MAE/ECE 5320 Mechatronics

2025 Spring semester

Lecture 07

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Content

- Background
- Encoder as position and speed sensor
- Accelerometer and angular rate gyro sensors
  - Accelerometer
  - Angular rate gyro
- Sensor fusion
- Other sensors/GPS, temperature.

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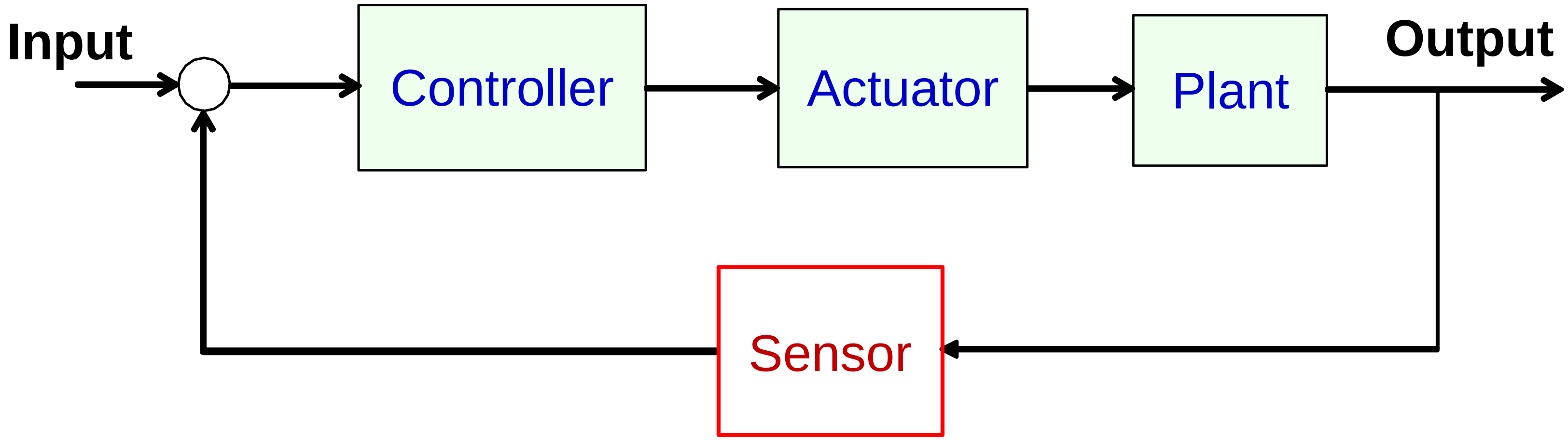
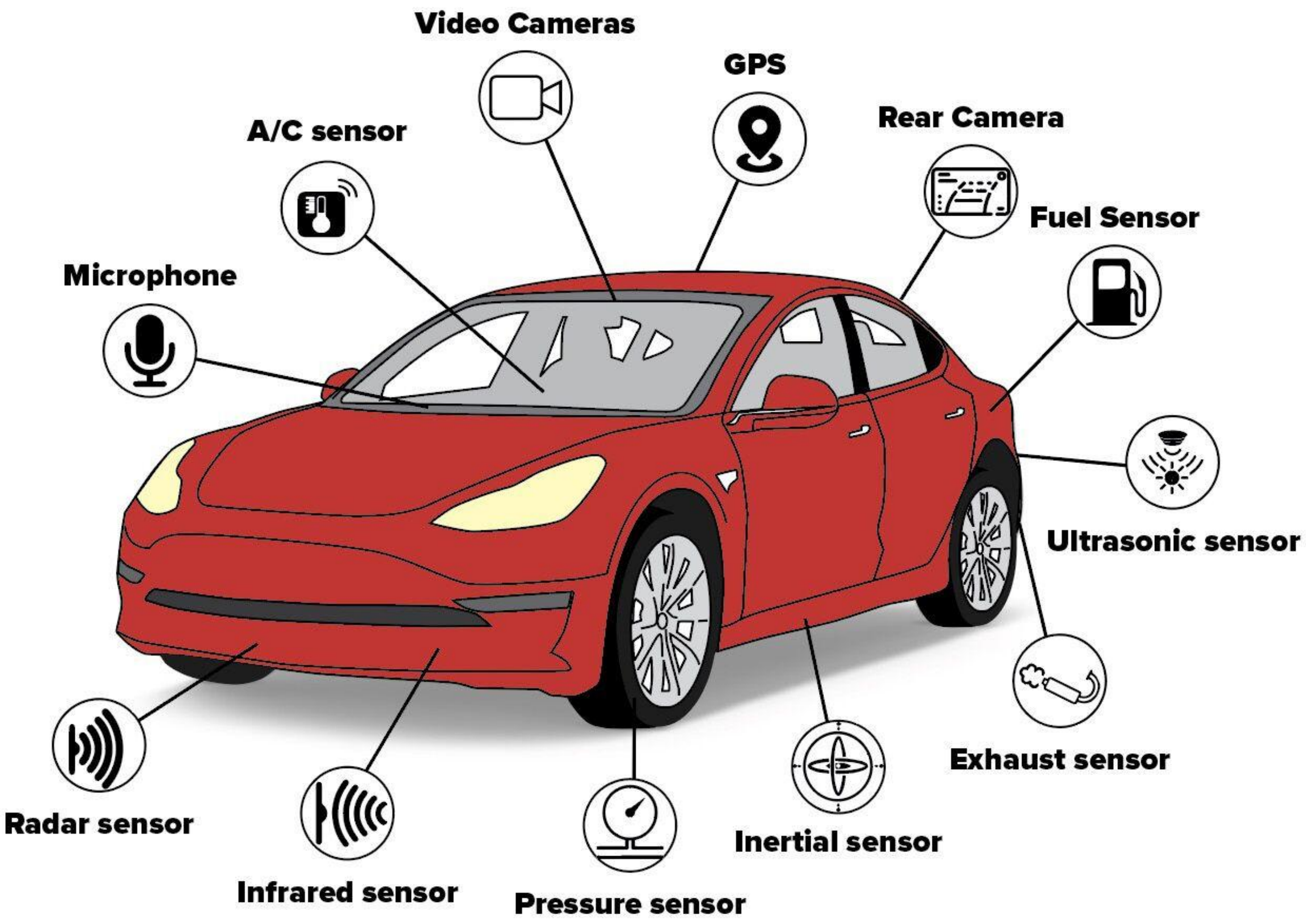
Background (1)

Sensors used in a closed-loop control system



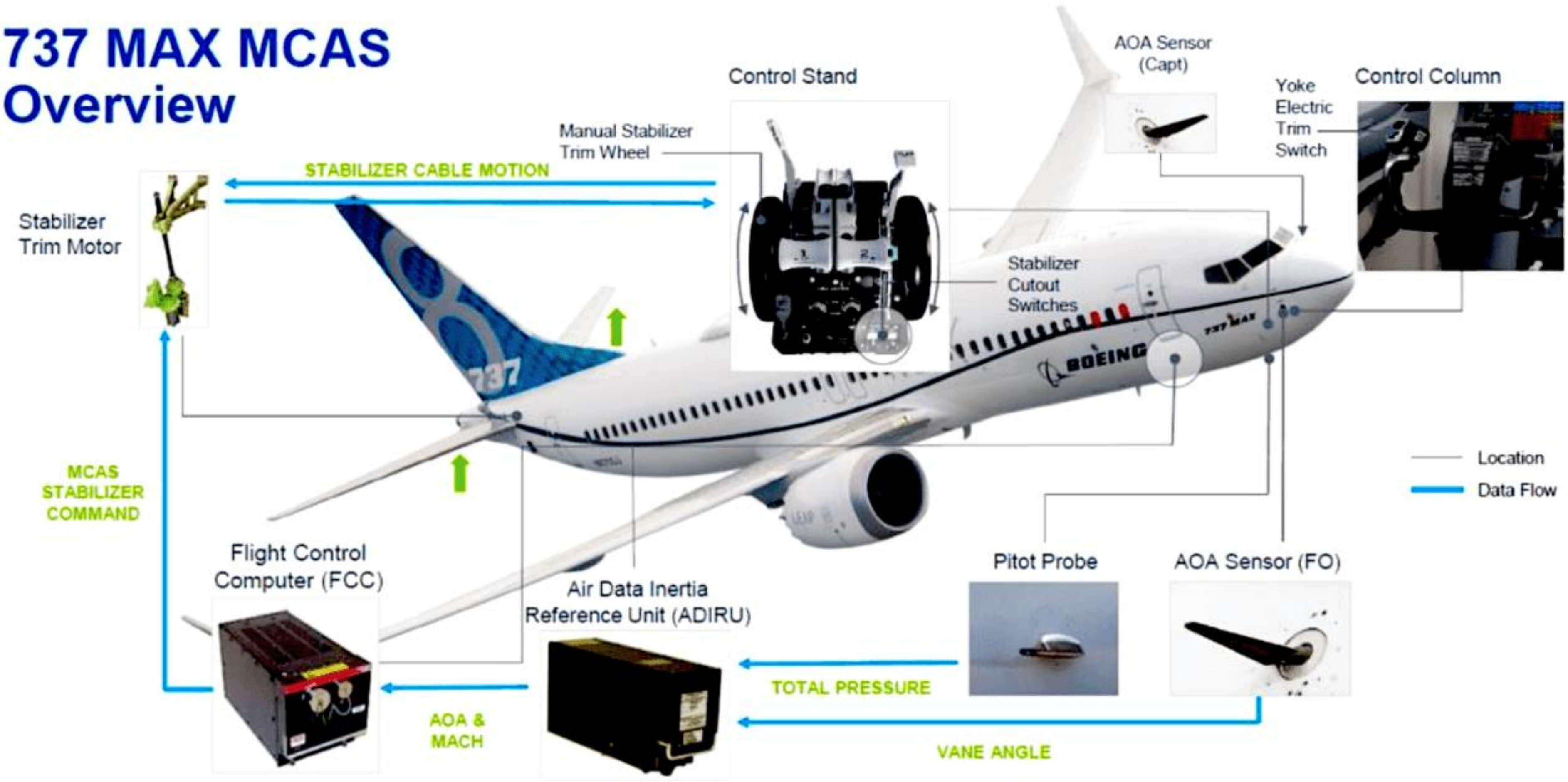
- ❑ Sensors are critical in a closed-loop control system, which are used to measure the system physical variables (e.g., speed, position, etc.) and provide feedback for the closed-loop controller.
- ❑ In this lecture, these sensors used in class mini Segway are introduced; their physical principles are illustrated; and how they interact with Arduino microcontroller are discussed.

Example: sensors installed on a vehicle



Sensors installed on an aircraft

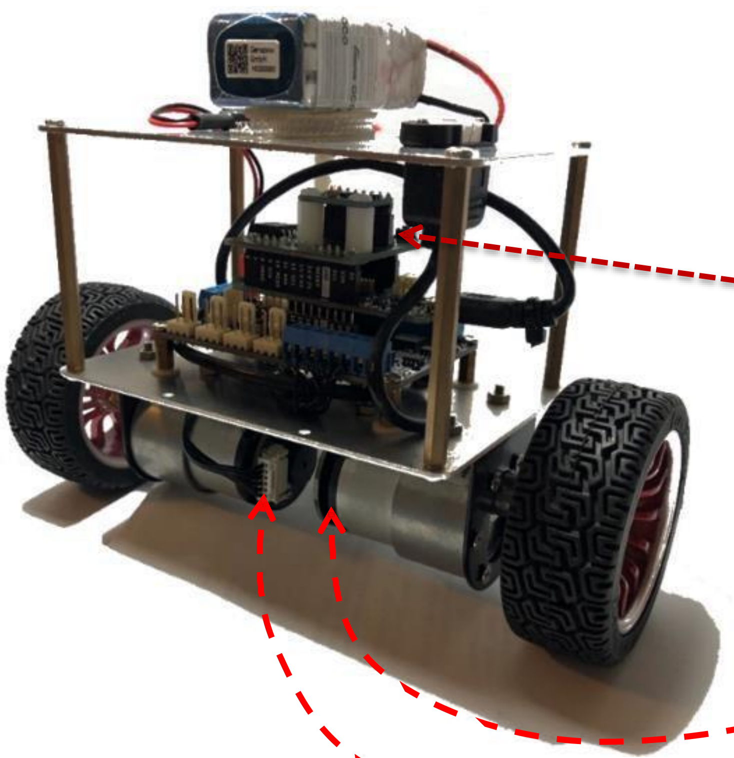
Revisit-Boeing 737MAX

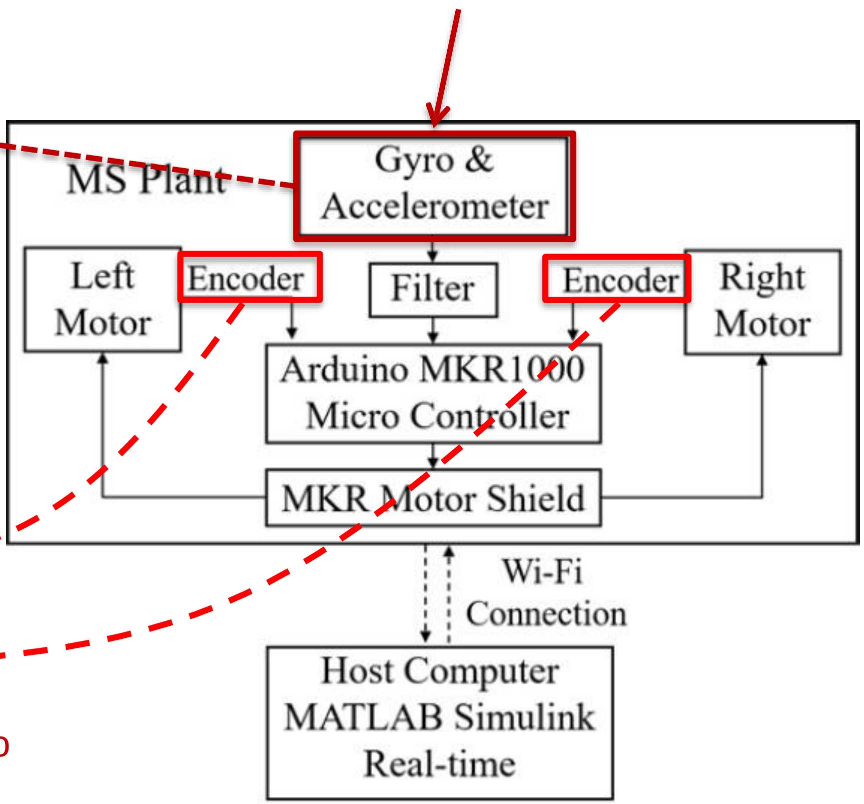


Background (2)

Sensors used in the mini Segway

Three dimensional Gyro and Accelerometer are integrated in MPU 6050 to measure pitch, yaw, roll angles through sensor fusion.





Encoders on two wheel axils are used to measure angular velocities.

Background (3)

Analog and digital Sensors

- In a mechatronic system, sensors use physical laws (hall-effect, optics, electromagnetism) to convert physical parameters to electrical outputs (voltage, current), and feedback to microcontrollers.
- Analog sensors output **continuous (analog)** signals and digital sensors output **discrete (logic high and low)** signals.

Analog sensor features:

- Limited dynamic range of measured parameter
- Direct measurement, however, subject to noise pollution
- No need of further signal processing or conversing

Digital sensor features:

- Possible to achieve high dynamic range
- Output digital signal with high or low voltage/current, thus, with high noise rejection capability
- Need further signal processing to convert measured to true value
- Accuracy often depends on digital (clock) resolution

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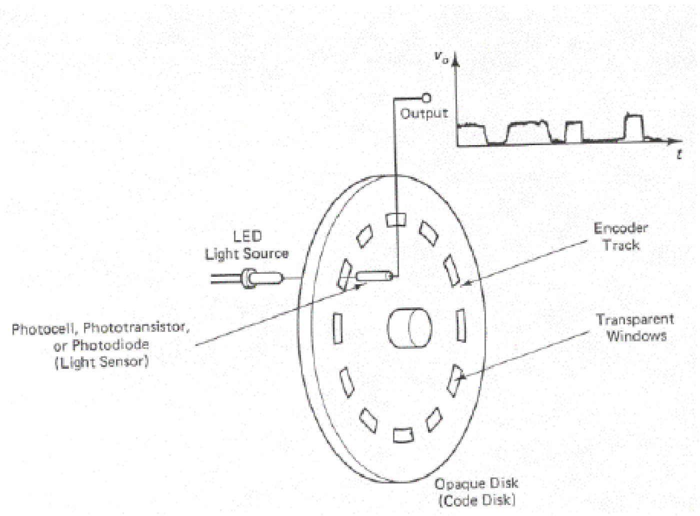
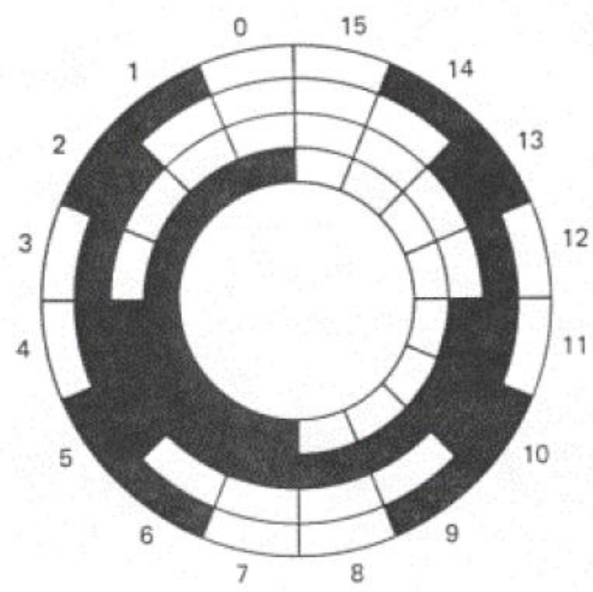
Encoder-category

❑ **Incremental encoder: counting number of pulses**

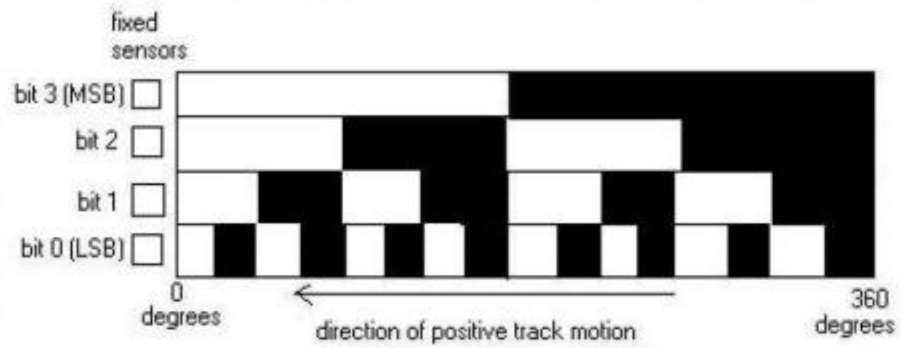
Hall-effect encoder
Optical encoder

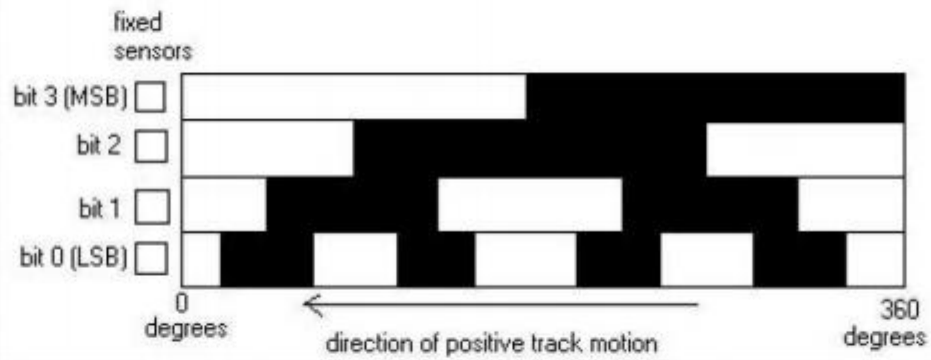
❑ **Absolute encoder: decode digital reading**





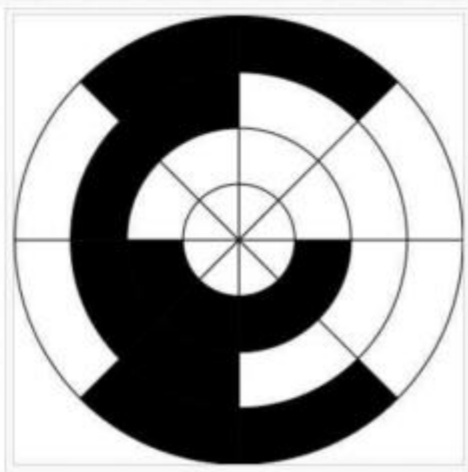
Absolute encoder (1)





Gray Coding

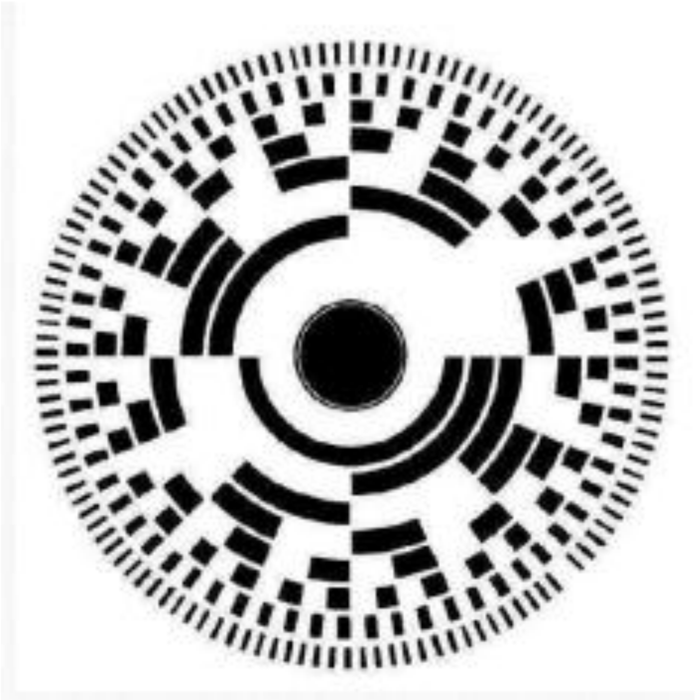
Sector	Contact 1	Contact 2	Contact 3	Angle
0	off	off	off	0° to 45°
1	off	off	ON	45° to 90°
2	off	ON	ON	90° to 135°
3	off	ON	off	135° to 180°
4	ON	ON	off	180° to 225°
5	ON	ON	ON	225° to 270°
6	ON	off	ON	270° to 315°
7	ON	off	off	315° to 360°



3 bit, resolution 360/8 = 45

Absolute encoder (2)

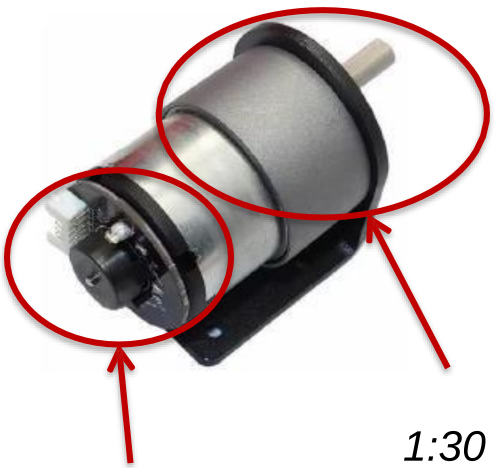
Increase to 8 bits
Resolution is 360/2^8 = 1.40625



Tradeoff between fabrication complexity and resolution

Hall-effect encoder (1)

Hall-effect based incremental encoder

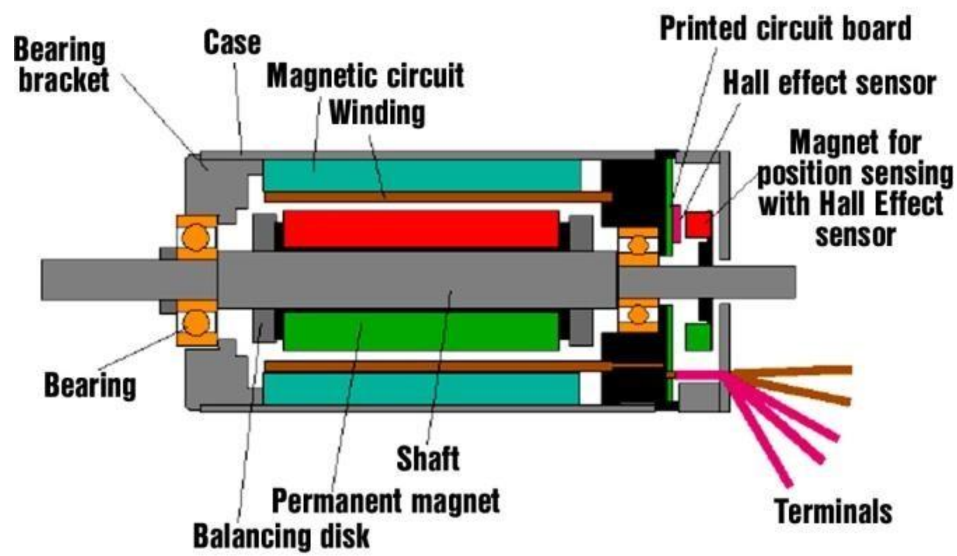


Hall-effect encoder

1:30 gear ratio motor

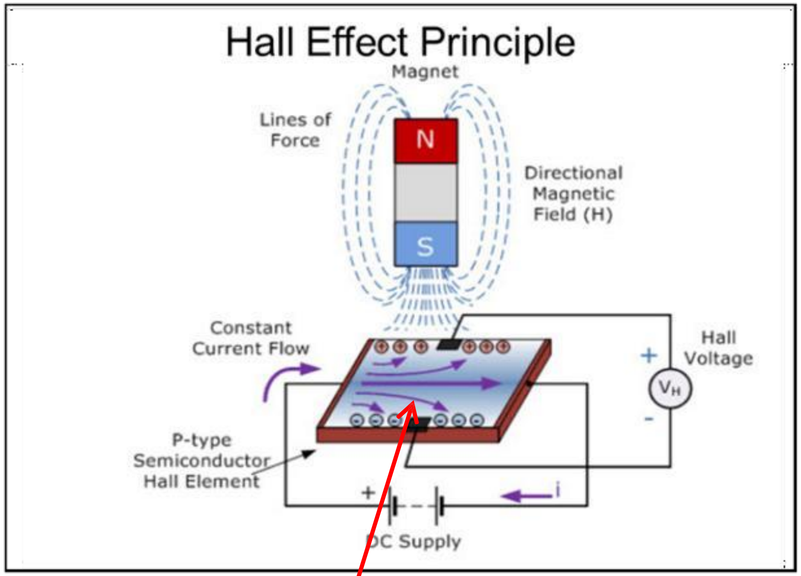
hall-effect based encoder is integrated with the DC motor.

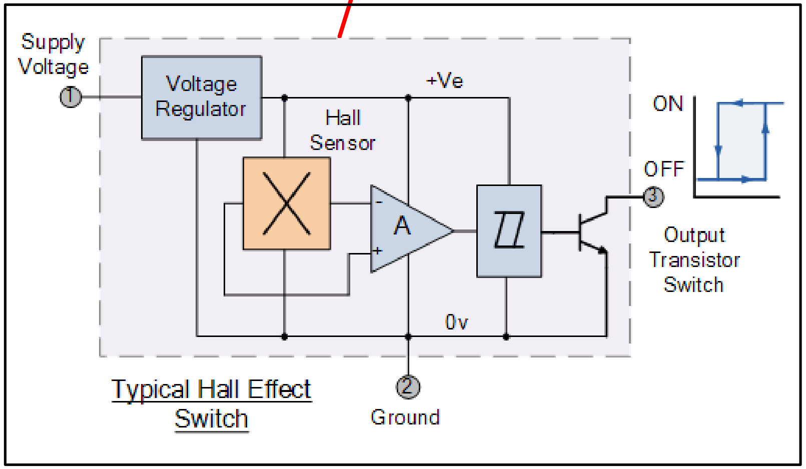
One encoder demonstration example



Hall-effect encoder (2)

Hall-effect based incremental encoder





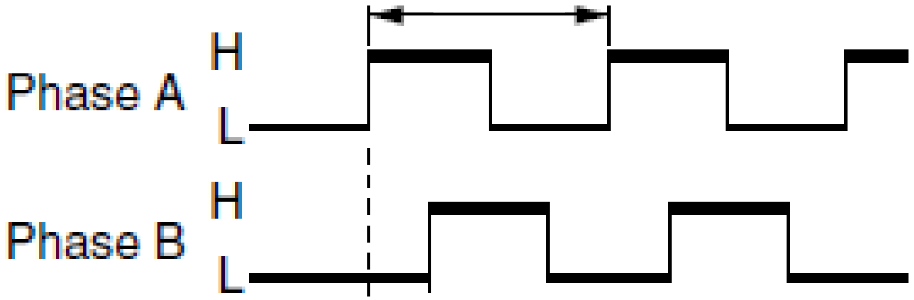
- ❑ Hall-effect sensors are devices which are activated by an external magnetic field.
- ❑ The output signal from a hall-effect sensor is the function of magnetic field density around the device. When the magnetic flux density around the sensor exceeds a certain pre-set threshold, the sensor detects it and generates an output voltage called the Hall Voltage.
- ❑ Hall-effect sensors consist basically of a thin piece of rectangular p-type semiconductor material such as gallium arsenide (GaAs), indium antimonide (InSb) or indium arsenide (InAs) passing a continuous current through itself. .

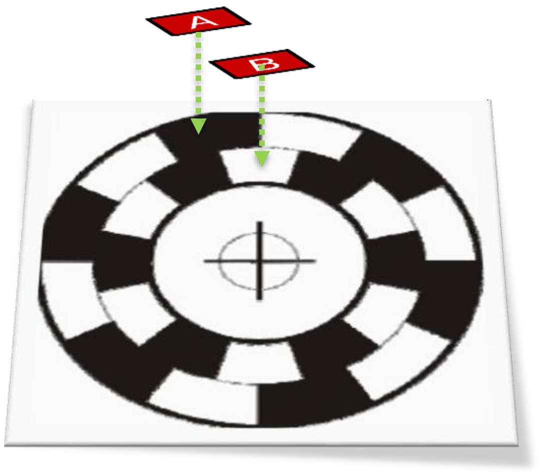
Hall-effect encoder (3)

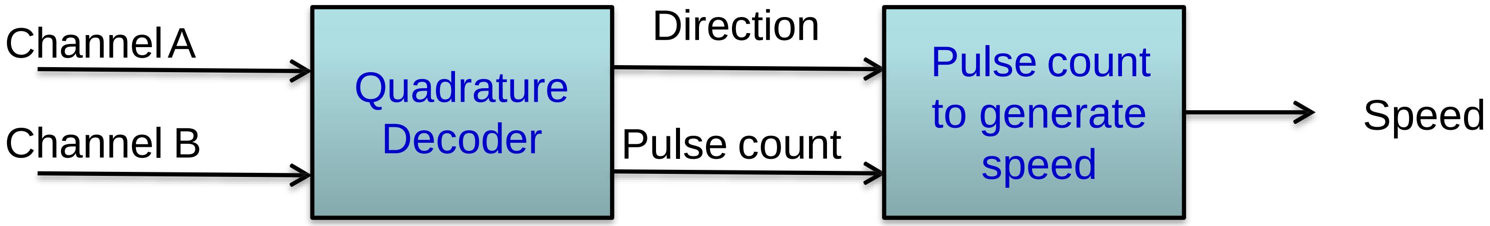
Hall-effect based incremental encoder

Incremental encoder uses quadrature encoding:

1) Two hall-effect sensors generate two voltage signals (A and B), if disk is rotated to magnetic sections, with a 90° phase.
2) Rotation speed and direction can be obtained by reading two pulse signals.

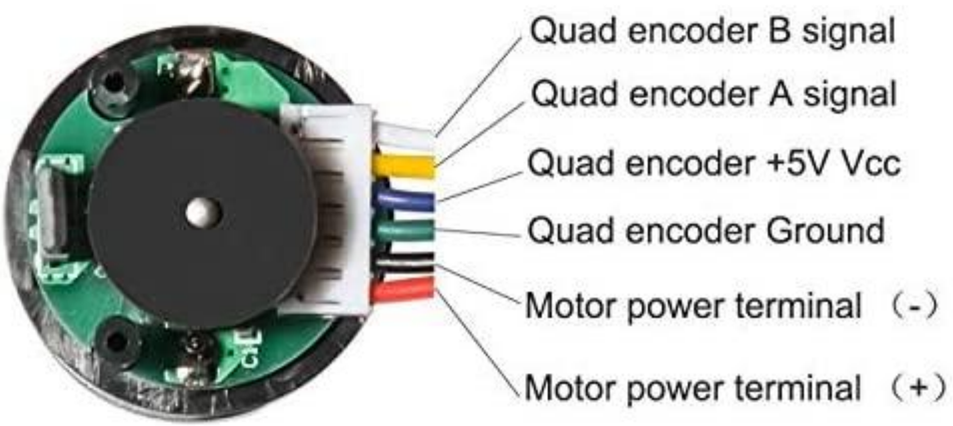


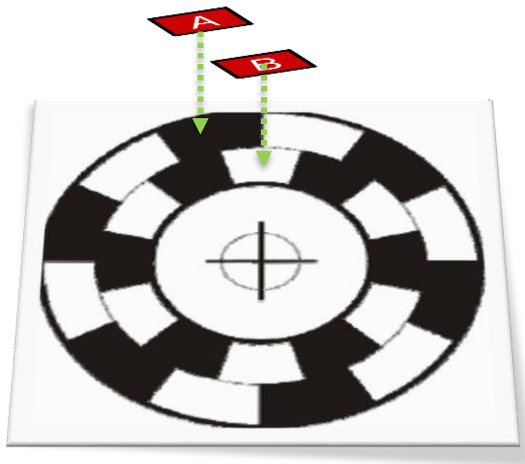


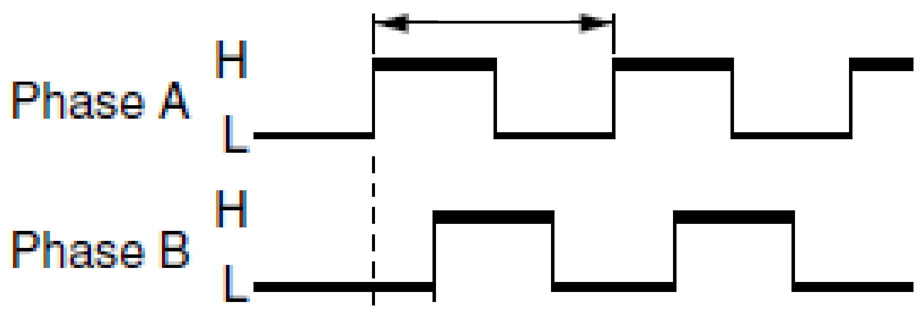


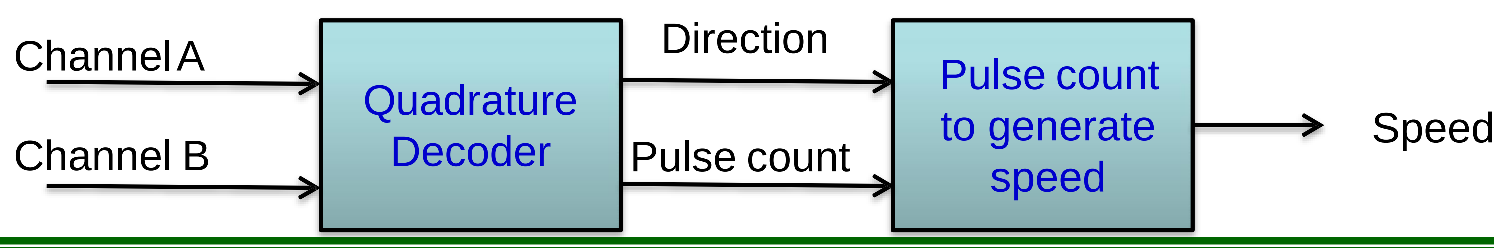
Encoder wiring

Wiring Diagram





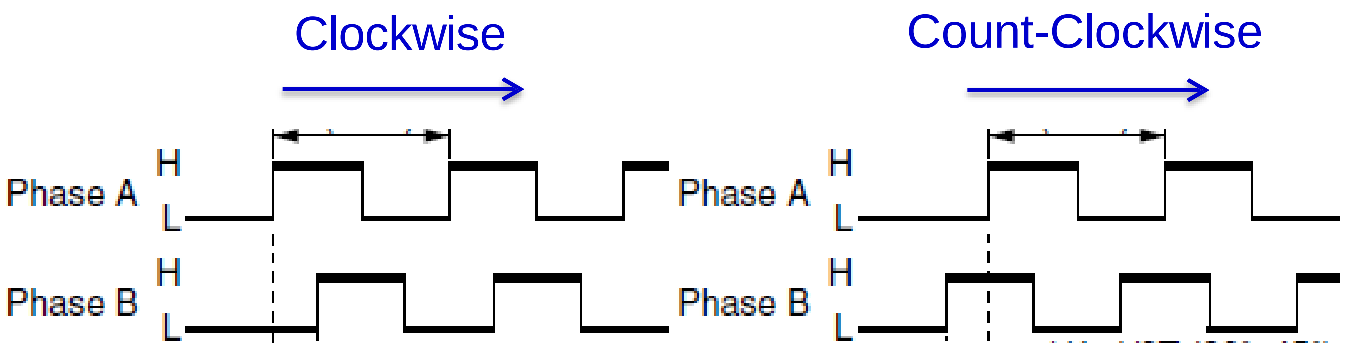




Hall-effect encoder (4)

How to decode two channel voltage signals (A and B)?

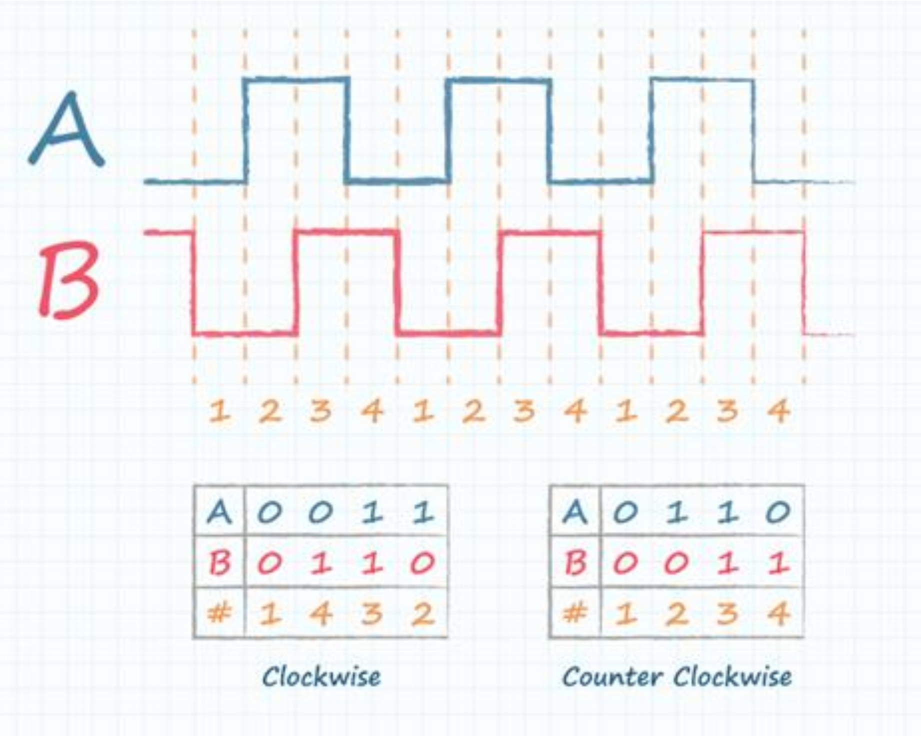
Rotation direction



- Clockwise case: let phase A signal be the reference, signal B has a 90° phase delay comparing with signal A. Then, the rotation direction is clockwise ;
- Count clockwise: If signal B has a 90° phase advance comparing with signal A, the rotation direction is count-clockwise.

Quadrature encoder

1 pulse generates 4 states



Hall-effect encoder (5)

How to decode two channel voltage signals (A and B)?

Rotation revolution and speed

Count per revolution = encoder gear ratio × motor gear ratio × lines on disk For

example, the motor JGB37-520

Number of pulse per revolution (PPR) = 4*30*11 = 1320/cycle

One cycle rotation generates 1320 counts (pulses).

The accuracy is $\frac{360^{\circ}}{1320} = \frac{0.2727^{\circ}}{\text{pulse}}$, that is, each pulse represents 0.27°

Rotation speed can be obtained by converting number of pulses per sampling period to rotation per minute (RPM).

$$RPM = \frac{\#\,of\ pulses}{T_{sampl}} * \frac{1}{PPR} * \frac{T_{sample}}{minute} = \frac{\#\,of\ pulses}{T_{sample}} * \frac{1}{1320} * \frac{T_{sample}}{minute}$$

Optical Encoder

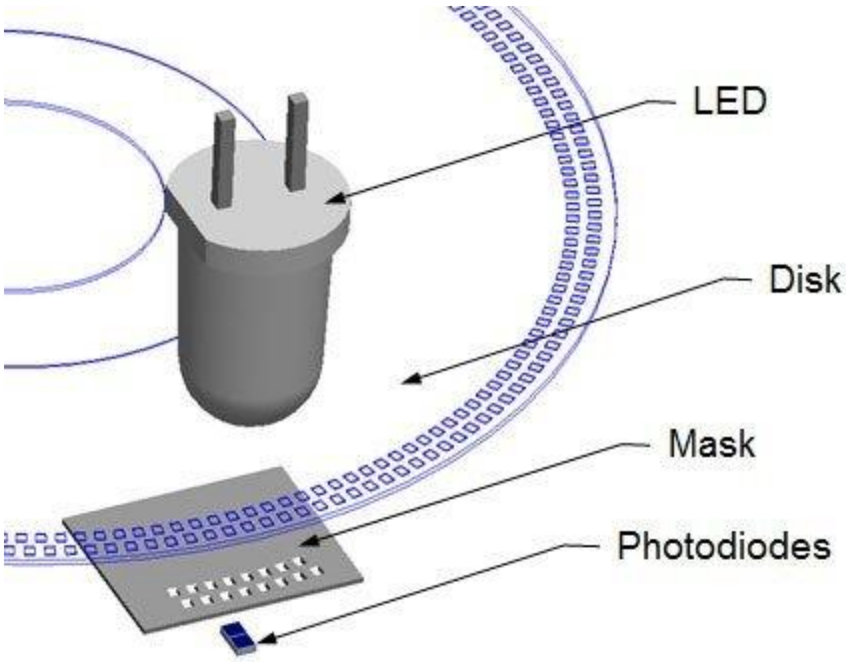
Components:

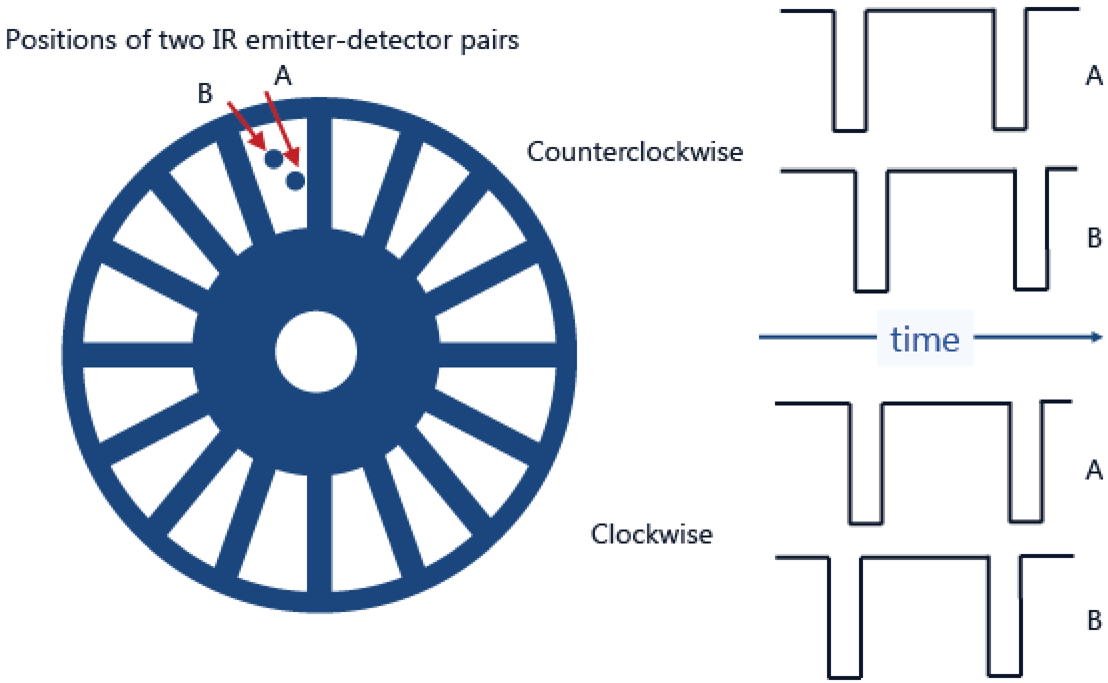
(1) Light source

(2) Light sensor

(3) Moving disk

(4) Fixed mask





Compared to absolute encoder, fabrication complexity is reduced, but may have missing pulses counting.

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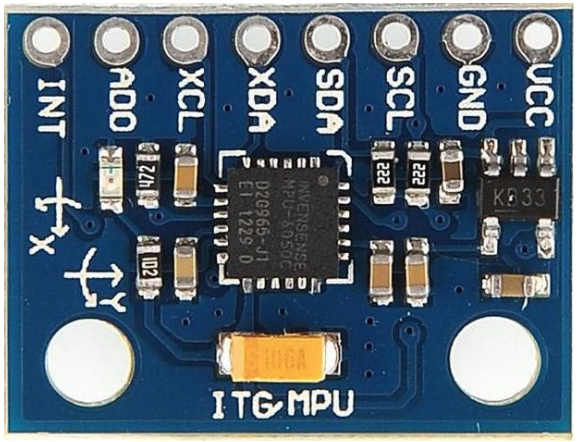
- Sensor fusion

Accelerometer and rate gyro (1)

MPU 6050: Gyro & Accelerometer



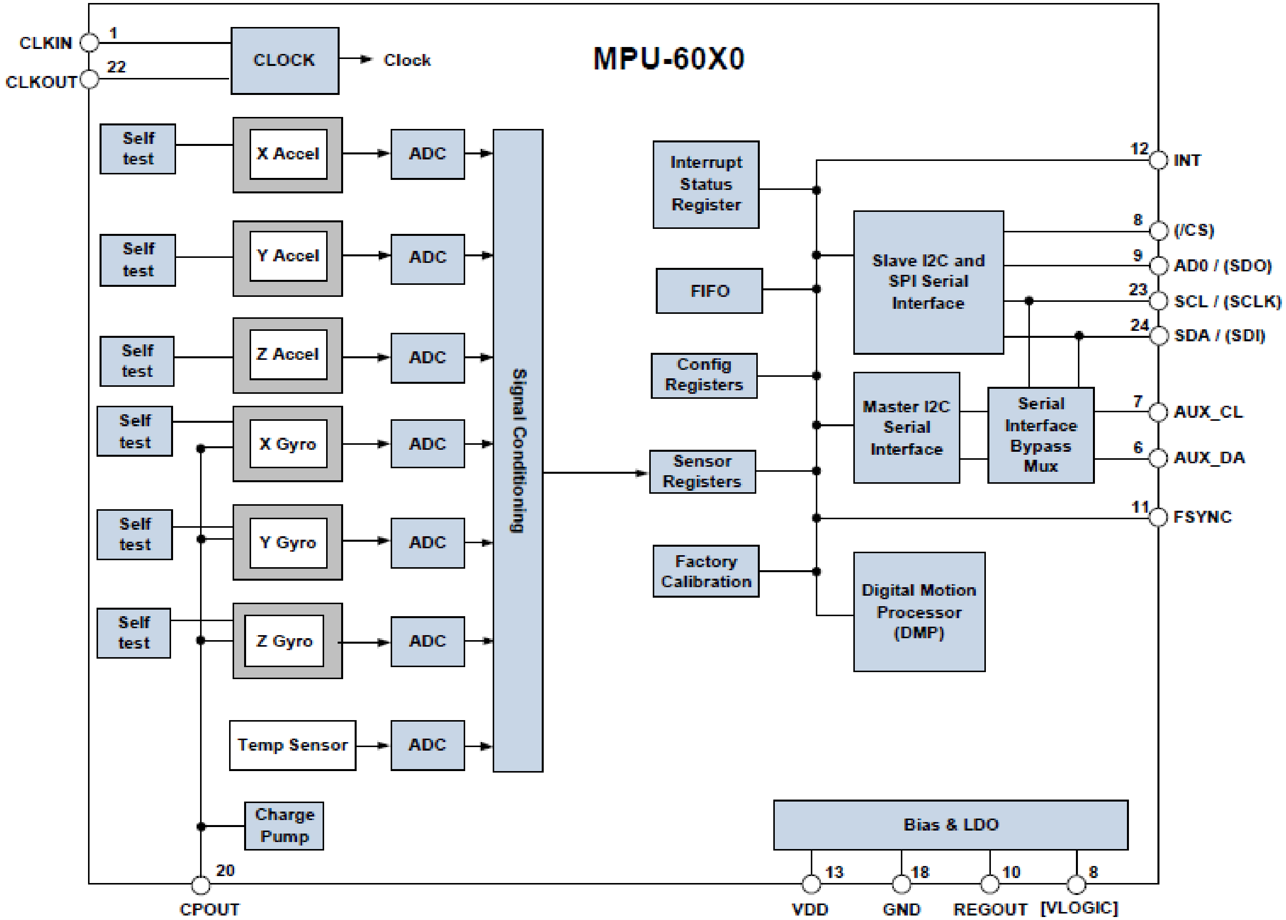
- **MPU-6050** is a single chip integrating a three-axis rate gyroscope sensor and a three-axis accelerometer sensor.

- Three 16-bit ADCs and three 16-bit ADCs are used for digitizing the gyroscope and accelerometer outputs, respectively

MEMS MEMS (Micro-ElectroMechanical Systems) advanced sensor technology is used to integrate micro-scale sensors into a single chip.

Accelerometer and rate gyro (2)

the integrated chip MPU 6050 with three-axis gyro and three-axis accelerometer will provide orientation angles of an object.



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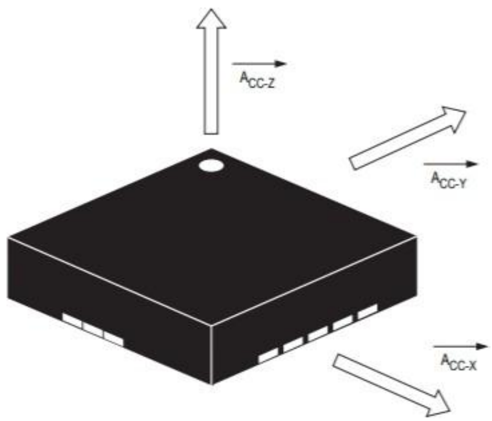
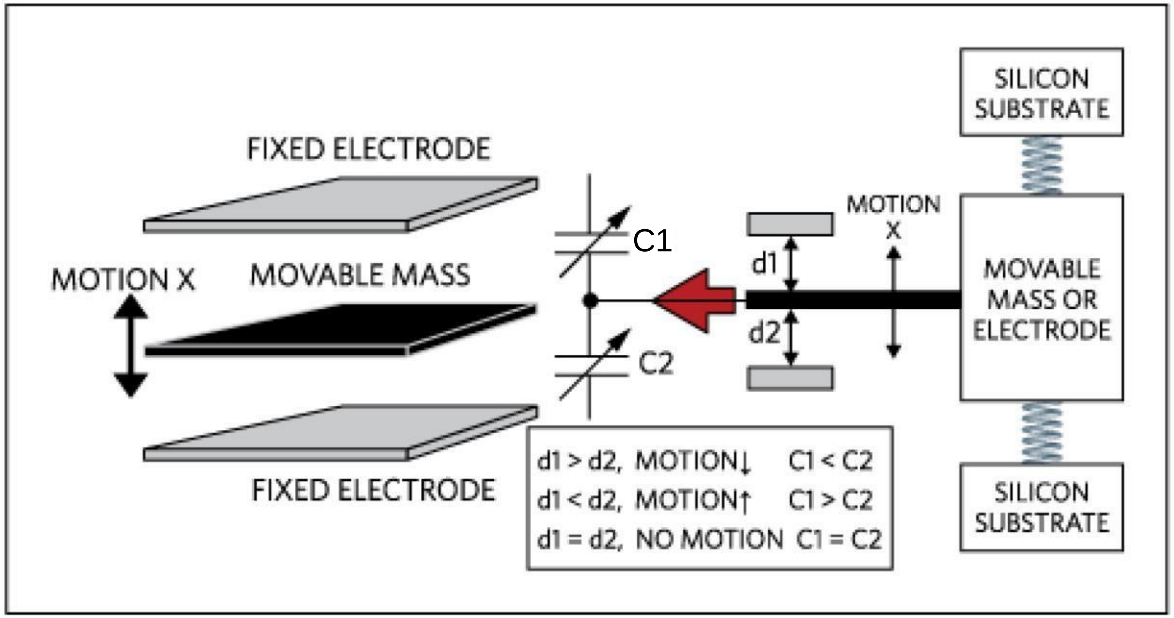
Accelerometer (1)



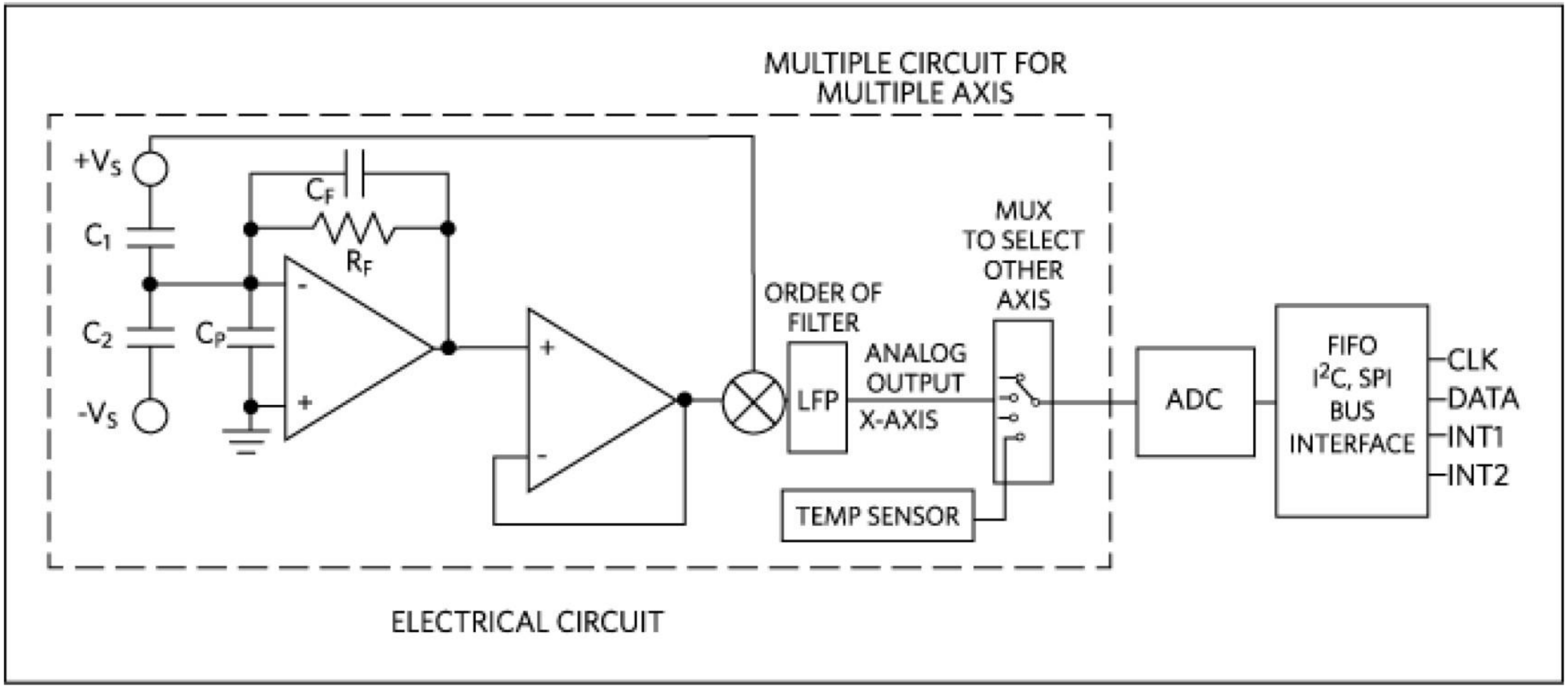
- Accelerometer measures the **three-axis** acceleration simultaneously.
- The three-axis mass acceleration is converted to voltage signals by measuring, for example, capacitance.

An single axis example



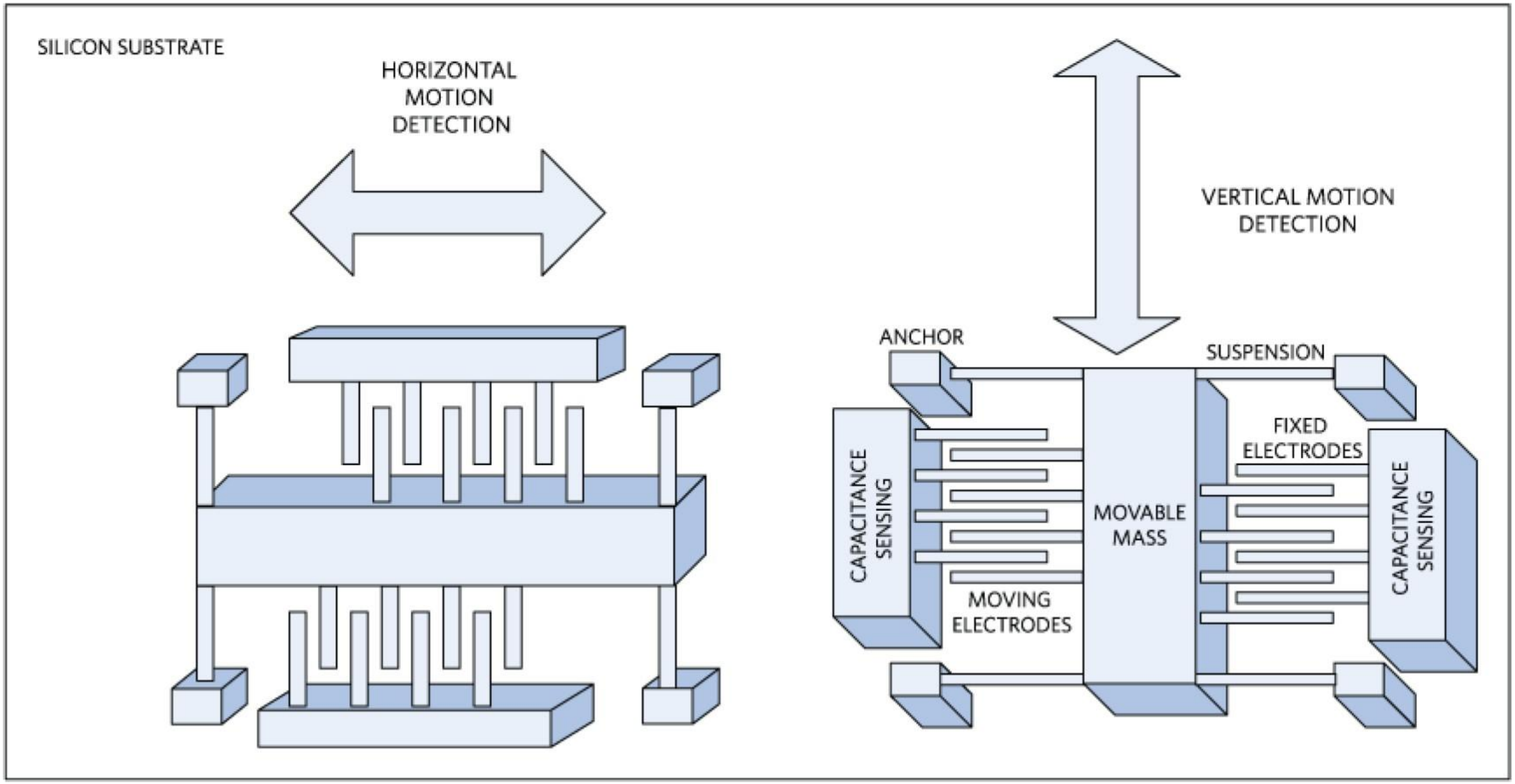
Accelerometer (2)

An example of measurement electronic circuit



Accelerometer (3)

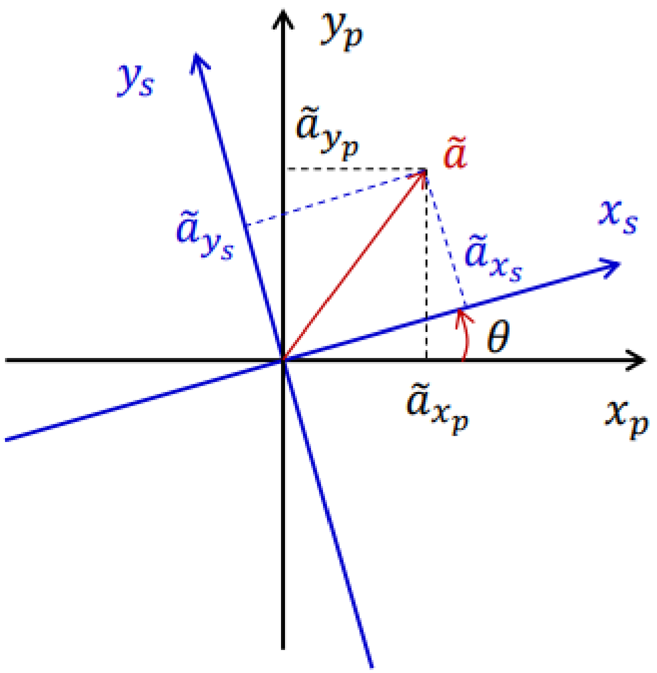
A two-axis example



Accelerometer (4)

Physical coordinate and sensor coordinate conversion (two-axil case)

Note that the sensor direction (body-coordinate) installed may not be in-line with the physical coordinate and a conversion is often needed.



Consider the 2-D case(x-y plane) and an acceleration vector $\tilde{a}$. Note that the sensor measure $(\tilde{a}_{x_s}, \tilde{a}_{y_s})$ in the sensor coordinate, and $(\tilde{a}_{x_p}, \tilde{a}_{y_p})$ in the physical coordinate can be calculated by

$$\begin{bmatrix} \tilde{a}_{x_p} \\ \tilde{a}_{y_p} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \tilde{a}_{x_s} \\ \tilde{a}_{y_s} \end{bmatrix}$$

and

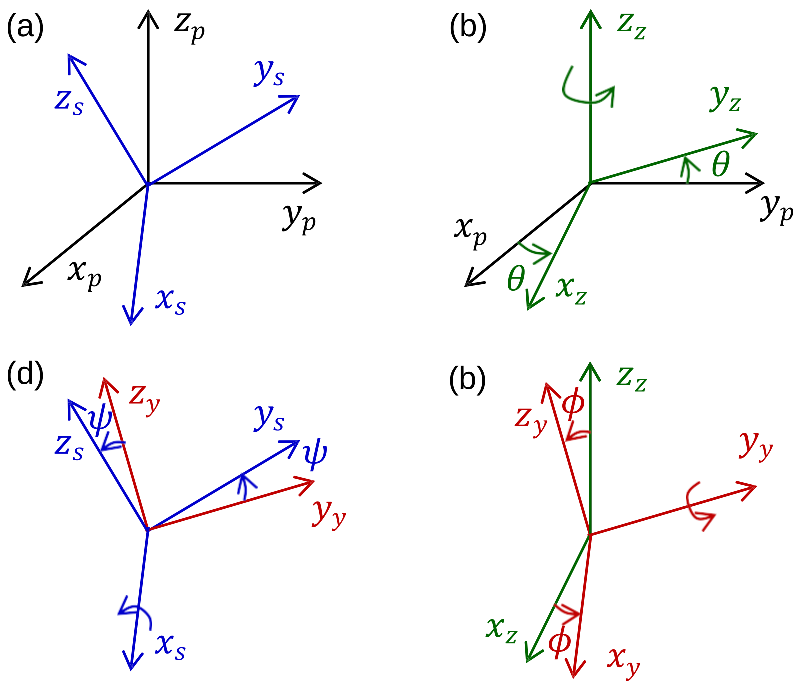
$$\tilde{a} = \sqrt{\tilde{a}_{x_s}^2 + \tilde{a}_{y_s}^2} = \sqrt{\tilde{a}_{x_p}^2 + \tilde{a}_{y_p}^2}$$

Note that θ is positive if the sensor coordinate rotates anti-clockwise relatively to the physical coordinate.

Accelerometer (5)

Physical coordinate and sensor coordinate conversion (three-axil case)

For the 3-D case, the coordinate transformation can be divided into three sequential rotations: around z axis (θ), y axis (ϕ), and x axis (ψ), and the coordinate transformation can be obtained as below:



Rotation around z axis:

$$\begin{bmatrix}\tilde{a}_{x_z}\\ \tilde{a}_{y_z}\\ \tilde{a}_{z_z}\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{a}_{x_p}\\ \tilde{a}_{y_p}\\ \tilde{a}_{z_p}\end{bmatrix} = R_{xy}\begin{bmatrix}\tilde{a}_{x_p}\\ \tilde{a}_{y_p}\\ \tilde{a}_{z_p}\end{bmatrix}$$

Rotation around y axis:

$$\begin{bmatrix}\tilde{a}_{x_y}\\ \tilde{a}_{y_y}\\ \tilde{a}_{z_y}\end{bmatrix} = \begin{bmatrix}\cos\phi & 0 & \sin\phi\\ 0 & 1 & 0\\ -\sin\phi & 0 & \cos\phi\end{bmatrix}\begin{bmatrix}\tilde{a}_{x_z}\\ \tilde{a}_{y_z}\\ \tilde{a}_{z_z}\end{bmatrix} = R_{zx}\begin{bmatrix}\tilde{a}_{x_z}\\ \tilde{a}_{y_z}\\ \tilde{a}_{z_z}\end{bmatrix}$$

Rotation around z axis:

$$\begin{bmatrix}\tilde{a}_{x_s}\\ \tilde{a}_{y_s}\\ \tilde{a}_{z_s}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\psi & -\sin\psi\\ 0 & \sin\psi & \cos\psi\end{bmatrix}\begin{bmatrix}\tilde{a}_{x_y}\\ \tilde{a}_{y_y}\\ \tilde{a}_{z_y}\end{bmatrix} = R_{yz}\begin{bmatrix}\tilde{a}_{x_y}\\ \tilde{a}_{y_y}\\ \tilde{a}_{z_y}\end{bmatrix}$$

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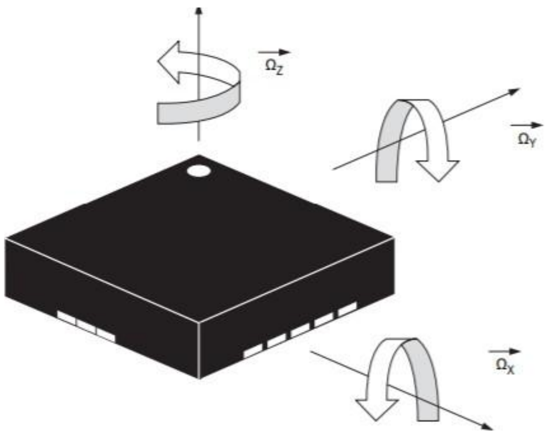
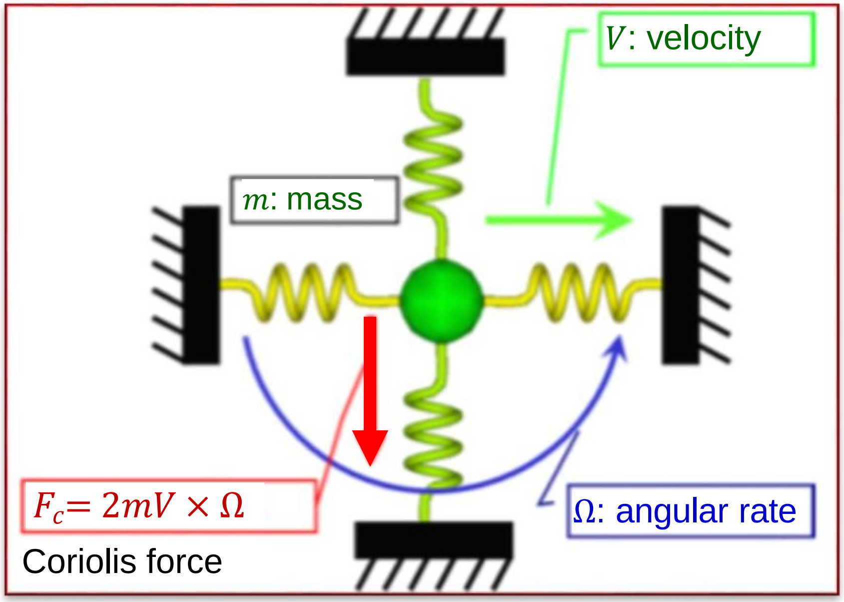
Rate gyro sensor (1)

Rate Gyro sensor



The **Coriolis force** is an inertial force that acts on objects that are in motion within a frame of reference that rotates with respect to an inertial frame.

Rate gyroscope uses Coriolis effect determine angular velocity Ω.

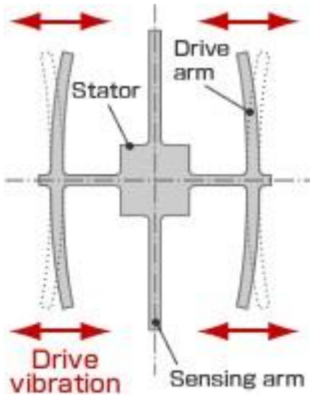


The mass and springs are placed on spinning frame with angular velocity Ω, when the mass moves with velocity V, then the mass moves due to Coriolis force F_c and capacitive measurement (like the accelerometer) is used to find Ω.

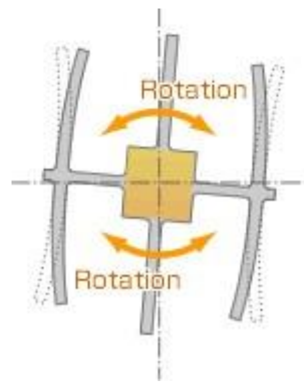
Rate gyro sensor (2)

Vibration Gyro Sensor

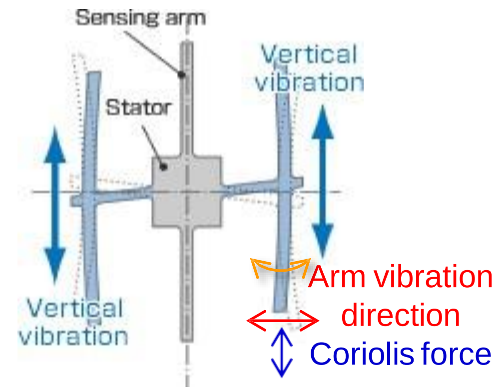
Vibration gyros sense angular velocity from the Coriolis force applied to a vibrating object.



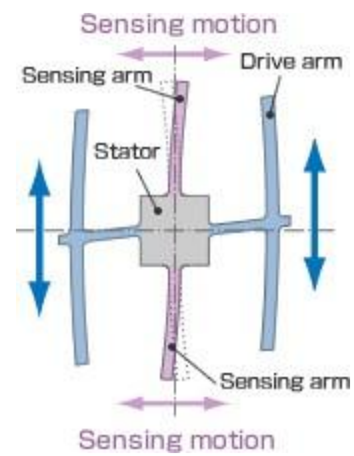
a) A drive arm vibrates in a certain direction.



b) Gyro sensor rotates.



c) When the gyro is rotated, the Coriolis force acts on the drive arm tips, producing vertical vibration due to rotation.



c) The stationary part bends due to vertical drive arm vibration, producing a sensing motion in the sensing arms.

e) The motion of a pair of sensing arms produces a potential difference from which angular velocity is sensed. The angular velocity is converted to, and output as, an electrical signal.

Rate gyro sensor (3)

Calculating angular displacement from gyro signal

Angular displacement θ can be calculated from gyro output (angular velocity Ω) , by integrating Ω in a digital sense with sample time T_s.

Taylor expansion of angular displacement θ:

$$\theta(t+\Delta T) \approx \theta(t) + \frac{d\theta(t)}{dt} \cdot \Delta T + \mathcal{E}(t)$$

High Order Terms

$$\theta[(k+1)T_s] \approx \theta[kT_s] + \Omega \cdot T_s$$

Updated value Last value Current angular velocity

Consider the gyro sensor has a DC Ω_{bias} bias such that $\Omega = \Omega_{true} + \Omega_{bias}$, then,

$$\theta[(k+1)T_s] \approx \theta[kT_s] + \Omega_{true}T_s + k\Omega_{bias}T_s$$

Note that term $k\Omega_{bias}T_s$ increases as time index k increases causing angular displacement drift due to accumulated error. As a result, angular displacement calculated from gyro signal is not accurate over time, and a new method is needed through sensor fusion (combining gyro and accelerometer sensor signals).

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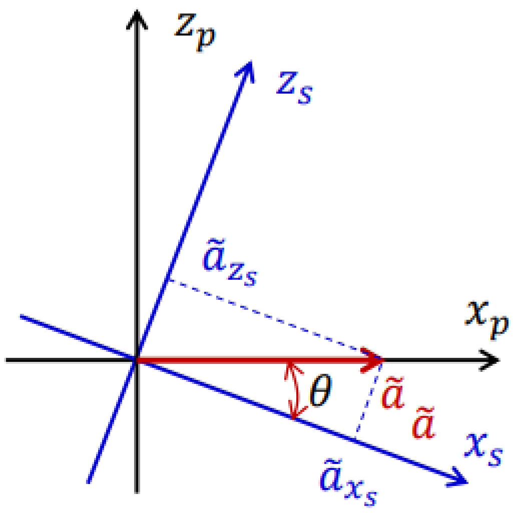
Sensor fusion (1)

Calculating the angular displacement from accelerometer

- One key advantage of using the accelerometer signal to calculate angular displacement is that it does not use integration and thus will **no drift**.
- However, accelerometer signals are subject to high-frequency noise.
- Therefore, the angular displacement calculation work well at low frequency. use.

How to calculate the pitch angle using accelerometer signals?



Let $(\tilde{a}_{x_s}, \tilde{a}_{z_s})$ and $(\tilde{a}_{x_p}, \tilde{a}_{z_p})$ be the sensor and physical coordinates, respectively, and assume that the vehicle accelerates in a forward direction, resulting $\tilde{a}$. Then,

$$\tilde{a}_{x_s} = \tilde{a}\cos\theta\,,\tilde{a}_{z_s} = \tilde{a}\sin\theta$$

As a result, we have

$$\tan\theta = \frac{\tilde{a}_{z_s}}{\tilde{a}_{x_s}} \quad \rightarrow \quad \theta = \operatorname{atan}\left(\frac{\tilde{a}_{z_s}}{\tilde{a}_{x_s}}\right)$$

Sensor fusion (2)

- Note that accelerometer based angular displacement calculation is accurate at low frequency and gyro based is accurate at high frequency
- Sensor fusion technique is used to combine low-pass filtered accelerometer-based displacement signal and high-pass filtered gyro-based signal to have accurate angular displacement.

Complementary filter of gyro & accelerometer

Let $\theta_a = \operatorname{atan}\left(\tilde{a}_{z_s}/\tilde{a}_x\right)$ and $\Theta_a(z)$, $\Theta_g(z)$ and $\Omega_g(z)$ are z-transform of θ_a, θ_g, and $\dot{\theta}_g$, respectively. Then,

$$\Theta_g(z) = \frac{z}{z-1}\Omega_g(z)$$

and sensor fusion leads to

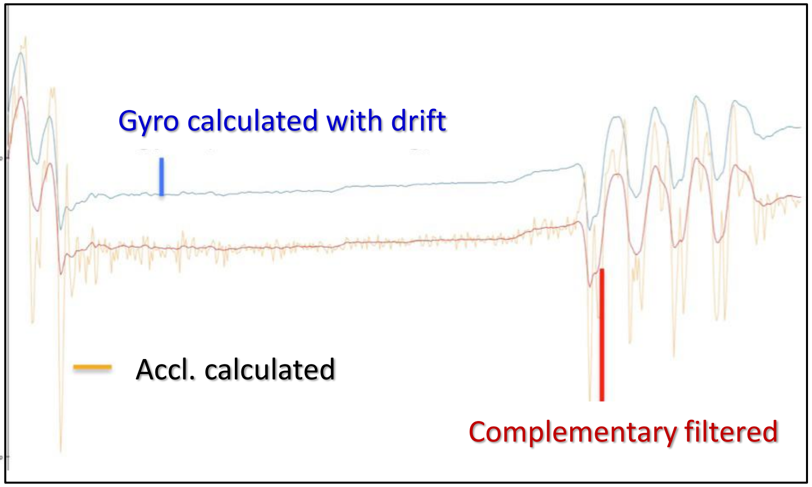
$$\Theta(z) = \underbrace{\frac{1-\alpha}{z-\alpha}}_{\text{low-pass}}\Theta_a(z) + \underbrace{\frac{\alpha(1-z^{-1})}{z-\alpha}}_{\text{high-pass}}\Theta_g(z)$$

or

$$\Theta(z) = \frac{1-\alpha}{z-\alpha}\Theta_a(z) + \frac{\alpha}{z-\alpha}\Omega_g(z)$$

where $\alpha = 0.98$

Simulation results



Sensor fusion (3)

Simulink implementation of sensor fusion (gyro & accelerometer)