PHYS 512 - Homework 7
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Problem 1:

The leapfrog scheme is given by:

$$\frac{f(t+dt,x)-f(t-dt,x)}{2dt} = -\frac{v\big(f(t,x+dx)-f(t,x-dx)\big)}{2dx}$$

By plugging in a solution of $f(x,t) = \xi^t e^{ikx}$ we can check to see if the CFL condition will be meet. The factor of ξ^t is assumed to be complex and a function of *k* meaning it also has no dependence on *x,* only *t.* Plugging in, and isolating leads to:

$$\frac{e^{ikx}[\xi^{t+dt}-\xi^{t-dt}]}{\xi^t} = -\frac{vdt}{dx}e^{ikx}(e^{ikdx}-e^{-ikdx})$$

Where $(e^{ikdx}-e^{-ikdx}) = isin(dxk)$ and the factor of e^{ikx} can be cancelled on both sides. This results in,

$$\frac{[\xi^{t+dt}-\xi^{t-dt}]}{\xi^t} = -\frac{vdt}{dx}isin(dxk)$$

Now the CFL condition says that in order for it to be stable $\frac{vdt}{dx} \leq 1$. Looking at the magnitude of the right-hand side, it will obey the condition as long as $-\frac{vdt}{dx}$ is smaller than one. On the left hand-side, the factor is dependent on *k* so as long as the numerator remains small, which should be the case being that the function is complex meaning it will most likely lead to a sine or cosine solution as with the right hand side, we can assume that the left hand side will also remain smaller than one. This means the only factor that can violate the CFL condition is the spatial derivative itself, therefore the leapfrog scheme conserves energy and is stable as long as the condition is met.

Problem 2:

Unfortunately, the correct answer was not obtained for section A, where we are setting up Green's function, meaning the remain graphs are not perfectly correct either. For reference, I was receiving a value of -3.024 for V[5,0] rather than the expected -1.05 approximately. This lead to a value of -2.582294696690568 for V[1,0] and -5.600594844687763 for V[2,0]. This obviously means that all my values obtained with Green's function are larger than what they should be lea ding to issues when the charge density and potential are calculated later. Looking at the plot of the potential [Figure 1 (right)] from part C, it is clear that the potential inside the box is not perfectly constant and looking at the values there are changes in the potential, but it is hovering generally around 2000 ± 200. The graph does seem to show that the potential is stronger closer to the box edges which makes sense since the potential should be stronger as we get closer to a point charge. The charge density on one side of the box is plotted below [Figure 2]. There is a bit of a curve to the plot but overall looks rather linear. An oddity encountered when calculating the conjugate gradient was the fact that the residual was actually increasing with each iteration rather than decreasing as it should have been. It is hard to tell if the issue is coming from anything other than the Green's function since that function is what

we use to create the original Ax which directly leads to the calculation of the residual. It is safe to say that the rho calculated then is not correct since the conjugate gradient should only be cut off once the value for the residual becomes small enough, and this shows no sign of shrinking.

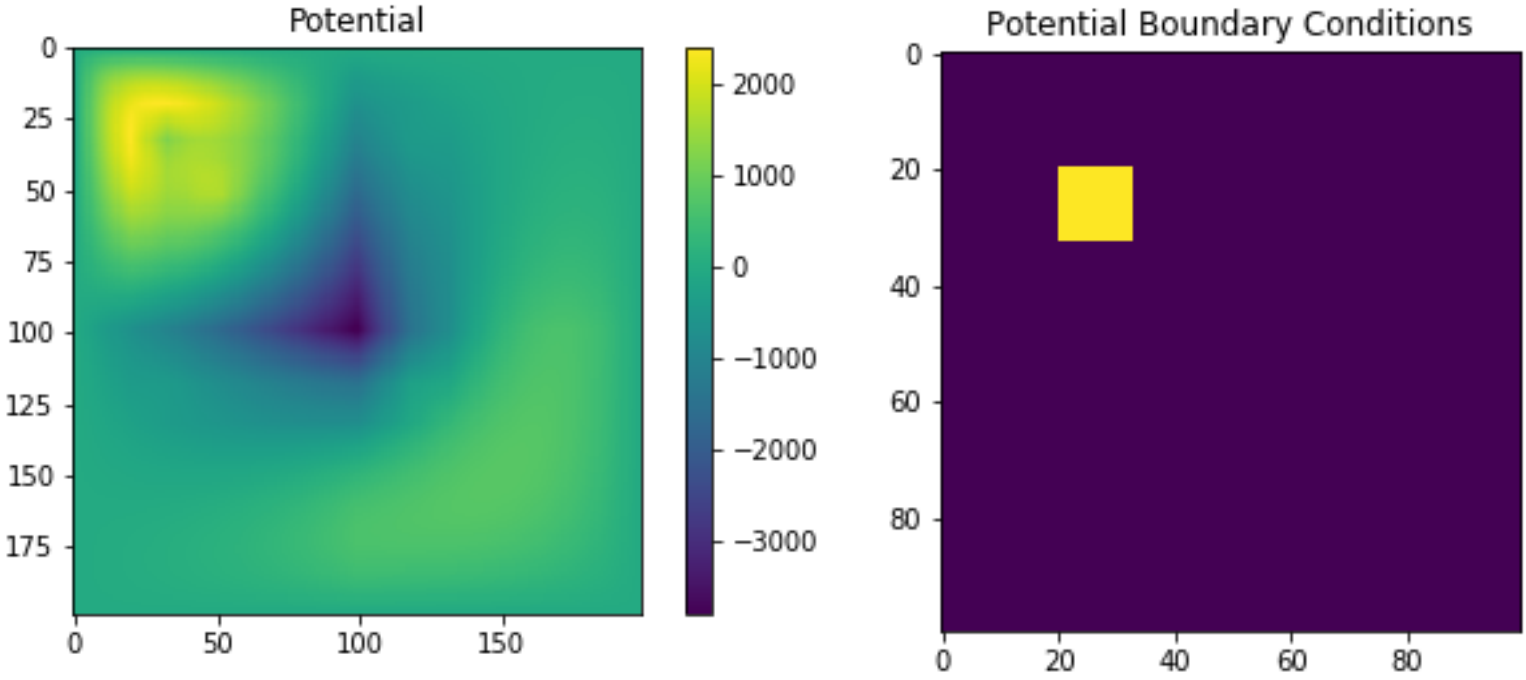

Figure 1: (Left) The graph of the potential energy calculated in part C. (Right) For reference, the boundary conditions used for the square being held at constant potential.

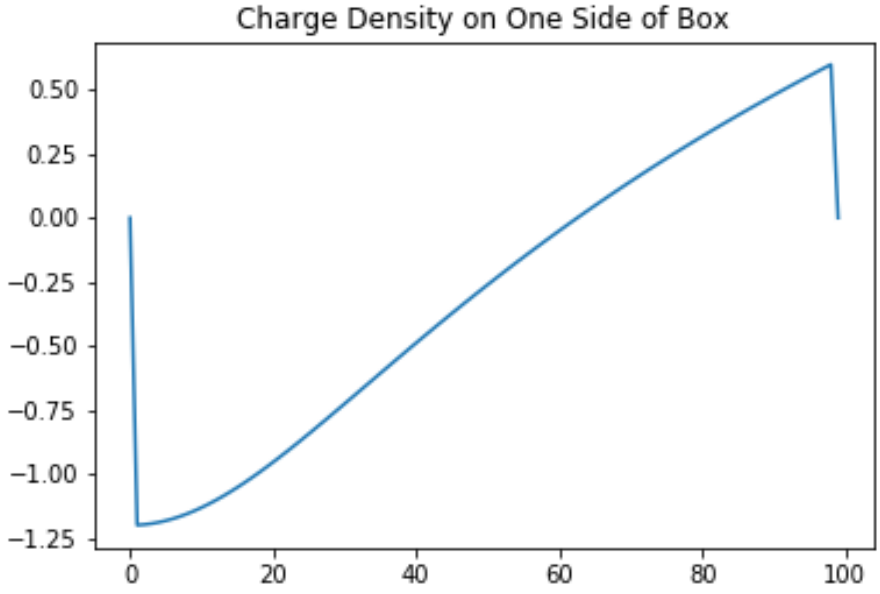

Figure 2: Graph of charge density for one side of the box.