DIY 2: Parallax Curve Fit Report

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1 Problem Description

Given the existing VLBI astrometric data of the Pleiades member hii_625, we can derive its distance with parallax. To obtain the parallax, we can implement the least squares fitting on the Julian date and the declination degree and derive the parallax by $parallax = A/3600$. Aditionally, the distance can then be derived following: $distance = 1/parallax$.

To better fit the sine curve, the Earth orbital parameters should be taken into consideration since we can correct the declination over time due to the Earth's orbit using the following formulas.

$$n = \frac{2\pi}{orbital\_period}$$
$$M = n \times (t - (perihelion - ve))$$
$$\delta = \theta e sin(M)$$

where e corresponds to eccentricity and θ corresponds to earth obliquity

2 Fitting Algorithm

In this section we follow the Gauss-Newton method to fit the sine curve of the time-declination relation. First we construct a function model:

$$fitfunction : Asin(\omega t + \phi) + y_0$$

Then we can expand the sin function using the auxiliary angle formula:

$$\begin{cases} y_1 = p_1 sin\omega x_1 + p_2 cos\omega x_1 + p_3 \\ y_2 = p_1 sin\omega x_2 + p_2 cos\omega x_2 + p_3 \\ \quad \vdots \\ y_n = p_1 sin\omega x_n + p_2 cos\omega x_n + p_3 \end{cases}$$

From which we obtain the matrix form:

$$\begin{bmatrix} sin\omega x_1 & cos\omega x_1 & 1 \\ sin\omega x_2 & cos\omega x_2 & 1 \\ \vdots & & \\ sin\omega x_n & cos\omega x_n & 1 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Then we can solve the linear function set using matrix operation (if invertible). During the iteration, we update the coefficients using the Gauss-Newton formula.

3 Results

图 1: Sine curve fitted by origin

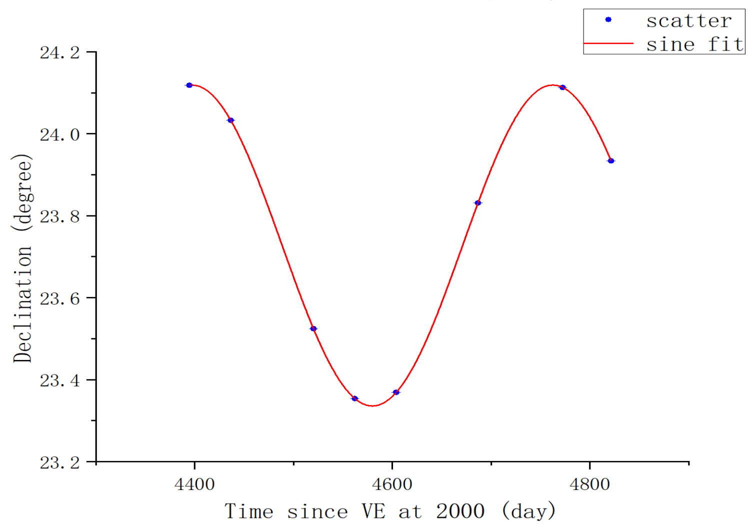
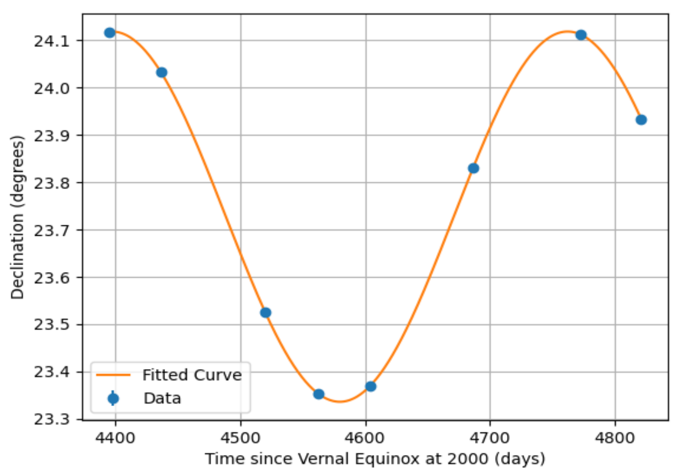


图 2: Sine curve fitted with least squares implemented from scratch



We can obtain the fitted parameter set as:

$$A = 0.39144779934999385$$

$$\omega = 0.01720252575646069$$
$$\phi = -11.24040760653467$$
$$y_0 = 23.727309736577485$$

Estimated Parallax: 0.00010873549981944273 arcseconds
Estimated Distance: 9196.628531257207 parsecs

From the above analysis we find out that the data provided is very good, since the fitting process converges in no more than 10 iteration, which is surprising!