MAE/ECE 5320 Mechatronics

2025 Spring semester

Lecture 01

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Course introduction

1. Syllabus

2. Lab sessions: in person (7 sessions) or remote (do it yourself)

A) Experiment stations (ENLAB 112), or

B) Option: buy an Arduino package (~$100); will post the link on Arduino.

3. Final project: a team with 1-3 members

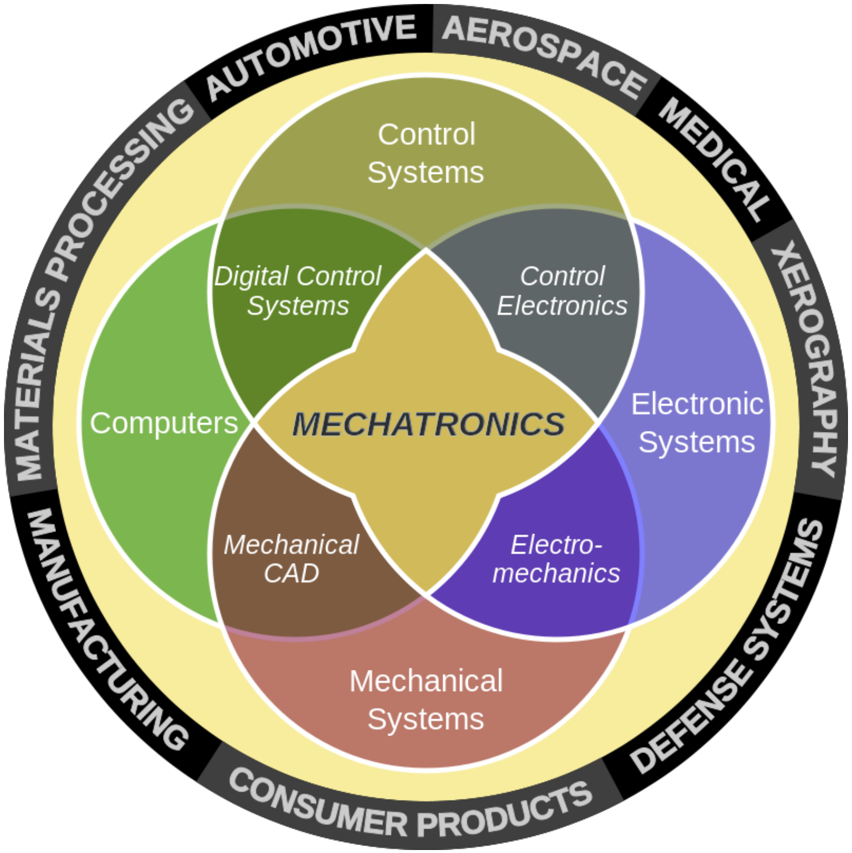
A) The last 3-4 weeks are reserved to work on final project.

What is covered in this class?

- Modeling physical systems
- Hardware components, sensors, actuators, Arduino (micro-controller)
- Basic signal processing, digitalization
- Control (PID) tuning
- Programming on Arduino using Matlab/Simulink
- Final project

Mechatronic System Overview

Mechatronic = mechanical + electronic



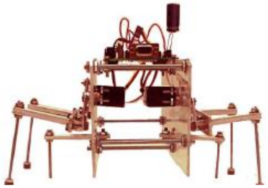
Mechatronic system is a multidisciplinary engineering field, including

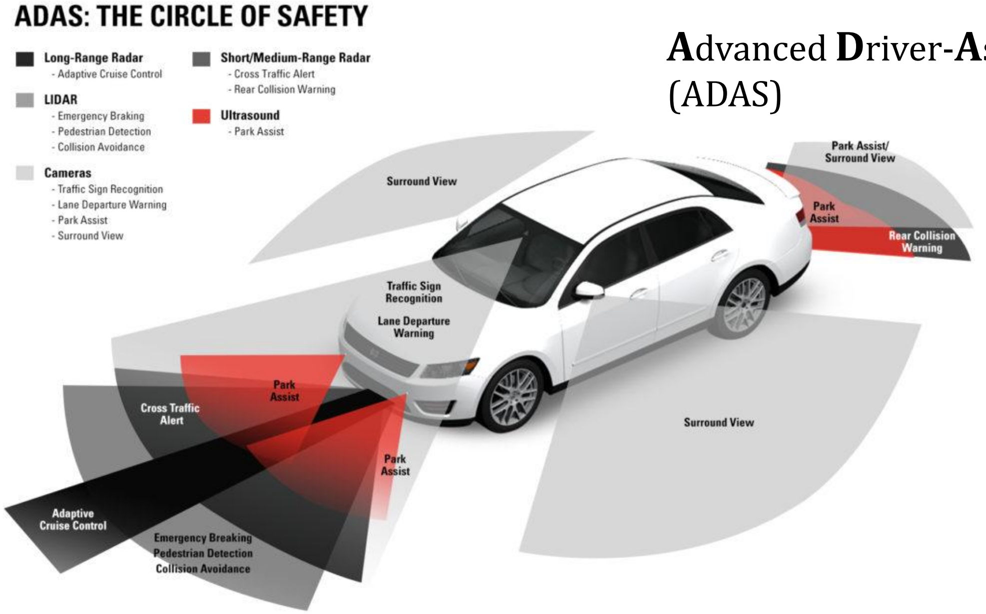
- Dynamic systems and Controls

(MAE/ECE-5310)
- Systems and signals
(ECE-3620-3640)
- Mechanical systems
(ENGR-2030 Dynamics)
(MAE-5300 Vibrations)
- Digital system
(ECE-3700)
- Computer engineering

Examples

Robotics





Advanced **D**river-**A**ssistance **S**ystems (ADAS)

Example 1: self-balancing two-wheel robot

Controller/microchip: Arduino MKR 1000 with WiFi

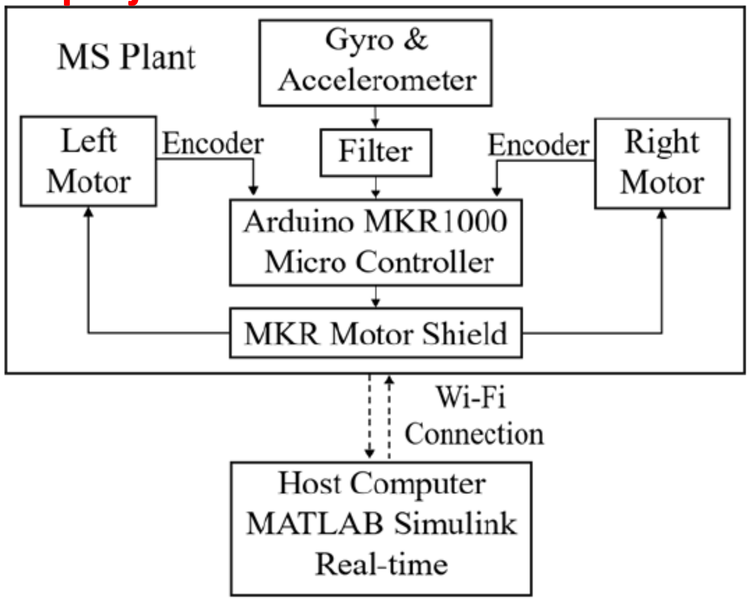
Motor: GB37 12V 1:4 gear DC motor with hall effect encoder with a resolution of 1320

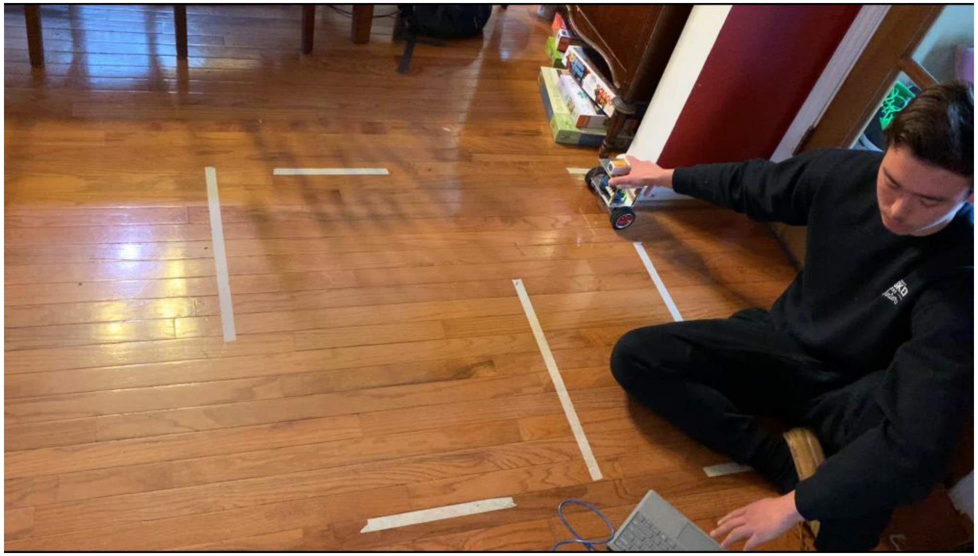
Gyro: MPU 6050 three-axle accelerometer and gyro chip

Motor Carrier: 12V PWM DC motor driver with 2 encoder reading channels

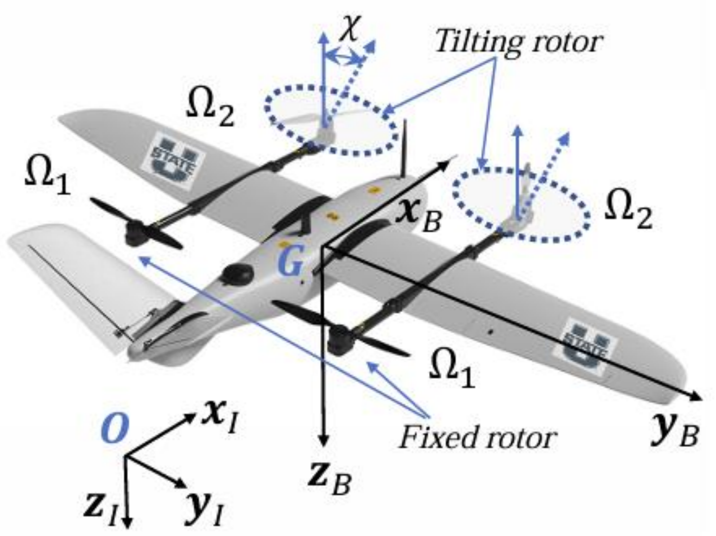
Accessories: Wheels, screws, motor mounts, controller mounts, and cart body and tools

Hardware available for final project !!!





Example 2: VTOL aircraft



Hardware available for final project !!!

https://usu.box.com/s/ir5e4axf0wmopk2ilb86e0a5l3hm53su

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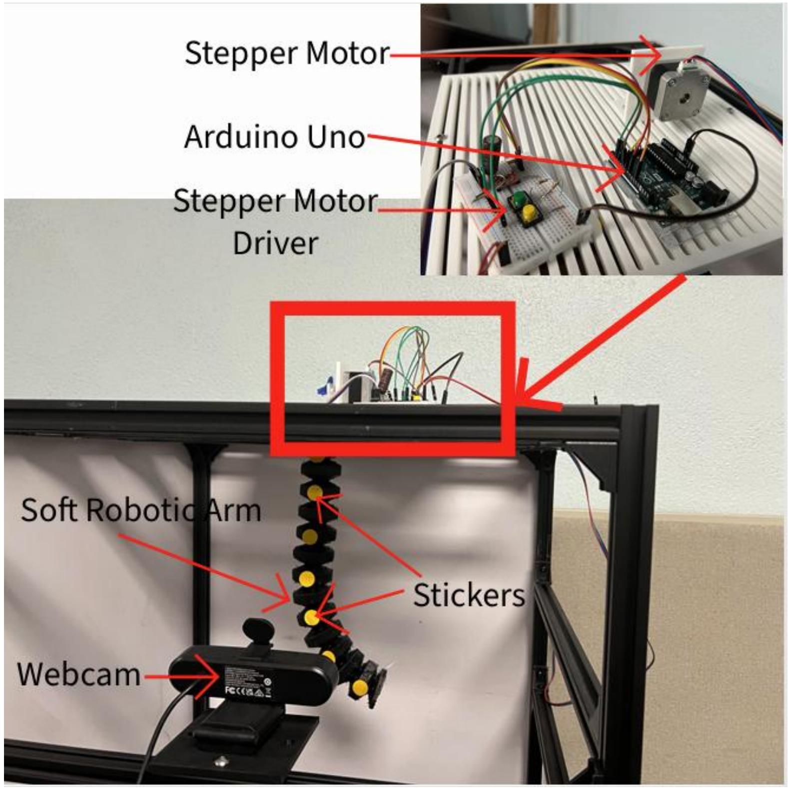
Example 3:Soft robotic arm

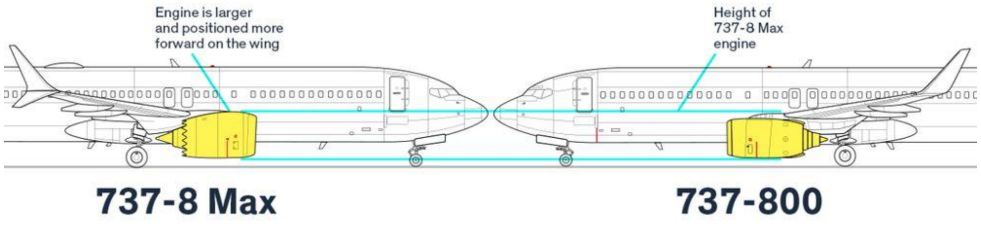


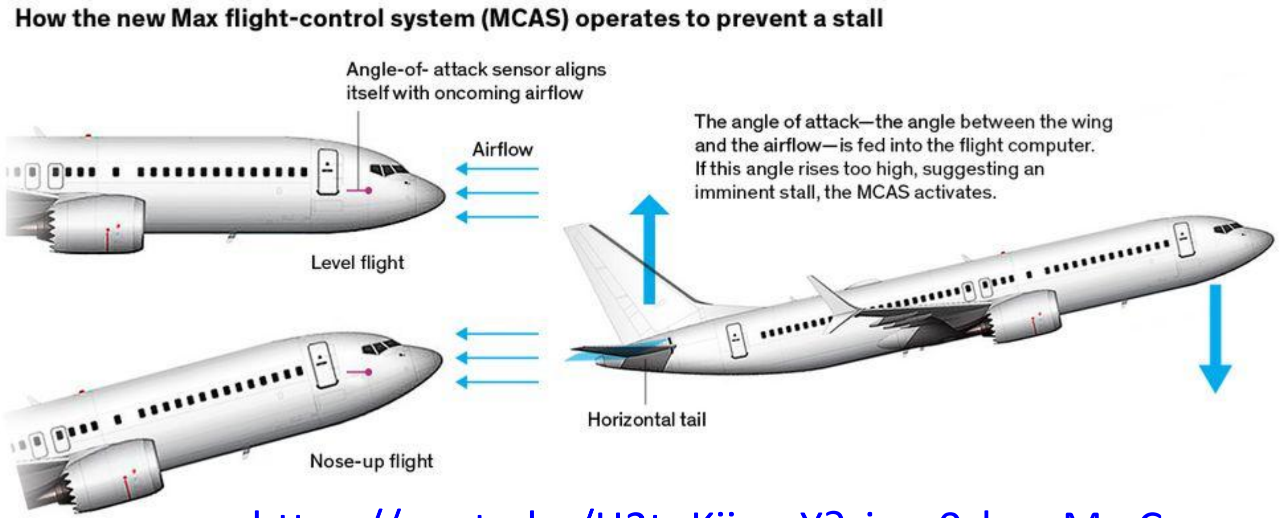
Hardware available for final project !!!

https://usu.box.com/s/7qa64tks2wukc26qapzzq1pvhr0y73c5

Example 4: How a disaster happens

Ethiopian Airlines Flight ET302, a Boeing 737 Max airliner that crashed on 11 March in Bishoftu, Ethiopia, killing all 157

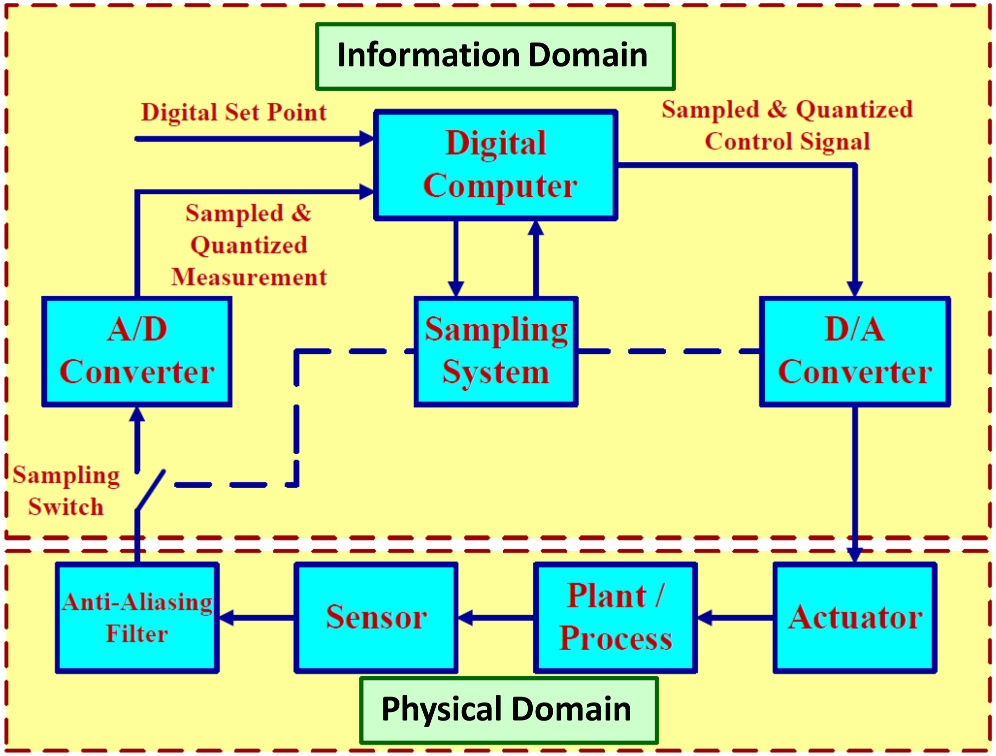


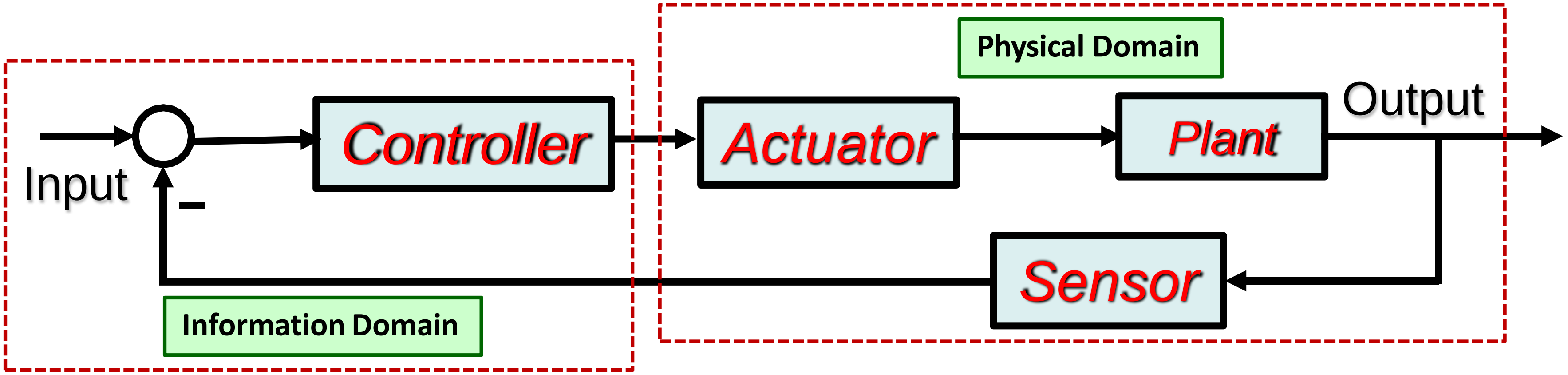


https://youtu.be/H2tuKiiznsY?si=m0skxzcMwC_8g7ts

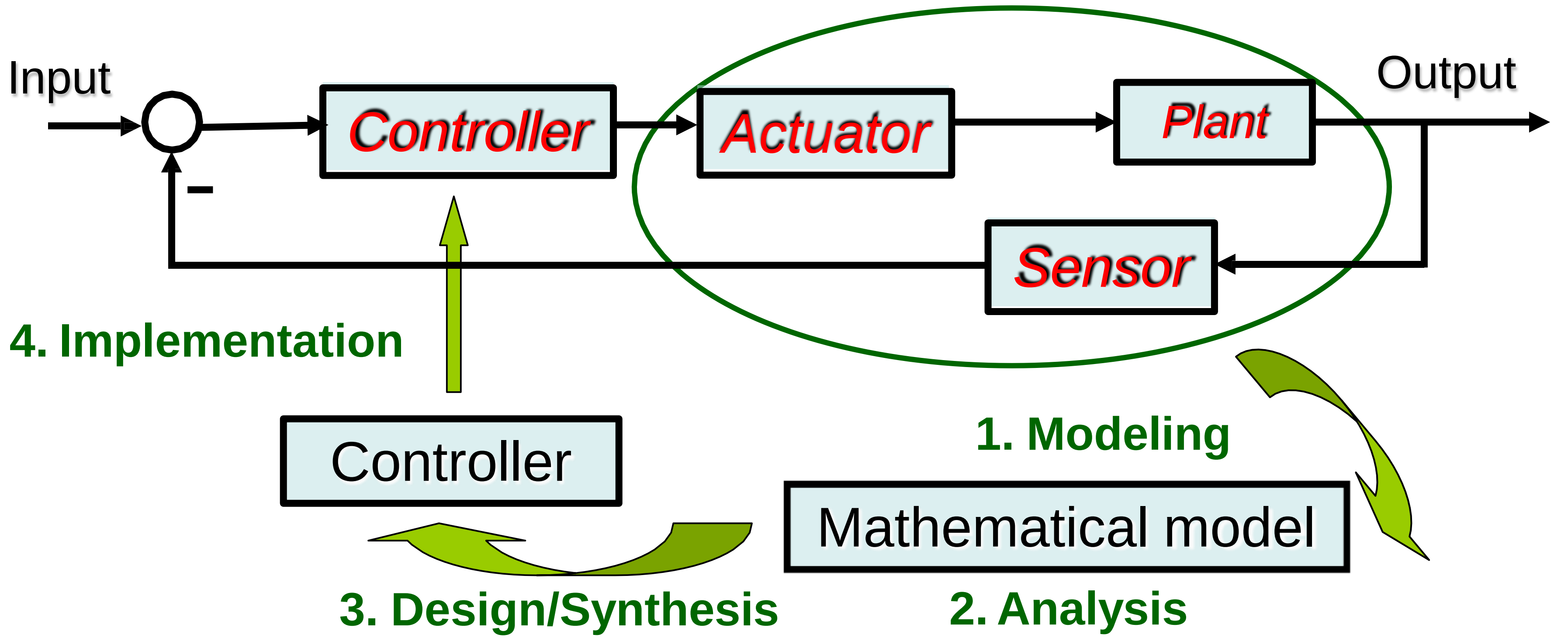
Read more: https://spectrum.ieee.org/how-the-boeing-737-max-disaster-looks-to-a-software-developer

Information and Physical Domains





Modeling – Part of Control Design Process



develop mathematical models for

- Electrical systems
- Mechanical systems
- Electro-mechanical system

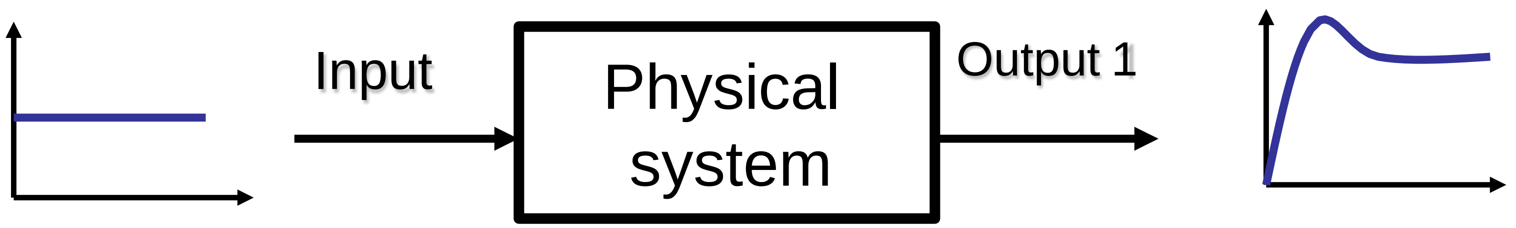
Electrical Systems:

- Kirchhoff's voltage & current laws

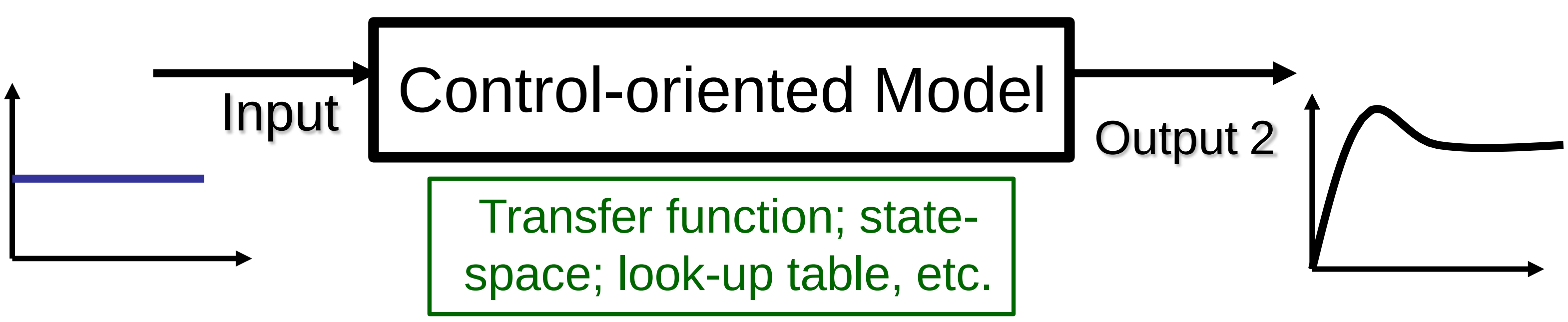
Mechanical systems:

- Newton's laws

Modeling Approaches

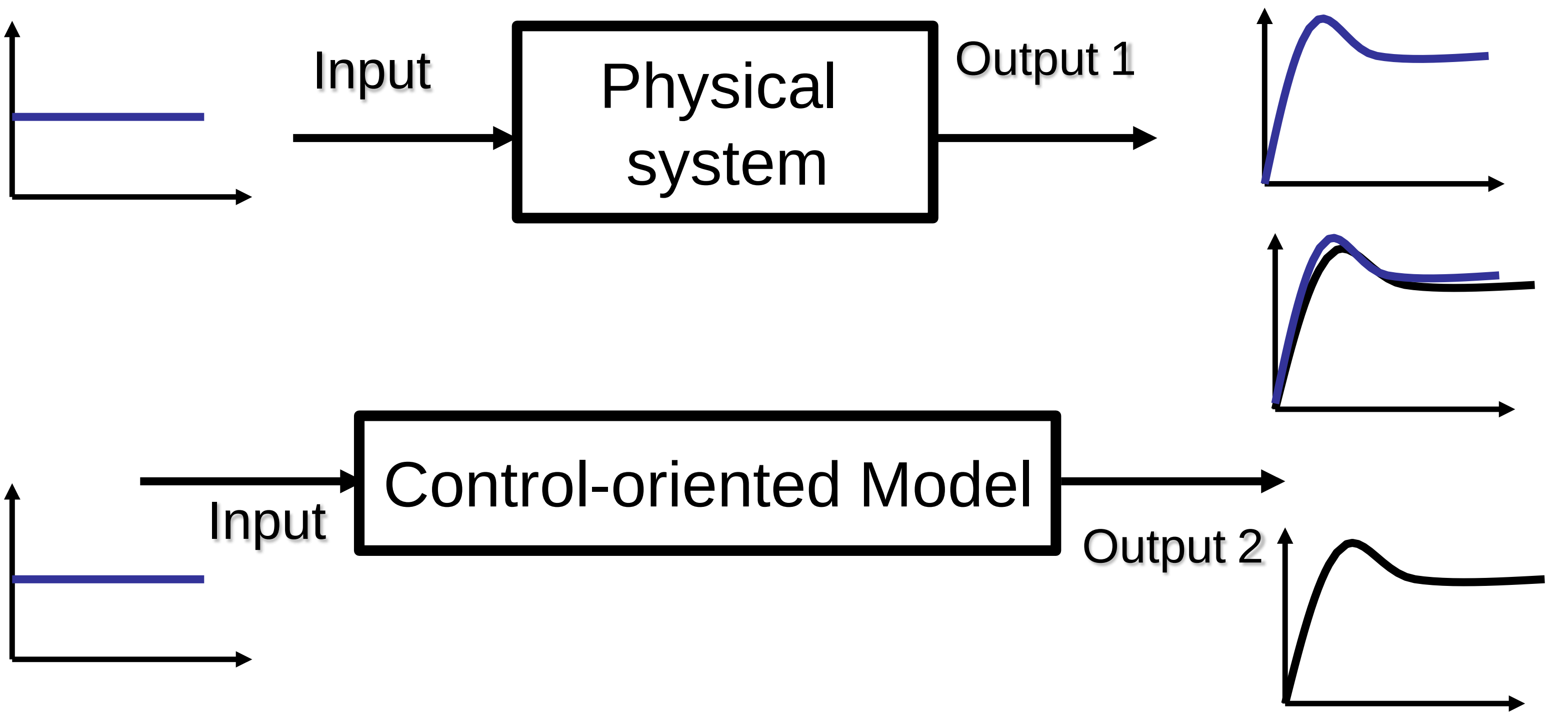


Physics-based model	Hybrid approach	Black box approach
Kirchhoff's Newton's Law Newton-Euler equations	System parameter identification with known structure	System identification to get System order & system parameter
Partial Differential Equations Ordinary Differential Equations		Neural-network AI (Artificial Intelligence) Machine learning



Modeling Discussion

- A control-oriented model is used for **analysis, controller design** and **simulation validation**
- No model is exact, but some are useful
- A good model is simple but captures the essential dynamics



Modeling – Transfer Function

- A transfer function is defined by its Laplace transform

$$G(s) := \frac{Y(s)}{U(s)}$$

Laplace transform of system output

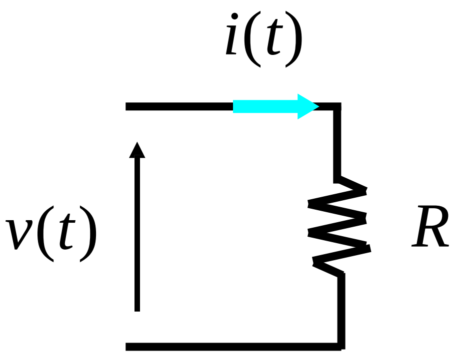
Laplace transform of system input

Time-domain	$u(t)$	$\rightarrow \boxed{g(t)} \rightarrow$	$y(t)$
Frequency-domain	$U(s)$	$\rightarrow \boxed{G(s)} \rightarrow$	$Y(s)$

- Assumption: zero initial condition

Modeling – Electrical Elements

Resistance

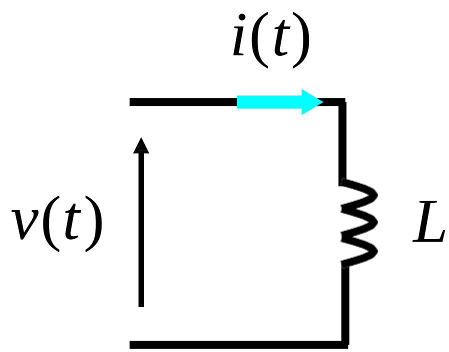


$$v(t) = R\,i(t)$$

Laplace transform

$$\frac{V(s)}{I(s)} = R$$

Inductance

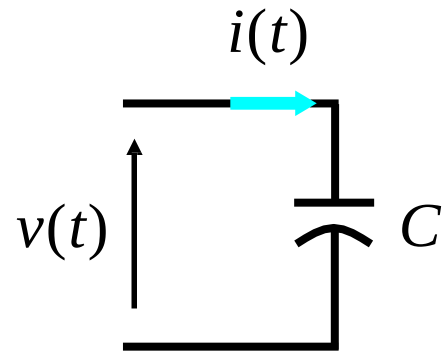


$$v(t) = L\frac{di(t)}{dt}$$

$$(i(0) = 0)$$

$$\frac{V(s)}{I(s)} = sL$$

Capacitance



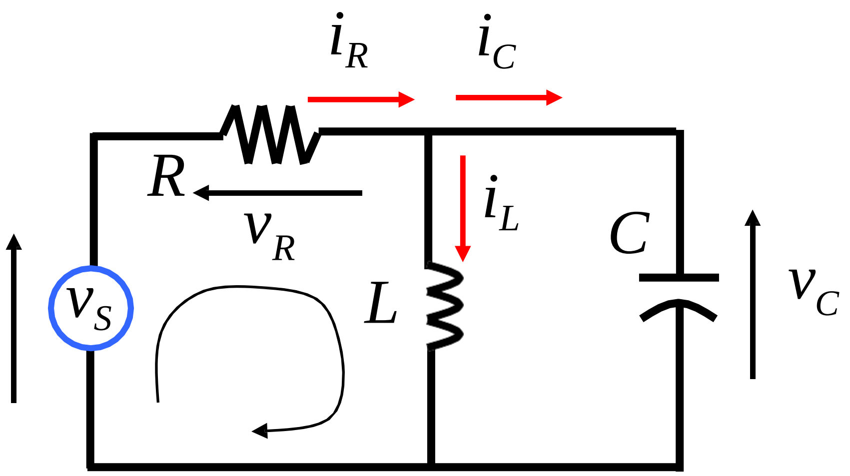
$$v(t) = v(0) + \frac{1}{C}\int_0^t i(t)dt$$

$$(v(0) = 0)$$

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

Modeling – Kirchhoff's Voltage & Current Laws

- The algebraic sum of voltage drops around any loop is zero.
- The algebraic sum of currents into any junction is zero.



Zero sum voltage:

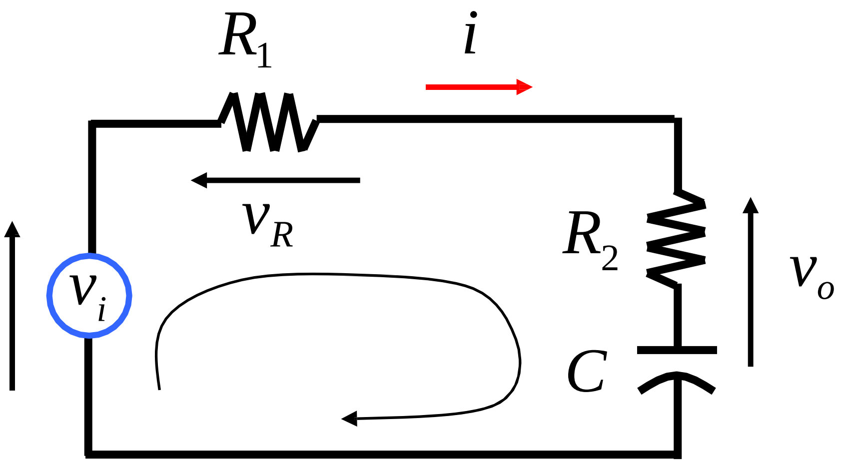
$$-v_S + v_R + v_C = 0$$

$$-v_S + v_R + v_L = 0$$

Zero junction current:

$$i_R - i_C - i_L = 0$$

Modeling – Electrical Example 1



Kirchhoff voltage law
(zero initial conditions):

$$v_i = (R_1 + R_2)i(t) + \frac{1}{C}\int_0^t i(\tau)d\tau$$

$$v_o = R_2 i(t) + \frac{1}{C}\int_0^t i(\tau)d\tau$$

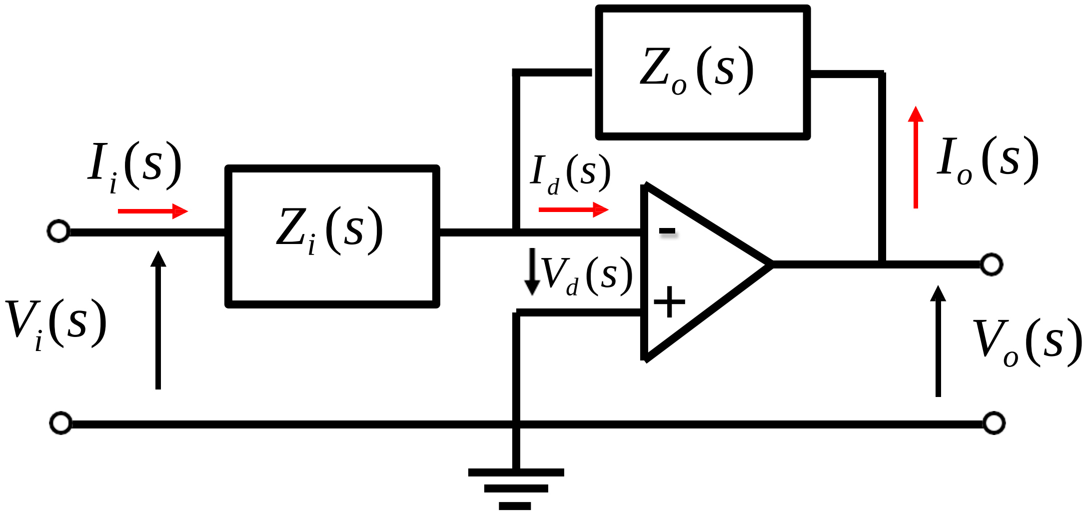
Laplace Transformation

$$V_i(s) = (R_1 + R_2)I(s) + \frac{1}{sC}I(s) \qquad V_o(s) = R_2 I(s) + \frac{1}{sC}I(s)$$

Transfer Function

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 I(s) + \frac{1}{sC}I(s)}{(R_1 + R_2)I(s) + \frac{1}{sC}I(s)} = \frac{1 + R_2Cs}{1 + (R_1 + R_2)Cs}$$

Modeling – Electrical Example 2 (OP Amp)



OP Amp Assumptions:

Infinity gain: $V_d(s) = 0$

Infinity input impedance:

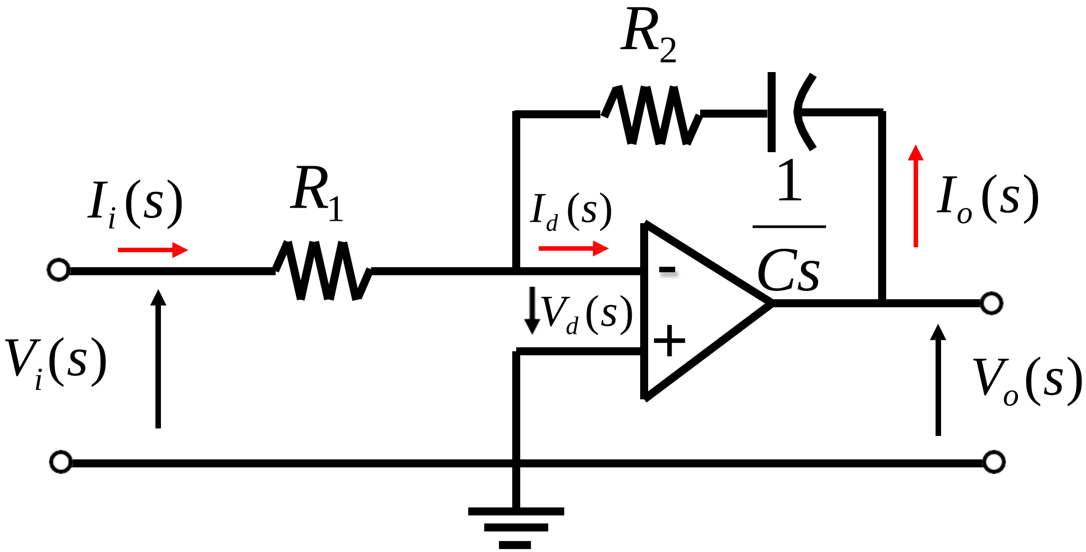
$$I_d(s) = 0$$

$$V_i(s) = Z_i(s)I_i(s) \qquad V_o(s) = Z_o(s)I_o(s) \qquad I_i(s) = -I_o(s)$$

Transfer Function

$$\boxed{G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_o(s)I_o(s)}{Z_i(s)I_i(s)} = -\frac{Z_o(s)}{Z_i(s)}}$$

Modeling – Electrical Example 3



$$Z_i(s) = R_1$$

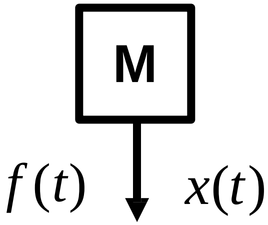
$$Z_o(s) = R_2 + \frac{1}{Cs} = \frac{1}{Cs}(R_2 Cs + 1)$$

Transfer Function

$$G(s) = \frac{V_o(s)}{V_i(s)} = -\frac{Z_o(s)}{Z_i(s)} = -\frac{R_2 Cs + 1}{R_1 Cs}$$

Modeling – Translational Mechanism

Mass



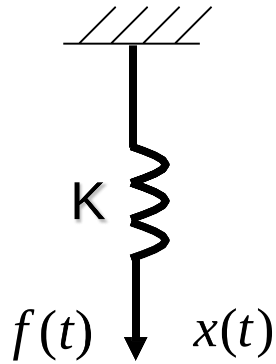
$$f(t) = M\ddot{x}(t)$$

$$x(0)=0$$

$$\dot{x}(0)=0$$

$$F(s) = Ms^2X(s)$$

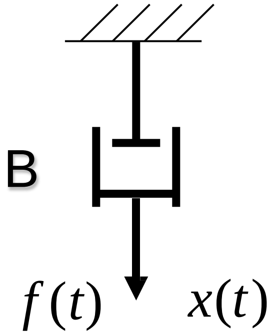
Spring



$$f(t) = K\,x(t)$$

$$F(s) = KX(s)$$

Damper

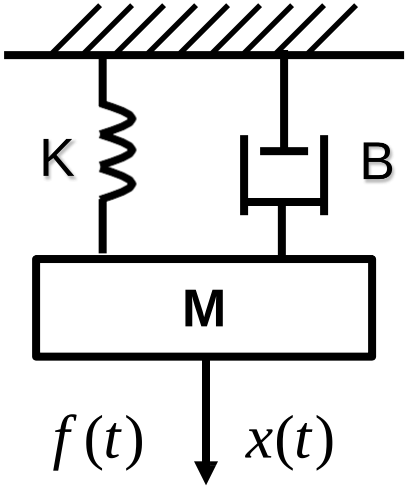


$$f(t) = B\,\dot{x}(\mathrm{t})$$

$$x(0)=0$$

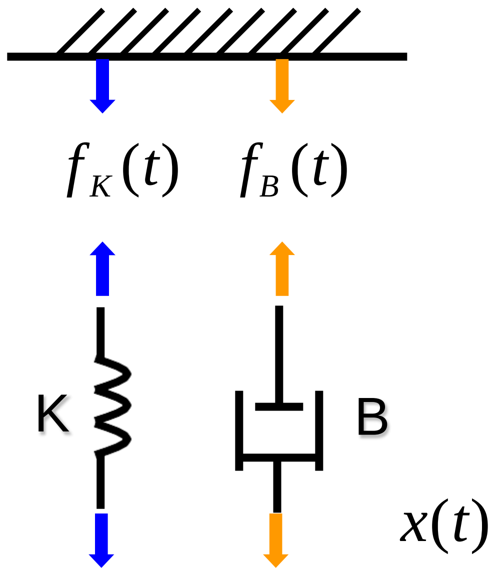
$$F(s) = BsX(s)$$

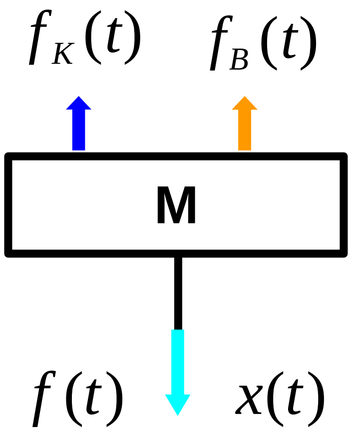
Modeling – Spring-Mass-Damper Systems



$$\boxed{M\ddot{x}(t) + Kx(t) + B\dot{x}(t) = f(t)}$$

Modeling – Free Body Diagram





$$f_K(t) = Kx(t) \qquad f_B(t) = B\dot{x}(t)$$

Note: $x(t)$ represents the displacement change for spring resting position

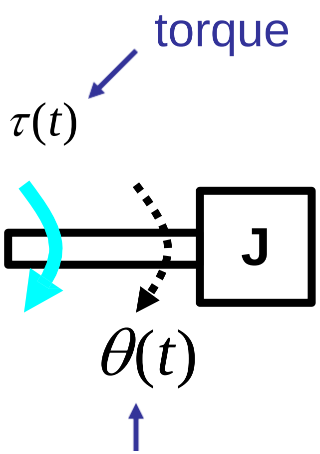
Using Newton's Law : $F(t) = M\ddot{x}(t)$

$$M\ddot{x}(t) = f(t) - f_K(t) - f_B(t) = f(t) - Kx(t) - B\dot{x}(t)$$

$$\boxed{M\ddot{x}(t) + Kx(t) + B\dot{x}(t) = f(t)}$$

Modeling – Rotational Mechanism

Moment of inertia



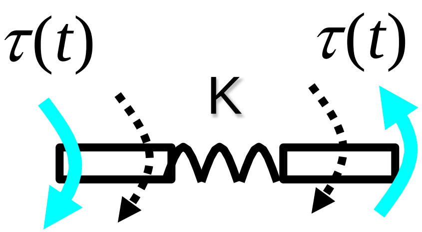
rotation angle

$$\tau(t) = J\ddot{\theta}(t)$$

$$\theta(0) = 0$$
$$\dot{\theta}(0) = 0$$

$$T(s) = Js^2\Theta(s)$$

Rotational spring



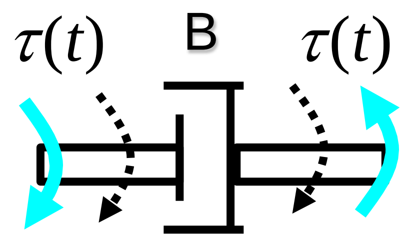
$$\theta(t) = \theta_1(t) - \theta_2(t)$$

$$\tau(t) = K\theta(t)$$

$$T(s) = K\Theta(s)$$

Friction

Friction of air/fluid resistance



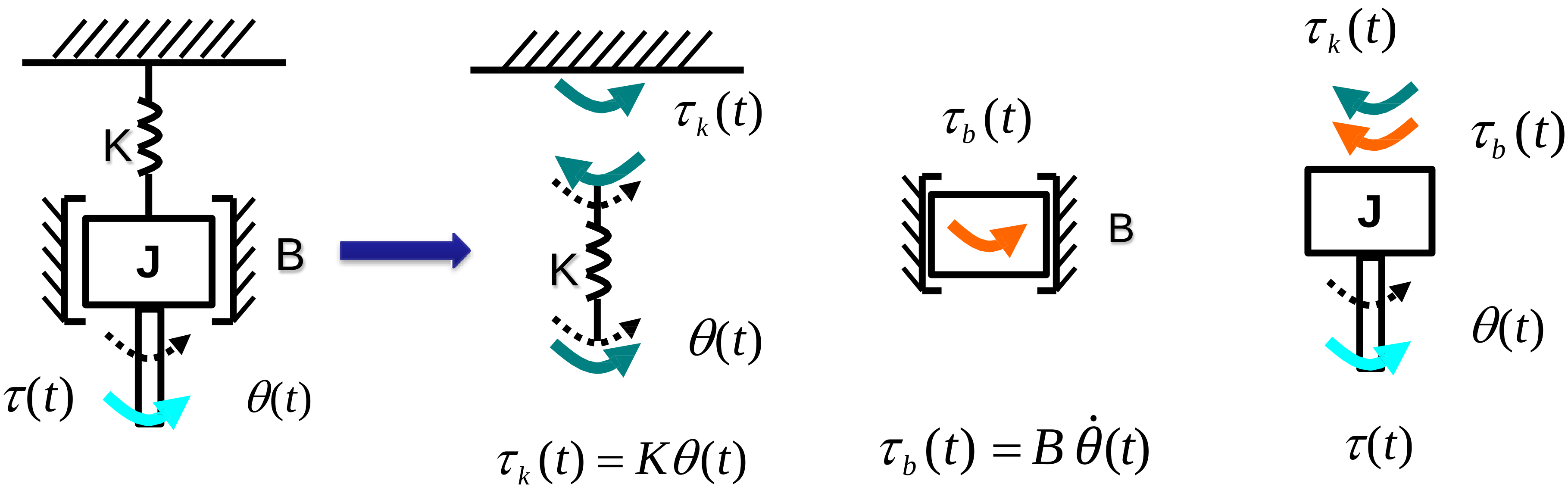
$$\theta(t) = \theta_1(t) - \theta_2(t)$$

$$\tau(t) = B\,\dot{\theta}(t)$$

$$\theta(0) = 0$$

$$T(s) = Bs\Theta(s)$$

Modeling – Torsional Pendulum System (Ex. 2.12)

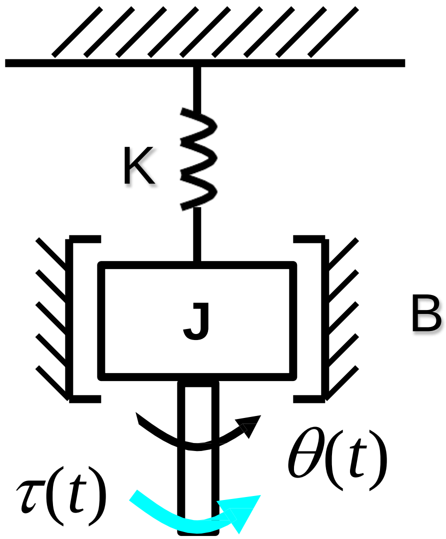


Newton's law

$$J\ddot{\theta}(t) = \tau(t) - \tau_k(t) - \tau_b(t) = \tau(t) - K\theta(t) - B\dot{\theta}(t)$$

$$\boxed{J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau(t)}$$

Modeling – Torsional Pendulum System (cont'd)



Friction of air/fluid resistance

- Equation of Motion

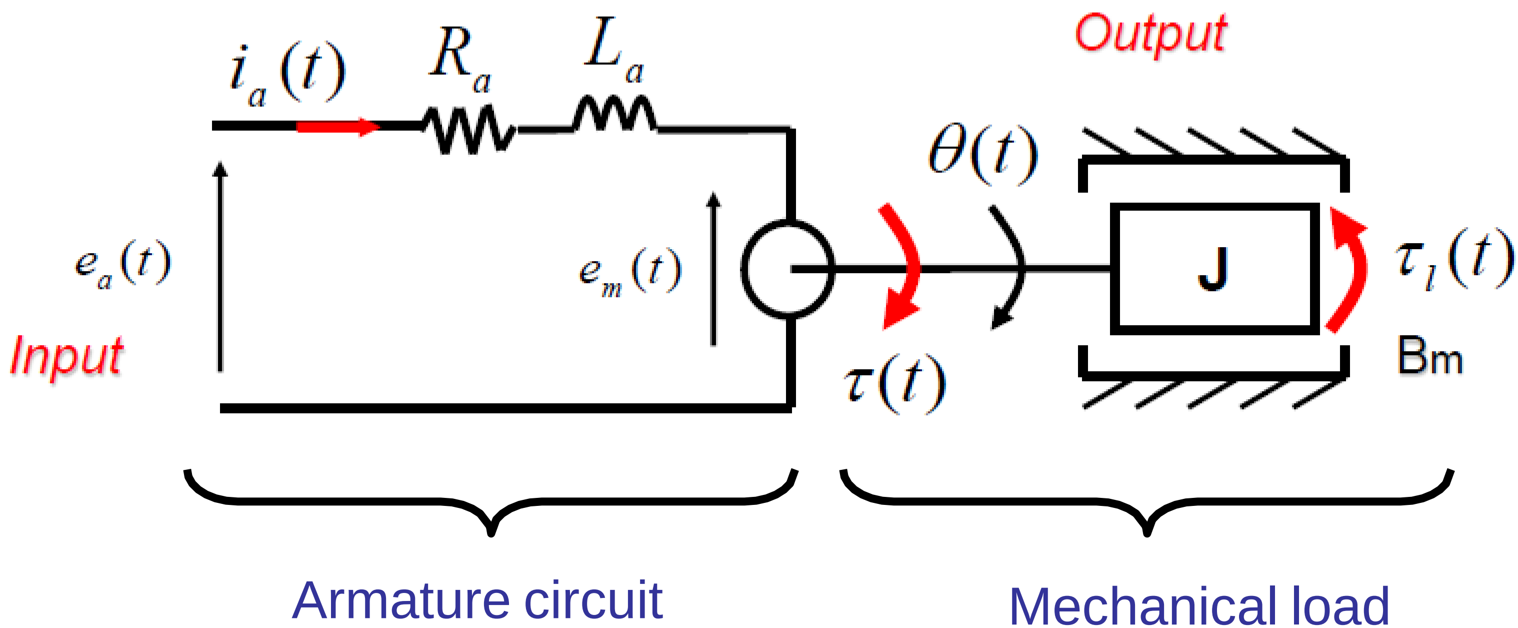
$$J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau(t)$$

- By Laplace transform (with zero ICs),

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

(2nd order system)

Modeling – Model of DC Motor



e_a :	applied voltage	θ :	angular position
i_a :	armature current	ω :	angular velocity
e_m	back EMF voltage	J :	rotor inertia
(counter-electromotive force/voltage)		B :	viscous fricton

Modeling – Model of DC Motor in Time Domain

- Armature circuit

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_m(t)$$

  - Motor torque
  - Back EMF

$$\begin{aligned} \tau(t) &= K_\tau i_a(t) \\ e_m(t) &= K_m \omega(t) \end{aligned}$$

- Mechanical load

Load torque

$$J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_l(t)$$

- Rigid connection between mechanical/electrical parts

$$\omega(t) = \dot{\theta}(t)$$

Modeling – Model of DC Motor in "s" Domain

- Armature circuit

$$I_a(s) = \frac{1}{R_a + L_a s}(E_a(s) - E_m(s))$$

- Connection between mechanical/electrical parts
  - Motor torque
  - Back EMF

$$\begin{aligned} T(s) &= K_\tau I_a(s) \\ E_m(s) &= K_m \Omega(s) \end{aligned}$$

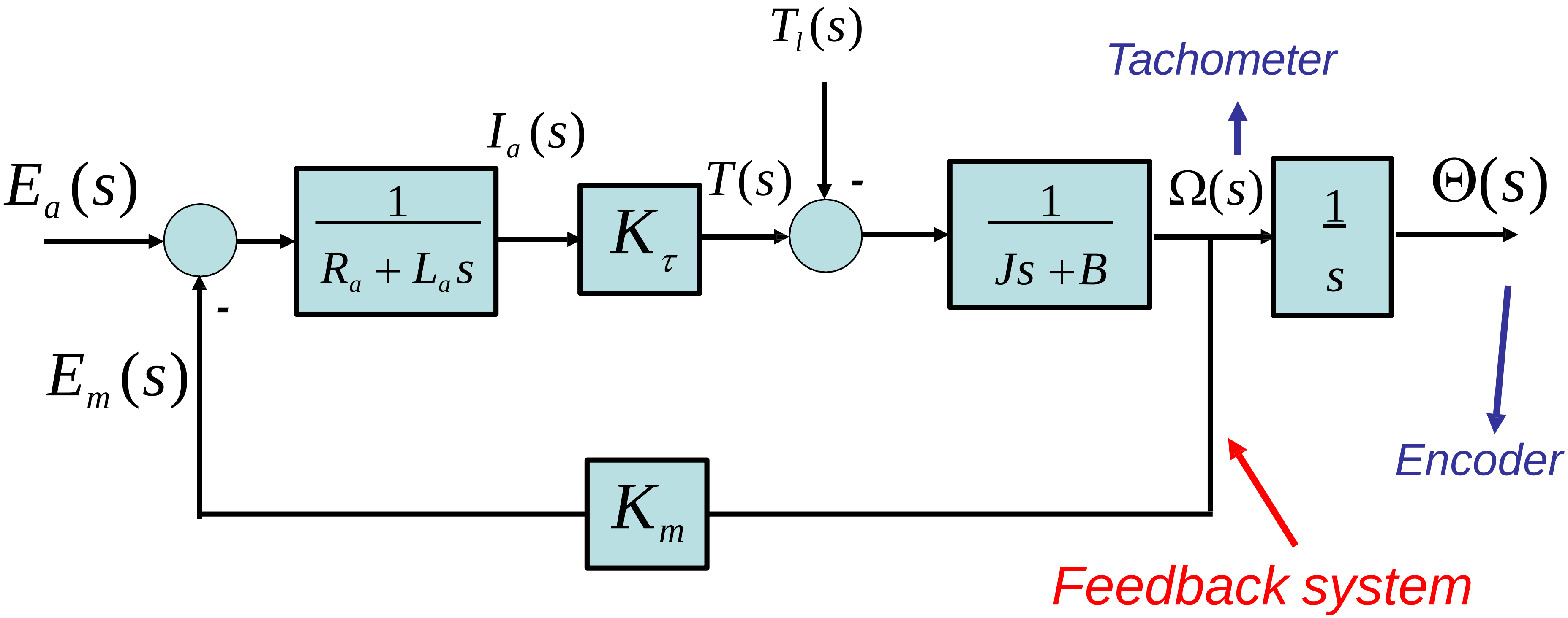
- Mechanical load

$$\Omega(s) = \frac{1}{Js + B}(T(s) - T_L(s))$$

- Angular position (zero initial condition)

$$\Omega(s) = s\Theta(s)$$

Modeling – DC Motor Block Diagram



$$\Omega(s) = \frac{K_\tau}{(L_a s + R_a)(Js + B)}(E_a(s) - K_m \Omega(s)), \quad T_l(s) = 0$$

$$\Omega(s) = \frac{1}{(Js + B)}\left(-T_l(s) - \frac{K_m K_\tau}{L_a s + R_a}\Omega(s)\right), \quad E_a(s) = 0$$

Modeling – DC Motor Transfer Function

$$\frac{\Omega(s)}{E_a(s)} = \frac{\dfrac{K_\tau}{(L_a s + R_a)(Js + B)}}{1 + \dfrac{K_\tau K_m}{(L_a s + R_a)(Js + B)}} = \frac{K_\tau}{(L_a s + R_a)(Js + B) + K_\tau K_m} =: G_1(s)$$

2nd order system

$$\frac{\Omega(s)}{T_l(s)} = \frac{-\dfrac{1}{(Js + B)}}{1 + \dfrac{K_\tau K_m}{(L_a s + R_a)(Js + B)}} = \frac{-(L_a s + R_a)}{(L_a s + R_a)(Js + B) + K_\tau K_m} =: G_2(s)$$

$$\Rightarrow \quad \Omega(s) = G_1(s) E_a(s) + G_2(s) T_l(s)$$

$$\Rightarrow \quad \Theta(s) = \frac{1}{s}\Omega(s) = \frac{1}{s}\left(G_1(s) E_a(s) + G_2(s) T_l(s)\right)$$

Modeling – DC Motor Transfer Function (cont'd)

Note: In many cases $L_a \ll R_a$. Then, an approximated TF is obtained by setting $L_a = 0$.

$$\frac{\Omega(s)}{E_a(s)} = \frac{K_\tau}{(L_a s + R_a)(Js + B) + K_\tau K_m} \approx \frac{K_\tau}{R_a(Js + B) + K_\tau K_m}$$

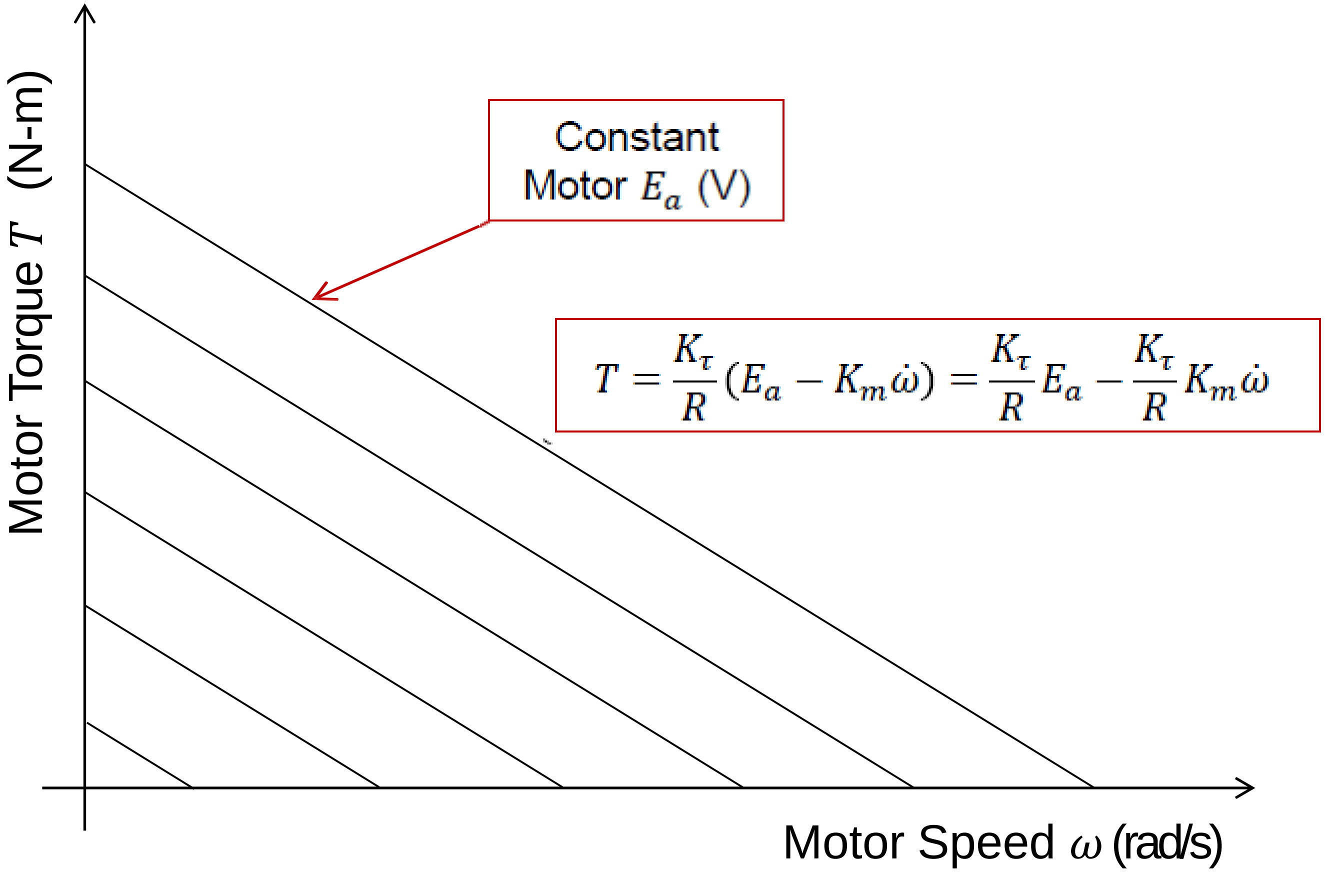
$$=: \frac{K}{Ts+1} \quad \left(K := \frac{K_\tau}{R_a B + K_m K_\tau},\ T = \frac{R_a J}{R_a B + K_m K_\tau} \right)$$

$$\frac{\Omega(s)}{T_l(s)} = \frac{-(L_a s + R_a)}{(L_a s + R_a)(Js + B) + K_m K_\tau} \approx \frac{-R_a}{R_a(Js + B) + K_m K_\tau}$$

2nd order system ➡ *1st order system*

$$\frac{\Theta(s)}{E_a(s)} = \frac{K}{s(Ts+1)}$$

Modeling – DC Motor SS Characteristics



Modeling – What Is a Linear System?

- A linear system satisfies ***Principle of Superposition***

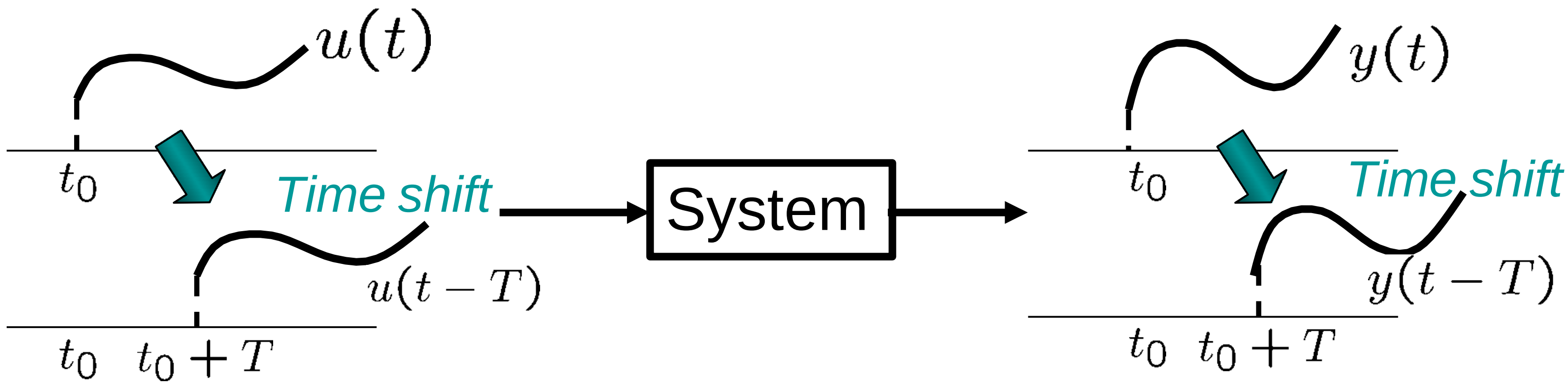
$$u(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

$$\boxed{\begin{array}{c} \left.\begin{array}{ll} u_1(t) & \to y_1(t) \\ u_2(t) & \to y_2(t) \end{array}\right\} \Rightarrow \alpha_1 u_1(t) + \alpha_2 u_2(t) \to \alpha_1 y_1(t) + \alpha_2 y_2(t) \\ \\ \forall \alpha_1, \alpha_2 \in \Re \end{array}}$$

A nonlinear system does not satisfy the principle of superposition.

Modeling – Time Invariant and Varying Systems

- A system is called ***time-invariant*** ***(time-varying)*** if system parameters ***do not*** (***do***) change in time.
- Example: vehicle/rocket $M\,\ddot{x}(t) = f(t)$ and $M(t)\ddot{x}(t) = f(t)$
- For time-invariant systems:



- This course deals with time-invariant systems.

Modeling – Why Linear System?

- Easy to understand and obtain solutions
- Linear ordinary differential equations (ODEs),
  - Homogeneous solution and particular solution
  - Transient solution and steady-state solution
  - Solution caused by initial values and forced solution
- Add many simple solutions to get more complex ones (using superposition!)
- Easy to check the Stability of stationary states (Laplace Transform)

Modeling – Why Linearization

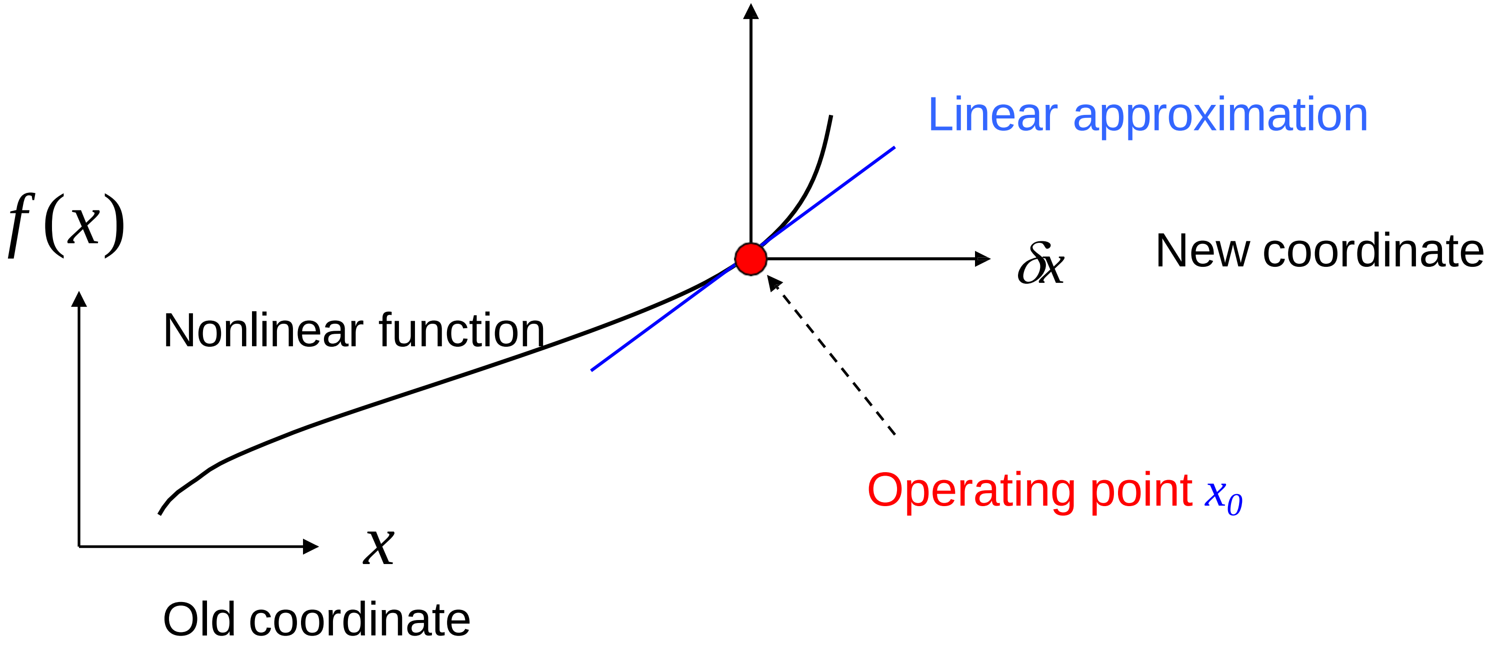
- Actual physical systems are inherently nonlinear. (linear system rarely exists!)
- TF models are only for Linear Time-Invariant (LTI) systems.
- Many control analysis/design techniques are available only for linear systems.
- Nonlinear systems are difficult to deal with mathematically.
- Often, we linearize nonlinear systems before analysis and design. How?

Modeling – How to Linearize It?

- Nonlinearity can be approximated by a linear function for small deviations δx around an operational point x_0

- Use a Taylor series expansion at x_0

$$\delta f = f(x_0 + \delta x) - f(x_0)$$



Modeling – Summary: 6 Steps

- Step 1: identify the system model's input $r(t)$ and ouput $y(t)$

With initial conditions $\dot{r}_0 = 0, \dot{y}_0 = 0, \ddot{r}_0 = 0, \ddot{r}_0 = 0$

- Step 2: express model in the form $f(r, \dot{r}, \dots, y, \dot{y}, \dots) = 0$

- Step 3: define equilibrium operation point r_0, y_0 and set all derivatives to zeros at the equilibrium points

- Step 4: perform Taylor expansion about (r_0, y_0) and only retain 1st order derivative

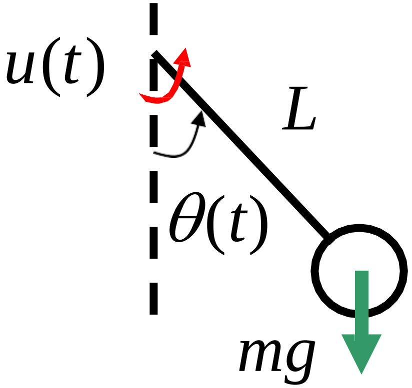
- Step 5: change variables to derivatives

$$(\tilde{r} = r - r_0, \dot{\tilde{r}} = \dot{r} - \dot{r}_0, \dots, \tilde{y} = y - y_0, \dot{\tilde{y}} = \dot{y} - \dot{y}_0)$$

- Step 6: rewrite the function in step 5 into a standard ODE

Modeling – Pendulum Linearization

- Motion of the pendulum



$$mL^2\ddot{\theta}(t) + mgL\sin\theta(t) = u(t)$$

$$\underbrace{\ddot{\theta}(t) + \frac{g\sin\theta(t)}{L} - \frac{u(t)}{mL^2}}_{f(\theta,\ddot{\theta},u)} = 0$$

- Linearize it at $\theta_0 = \pi$

- Find u_0

$$\ddot{\pi} + \frac{g\sin\pi}{L} - \frac{u_0}{mL^2} = 0 \rightarrow u_0 = 0$$

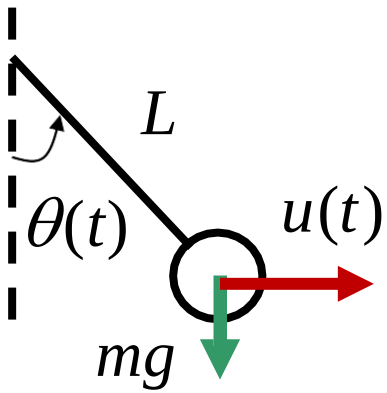
- New coordinates:

$$\theta = \theta_0 + \delta\theta = \pi + \delta\theta$$

$$u = u_0 + \delta u = 0 + \delta u$$

Modeling – Pendulum Linearization

- Motion of the pendulum



$$mL^2\ddot{\theta}(t) + mgL\sin\theta(t) = u(t)$$

$$\underbrace{\ddot{\theta}(t) + \frac{g\sin\theta(t)}{L} - \frac{u(t)}{mL^2}}_{f(\theta,\ddot{\theta},u)} = 0$$

- Linearize it at $\theta_0 = \pi$

- Find u_0 $\quad \ddot{\pi} + \frac{g\sin\pi}{L} - \frac{u_0}{mL^2} = 0 \rightarrow u_0 = 0$

- New coordinates:

$$\theta = \theta_0 + \delta\theta = \pi + \delta\theta$$

$$u = u_0 + \delta u = 0 + \delta u$$

Modeling – 6 Steps (Example 1)

System ODE:

$$mL^2\ddot{\theta}(t) + mgL\sin\theta(t) = u(t)$$

Step One (input/output):

$$\text{System input}: u(t), \text{system output}: \theta(t)$$

Step Two ($f(\ldots)=0$):

$$\underbrace{\ddot{\theta}(t) + \frac{g\sin\theta(t)}{L} - \frac{u(t)}{mL^2}}_{f(\theta,\ddot{\theta},u)} = 0$$

Step Three (θ_0, u_0):

$$\text{Let } \theta_0 = \pi, \text{ and } \dot{\theta}_0 = \ddot{\theta}_0 = 0,$$

$$\ddot{\theta} + \frac{g\sin\pi}{L} - \frac{u_0}{mL^2} = 0 \rightarrow u_0 = 0$$

Modeling – 6 Step Example 1 (cont'd)

Step Four (Taylor expansion):

$$f(\theta,\ddot{\theta},u) \cong \underbrace{f(\theta_0,\ddot{\theta}_0,u_0)}_{0} + \left.\frac{\partial f}{\partial \theta}\right|_{\substack{\theta=\theta_0\\ \ddot{\theta}=\ddot{\theta}_0\\ u=u_0}} (\theta-\theta_0) + \left.\frac{\partial f}{\partial \ddot{\theta}}\right|_{\substack{\theta=\theta_0\\ \ddot{\theta}=\ddot{\theta}_0\\ u=u_0}} (\ddot{\theta}-\ddot{\theta}_0) + \left.\frac{\partial f}{\partial u}\right|_{\substack{\theta=\theta_0\\ \ddot{\theta}=\ddot{\theta}_0\\ u=u_0}} (u-u_0)$$

$$= \left.\frac{g}{L}\cos\theta\right|_{\theta=\pi} (\theta-\pi) + 1\cdot(\ddot{\theta}-0) - \frac{1}{mL^2}(u-0)$$

$$= -\frac{g}{L}(\theta-\pi) + (\ddot{\theta}-0) - \frac{1}{mL^2}(u-0)$$

Step Five ($\delta\theta_0$, δu_0):

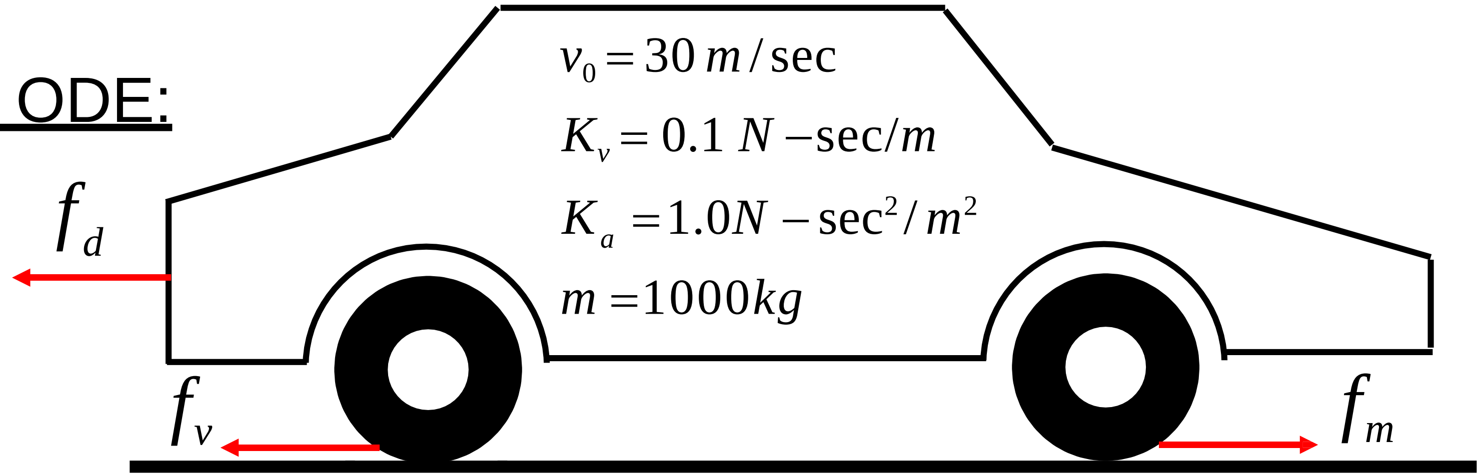
$$f(\theta,\ddot{\theta},u) \cong \delta\ddot{\theta} - \frac{g}{L}\delta\theta - \frac{1}{mL^2}\delta u = 0$$

Step Six (rewrite):

$$\delta\ddot{\theta} - \frac{g}{L}\delta\theta = \frac{1}{mL^2}\delta u \Rightarrow \frac{\Delta\Theta(s)}{\Delta U(s)} = \frac{1}{mL^2 s^2 - mLg}$$

Modeling – 6 Steps (Example 2)

System ODE:



Tractive force input : $f_m = f_m(t)$

Viscous friction force: $f_v = f_v(t) = K_v v$

Aerodynamic drag force: $f_d = f_d(t) = K_a v^2$

$$m\dot{v} = f_m - f_d - f_v = f_m - K_v v - K_a v^2$$

$$\boxed{m\dot{v} + K_v v + K_a v^2 = f_m}$$

Modeling – 6 Step Example 2 (cont'd-1)

Vehicle System:

$$\boxed{m\dot{v} + K_v v + K_a v^2 = f_m}$$

Step One (input/output):

System input (tractive force): $f_m(t)$, system output (velocity): $v(t)$

Step Two ($f(\ldots)=0$):

$$\underbrace{m\dot{v} + K_v v + K_a v^2 - f_m}_{f(f_m, v, \dot{v})} = 0$$

Step Three ($v_0, \dot{v}_0, u_0$):

Let $v_0 = 30$ and $\dot{v}_0 = 0$, and find f_{m0}

$$f_{m0} = K_v v_0 + K_a v_0^2 = 0.1 \times 30 + 1 \times 30^2 = 903N$$

Modeling – 6 Step Example 2 (cont'd-2)

Step Four (Taylor expansion):

$$f(f_m, v, \dot{v}) \cong \underbrace{f(f_{m0}, v_0, \dot{v}_0)}_{0} + \left.\frac{\partial f}{\partial f_m}\right|_{\substack{f_m = f_{m0} \\ v = v_0 \\ \dot{v} = \dot{v}_0}} (f_m - f_{m0}) + \left.\frac{\partial f}{\partial v}\right|_{\substack{f_m = f_{m0} \\ v = v_0 \\ \dot{v} = \dot{v}_0}} (v - v_0) + \left.\frac{\partial f}{\partial \dot{v}}\right|_{\substack{f_m = f_{m0} \\ v = v_0 \\ \dot{v} = \dot{v}_0}} (\dot{v} - \dot{v}_0)$$

$$= 0 - 1(f_m - f_{m0}) + (K_v + 2K_a v_0) \cdot (v - v_0) + m(\dot{v} - \dot{v}_0) = 0$$

Step Five $(\tilde{f}_m = f_m - f_{m0}, \tilde{v} = v - v_0, \tilde{\dot{v}} = \dot{v} - \dot{v}_0)$:

$$\underbrace{m}_{1000} \tilde{\dot{v}} + \underbrace{(K_v + 2K_a v_0)}_{0.1+2\times 1\times 30=60.1} \cdot \tilde{v} - \tilde{f}_m = 0$$

Step Six (rewrite):

$$1000\tilde{\dot{v}} + 60.1\tilde{v} = \tilde{f}_m \Rightarrow \frac{V(s)}{F_m(s)} = \frac{1}{1000s + 60.1}$$