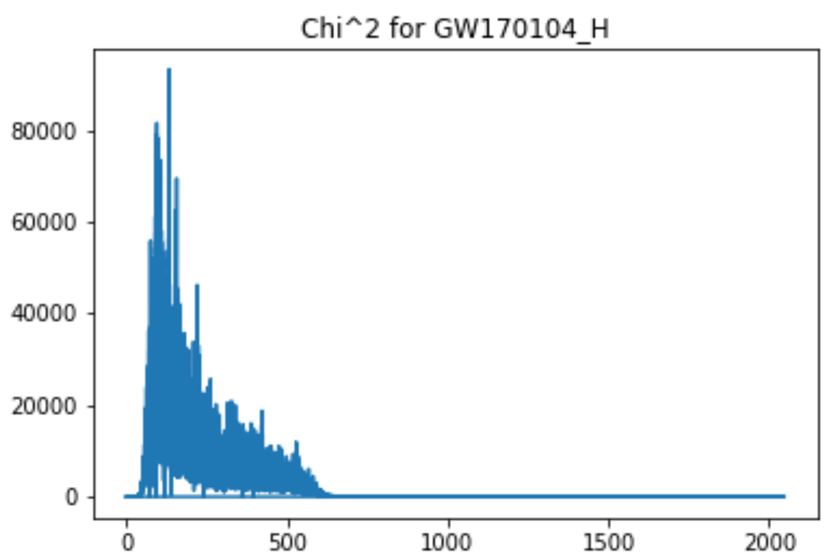
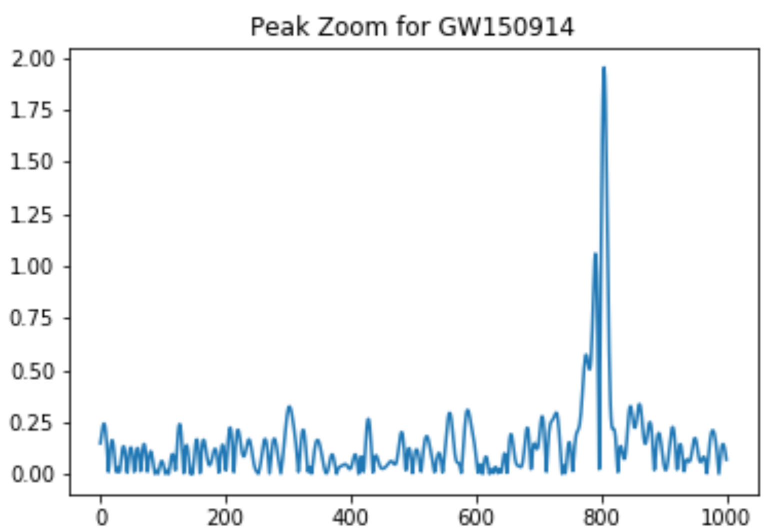


Figure 2: χ^2 plotted for both Hanford and Livingston during event GW170104

F –

Look at FWHM of each event. What do we use as difference in arrival time (aka L)?

In order to localize how well the time of arrival can be localized, the gravitational wave peak was zoomed in on and the full width at half maximum was approximated to get an idea of the uncertainty. There were certain events which are easier to see, meaning the gravitational wave came in more like a delta function and therefore had a very small width at half max, to just a few dt. Others were slightly more blurred and therefore had higher FWHM. For the event GW150914 at Hanford, the peak was nearly a delta function therefore it was very easy to localize the arrival time, where the peak occurred at [1804,1.954]. The FWHM was approximately 11 indexes wide, which is quite narrow compared to the whole matched filter plot. In comparison, the least clear event was GW151226 at Livingston. The peak in this case was [2655, 0.176] and the FWHM was approximately 13 indexes wide. It is still quite narrow but slightly less precise. For comparison the plots for both events zoomed in is displayed below [figure 3].



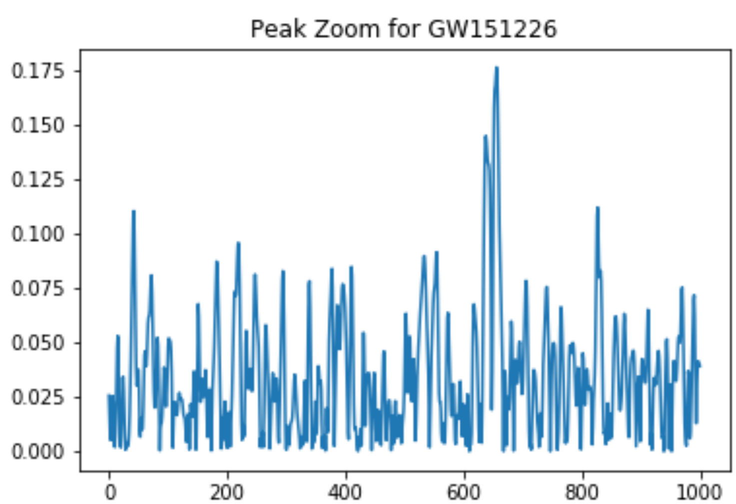


Figure 3: Comparison between two events for localization of GW (Upper) Hanford detector for GW150914, (Lower) Livingston detector for event GW151226

Using the interferometer geometry given by the image below [figure 4], the angular positional uncertainty was approximated for the two detectors approximately 2000km apart (This number was used since it is estimated the LIGO interferometers are a few 1000 km apart). Using the dt value as the time difference error, which is 0.0002441406255 no matter the interferometer choice the angular positional error was found to be approximately 2 degrees. This means we have a circle on the sky within which the gravitational wave event occurred, and it has an uncertainty of approximately 2 degrees or 0.0349 radians.

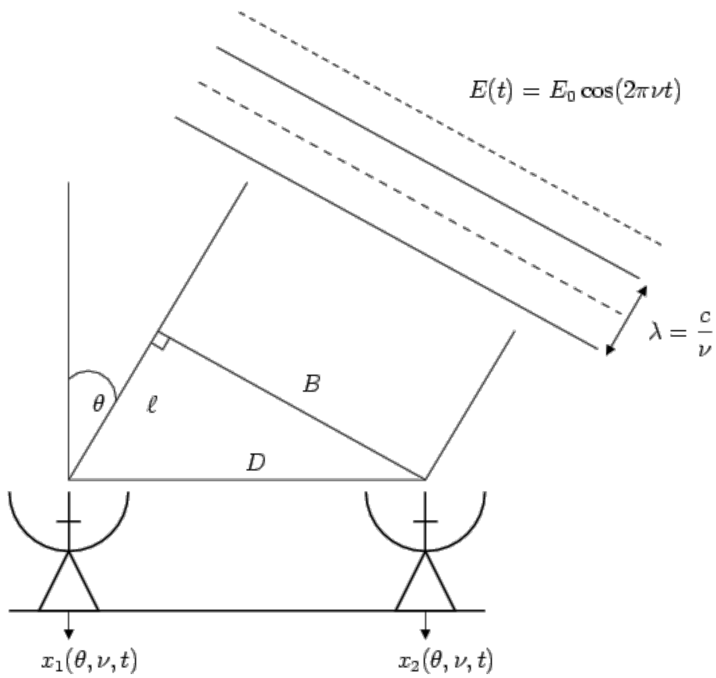


Figure 4: Geometry used to solve for angular positional uncertainty