Identity $(i(x))$

$$i(x) = x$$

$$i'(x) = \frac{d\,i(x)}{dx} = \frac{dx}{dx} = 1$$

Sigmoid $(s(x))$

$$s(x) = \frac{1}{1+e^{-x}}$$

$$s'(x) = \frac{d\,s(x)}{dx} = \frac{d}{dx}\left(1+e^{-x}\right)^{-1} =$$

$$= -1 \cdot \left(1+e^{-x}\right)^{-2} \cdot \frac{d}{dx}\left(1+e^{-x}\right)$$

$$= -\left(\frac{1}{1+e^{-x}}\right)^{2} \cdot -e^{-x} =$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} =$$

$$= s(x) \cdot \frac{e^{-x} + (1-1)}{1+e^{-x}} =$$

$$= s(x) \cdot \left(\frac{e^{-x}+1}{1+e^{-x}} - \frac{1}{1+e^{-x}} \right) =$$

$$s'(x) = s(x) \cdot (1 - s(x))$$

ReLU $(r(x))$

$$r(x) = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

- $x > 0 \; (r(x) > 0)$

$$\frac{dr(x)}{dx} = \frac{dx}{dx} = 1$$

- $x < 0 \ (r(x) = 0)$

$$\frac{d\,r(x)}{dx} = \frac{d0}{dx} = 0$$

- $x = 0 \ (r(x) = 0)$

$$r'_+(0) \neq r'_-(0)$$

$$\frac{d\,r(x)}{dx} \rightarrow \text{Undefined}$$

$$r'(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

Softmax $(sm(x_k))$

$$x_k \in S = \{x_0, x_1, \ldots, x_n\}$$

$$sm(x_k) = \frac{e^{x}}{\sum_{i=0}^{n} e^{x_i}}$$

$$t = \sum_{i=0}^{n} e^{x_i}$$

$$\frac{\partial sm(x_k)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{e^{x_k}}{t}\right)$$

$$= \frac{\frac{\partial (e^{x_k})}{\partial x_j} \cdot t - \frac{\partial (t)}{\partial x_j} \cdot e^{x_k}}{t^2} =$$

- $\boxed{k = j}$

$$= \frac{e^{x_j} \cdot t - e^{x_j} \cdot e^{x_j}}{t^2}$$

$$= \frac{e^{x_j} \cdot (t - e^{x_j})}{t^2} =$$

$$= \frac{e^{x_j}}{\Sigma} \cdot \frac{\Sigma - e^{x_j}}{\Sigma} =$$

$$= sm(x_j) \cdot \left(\frac{\Sigma}{\Sigma} - \frac{e^{x_j}}{\Sigma} \right) =$$

$$\frac{\partial sm(x_k)}{\partial x_j} = sm(x_j)(1 - sm(x_j))$$

- $k \neq j$

$$= \frac{0 - e^{x_j} \cdot e^{x_k}}{\Sigma^2} =$$

$$= -\frac{e^{x_j}}{\Sigma} \cdot \frac{e^{x_k}}{\Sigma} =$$

$$\frac{\partial sm(x_k)}{\partial x_j} = -sm(x_k) \cdot sm(x_j)$$

$$\frac{\partial sm(x_k)}{\partial x_j} = \begin{cases} sm(x_j) \cdot (1 - sm(x_j)), & j = k \\ -sm(x_j)\, sm(x_k), & j \neq k \end{cases}$$