MAE/ECE 5320 Mechatronics

2025 Spring semester

Lecture 06

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Content

- Background
- Motor drive and control
- Motor speed control
- Pulse width modulation (PWM)
- Closed-loop motor speed control

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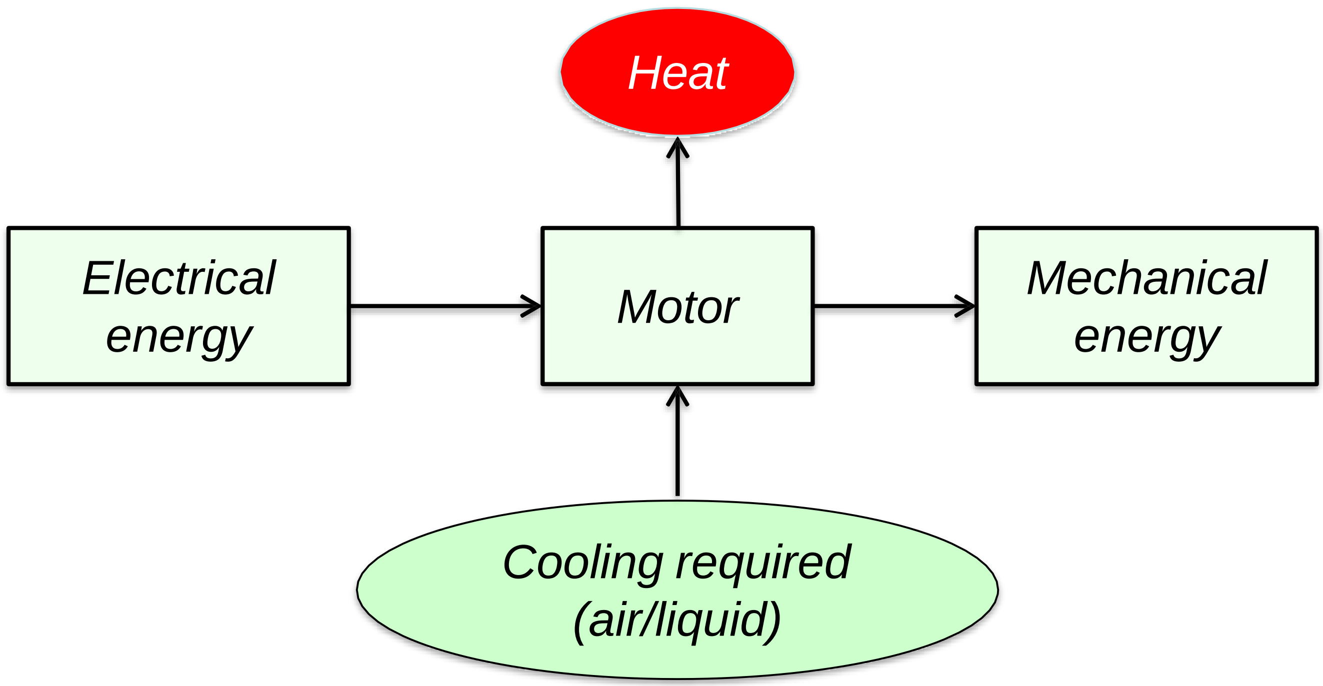
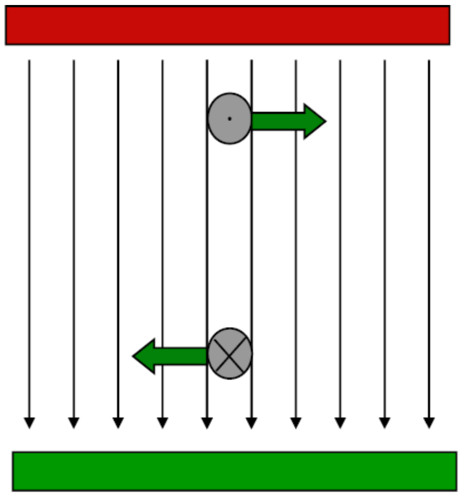
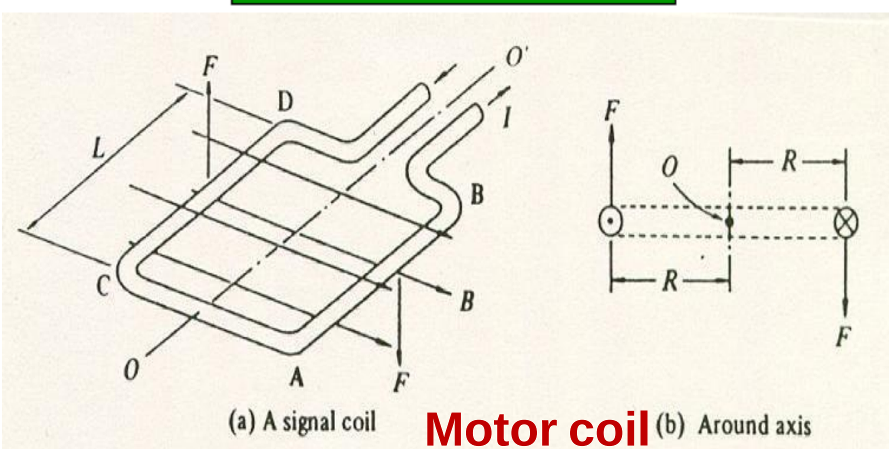
Background (1)

- Electric motor converts electric energy to mechanical energy. Motors operate through electromagnetism, e.g. the interaction between magnetic filed and electric current.
- Heat is generated during this process since conversion efficiency is not 100%.
- This class focuses on the permanent magnetic DC motors.



Magnetic force law





(a) A signal coil **Motor coil** (b) Around axis

The total torque is

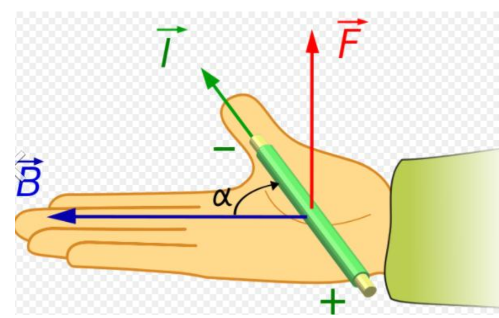
$$\tau = 2 \cdot i \cdot L \cdot B \cdot R$$

Lorentz force:

In a magnetic filed with flux density B, the magnetic force F applied to the cable with length L supply current i can *be calculated by integrating over the coil length,*

$$F = i \cdot L \cdot B$$

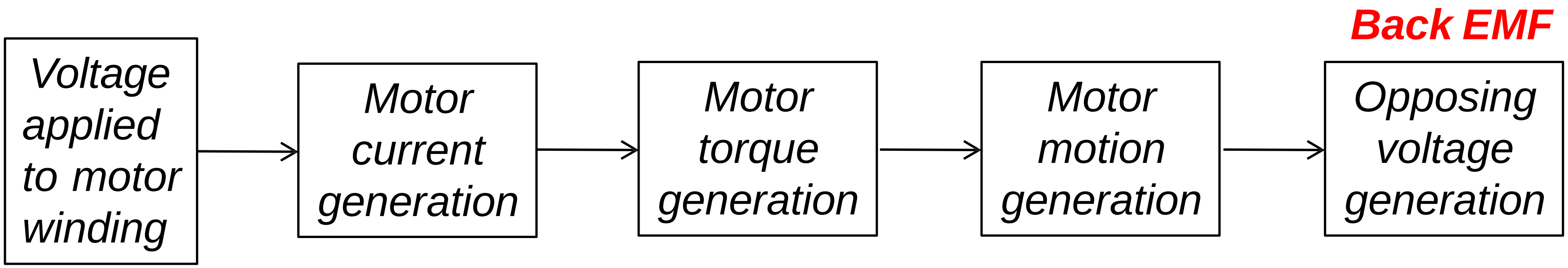
The direction of force can be determined by right-hand-rule



Background (3)

Magnetic force law

Back EMF (ElectroMotive Force) or counter EMF voltage:



The voltage generated by back EMF effect can be calculated by

$$V_{emf} = \underbrace{2 \cdot N \cdot R \cdot B \cdot L}_{K_{emf}:\ emf\ constant} \cdot \omega = K_{emf} \cdot \omega$$

where N is number of winding.

Note that due to EMF as the motor speed increased the motor output torque reduces, making it easy to control motor speed.

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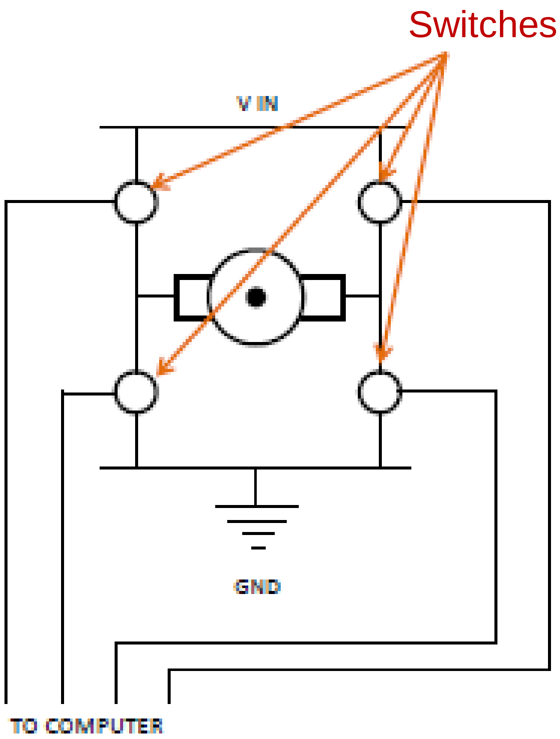
Motor drive and control (1)

The speed and rotational direction of a DC motor can be easily controlled using a four-switch H-bridge circuit below.



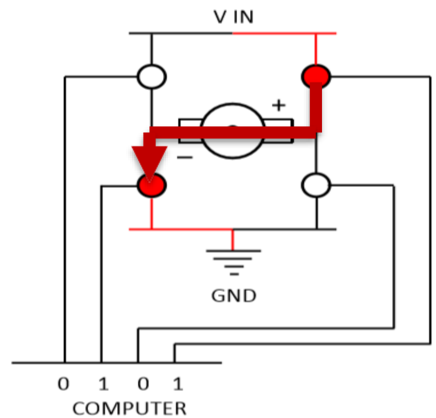
Motor direction is controlled by how the switches in the H-bridge circuit are turn on

Motor speed is controlled by the applied voltage/current to the H-bridge circuit using pulse width modulation (PWM)

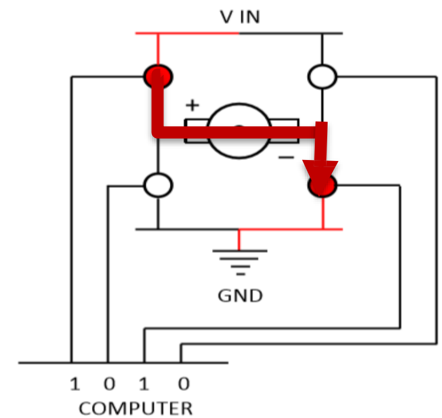
Motor torque is determined by current motor speed (due to back EMF) and PWM duty cycle

Motor drive and control (2)

Connecting motor into circuit with supply voltage, motor will be driven and convert electric energy to mechanical energy. The current/voltage direction will realize forward/reverse operation.

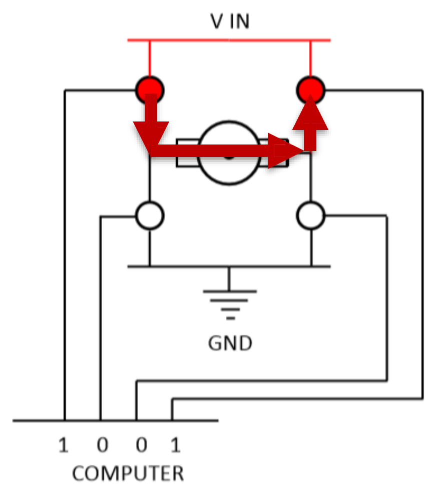


Forward Operation

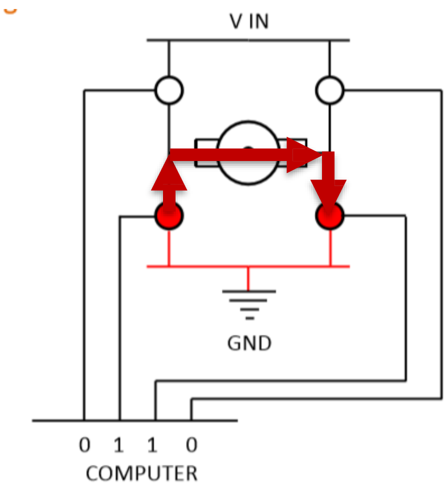


Reverse Operation

Connecting motor into circuit with no supply voltage, motor will become generator and convert mechanical energy to electric energy, thus realize braking operation.

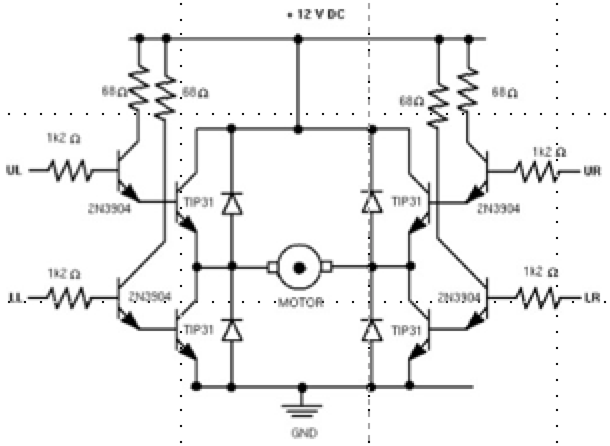


Braking Operation



Braking Operation

Motor drive and control (3)



Upper Left	Upper Right	Lower Left	Lower Right	Description
On	Off	Off	On	Forward Running
Off	On	On	Off	Backward Running
On	On	Off	Off	Braking
Off	Off	On	On	Braking

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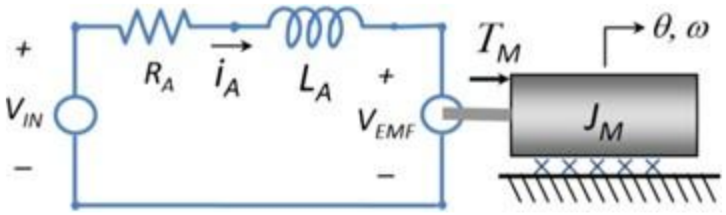
Motor speed control (1)

DC motor speed can be controlled by altering supply voltage. Why ?

Speed-Voltage characteristics

Equivalent circuit



$$V_{IN} = i_A R_A + V_{EMF} + L_A \cancel{\frac{di_A}{dt}}$$

At steady-state

$$V_{IN} = i_A R_A + V_{EMF} = \frac{T_M}{K_T} R_A + K_{EMF}\,\omega$$

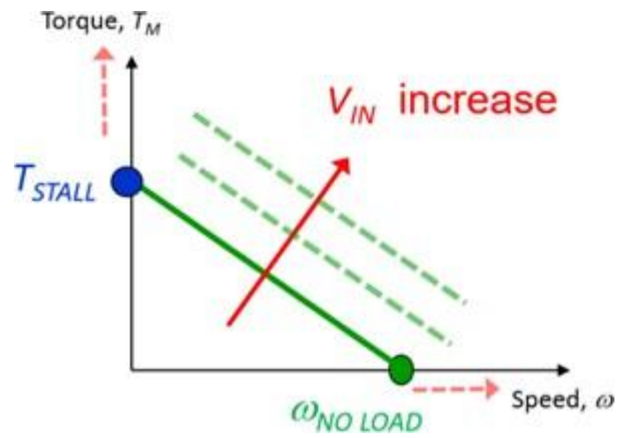
$$\rightarrow \quad T_M = \frac{K_T}{R_A}\left(V_{IN} - K_{EMF}\,\omega\right)$$

Stall Torque

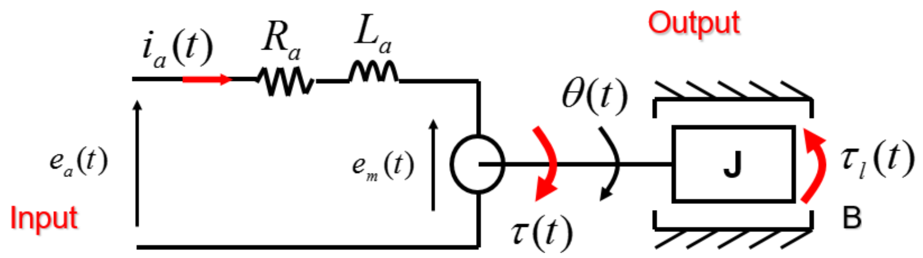
$$T_{STAL} = \frac{K_T}{R_A} V_{IN}$$

No load speed

$$\omega_{NO-LOAD} = \frac{V_{IN}}{K_{EMF}}$$



Motor speed control (2)



$$\Omega(s) = \frac{K_\tau}{(L_a s + R_a)(Js + B)}(E_a(s) - K_m\Omega(s)),\ T_l(s) = 0$$

$$\Omega(s) = \frac{K_\tau E_a(s)}{K_\tau K_m L_a J s^2 + K_\tau K_m (B L_a + J R_a) s + (K_\tau K_m B R_a + 1)}$$

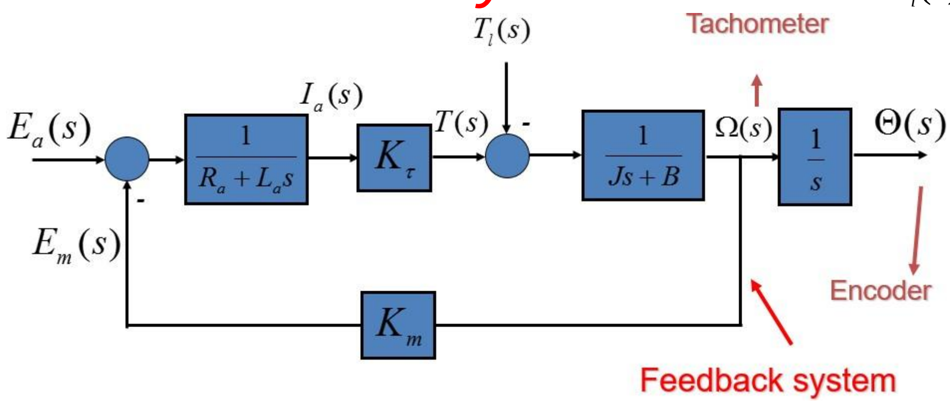
$$\Omega(s) = \frac{1}{(Js + B)}\left(-T_l(s) - \frac{K_m K_\tau}{L_a s + R_a}\Omega(s)\right),\quad E_a(s) = 0$$

$$\frac{\Omega(s)}{T_l(s)} = \frac{-\dfrac{1}{(Js+B)}}{1 + \dfrac{K_\tau K_m}{(L_a s + R_a)(Js + B)}} = \frac{-(L_a s + R_a)}{(L_a s + R_a)(Js + B) + K_\tau K_m} =: G_2(s)$$

2nd order system

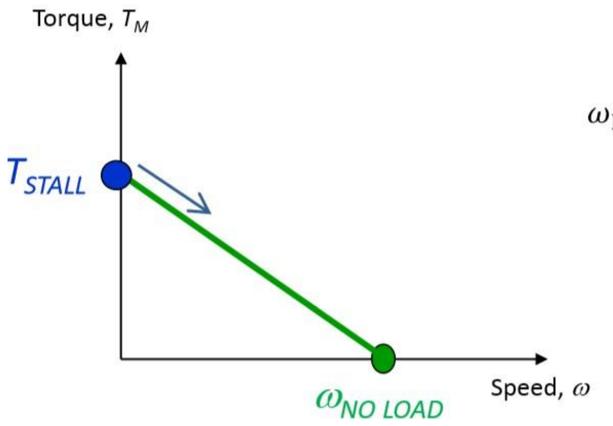
→

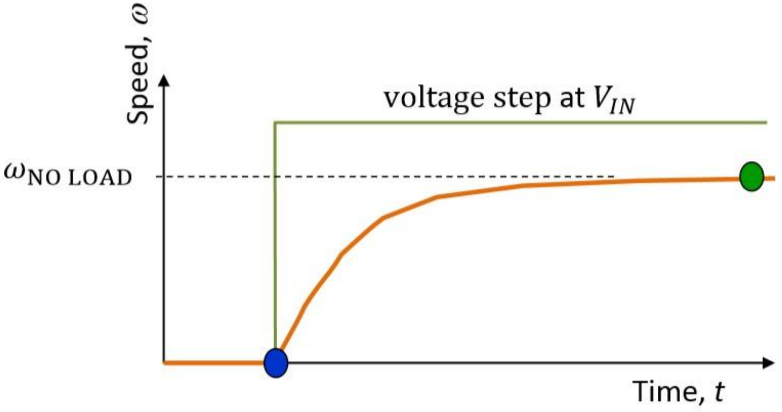
1st order system



Motor speed control (3)

DC motor transient responses with NO LOAD

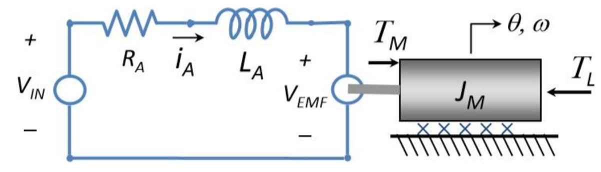




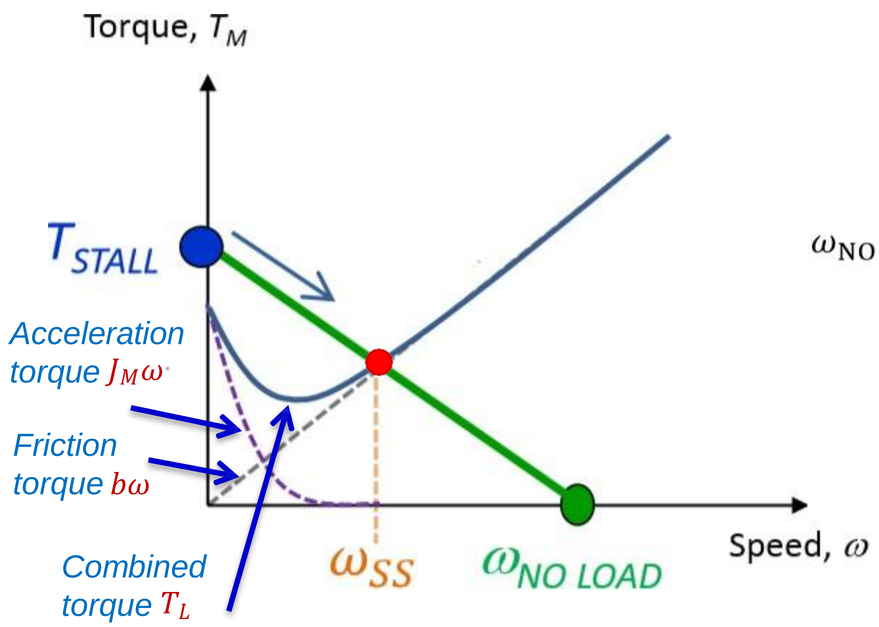
Motor speed control (4)

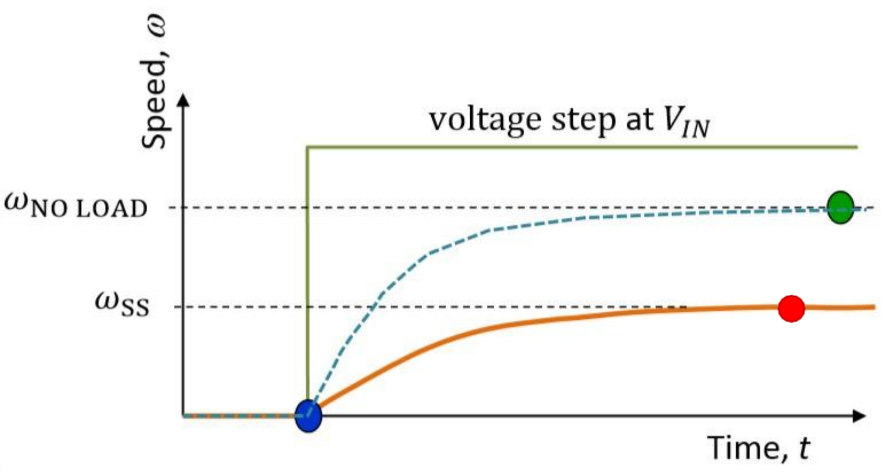
DC motor transient responses with LOAD T_L

Assuming a constant angular acceleration $\dot{\omega} = c$ *and friction torque is proportional to angular velocity* $T_f = b\omega$*, the following is the transient response*



$$T_L = J_M\,\dot{\omega} + b\omega$$



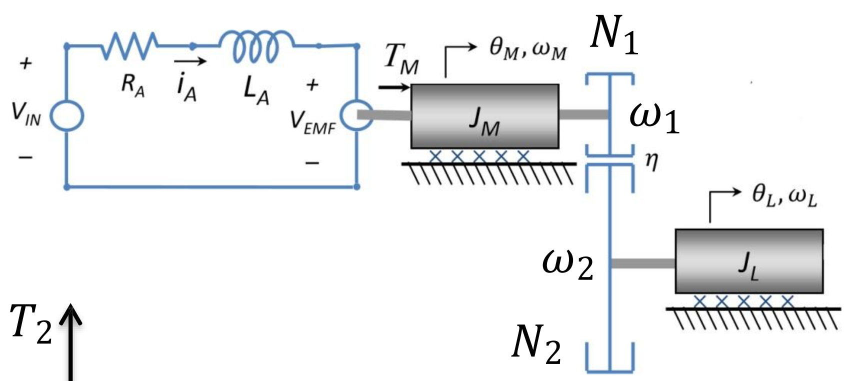


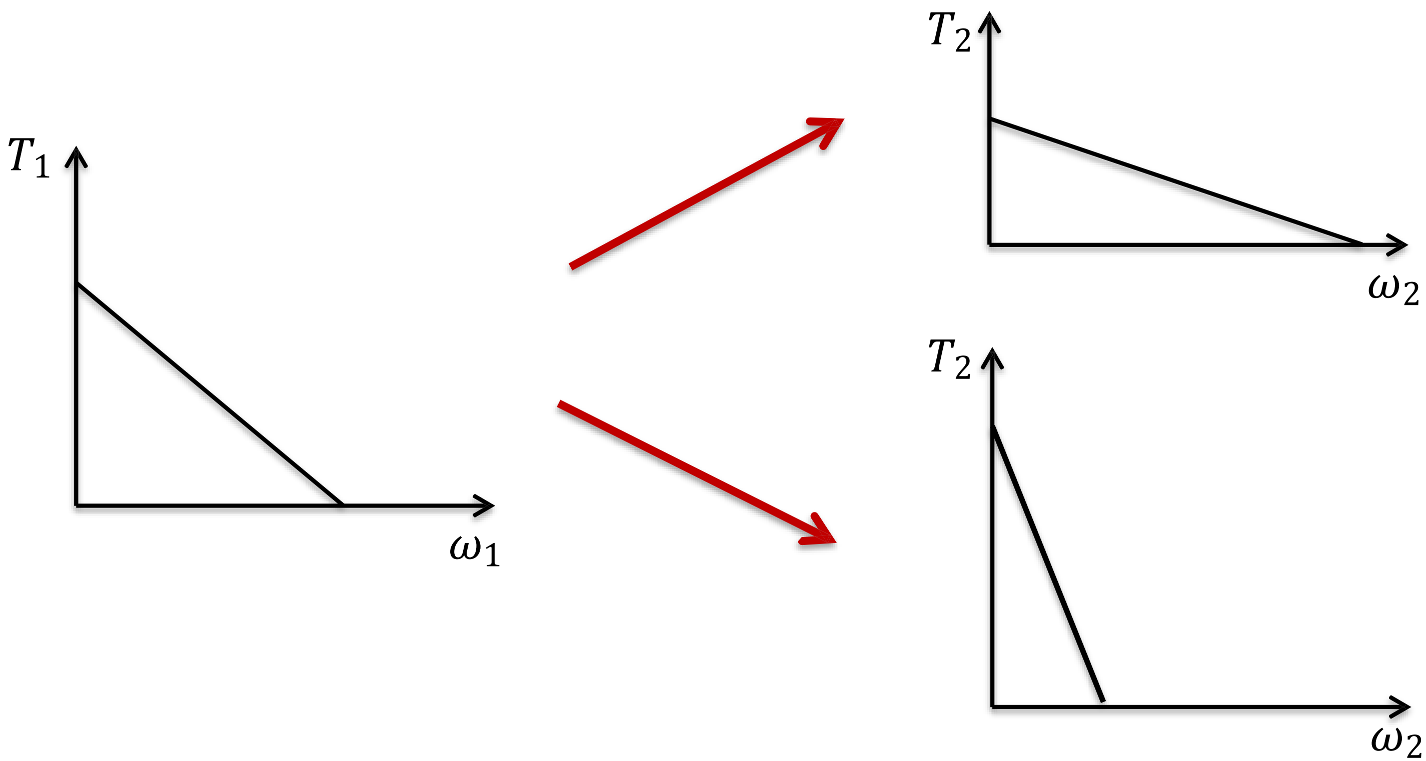
Motor speed control (5)

Gearbox is often used to reduce or increase angular velocity (or torque).

Assuming the gearbox does not have power loss (normally there is a few percent), power $P = T \cdot \omega$ *and*

$$\eta = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$





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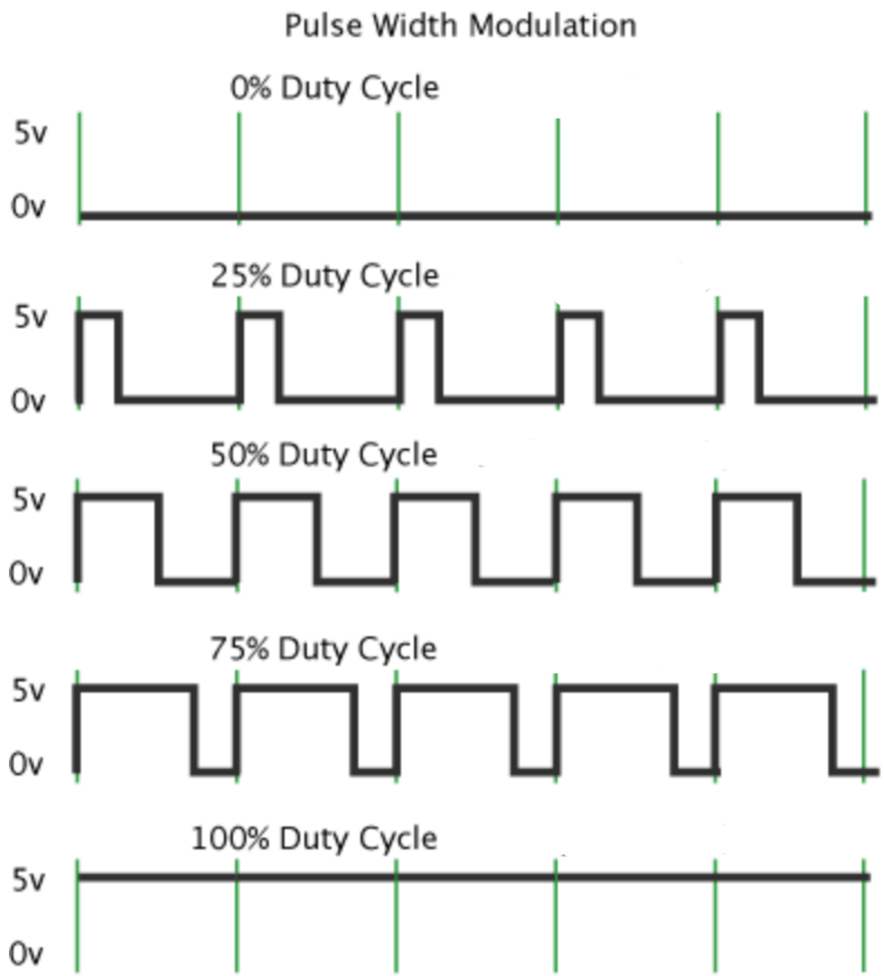
Pulse width modulation (PWM) (1)



PWM (pulse width modulation) is a digital control to realize analog control results.

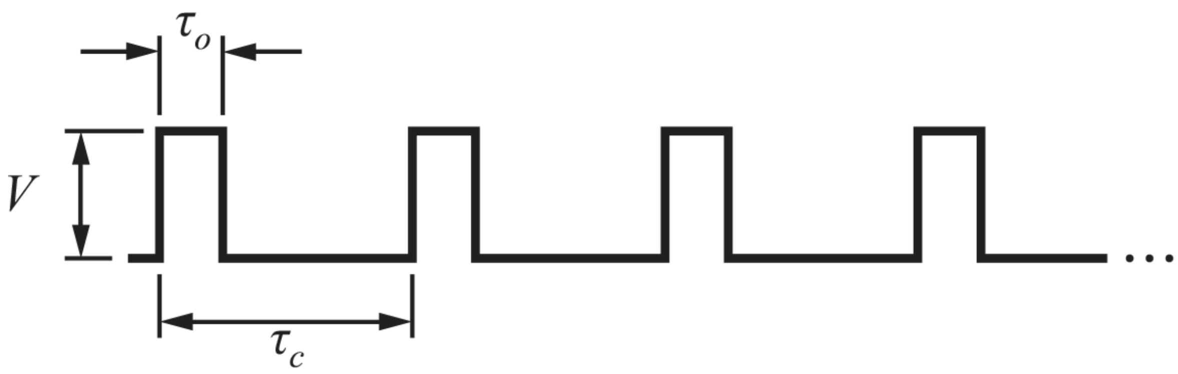
Advantages:

1. *Easy to develop and implement*
2. *High Efficiency in energy conversion and higher power output of given motor because of short-period duty period.*
3. *Fast response,*

Disadvantage:

1. EMF generated by fast switching on and off

Pulse width modulation (PWM) (2)



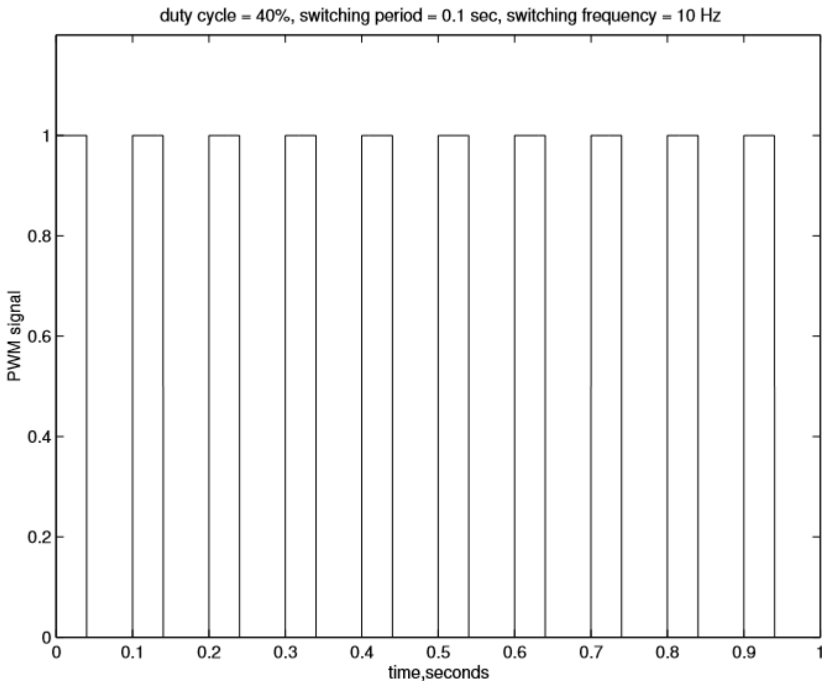
Duty cycle $\alpha = \tau_0/\tau_c$
Power $\approx \alpha P_{max}$

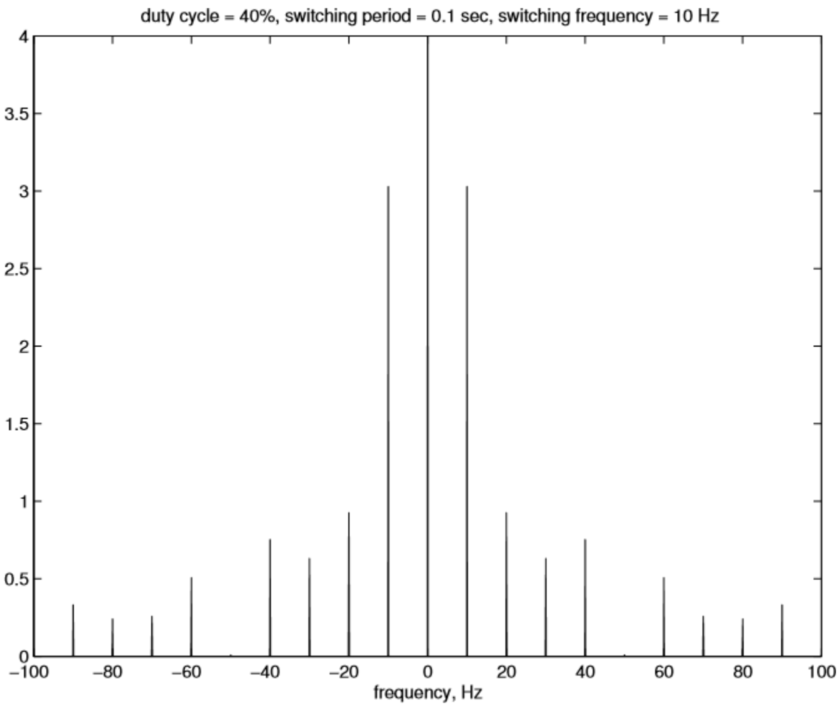
A square wave, a switch signal are used as control signals. The duration of "switch on" is called the pulse width (duty cycle). During each PWM period, the average voltage approximating analog voltage signal is

$$U = duty\_cycle \times V$$

To get varying analog values, change pulse width to approximate analog signal.

Pulse width modulation (PWM) (3)

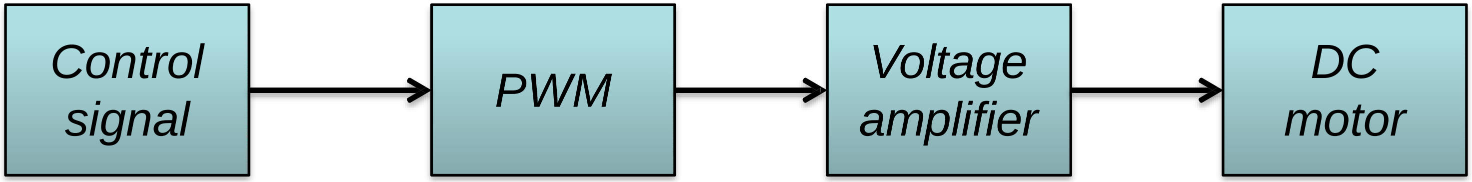


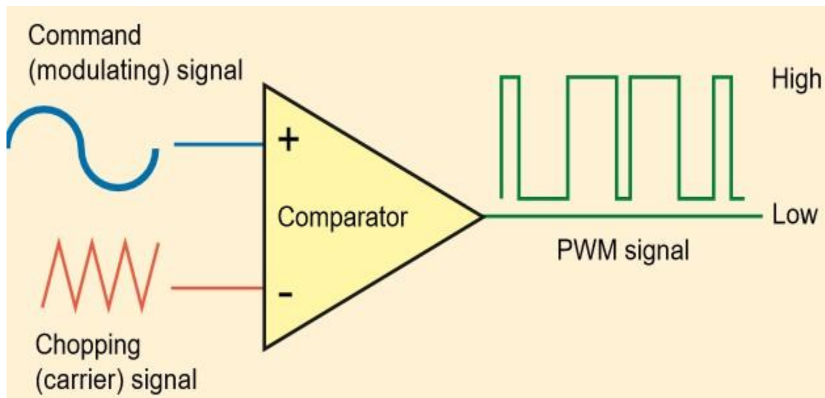


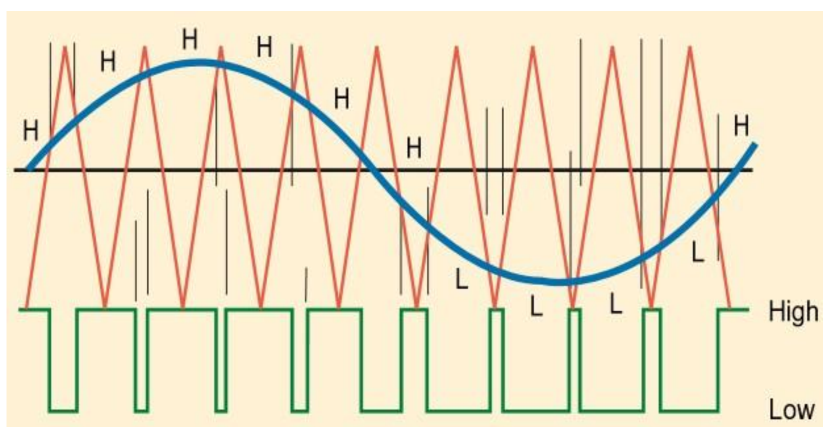
Switching frequency 10Hz

Frequency component multiplier of 10Hz

Pulse width modulation (PWM) (4)

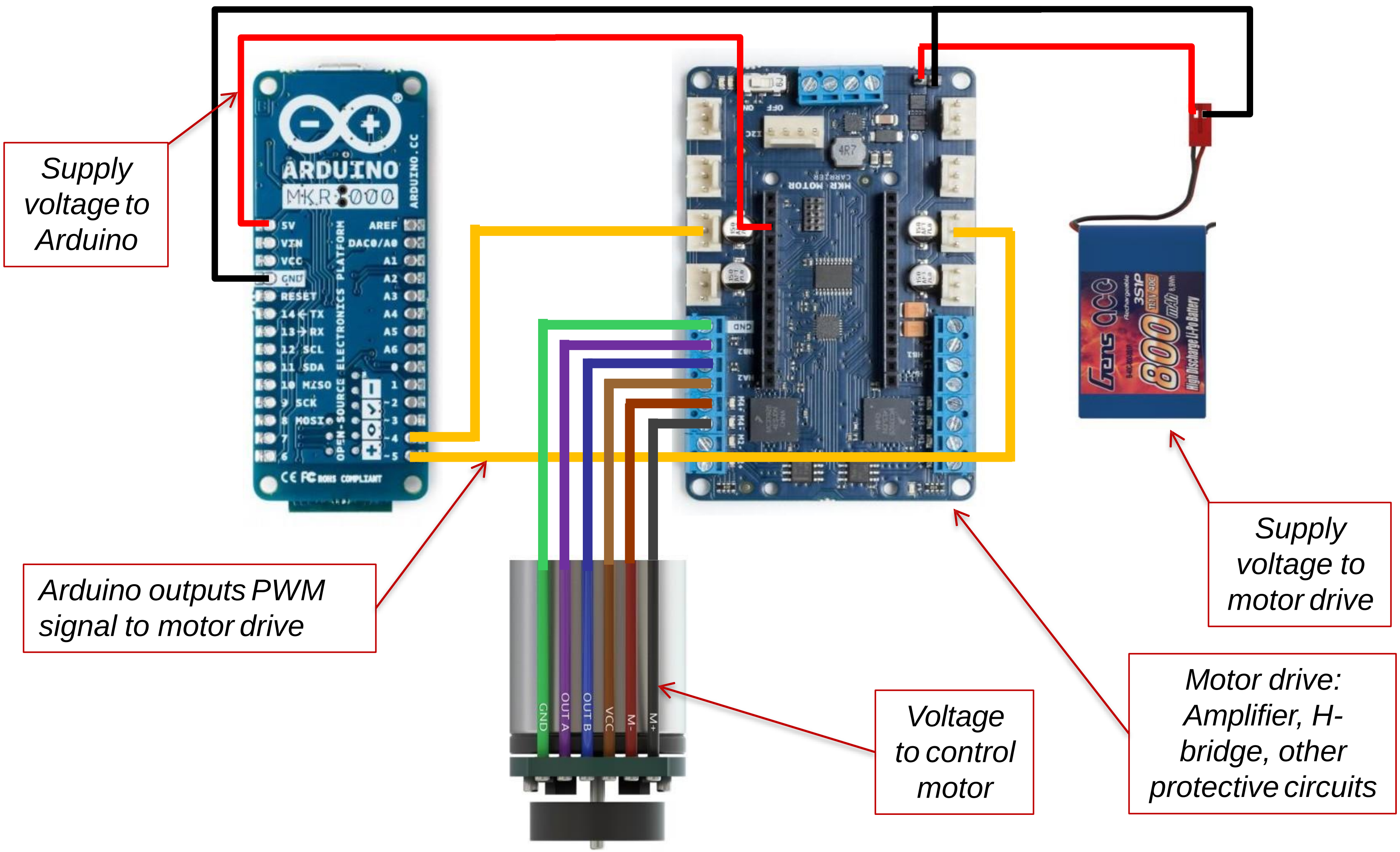






Pulse width modulation (PWM) (5)

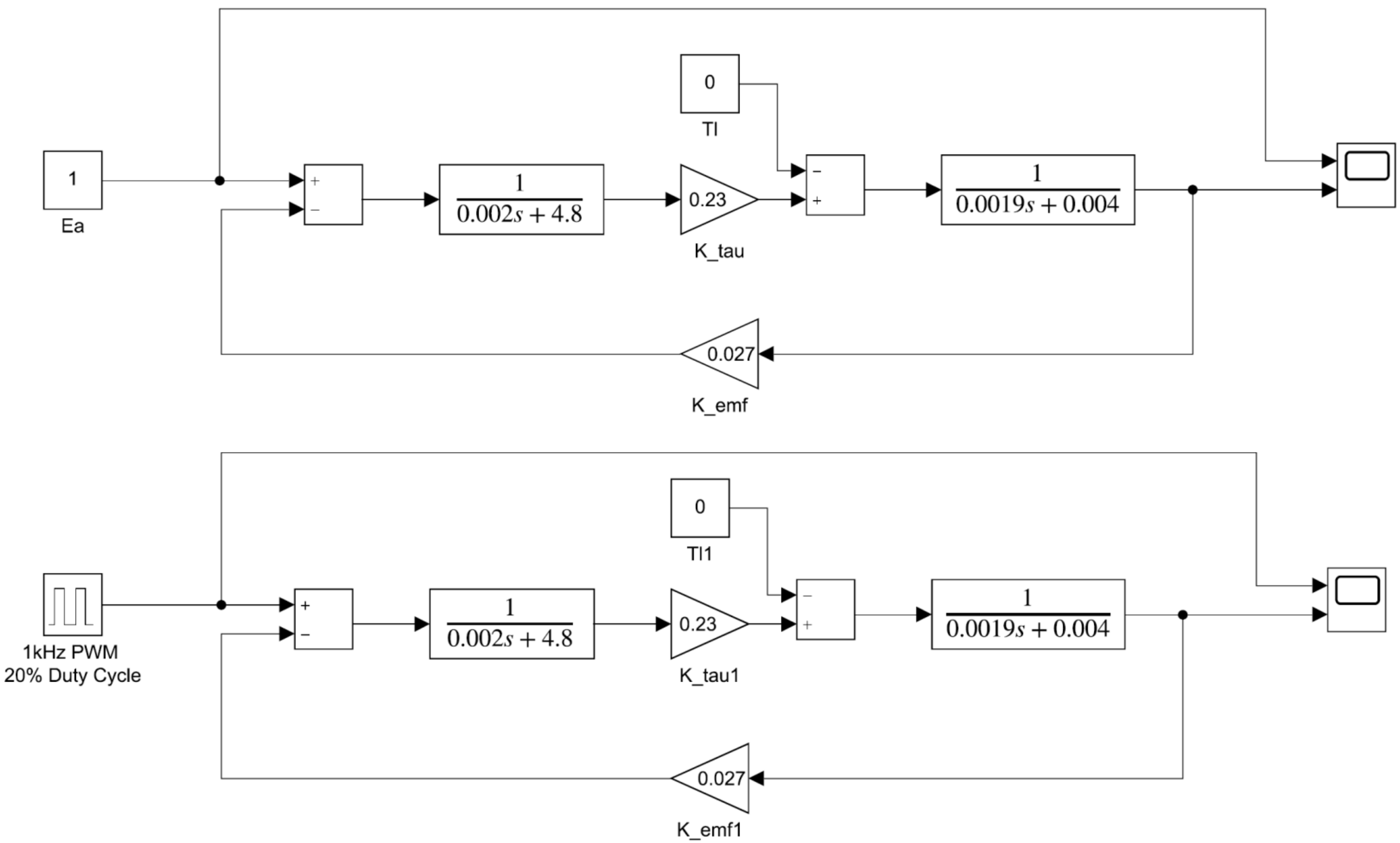
PWM control of DC motor by
Arduino MKR1000 and motor drive



Pulse width modulation (PWM) (6)

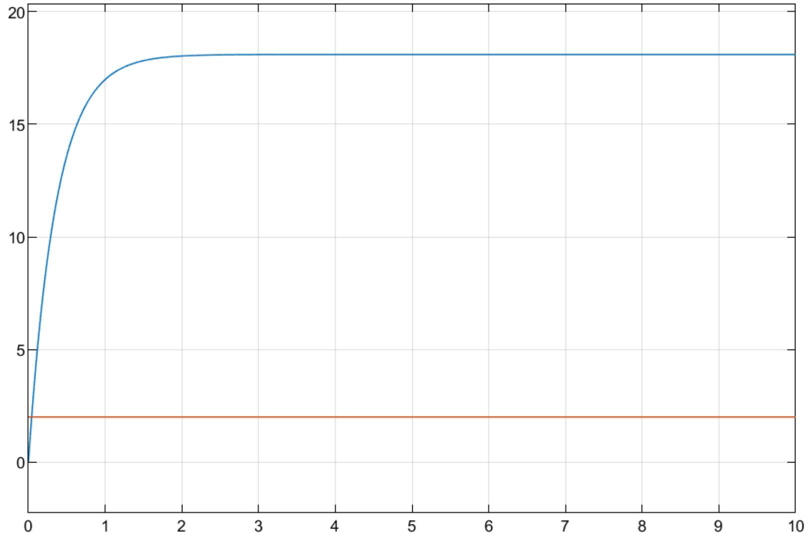
Simulink model of a DC motor with DC voltage and PWM control

(Motor parameters are configured using our Segway robot – Chp06Code01)

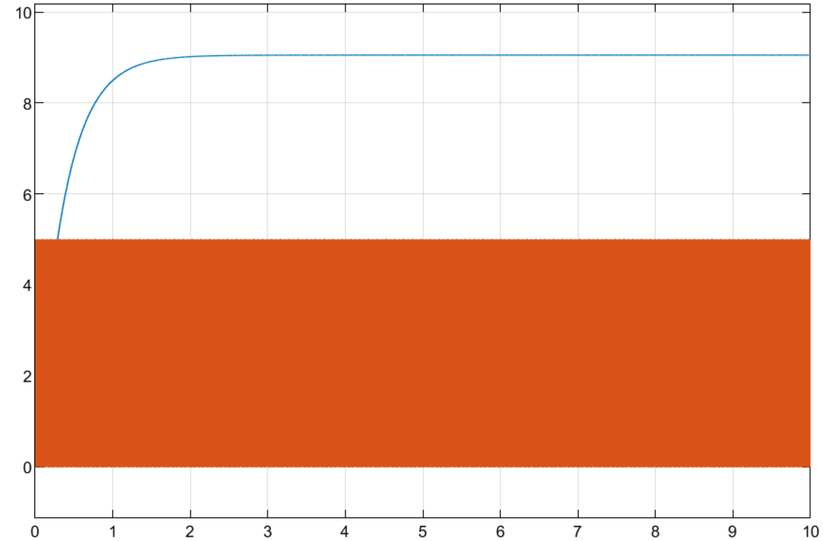


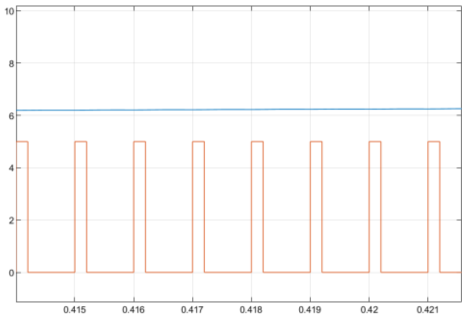
Pulse width modulation (PWM) (6)

DC voltage control



PWM control





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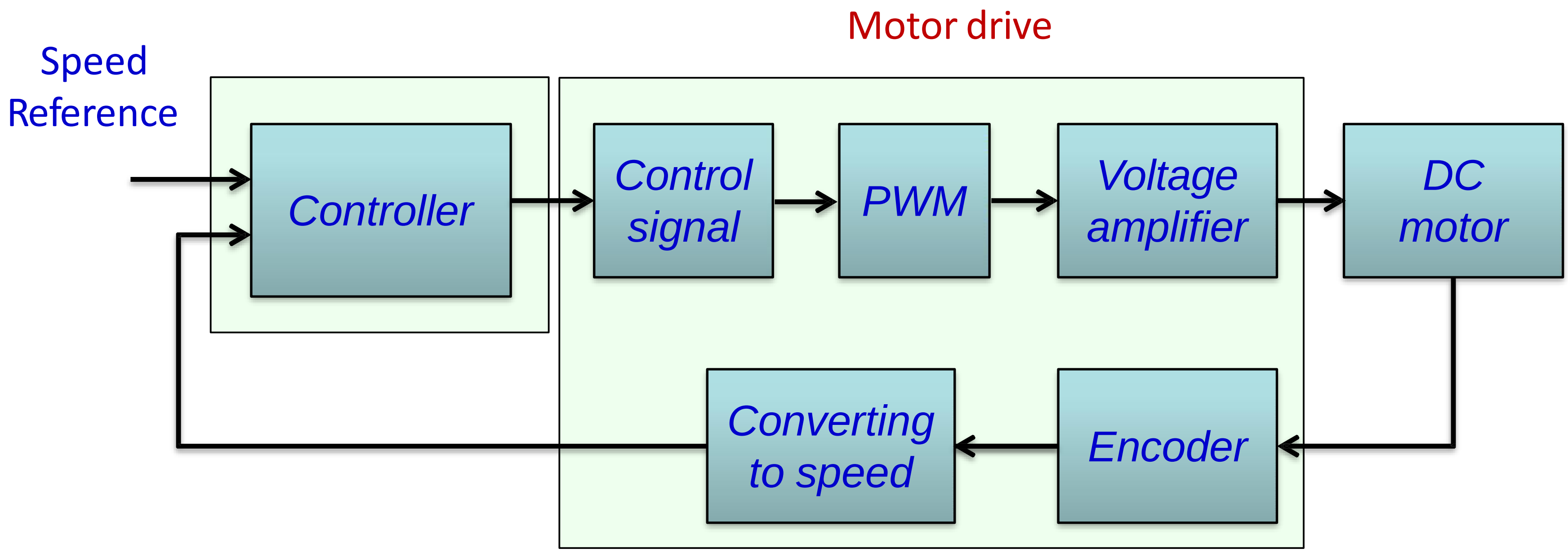
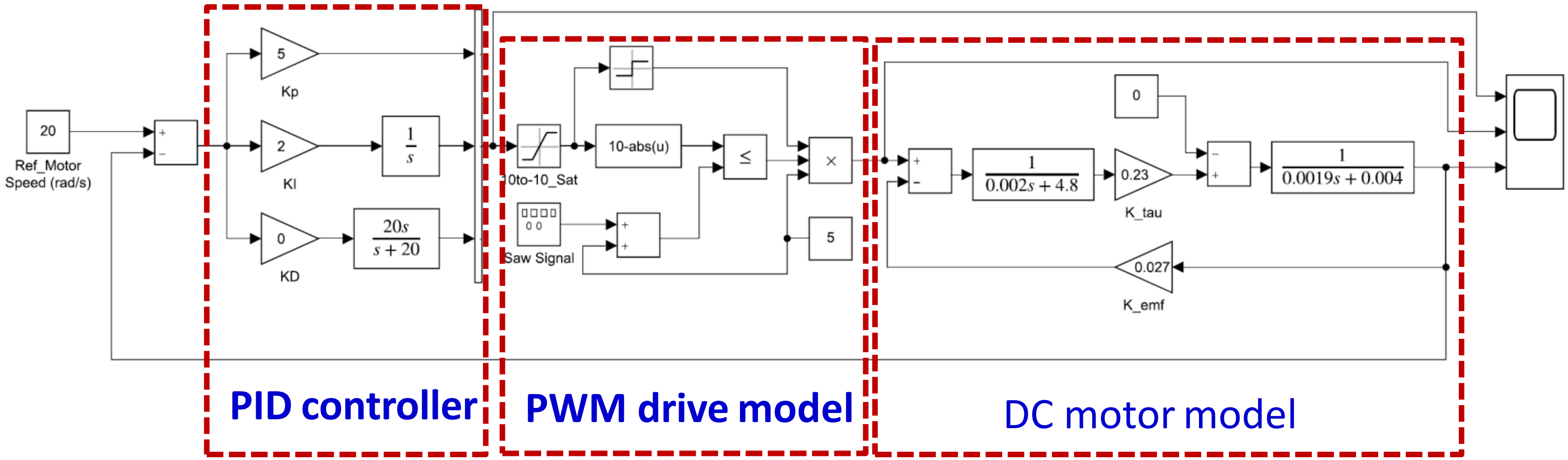
Closed-loop motor speed control (1)



Closed-loop motor speed control (2)

Simulink model of closed-loop PWM motor speed control

(Chp06Code02)



Closed-loop motor speed control (3)

Closed-loop PWM motor speed control